

EC0607 - (lec 19-28)

$\Rightarrow$ Dynamic sense of equilibrium

$$P(t) = P_c + P_p$$

$$P(t) = P_c + P^*$$

$$\Rightarrow P(0) = P^*$$

requires no adjustment over time

$$\Rightarrow P(0) > P^*$$

requires adjustment over time

$$\Rightarrow P(0) < P^*$$

requires adjustment over time.

$\Rightarrow$ Price adjustments are directly proportional to excess demand

$$\frac{dP}{dt} \propto (Q_d - Q_s)$$

OR

$$\frac{dP}{dt} = j(Q_d - Q_s)$$

adjustment coefficient

$\Rightarrow$ First order differential eq:- (Standard)

$$\frac{dy}{dt} + u(t) \cdot y = w(t)$$

Formula for

Non-homogeneous case $a \neq 0, b \neq 0$.

$$y(t) = \left\{ y(0) - \frac{b}{a} \right\} e^{-at} + \frac{b}{a}.$$

⇒ Definite time path for market equilibrium in dynamic system -

$$P(t) = \left\{ P(0) - P^* \right\} e^{-kt} + P^*$$

⇒ For dynamic stability of time path of FODF it should converge towards equilibrium.

⇒ Convergence happens when deviation from equilibrium goes to zero.

⇒ equation of exchange.

$$M_s \times V = P \times Y$$

⇒ Hotelling's rule - (named after Harold Hotelling)

$$\frac{\pi'(t)}{\pi(t)} = \gamma$$

$$\pi(t)$$

Formula for homogeneous case with $a \neq 0$.

$$y(t) = \{y(0)\} e^{-at}$$

For homogeneous case standard form of ~~homogeneous~~ differential eq becomes.

$$y(t) = Ae^{-\int u(t) dt} \quad \text{(General sol)}$$

General sol for non-homogeneous case.

$$y(t) = e^{-\int u(t) dt} \left\{ A + w(t) \int e^{\int u(t) dt} dt \right\}$$

Exact differential eq:-

$$F(y, t) = K$$

and

$$\left(\frac{\partial F}{\partial y} \right) dy + \left(\frac{\partial F}{\partial t} \right) dt = 0.$$

Exactness test:- to check whether

the eq is exact or not.

$$\frac{\partial}{\partial x} \left(\frac{\partial F}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial F}{\partial x} \right)$$

also is.
Hence's theorem $F_{xy} = F_{yx}$.

$$\frac{\partial}{\partial t} \left(\frac{\partial F}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial F}{\partial t} \right)$$

$$\frac{\partial F}{\partial y} = M, \quad \frac{\partial F}{\partial t} = N.$$

$(My + (N)) \partial t = 0$ (also an exact differential eq).

2) Integrating Factor can be found by two methods.

1) Trial & error method

2) By formula.

2-Rules to find integrating factor by formula:-

Rule 1:- $\frac{1}{M} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial t} \right) = f(y)$

above then $e^{\int f(y) dy}$ is int. factor

Rule 2:- $\frac{1}{N} \left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial t} \right) = g(t)$ or

then $e^{\int g(t) dt}$ is (First).

Bernoulli's equation :-

$$\frac{dy}{dt} + Ry = Ty^m \rightarrow \text{power of } y \text{ is } m$$

linearized form of Bernoulli's equation:-

$$dz + \{ (1-m) \times (Rz - T) \} dt = 0$$

Solow growth model

$$I = S \cdot \Phi(K) - \delta K$$

2nd order differential eq:-

$$\frac{d^2 y}{dt^2} = Ky$$

Leibniz notation:-

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + a_2 \frac{d^{n-2} y}{dt^{n-2}} + \dots +$$

$$a_{n-1} \frac{dy}{dt} + a_n(y) =$$

Lagrange's notation:-

$$y^{(n)}(t) + a_1 y^{(n-1)}(t) + a_2 y^{(n-2)}(t) + \dots + a_n y = b$$

2nd order differential eq:-

$$\frac{d^2y}{dt^2} + a_1 \cdot \frac{dy}{dt} + a_2 \cdot y = b$$

$$y''(t) + a_1 y'(t) + a_2 y(t) = b$$

Non-homogeneous case 1- $b \neq 0$
Sub-cases:-

$$\begin{array}{l} a_2 = 0 \\ a_1 = 0 \end{array} \quad \text{Then} \quad y_p = \frac{b}{2} t^2$$

$$\begin{array}{l} a_2 = 0 \\ a_1 \neq 0 \end{array} \quad \text{Then} \quad y_p = \frac{b}{a_1} t$$

$$\begin{array}{l} a_2 \neq 0 \\ \{a_1 \neq 0 \\ \text{or} \\ a_1 = 0\} \end{array} \quad \text{Then} \quad y_p = \frac{b}{a_2}$$

Homogeneous case:- ($b=0$) $y_p=0$
Sub-cases:-

$$\begin{array}{l} a_2 = 0 \\ a_1 = 0 \end{array} \quad \text{Then} \quad y_p = \frac{0}{2} t^2$$

$$\begin{array}{l} a_2 = 0 \\ a_1 \neq 0 \end{array} \quad \text{Then} \quad y_p = \frac{(0)}{a_1} t$$

$$a_2 \neq 0$$

then

$$y_p = \frac{0}{a_2}$$

$$\begin{cases} a_1 \neq 0 \\ \text{or} \\ a_1 = 0 \end{cases}$$

$$y = -a_1 \pm \sqrt{a_1^2 - 4a_2}$$

case I

$$a_1^2 - 4a_2 > 0$$

$$a_1^2 > 4a_2$$

Distinct real roots

2

case II

$$a_1^2 - 4a_2 = 0$$

$$a_1^2 = 4a_2$$

Repeated real roots

case III

$$a_1^2 - 4a_2 < 0$$

$$a_1^2 < 4a_2$$

Complex roots