

Mth 101 (Main Points) (23-40+5)

→ To find max or min value on interval:

- find critical points $\rightarrow f'(x)=0$
- Evaluate $f(x)$ on critical & end points.
= where end points $f(x)=0$ (if not given)
- Smallest value $\rightarrow$ Minimum • Largest value $\rightarrow$ Maximum

* $\lim_{x \rightarrow \infty} f(x) = +\infty$ (No max value but have min value)

* $\lim_{x \rightarrow \infty} f(x) = -\infty$ (No min value but have max value)

→ Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (\text{for finding the approximate value})$$

→ Rolle's Theorem:

Differentiable on (a, b) & continuous

on $[a, b]$. If $f(a) = f(b) = 0$, then

there is at least one point c where

$$f'(c) = 0$$

→ Mean Value Theorem:-

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

→ Absolute Maximum $\rightarrow$ Absolute Minimum

$$f(x_0) > f(x)$$

$$f(x_0) < f(x)$$

Integration:

(process of finding antiderivative)

$$\int f(x) dx = F(x) + C$$

Annotations:
- $\int$: integral sign
- $f(x)$: Integrand
- C : constant of integration

"Indefinite integral of $f(x)$ equals $F(x)$."

Sigma Notation:

Σ $\rightarrow$ upper case letter in Greek

$\hookrightarrow$ sigma / summation

Properties of Sigma Notation:-

Theorems:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2} \right]^2$$

Area as Limits:

$$\Delta x = \frac{b-a}{n}$$

$$\text{Area} = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k^*) \Delta x$$

$$= f(x_1^*) \Delta x + f(x_2^*) \Delta x + \dots + f(x_n^*) \Delta x$$

Left End point: $x_k^* = x_{k-1} = a + (k-1) \Delta x$

Right End point: $x_k^* = x_k = a + k \Delta x$

Mid point: $x_k^* = \frac{1}{2} (x_{k-1} + x_k) = a + (k - \frac{1}{2}) \Delta x$

Used to find
approx area
under curve.

Riemann Sum: $\lim_{\max \Delta x_k \rightarrow 0} \sum_{k=1}^n f(x_k^*) \Delta x_k$

if we take limits, it becomes definite integral — & if we don't take limits then it approx equal to definite integral.

Mean Value Theorem (MVT): $\int_a^b f(x) dx = f(x^*) (b-a)$

Average Value Theorem (AVT): $f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$

Second fundamental theorem of Calculus: $\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$

→ Every continuous function on an interval has an antiderivative on that interval.

Area of Region: $A = \int_a^b [f(x) - g(x)] dx$

Volume formula: $V = \int_a^b A(x) dx$

Volume of Solid (Disk): $V = \int_a^b \pi [f(x)]^2 dx$
perpendicular to x-axis

Volume of washer: $V = \int_a^b \pi [(f(x))^2 - (g(x))^2] dx$
perpendicular to x-axis

Volume of cylindrical shell: $V = \int_a^b 2\pi x f(x) dx$

Arc length: $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$

Surface Area: $S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$

Work done:

$W = F \cdot d$ ($d \rightarrow$ meter) ($F \rightarrow$ pounds (lbs), Dynes or Newton (N))

- One Dyne - mass of 1 gram & acceleration of 1 cm/s^2 .
- 1 Newton - mass of 1 kg & acceleration of 1 m/s^2 .

Common units of Workdone:

- * foot-pounds (ft-lbs)
- * dyne-centimeters (dyne cm)
- * newton-meter (Nm)

$1 \text{ Nm} \rightarrow \text{Joule (J)}$

$W = \int_a^b F(x) dx$

$\therefore \text{density of } H_2O = 62.4$

Pressure:

$P = \frac{F}{A} = \rho h$

Fluid pressure:

$F = \rho h A$

Fluid force: $F = \int_a^b \rho h(x) w(x) dx$

Improper Integral:

interval is infinite $[a, \infty)$ the integral will be $\int_a^{\infty} f(x) dx = \lim_{l \rightarrow \infty} \int_a^l f(x) dx$

- If limit exists $\rightarrow$ converge
- If limit does not exist $\rightarrow$ diverges

L'Hopital Rule & Indeterminant:

Indeterminant form:-

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 0^0, \infty^0, 1^\infty, \infty \cdot \infty$$

L'Hopital Rule for $0/0$ form:

$$\lim f(x) = 0, \quad \lim g(x) = 0$$

$$\lim \left[\frac{f(x)}{g(x)} \right] = \lim \left[\frac{f'(x)}{g'(x)} \right]$$

L'Hopital Rule for ∞/∞ form:

$$\lim f(x) = \infty, \quad \lim g(x) = \infty$$

$$\lim \left[\frac{f(x)}{g(x)} \right] = \lim \left[\frac{f'(x)}{g'(x)} \right]$$

L'Hopital Rule for $0 \cdot \infty$:

$$\lim f(x) = 0, \quad \lim g(x) = \infty$$

$$\lim f(x) \cdot g(x) = 0 \cdot \infty$$

1st convert it into $0/0$ form & then

apply L'Hopital Rule:-

$$f(x) \cdot g(x) = \frac{f(x)}{1/g(x)} = \frac{0}{1/\infty} = \frac{0}{0}$$

L'Hopital Rule for 0° , ∞° , 1° :

$$\lim_{x \rightarrow a} f(x)^{g(x)}$$

let, $y = f(x)^{g(x)}$

$$\ln y = \ln f(x)^{g(x)} \quad (\text{Apply } \ln)$$

$$\ln y = g(x) \ln f(x)$$

$$\lim_{x \rightarrow a} \ln y = \lim_{x \rightarrow a} [g(x) \ln f(x)] \quad (\text{Apply Limit})$$

Sequence:

Succession of (number) $\leftarrow$ converge
numbers $(-\infty, +\infty) \leftarrow$ diverge

Monotone:

Sequence that is either non-decreasing or non-increasing.

Strictly Monotone:

Sequence which is increasing or decreasing.

$\therefore$ "Strictly monotone" is monotone but not conversely.

Monotonicity & Testing for it:

A sequence $\{a_n\}$ is called

increasing if $\rightarrow a_1 < a_2 < a_3 < \dots < a_n < \dots$

non-decreasing if $\rightarrow a_1 \leq a_2 \leq a_3 \leq \dots \leq a_n \leq \dots$

decreasing if $\rightarrow a_1 > a_2 > a_3 > \dots > a_n > \dots$

non-increasing if $\rightarrow a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \geq \dots$

Testing for Monotonicity:

(a) Difference b/w successive terms:

- $a_{n+1} - a_n > 0$ Increasing
- $a_{n+1} - a_n < 0$ Decreasing
- $a_{n+1} - a_n \geq 0$ Non-decreasing
- $a_{n+1} - a_n \leq 0$ Non-increasing

(b) Ratio of successive terms:

- $\frac{a_{n+1}}{a_n} > 1$ Increasing
- $\frac{a_{n+1}}{a_n} \geq 1$ Non-decreasing
- $\frac{a_{n+1}}{a_n} < 1$ Decreasing
- $\frac{a_{n+1}}{a_n} \leq 1$ Non-increasing

(c) Derivative Test:

- $f'(x) > 0$ Increasing
- $f'(x) < 0$ Decreasing
- $f'(x) \geq 0$ Non-decreasing
- $f'(x) \leq 0$ Non-increasing

Upper Bound: $a_n \leq M$ satisfy $L \leq M$

Lower Bound: $a_n \geq M$ satisfy $L \geq M$