

MTH301 FINAL TERM NOTES

LECTURE 23 TO 45 PROVIDED BY

ORANGE MONKEY TEAM

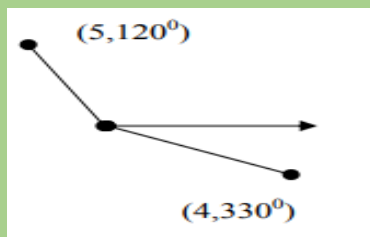
Lecture No – 23 Polar Coordinate Systems

POLAR COORDINATE SYSTEMS

To form a polar coordinate system in a plane, we pick a fixed point O , called the origin or pole, and using the origin as an endpoint we construct a ray, called the polar axis.

WHAT IS POLAR ANGLE?

The number r is called the radial distance of P and θ is called a polar angle of P



$(5, 120^\circ)$, and $(4, 330^\circ)$ are plotted in polar coordinate systems.

Using formulas

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x^2 + y^2 = r^2$$

$$y/x = \tan \theta$$

LINES IN POLAR COORDINATES

A line perpendicular to the x -axis and passing through the point t with 'XY' coordinates with $(a, 0)$ has the equation $x = a$.

We obtain the general polar form of the line.

$$r (A \cos \theta + B \sin \theta) + C = 0$$

CIRCLES IN POLAR COORDINATES

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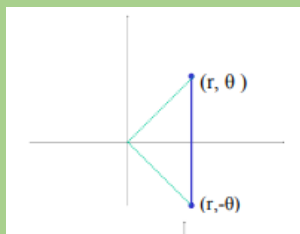
Lecture No -24

Sketching

Sketching of Curves in Polar Coordinates

➤ Symmetry about the Initial Line

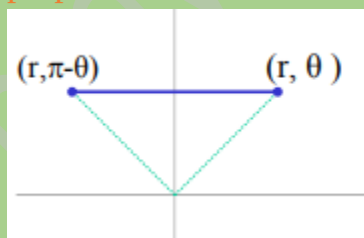
If the equation of a curve remains unchanged when (r, θ) is replaced by



either $(r, -\theta)$ in its equation, then the curve is **symmetric about initial line**.

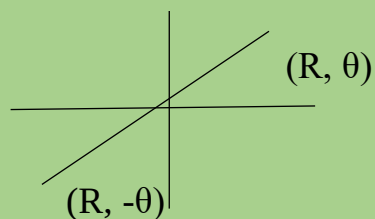
➤ Symmetry about the y-axis

If when (r, θ) is replaced by either $(r, \pi - \theta)$ in The equation of a curve and an equivalent equation is obtained, then the curve is symmetric about the **line perpendicular to the initial i.e, the y-axis**.



➤ Symmetry about the Pole

If the equation of a curve remains unchanged when either $(-r, \theta)$ or is substituted for (r, θ) in its equation, then the curve is symmetric about the pole. In such a case, the center of the curve.



Position of the Pole Relative to the Curve

See whether the pole on the curve by putting $r=0$ for in the equation of the curve and solving θ .

Table of Values

- Table of Values Construct a sufficiently complete table of values. This can be of great help in sketching the graph of a curve.
- When $r = 0$, $\theta = 0$. Hence the curve passes through the pole.

Difference b/w CARDIDOIDS AND LIMACONS

The equations of above form produce polar curves called **limacons**. Because of the heart shaped appearance of the curve in the case $a = b$, limacons of this type are called **cardioids**.

$$r=a+b \sin \theta, r = a-b\sin \theta$$

$$r =a+b \cos \theta, r = a-b\cos \theta$$

LEMINSKATE

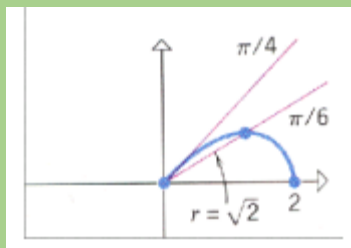
If $a > 0$, then

Equation of the form

$$r^2 = a^2 \cos 2\theta, r^2 = -a^2 \cos 2\theta$$

$$r^2 = a^2 \sin 2\theta, r^2 = -a^2 \sin 2\theta$$

Represent propeller-shaped curves, called **lemiscates**

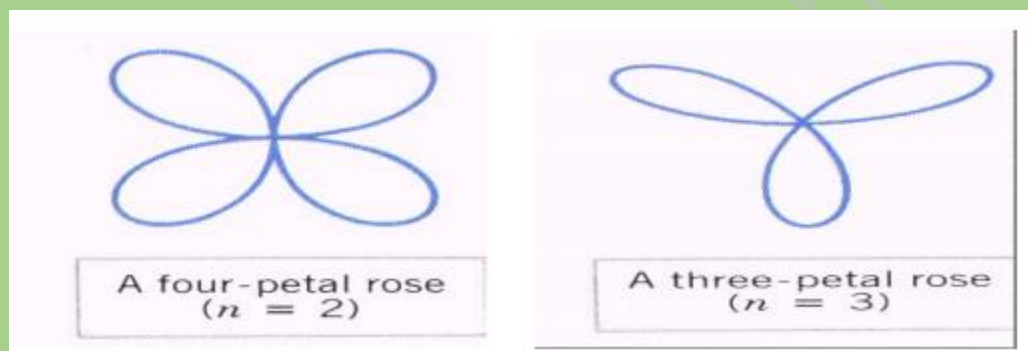


$$r^2 = 4 \cos 2 \theta$$

The equation represents a lemniscate. The graph is symmetric about the x-axis and the y-axis.

