

Practice Exercise lecture no. 4

Q1. Find the slopes of the sides of the triangle with vertices $(-1, 3)$, $(5, 4)$ and $(2, 8)$.

Solution: Let $A(-1,3)$, $B(5,4)$ and $C(2,8)$ be the given points, then

$$\text{Slope of side AB} = \frac{4-3}{5+1} = \frac{1}{6}$$

$$\text{Slope of side BC} = \frac{8-4}{2-5} = \frac{-4}{3}$$

$$\text{Slope of side CA} = \frac{3-8}{-1-2} = \frac{5}{3}$$

Q2. Find equation of the line passing through the point $(1,2)$ and having slope 3.

Solution:

Point-slope form of the line passing through $P(x_1, y_1)$ and having slope m is given by the equation:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 2 = 3(x - 1)$$

$$\Rightarrow y - 2 = 3x - 3$$

$$\Rightarrow y = 3x - 1$$

Q3. Find the slope-intercept form of the equation of the line that passes through the point $(5, -3)$ and perpendicular to line $y = 2x + 1$.

Solution:

The slope-intercept form of the line with y -intercept b and slope m is given by the equation: $y = mx + b$

The given line has slope 2, so the line to be determined will have slope $m = -\frac{1}{2}$.

Substituting this slope and the given point in the point-slope form: $y - y_1 = m(x - x_1)$, yields

$$y - (-3) = -\frac{1}{2}(x - 5)$$

$$\Rightarrow y + 3 = -\frac{1}{2}(x - 5)$$

$$\Rightarrow y = -\frac{1}{2}x + \frac{5}{2} - 3 \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

- Q4. Find the slope and angle of inclination of the line joining the points (2, 3) and (-1, 2).

Solution: If m is the slope of line joining the points (2, 3) and (-1, 2) then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{-1 - 2} = \frac{1}{3} \text{ is the slope}$$

Now angle of inclination is:

$$\tan \theta = m$$

$$\tan \theta = \frac{1}{3}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43^\circ$$

- Q5. By means of slopes, Show that the points lie on the same line

A (-3, 4); B (3, 2); C (6, 1)

$$\text{Solution: Slope of line through A(-3, 4); B(3, 2)} = \frac{2 - 4}{3 - (-3)} = \frac{-2}{6} = -\frac{1}{3}$$

$$\text{Slope of line through B(3, 2); C(6, 1)} = \frac{1 - 2}{6 - 3} = \frac{-1}{3}$$

$$\text{Slope of line through C(6, 1); A(-3, 4)} = \frac{4 - 1}{-3 - 6} = \frac{3}{-9} = -\frac{1}{3}$$

Since all slopes are same, so the given points lie on the same line.

Practice Exercise

Lecture No.5: Distance, Circles, Equations

Solution 1:

The formula to find the distance between any two points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

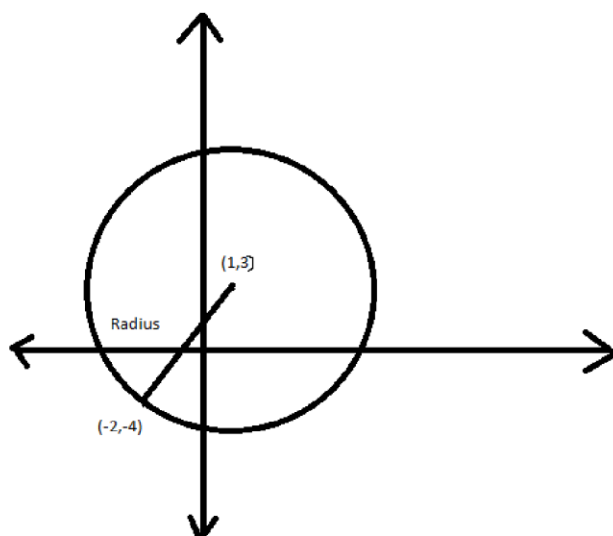
The given points are (5, 6) and (2, 4), so the distance between these two points will be

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$$\begin{aligned}d &= \sqrt{(2-5)^2 + (4-6)^2} \\&= \sqrt{(-3)^2 + (-2)^2} \\&= \sqrt{9+4} \\&= \sqrt{13}\end{aligned}$$

Solution 2:

It is given that center of the circle is (1,3). We are also given a point on the circle that is (-2,-4) as shown below.



