MTH202
PRACTICE QUESTION
WITH
SOLUTION
FOR FINAL TERM
LECTURE (23 TO 45)

Solution File Lecture No 23-25

Question No.1:

Use mathematical induction to prove that $4^n > 3^n + 4$ is true for integral values of $n \geq 2$

Solution:

Let P(n) = $4^n > 3^n + 4$

Let $P(n) = 4^n > 3^n + 4$

BASIS STEP:

$n = 2$

$4^2 > 3^2 + 4 = 16 > 13$

So $P(2)$ is true

INDUCTIVE STEP:

Suppose $P(k)$ is true for all $k \geq 2$

$$4^k > 3^k + 4 \qquad (1)$$

we are to prove $P(k+1)$ is true i.e.

$$4^{k+1} > 3^{k+1} + 4 \qquad (2)$$

Now

LHS $= 4^{k+1} = 4^k.4 > 4.(3^k + 4) = 4.3^k + 16$ *by* (1) *for* $k \geq 2$.

$4^{k+1} > 3^k.(3+1) + 16$

$4^{k+1} > 3^k.3 + 3^k + 16$

$4^{k+1} > 3^{k+1} + 3^k + 4 + 12$

$4^{k+1} > 3^{k+1} + 4 + 12 + 3^k$

$4^{k+1} > 3^{k+1} + 4$ *ignoring* $12 + 3^k$ *as if* $a > b + c$ *then* $a > b$

Hence it is proved by mathematical induction that $4^n > 3^n + 4$ is true for integral values of $n \geq 2$.

Question No.2

Use mathematical induction to prove that $3+6+9+\ldots+3n=\frac{3n(n+1)}{2}$ for all positive integers n.

Solution:

Let $P(n)$: $$3+6+9+\ldots+3n=\frac{3n(n+1)}{2}$$

***BASIS* STEP:**

$$LHS = 3(1) = 3$$

$$RHS = \frac{3(1+1)}{2} = 3$$

So $P(1)$ is

true INDUCTIVE STEP:

Suppose $P(k)$ is true for all $k \geq 1$

$$3+6+9+\ldots+3k=\frac{3k(k+1)}{2} \tag{1}$$

we are to prove $P(k+1)$ is true i.e.

$$P(k+1): \quad 3+6+9+\ldots+3(k+1)=\frac{3(k+1)((k+1)+1)}{2} \tag{2}$$

Now

$$\text{LHS} = 3+6+9+\ldots+3k+3(k+1) = (3+6+9+\ldots+3k)+3(k+1)$$

$$=\frac{3k(k+1)}{2}+3(k+1)$$

$$=3(k+1)\left(\frac{k}{2}+1\right)$$

$$=3(k+1)\frac{k+2}{2}$$

$$=\frac{3(k+1)[(k+1)+1]}{2} = \text{RHS}$$

Question No.3

Suppose $n^3 - n$ is divisible by 6 is true for any positive integer. Justify the statement for n = k+1.

Solution:

Inductive Step:

Suppose for n=k, $k^3 - k$ is dividible by 6 then $k^3 - k = 6.q$

we need to prove $(k+1)^3 - (k+1)$ is dividible by 6

$$
\begin{aligned}
(k+1)^3 - (k+1) &= (k^3 + 3k^2 + 3k + 1) - (k+1) \\
&= k^3 - k + 3k(k+1) \\
&= 6.q + 3k(k+1)
\end{aligned}
$$

As the product $k(k+1)$ of integers is even so we can take as $2r$

$$
\begin{aligned}
&= 6.q + 3k(k+1) \\
&= 6.q + 3.2r \\
&= 6(q+r)
\end{aligned}
$$

So $(k+1)^3 - (k+1)$ is dividible by 6

Hence by induction method, $n^3 - n$ is divisible 6

Question No 4:

Suppose $5^n - 1$ is divisible by 4 is true for $n = k$. Justify the statement for $n = k + 1$.

Solution:

Inductive Step:

Let for $n = k$, $5^k - 1$ is divisible by 4

then $5^k - 1 = 4.q$

we need to prove that $5^{k+1} - 1$ is divisible by 4

$$5^{k+1} - 1 = 5.5^k - 1$$
$$= 5^k\left(4+1\right) - 1$$
$$= 5^k.4 + 5^k - 1$$
$$= 5^k.4 + 4.q$$
$$= 4\left(5^k + q\right)$$

Hence $5^{k+1} - 1$ is divisible by 4

Question No 5

Prove that if x divides y and y divides z then x divides z.

Solution:

By our assumptions, and the definition of divisibility, there are natural numbers k₁ and k₂ such that

$y = xk_1$ ***and*** $z = yk_2$.

Consequently,

$z = yk_2 = xk_1k_2$.

Let $k =$ Now k is a natural number and $z = xk$, so by the definition of divisibility, x
k_1k_2
divides z.

Question No 6:

Show that for all integers a and b, if $a + b$ is even then $a - b$ is also even.

Solution:

Take a+b =
2k a= 2k-b
Subtracting 'b' from both
sides a-b =2k-b-b
a-b = 2k -
2b a-b = 2(
k-b)
Hence a-b is also even.

Question No 7:

Assume that m and n are particular integers. Justify your answer to each of the following:

1) Is $4m + 6n$ even?

2) Is $8mn + 5$ odd?

Solution:

1) Yes: 4m+6n =2.(2m+3n) [by definition of even]
(2m+3n) is an integer because 2, 3, m, n are integers and products and sums of integers are integers.

2) Yes: 8mn+5 =2.(4mn+2)+1 [by definition of odd]
(4mn+2) is an integer because 4, 2, m, n are integers and products and sums of integers are integers.

Solution to Practice questions Lecture 26-28

Solution 1:

We are to prove by contradiction that $4 + 3\sqrt{2}$ is an irrational number. So, let's assume that $4 + 3\sqrt{2}$ is a rational number. Then by definition of rational number, it can be written in the following form

$$4 + 3\sqrt{2} = \frac{a}{b} \quad \text{for some integers a and b with } b \neq 0$$

$$4 + 3\sqrt{2} = \frac{a}{b}$$

$$3\sqrt{2} = \frac{a}{b} - 4$$

$$\sqrt{2} = \frac{a - 4b}{3b}$$

Since a and b are integers, so are a-4b and 3b with $3b \neq 0$. This shows that $\sqrt{2}$ is a rational number.

Which is a contradiction to the f act that $\sqrt{2}$ is an irrational number. This contradiction shows that our initial supposition was incorrect and thus, $4\sqrt{2}$ is not an irrational number that is $4 + 3\sqrt{2}$ is a rational number.

Solution 2:

We are to prove by contradiction that if 5*n* +1 is odd then *n* is even. Suppose for some integer *n*, 5*n* +1 is odd but *n* is not even. That is *n* is odd. So it can be written as

$$n = 2m + 1 \text{ for some integer } m.$$

$$5n + 1 = 5(2m + 1) + 1$$

$$= 10m + 6$$

$$= 2(5m + 3)$$

This shows that 5*n* +1 is equal to some multiple of 2 and so it is even which is a contradiction to our initial supposition. So, the correct supposition will be that if 5*n* +1 is odd then *n* is even.

Solution 3:

We are to prove by contraposition that if n^2 is odd then *n* is odd. That is we are to prove that if *n* is even then n^2 is even.

Let ssu *n* ver The ca be w tter s a ltip of 2 Tha s

$$\begin{aligned} n &= 2k \quad \text{for some integer } k \\ n^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2(2k^2) \end{aligned}$$

Clearly, n^2 is a multiple of 2 so n^2 is also even.

This Proves that if n^2 is odd then n is odd.

Solution 4:

The following are pre and post conditions.

Pre-condition: **The input variables a and b are positive integers.**
Post-condition: **The output variable q and r are positive integers such that**

$$a = b \cdot q + r \text{ and } 0 \leq r < b.$$

Solution 5:

We are to find the GCD (61114,

94). Divide 61114 by 94

$$61114 = 94 \times 650 + 14$$

Divide 94 by 14

$$94 = 14 \times 6 + 10$$

Divide 14 by 10

$$14 = 10 \times 1 + 4$$

Divide 10 by 4

$$10 = 4 \times 2 + 2$$

Divide 4 by 2

$$4 = 2 \times 2 + 0$$

Thus, the GCD of 61114 and 94 is 2.