The radius of the circle will be the distance between the points (1,3) and (-2,-4). That is

$$\begin{aligned}\text{Radius} = d &= \sqrt{[1 - (-2)]^2 + [3 - (-4)]^2} \\&= \sqrt{(3)^2 + (7)^2} \\&= \sqrt{9 + 49} = \sqrt{58}\end{aligned}$$

Solution 3:

The general form of the equation of circle is given as

$$4x^2 + 4y^2 - 16x - 24y + 51 = 0$$

It can be re-written as

$$(4x^2 - 16x) + (4y^2 - 24y) = -51$$

$$(2x)^2 - 2(8x) + (2y)^2 - 2(12y) = -51$$

In order to complete the squares on the left hand side, we have to add 16 and 36 on both sides, it will then become

$$(2x)^2 - 2(8x) + 16 + (2y)^2 - 2(12y) + 36 = -51 + 16 + 36$$

$$(2x)^2 - 2(2x)(4) + (4^2) + (2y)^2 - 2(2y)(6) + (6)^2 = 1$$

$$(2x-4)^2 + (2y-6)^2 = 1$$

$$(x-2)^2 + (y-3)^2 = \left(\frac{1}{2}\right)^2$$

Comparing it with the standard form of the equation, the center of the circle will be (2, 3) and the radius will be $\frac{1}{2}$.

Solution 4:

The general form of the equation of circle is given as

$$2x^2 + 2y^2 + 6x - 8y + 12 = 0$$

It can be re-written as

$$(2x^2 + 6x) + (2y^2 - 8y) = -12$$

$$(x^2 + 3x) + (y^2 - 4y) = -6$$

In order to complete the squares on the left hand side, we have to add $\frac{9}{4}$ and 4 on both sides, it will then become

$$(x^2 + 3x + \frac{9}{4}) + (y^2 - 4y + 4) = -6 + \frac{9}{4} + 4$$

$$(x + \frac{3}{2})^2 + (y - 2)^2 = \frac{1}{4}$$

$$\left(x + \frac{3}{2}\right)^2 + (y - 2)^2 = \frac{1}{4}$$

Comparing it with the standard form of the equation, the center of the circle will be $\left(-\frac{3}{2}, 2\right)$ and radius will be $\frac{1}{2}$.

Solution 5:

The general form of the equation of circle is given as

$$x^2 + y^2 - 4x - 6y + 8 = 0$$

This can be re-written as

$$(x^2 - 4x) + (y^2 - 6y) = -8$$

In order to complete the squares on the left hand side, we have to add 4 and 9 on both sides, it will then become

$$(x^2 - 4x + 4) + (y^2 - 6y + 9) = -8 + 4 + 9$$

$$(x)^2 - 2(x)(2) + (2)^2 + (y)^2 - 2(y)(3) + (3)^2 = 5$$

$$(x-2)^2 + (y-3)^2 = 5$$

Comparing it with the standard form of the equation, the center of the circle will be (2,3) and the radius will be $\sqrt{5}$.

SOLUTION 6:

$$\therefore 3x^2 + 6x + 3y^2 + 18y - 6 = 0,$$

$$\Rightarrow 3(x^2 + 2x + y^2 + 6y - 2) = 0, \quad (\because \text{taking 3 as common})$$

$$\Rightarrow x^2 + 2x + y^2 + 6y - 2 = 0, \quad (\because \text{dividing by 3 on both sides})$$

$$\Rightarrow x^2 + 2x + 1 + y^2 + 6y + 9 = 2 + 9 + 1,$$

$$\Rightarrow (x+1)^2 + (y+3)^2 = 12,$$

$$\Rightarrow (x+1)^2 + (y+3)^2 = (\sqrt{12})^2,$$

$$\Rightarrow (x-(-1))^2 + (y-(-3))^2 = (\sqrt{12})^2,$$

$$\therefore \text{Centre of the circle is } (-1, -3) \text{ and radius is } \sqrt{12}.$$

SOLUTION 7:

$$x^2 - 6x + y^2 - 8y = 0, \quad (\because \text{rearranging the term})$$

$$x^2 - 6x + y^2 - 8y + (3)^2 = (3)^2, \quad (\because \text{adding } (3)^2 \text{ on both sides})$$

$$(x^2 - 6x + 9) + y^2 - 8y = 9,$$

$$(x^2 - 6x + 9) + y^2 - 8y + (4)^2 = 9 + (4)^2, \quad (\because \text{adding } (4)^2 \text{ on both sides})$$

$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16,$$

$$(x-3)^2 + (y-4)^2 = 9 + 16,$$

$$(x-3)^2 + (y-4)^2 = (\sqrt{25})^2, \quad \text{eq.(1)}$$

$$\therefore (x-x_0)^2 + (y-y_0)^2 = r^2. \quad \text{eq.(2)}$$

The eq.(1) is now in the standard form of eq.(2). This equation represents a circle with the center at (3, 4) and with a radius equal to $\sqrt{25}$.

SOLUTION 8:

The standard form of equation of circle is

$$(x-h)^2 + (y-k)^2 = r^2 ,$$

$$\text{Here } h=3 , k=-2 , r=4 ,$$

$$(x-3)^2 + (y-(-2))^2 = 4^2 ,$$

$$x^2 - 6x + 9 + y^2 + 4 + 4y = 16 ,$$

$$x^2 + y^2 - 6x + 4y = 16 - 9 - 4 ,$$

$$x^2 + y^2 - 6x + 4y = 3.$$

SOLUTION 9:

The distance formula between two points (x_1, y_1) and (x_2, y_2) in a coordinate plane is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} ,$$

$$d = \sqrt{(8-2)^2 + (6-4)^2} ,$$

$$= \sqrt{(6)^2 + (2)^2} ,$$

$$= \sqrt{36+4} ,$$

$$= \sqrt{40} ,$$

$$= 2\sqrt{10}.$$

SOLUTION 10:

The radius is the distance between the center and any point on the circle, so find the distance:

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} ,$$

$$r = \sqrt{(3-(-1))^2 + (-2-(-3))^2} ,$$

$$= \sqrt{(3+1)^2 + (-2+3)^2} ,$$

$$= \sqrt{(4)^2 + (1)^2} ,$$

$$= \sqrt{16+1} ,$$

$$= \sqrt{17} ,$$

$$\approx 4.123.$$

Then the radius is $\sqrt{17}$, or about 4.123, rounded to three decimal places.

Practice Exercise
Lecture No.6: Functions

Q.No.1

Find the natural domain and the range of the given function

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$$h(x) = \cos^2(\sqrt{x}).$$

Solution:

As we know that the $\sqrt{x}$ is defined on non-negative real numbers $x \geq 0$. This means that the natural domain of $h(x)$ is the set of positive real numbers.

Therefore, the natural domain of $h(x) = [0, +\infty)$.

As we also know that the range of trigonometric function $\cos x$ is $[-1, 1]$.

The function $\cos^2 \sqrt{x}$ always gives positive real values within the range 0 and 1 both inclusive. From this we conclude that the range of $h(x) = [0, 1]$.

Q.No.2

Find the domain and range of function f defined by $f(x) = x^2 - 2$.

Solution:

$$\because f(x) = x^2 - 2$$

The domain of this function is the set of all real numbers.