Solution to practice questions for Lecture No. 29-31

Solution 1:

For a round trip, the person will travel from city A to B, B to C then C to B and then B to A i.e.,

$$A \to B \to C \to B \to A$$

The person can travel 3 ways from A to B and 5 ways from B to C and back that is

$$A \xrightarrow{3} B \xrightarrow{5} C \xrightarrow{5} B \xrightarrow{3} A$$

Thus, there are

$$3 \times 5 \times 5 \times 3 = 225$$

ways to have a round
trip.

Solution 2:

Each bit (binary digit) is either 0 or 1.
Hence, there are 2 ways to choose each bit. Since we have to choose seven bits therefore, the product rule shows, there are a total of $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 = 128$

Solution 3:

Given that

$$p(n,2)=42$$

That is

$$\frac{n!}{(n-2)!} = 42$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 42$$

$$n(n-1) = 42$$

$$n^2 - n - 42 = 0$$

$$n^2 - 7n + 6n - 42 = 0$$

$$n(n-7) + 6(n-7) = 0$$

$$(n+6)(n-7) = 0$$

$$n = -6,\ n = 7$$

Ignoring the negative value, the desired value of n is 7.

Solution 4:

5 boys can be selected from 10 in following number of ways

$${}^{10}C_5 = 252$$

5 girls can be selected from 15 in following number of ways

$${}^{15}C_5 = 3003$$

Total number of ways a team of 10 students be selected = 252 × 3003=756756

Solution 5:

There are 13 Spade cards in a deck of cards so C(13,2) = 78 There are 13 Heart cards in a deck of cards so C(13,2) = 78

$$C\left(13, 2\right) \times C\left(13, 2\right) = 78 \times 78 = 6084$$

Practice questions Lectures 32-34

Question No 1:

How many distinguishable ways can the letter of the word HULLABALOO be arranged?

Question No 2:

How many signals can be given by 7 flags of different colors using 3 flags at a time?

Question No 3:

How many 4-digit numbers can be formed by using each of the digits1, 2, 3,4,5,6 only once?

Question No 4:

Suppose that there are 2000 students at your university. Of these 500 are taking a course in computer science, 670 are taking a course in mathematics and 300 are taking course in both computer science and mathematics. How many are taking a course either in computer science or in mathematics?

Question No 5:

There are 24 students in a class, only 13 are willing to go to a tour for Murree. In how many ways can these be chosen?

Question No 6:

In an office, there are 75 faculty members can speak Malay(M) and 25 can speak Chinese (C), while only 10 can speak both Malay and Chinese. Using Inclusion-exclusion Principle find how many faculty members can speak either Malay (M) or Chinese(C)?

Practice Questions for Lecture No. 35 to 37

Question 1:

A box contains seven discs numbered 1 to 7. Find for each integer k (k is from 3 to 11) , the probability that the numbers on two discs drawn without replacement have a sum equal to k.

Question 2:

A die is weighted so that the outcomes produce the following probability distributio

Outcome	1	2	3	4	5	6
Probability	0.1	0.3	0.2	0.1	0.1	0.2

Consider the event A = {odd number occur on die} then find $P(\text{A})$ and $P(\text{A}^c)$.

Question 3:

There are 20 girls and 40 boys in a class. Half of the boys and half of the girls have blue eyes. Find the probability that one student chosen as monitor is either a girl or a student having blue eyes.

Question 4:

A pair of fair dice is thrown. Find the probability P that the sum is 9 or greater if

(i) 5 appear on first die

(ii) 5 appear on at least one die

Prcatice Question Lecture No 39-41

Question No 1:

Find the maximum number of edges for a complete graph $k_n$by using handshaking theorem.

Solution

The total degrees in graphK_n = 2 (number of edges in K_n)

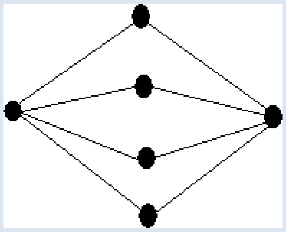
⇒ n(n-1) = 2(number of edges in K_n)