The range is the set of values that $f(x)$ takes as x varies. If x is a real number, x^2 is either positive or zero. Hence we can write the following:

$$x^2 \geq 0,$$

Subtract -2 on both sides to obtain

$$x^2 - 2 \geq -2.$$

The last inequality indicates that $x^2 - 2$ takes all values greater than or equal to -2 . The range of function f is the set of all values of $f(x)$ in the interval $[-2, +\infty)$.

Q.No.3

Determine whether $y = \pm \sqrt{x+3}$ is a function or not? Justify your answer.

Solution:

$$\because y = \pm \sqrt{x+3}$$

This is not a function because each value that is assigned to 'x' gives two values of y. So this is not a function. For example, if $x=1$ then

$$y = \pm \sqrt{1+3},$$

$$y = \pm \sqrt{4},$$

$$y = \pm 2.$$

Q.No.4

Determine whether $y = \frac{x+2}{x+3}$ is a function or not? Justify your answer.

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Solution:

$$\therefore y = \frac{x+2}{x+3}$$

This is a function because each value that is assigned to 'x' gives only one value of y
So this is a function. For example if $x=1$ then

$$y = \frac{1+2}{1+3},$$

$$y = \frac{3}{4},$$

$$y = 0.75.$$

Q.No.5

(a) Find the natural domain of the function $f(x) = \frac{x^2-16}{x-4}$.

(b) Find the domain of function f defined by $f(x) = \frac{-1}{(x+5)}$.

Solution:

(a)

$$\begin{aligned}\therefore f(x) &= \frac{x^2-16}{x-4}, \\ \Rightarrow f(x) &= \frac{(x+4)(x-4)}{(x-4)}, \\ &= (x+4) \quad ; \quad x \neq 4.\end{aligned}$$

This function consists of all real numbers x , except $x=4$.

(b)

$$\therefore f(x) = \frac{-1}{(x+5)}$$

This function consists of all real numbers x , except $x=-5$. Since $x=-5$ would make the denominator equal to zero and the division by zero is not allowed in mathematics.

Hence the domain in interval notation is given by $(-\infty, -5) \cup (-5, +\infty)$.

Practice Exercise

Lecture No.8: Graphs of Functions

Lecture No.9: Limits

Choose the correct option for the following questions:

- 1) If a vertical line intersects the graph of the equation at two points, then which of the following is true.
 - I. It represents a function.
 - II. It represents a parabola.
 - III. It represents a straight line.

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IV. It does not represent a function.

2) Which of the following is the reflection of the graph of $f(x)$ about y-axis?

I.

II.

III.

IV.

3) Given the graph of a function $f(x)$ and a constant c , the graph of $f(x+c)$ can be obtained by

I. Translating the graph of $f(x)$ up by c units.

II. Translating the graph of $f(x)$ down by c units.

III. Translating the graph of $f(x)$ right by c units.

IV. Translating the graph of $f(x)$ left by c units.

4) Given the graph of a function $f(x)$ and a constant c , the graph of $f(x-c)$ can be obtained by

I. Translating the graph of $f(x)$ up by c units.

II. Translating the graph of $f(x)$ down by c units.

III. Translating the graph of $f(x)$ right by c units.

IV. Translating the graph of $f(x)$ left by c units.

5) Which of the following is the reflection of the graph of $f(x)$ about x-axis?

V.

VI.

VII.

VIII.

Q.No.6

If $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$ then find the value of 'c' so that

Answer:

Q.No.7

Find the limit by using the definition of absolute value

Answer:

Q.No.8

Find the limit by using the definition of absolute value

Answer: -1

Q.No.9

Evaluate: .

Answer: 0

Q.No.10

Evaluate: .

Answer:

Practice Exercise Lecture No.14: Rate of Change

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Q.No.1

Find the instantaneous rate of change of $f(x) = x^2 + 1$ at x_0 .