⇒

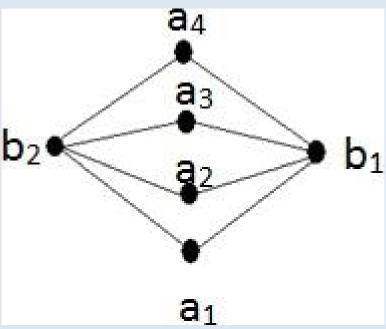
Number of edges in K_n $= \frac{n(n-1)}{2}$

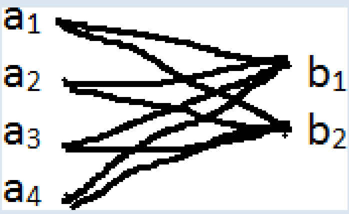
Question No 2:

Is the graph below is Complete Bipartite or not? Redraw the bipartite graph so that its bipartite nature is evident?



Solution:

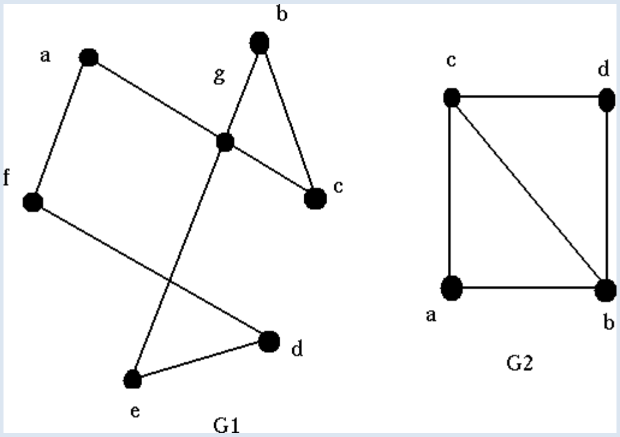




Yes it is a complete bipartite graph.

Question No 3:

Check whether the given graphs have an Euler circuit? If it does, find such a circuit, if it does not, give an argument to show why no such circuit exists.

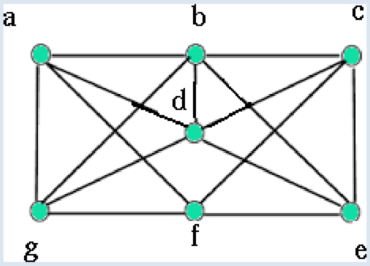


Solution:

In G1: each vertex has even degree so it an Euler circuit. In G2: vertices "c" and "b" has odd degree so it is not an Euler circuit

Question No 4:

Find two Hamiltonian circuits of the following graph



Solution:

(i) abcdefga ii) abcefgda

Question No 5:

Find the product AB and BA of the matrices (if not possible then give reason).

$$A=\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} \text{ an } \qquad B=\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 4 \end{bmatrix}$$

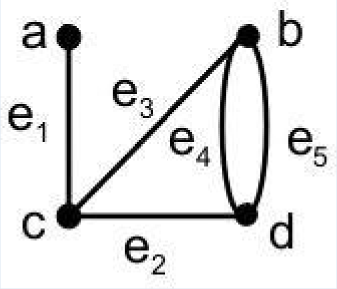
Solution:

$$AB=\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 4 \end{bmatrix}=\begin{bmatrix} 2+0 & 4+0 & 6+0 \\ 1-3 & 2+2 & 3-4 \end{bmatrix}=\begin{bmatrix} 2 & 4 & 6 \\ -2 & 4 & -1 \end{bmatrix}$$

BA is not possible as B has 3 columns and A has two rows, so multiplication is not possible.

Question No 6:

Find the adjacency matrix of the graph shown below.



Solution:

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 0 & 1 & 0 \\ b & 0 & 0 & 1 & 2 \\ c & 1 & 1 & 0 & 1 \\ d & 0 & 2 & 1 & 0 \end{array}$$

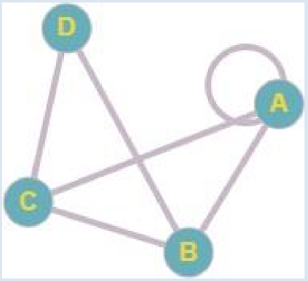
Question No 7:

Draw a graph with the adjacency matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

with respect to the ordering of vertices a, b, c, d.

Solution:



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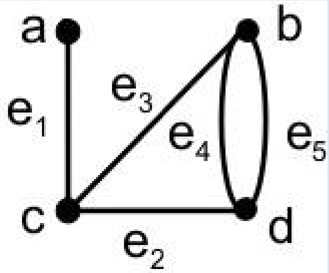
Question No 1:

Find the product AB and BA of the matrices (if not possible then give reason).

$$A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} \quad \text{an} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 4 \end{bmatrix}$$

Question No 2:

Find the adjacency matrix of the graph shown below.



Question No 3:

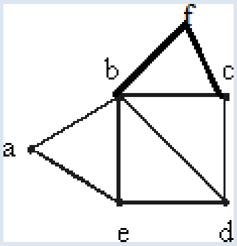
Draw a graph with the adjacency matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

with respect to the ordering of vertices a, b, c, d.

Question No 4 :

Find the degree sequence of the following graph.

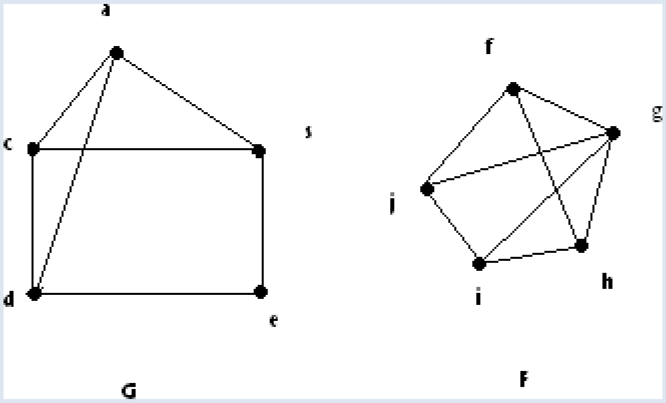


Question No 5:

Draw all possible simple graphs with 2 vertices, which are non-isomorphic to each other.

Question No 6:

Determine whether the given graphs are isomorphic? (justify your answer)

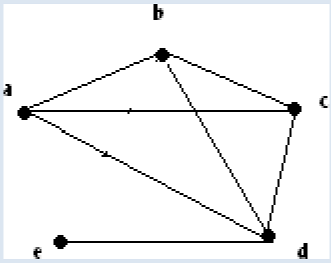


Question No 7:

Suppose that a connected planar simple graph has 22 edges. If you want to draw a graph having 11 faces, how many vertices does this graph have?

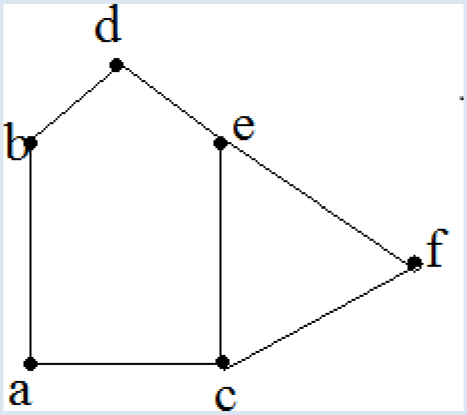
Question No 8:

Re-draw the given graph to prove it planar.



Question No 9:

Determine the chromatic number of the given graph.



Practice Questions for Lecture No. 44 and 45

Question 1:

Find the total number of internal and terminal vertices of a full binary tree with nine vertices

Question 2:

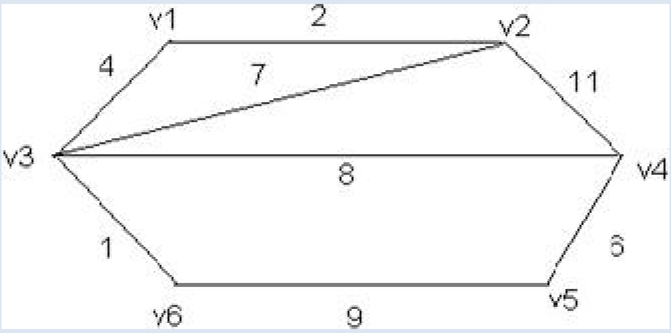
Draw a binary tree with height 3 and having seven terminal vertices.

Question 3:

Draw any two non-isomorphic trees of five vertices.

Question 4:

Use Kruskal's Algorithm to draw the minimal spanning tree for the graph given below. Indicate the order in which edges are added to form a tree.



Question 5:

Find a spanning tree for the graph $k_{1,5}$?