Answer: $2x_0$

Solution:

Since $f(x) = x^2 + 1$ at x_0 ,

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{f(x_0+h)-f(x_0)}{h} &= \lim_{h \rightarrow 0} \frac{((x_0+h)^2+1)-(x_0^2+1)}{h}, \\ &= \lim_{h \rightarrow 0} \frac{x_0^2+h^2+2x_0h+1-x_0^2-1}{h}, \\ &= \lim_{h \rightarrow 0} \frac{h^2+2x_0h}{h}, \\ &= \lim_{h \rightarrow 0} \frac{h(h+2x_0)}{h}, \\ &= \lim_{h \rightarrow 0} (h + 2x_0), \\ &= 2x_0 \text{ by applying limit, (Answer).}\end{aligned}$$

Q.No.2

Find the instantaneous rate of change of $f(x) = \sqrt{x+2}$ at an arbitrary point of the domain of f .

Answer: $\frac{1}{2\sqrt{a+2}}$

Solution:

Let a be any arbitrary point of the domain of f . The instantaneous rate of change of $f(x)$ at $x = a$ is

$$\begin{aligned}\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a} &= \lim_{x \rightarrow a} \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a}, \\ &= \lim_{x \rightarrow a} \frac{\sqrt{x+2}-\sqrt{a+2}}{x-a} \times \frac{\sqrt{x+2}+\sqrt{a+2}}{\sqrt{x+2}+\sqrt{a+2}} \text{ by rationalizing,} \\ &= \lim_{x \rightarrow a} \frac{x+2-a-2}{(x-a)\sqrt{x+2}+\sqrt{a+2}}, \\ &= \lim_{x \rightarrow a} \frac{x-a}{(x-a)\sqrt{x+2}+\sqrt{a+2}},\end{aligned}$$

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$$\begin{aligned} &= \lim_{x \rightarrow a} \frac{1}{\sqrt{x+2} + \sqrt{a+2}}, \\ &= \frac{1}{\sqrt{a+2} + \sqrt{a+2}} \text{ by applying limit,} \\ &= \frac{1}{2\sqrt{a+2}} \text{ (Answer).} \end{aligned}$$

Q.No.3

The distance traveled by an object at time t is $= f(t) = t^2$. Find the instantaneous velocity of the object at $t_0 = 4 \text{ sec}$.

Answer: 8

Solution:

$$\begin{aligned} v_{inst} = m_{tan} &= \lim_{t_1 \rightarrow t_0} \frac{f(t_1) - f(t_0)}{t_1 - t_0}, \\ &= \lim_{t_1 \rightarrow t_0} \frac{t_1^2 - 4^2}{t_1 - t_0}, \\ &= \lim_{t_1 \rightarrow t_0} \frac{t_1^2 - 16}{t_1 - t_0}, \\ &= \lim_{t_1 \rightarrow t_0} \frac{(t_1 + 4)(t_1 - 4)}{t_1 - t_0}, \\ &= \lim_{t_1 \rightarrow 4} \frac{(t_1 + 4)(t_1 - 4)}{(t_1 - 4)} \text{ because } t_0 = 4 \text{ sec,} \\ &= \lim_{t_1 \rightarrow 4} (t_1 + 4), \\ &= 4 + 4 \text{ by applying limit,} \\ &= 8 \text{ (Answer).} \end{aligned}$$

Q.No.4

Find the instantaneous rate of change of $f(x) = x^3 + 1$ at $x_0 = 2$.

Answer: 12

Solution:

$$\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{h \rightarrow 0} \frac{f(2 + h) - f(2)}{h},$$

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$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{((2+h)^3 + 1) - (2^3 + 1)}{h}, \\
 &= \lim_{h \rightarrow 0} \frac{(2^3 + 3(2)^2h + 3(2)h^2 + h^3 + 1) - (2^3 + 1)}{h}, \\
 &= \lim_{h \rightarrow 0} \frac{8 + 12h + 6h^2 + h^3 + 1 - (8 + 1)}{h}, \\
 &= \lim_{h \rightarrow 0} \frac{9 + 12h + 6h^2 + h^3 - 9}{h}, \\
 &= \lim_{h \rightarrow 0} \frac{12h + 6h^2 + h^3}{h}, \\
 &= \lim_{h \rightarrow 0} \frac{h(12 + 6h + h^2)}{h}, \\
 &= \lim_{h \rightarrow 0} (12 + 6h + h^2), \\
 &= 12 \text{ (Answer)}.
 \end{aligned}$$

Q.No.5

(a) The distance traveled by an object at time t is $s = f(t) = t^2$. Find the average velocity of the object between $t = 2 \text{ sec.}$ and $t = 4 \text{ sec.}$

(b) Let $f(x) = \frac{1}{x-1}$. Find the average rate of change of f over the interval $[5, 7]$.

Answer: (a) 14 (b) $\frac{5}{2} \text{ m/sec}$

Solution:

$$\begin{aligned}
 \text{(a) Average Velocity} &= \frac{\text{Distance travelled during interval}}{\text{Time Elapsed}}, \\
 v_{ave} &= \frac{f(t_1) - f(t_0)}{t_1 - t_0}, \\
 &= \frac{f(4) - f(2)}{4 - 2}, \\
 &= \frac{4^2 - 2^2}{2}, \\
 &= \frac{16 - 4}{2},
 \end{aligned}$$

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$$\begin{aligned} &= \frac{12}{2}, \\ &= 6 \text{ (Answer).} \end{aligned}$$

(b) Average Velocity = $\frac{\text{Distance travelled during interval}}{\text{Time Elapsed}}$,

$$\begin{aligned} m_{sec} &= \frac{f(x_1) - f(x_0)}{x_1 - x_0}, \\ &= \frac{f(7) - f(5)}{7 - 5}, \\ &= \frac{\frac{1}{7-1} - \frac{1}{5-1}}{2}, \\ &= \frac{\frac{1}{6} - \frac{1}{4}}{2}, \\ &= -\frac{1}{24} \text{ m/sec. (Answer).} \end{aligned}$$

Practice Exercise

Lecture No.17: Derivatives of Trigonometric Functions

Q.No.1

Find $\frac{dy}{dx}$ if $y = x^3 \cot x - \frac{3}{x^3}$.

Answer: $3x^2 \cot x - x^3 \operatorname{cosec}^2 x + \frac{9}{x^4}$

Solution:

Given $y = x^3 \cot x - \frac{3}{x^3}$,

$$\begin{aligned} \frac{dy}{dx} &= \cot x \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}(\cot x) - \frac{d}{dx}\left(\frac{3}{x^3}\right), \\ &= \cot x (3x^2) + x^3 (-\operatorname{cosec}^2 x) - 3 \frac{d}{dx}\left(\frac{1}{x^3}\right), \\ &= 3x^2 \cot x - x^3 \operatorname{cosec}^2 x + \frac{9}{x^4} \quad \text{(Answer).} \end{aligned}$$

Q.No.2

Find $\frac{dy}{dx}$ if $y = x^4 \sin x$ at $x = \pi$.

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Answer: $-\pi^4$

Solution:

$$\because \frac{d}{dx}(f \cdot g) = f \cdot \frac{d}{dx}(g) + g \cdot \frac{d}{dx}(f),$$

$$y = x^4 \sin x \text{ at } x = \pi,$$

$$\frac{d}{dx} = \sin x \frac{d}{dx}(x^4) + x^4 \frac{d}{dx}(\sin x),$$

$$= \sin x (4x^3) + x^4(\cos x),$$

$$= 4x^3 \sin x + x^4 \cos x,$$

$$= 4\pi^3 \sin \pi + \pi^4 \cos \pi, \text{ at } x = \pi,$$

$$= 4\pi^3(0) + \pi^4(-1),$$

$$= -\pi^4 \quad (\text{Answer}).$$

Q.No.3

$$\text{Find } f'(t) \text{ if } f(t) = \frac{2-8t+t^2}{\sin t}.$$

$$\text{Answer: } \frac{[(2t-8)(\sin t)] - [(t^2-8t+2)(\cos t)]}{\sin^2 t}$$

Solution:

$$\because \frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot \frac{d}{dx}(f(x)) - f(x) \cdot \frac{d}{dx}(g(x))}{[g(x)]^2},$$

$$f(t) = \frac{2-8t+t^2}{\sin t},$$

$$f'(t) = \frac{[(\sin t)(-8+2t)] - [(2-8t+t^2)(\cos t)]}{(\sin t)^2},$$

$$= \frac{[(2t-8)(\sin t)] - [(t^2-8t+2)(\cos t)]}{\sin^2 t} \quad (\text{Answer}).$$

Q.No.4

$$\text{Find } f'(y) \text{ if } (y) = \frac{\sin y + 3 \tan y}{y^3 - 2}.$$

$$\text{Answer: } \frac{[(y^3-2)(\cos y + 3 \sec^2 y)] - [(\sin y + 3 \tan y) + (3 y^2)]}{y^6 - 4y^3 + 4}$$

Solution:

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot \frac{d}{dx}(f(x)) - f(x) \cdot \frac{d}{dx}(g(x))}{[g(x)]^2},$$

$$f(y) = \frac{\sin y + 3 \tan y}{y^3 - 2}.$$

$$f'(y) = \frac{[(y^3-2)(\cos y + 3 \sec^2 y)] - [(\sin y + 3 \tan y) + (3 y^2)]}{(y^3-2)^2},$$

$$= \frac{[(y^3-2)(\cos y + 3 \sec^2 y)] - [(\sin y + 3 \tan y) + (3 y^2)]}{y^6 - 4y^3 + 4} \quad (\text{Answer}).$$

Q.No.5

(a) Find $\frac{dy}{dx}$ if $y = (5x^2 + 3x + 3)(\sin x)$.

(b) Find $f'(t)$ if $(t) = 5t \sin t$.

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Answer: (a) $(5x^2 + 3x + 3)(\cos x) + \sin x \cdot (10x + 3)$
(b) $5t \cos t + 5 \sin t$

Solution:

$$\begin{aligned} \text{(a)} \quad \because \frac{d}{dx}(f \cdot g) &= f \cdot \frac{d}{dx}(g) + g \cdot \frac{d}{dx}(f), \\ y &= (5x^2 + 3x + 3)(\sin x), \\ \frac{d}{dx}[(5x^2 + 3x + 3)(\sin x)] &= (5x^2 + 3x + 3)(\cos x) + \sin x (10x + 3) \quad (\text{Answer}). \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \because \frac{d}{dx}(f \cdot g) &= f \cdot \frac{d}{dx}(g) + g \cdot \frac{d}{dx}(f), \\ f(t) &= 5t \sin t, \\ \frac{d}{dx}(5t \sin t) &= 5t \cos t + (\sin t)(5), \\ &= 5t \cos t + 5 \sin t \quad (\text{Answer}). \end{aligned}$$

Practice Exercise

Lecture No.20: Derivatives of Logarithmic and Exponential Functions

Q.No.1

Differentiate: $y = (5-x)^{\sqrt{x}}$.

$$\text{Answer: } \frac{dy}{dx} = \left(\frac{\ln(5-x)}{2\sqrt{x}} - \frac{\sqrt{x}}{(5-x)} \right) \cdot (5-x)^{\sqrt{x}}$$

Solution:

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$$\begin{aligned}
 \because y &= (5-x)^{\sqrt{x}}, \\
 \text{taking log on both sides,} \\
 \Rightarrow \ln y &= \sqrt{x} \ln(5-x) \quad (\because \ln m^n = n \ln m), \\
 \Rightarrow \frac{1}{y} \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \cdot \ln(5-x) + \frac{1}{(5-x)} (-1) \cdot \sqrt{x}, \\
 \Rightarrow \frac{dy}{dx} &= \left(\frac{\ln(5-x)}{2\sqrt{x}} - \frac{\sqrt{x}}{(5-x)} \right) \cdot y, \\
 \Rightarrow \frac{dy}{dx} &= \left(\frac{\ln(5-x)}{2\sqrt{x}} - \frac{\sqrt{x}}{(5-x)} \right) \cdot (5-x)^{\sqrt{x}}.
 \end{aligned}$$

Q.No.2

Differentiate $y = (\cos x)^{8x}$ with respect to 'x'.

Answer: $\frac{dy}{dx} = \left(8 \ln(\cos x) - \frac{8x \sin x}{\cos x} \right) (\cos x)^{8x}$

Solution:

$$\begin{aligned}
 \because y &= (\cos x)^{8x}, \\
 \text{taking log on both sides,} \\
 \Rightarrow \ln y &= (8x) \ln(\cos x), \quad (\because \ln m^n = n \ln m), \\
 \Rightarrow \frac{1}{y} \frac{dy}{dx} &= 8 \cdot \ln(\cos x) + \frac{1}{(\cos x)} \cdot (-\sin x) \cdot (8x), \quad \left(\because \frac{d}{dx}(\ln x) = \frac{1}{x}; \frac{d}{dx}(\cos x) = -\sin x \right), \\
 \Rightarrow \frac{dy}{dx} &= \left(8 \ln(\cos x) - \frac{8x \sin x}{\cos x} \right) \cdot y, \\
 \Rightarrow \frac{dy}{dx} &= \left(8 \ln(\cos x) - \frac{8x \sin x}{\cos x} \right) (\cos x)^{8x}.
 \end{aligned}$$

Q.No.3

Differentiate $y = x^{\sin 5x}$ with respect to 'x'.

Answer: $\frac{dy}{dx} = \left(5(\cos 5x) \cdot \ln(x) + \frac{\sin 5x}{x} \right) (x^{\sin 5x})$

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Solution:

$$\because y = x^{\sin 5x},$$

Taking log on both sides ,

$$\Rightarrow \ln y = (\sin 5x) \ln(x), \quad (\because \ln m^n = n \ln m),$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = 5(\cos 5x) \cdot \ln(x) + \frac{1}{x} \cdot (\sin 5x), \quad \left(\because \frac{d}{dx}(\ln x) = \frac{1}{x} ; \frac{d}{dx}(\sin x) = \cos x \right),$$

$$\Rightarrow \frac{dy}{dx} = \left(5(\cos 5x) \cdot \ln(x) + \frac{\sin 5x}{x} \right) \cdot y,$$

$$\Rightarrow \frac{dy}{dx} = \left(5(\cos 5x) \cdot \ln(x) + \frac{\sin 5x}{x} \right) (x^{\sin 5x}).$$

Q.No.4

Differentiate $y = x e^{3x+4}$.

Answer: $\frac{dy}{dx} = e^{3x+4} + 3xe^{3x+4}$

Solution:

$$\begin{aligned} \because y &= x e^{3x+4}, \\ \Rightarrow \frac{dy}{dx} &= e^{3x+4} + x e^{3x+4} \frac{d}{dx}(3x+4), \\ \Rightarrow \frac{dy}{dx} &= e^{3x+4} + 3xe^{3x+4}. \end{aligned}$$

Q.No.5

Find the derivative of the function $y = \ln(2+x^5)$ with respect to 'x'.

Answer: $\frac{dy}{dx} = \frac{5x^4}{(2+x^5)}$

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Solution:

$$y = \ln(2 + x^5) ,$$

now taking the derivative of the function on both sides ,

$$\frac{dy}{dx} = \frac{d}{dx} \{ \ln(2 + x^5) \} ,$$

$$\frac{dy}{dx} = \frac{1}{(2 + x^5)} \frac{d}{dx} (2 + x^5) ,$$

$$\frac{dy}{dx} = \frac{1}{(2 + x^5)} (0 + 5x^4) ,$$

$$\frac{dy}{dx} = \frac{5x^4}{(2 + x^5)} .$$