ROSE CURVES

Equations of the form $r = a \sin n\theta$ and $r = a \cos n\theta$ represent flower-shaped curves called roses. The rose has n equally spaced petals or loops if n is odd and $2n$ equally spaced petals if n is even.



SPIRAL

A curve that “winds around the origin” infinitely many times in such a way that r increases (or decreases) steadily as θ increases is called a spiral.

$$r = a\theta \dots\dots\dots (\theta > 0)$$

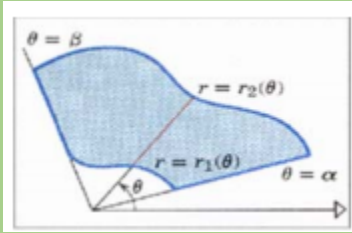
$$r = a\theta \dots\dots\dots (\theta < 0)$$

Sketch the curve $r = \theta$ ($\theta > 0$) in polar coordinates.

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What are Double integrals in polar co-ordinates?

Double integrals in which the integrand and the region of integration are expressed in polar coordinates are important for two reasons: First, they arise naturally in many applications, and second, many double integrals in rectangular coordinates are more easily evaluated if they are converted to polar coordinates. The function $z = f(r, \theta)$ to be integrated over the region R .



INTEGRALS IN POLAR COORDINATES

When we defined the double integral of a function over a region R in the xy -plane, we began by cutting R into rectangles whose sides were parallel to the coordinate axes. These were the natural shapes to use because their sides have either constant x -values or constant y -values. In polar coordinates, the natural shape is a “polar rectangle” whose sides have constant r and θ - values.

EXAMPLE

vmip

Question

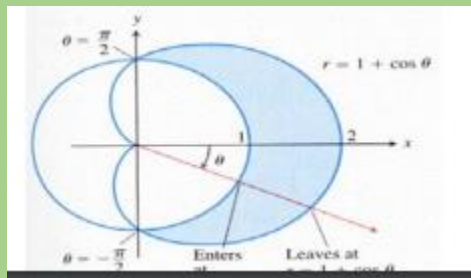
Find the limits of integration for integrating $f(r, \theta)$ over the region R that lies inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = 1$.

Solution:

Step 1.

We sketch the region and label the bounding curves.

Step 2.



The r-limits of integration. A typical ray from The origin enters R where $r = 1$ and leaves where $r = 1 + \cos \theta$.

Step 3.

The θ -limits of integration. The rays from the origin that intersect R run from $\theta = -\frac{\pi}{2}$ to $\theta = \frac{\pi}{2}$.

The integral is
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_1^{1+\cos\theta} f(r, \theta) r \, dr \, d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \int_1^{1+\cos\theta} f(r, \theta) r \, dr \, d\theta$$

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Lecture No – 26

EXAMPLES

ONLY Practices Question

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Lecture No -27 Vector Valued Functions

Difference b/w simply real valued functions and simply vector valued functions?

We have considered only functions for which the domain and range are sets of real numbers; such functions are called real-valued functions of a real variable or sometimes simply real-valued functions. In this section we shall consider functions for which the domain consists of real numbers and the range consists of vectors in 2-space or 3-space; such functions are called vector-valued functions of a real variable or more simply vector-valued functions.

In 2-space such functions can be expressed in the form.

$$\mathbf{r}(t) = (x(t), y(t)) = x(t)\mathbf{i} + y(t)\mathbf{j}$$

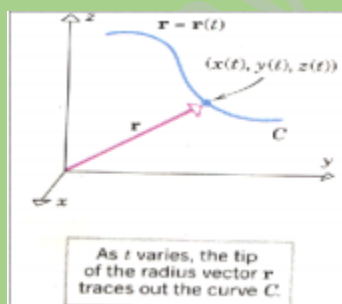
In 3-space such functions can be expressed in the form.

$$\mathbf{r}(t) = (x(t), y(t), z(t)) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

Where $x(t)$, $y(t)$, and $z(t)$ are real-valued functions of the real variable t . These real-valued functions are called the component functions or components of $\mathbf{r}$.

GRAPHS OF VECTOR-VALUED FUNCTIONS

The vector $\mathbf{r}$, when positioned in this way, is called the **radius vector** or **position vector** of C , and C is called the **graph of the function** $\mathbf{r}(t)$ or, equivalently, the graph of the equation $\mathbf{r} = \mathbf{r}(t)$. The vector equation $\mathbf{r} = \mathbf{r}(t)$ is equivalent to a set of parametric equations, so C is also called the **graph of these parametric equations**.



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Lecture No -28 Limits of Vector Valued Functions

Limits of Vector Valued Functions

The limit of a vector-valued function is defined to be the vector that results by taking t limit of each component. Thus, for a function $r(t) = x(t)i + y(t)j$ in

2-space we define $\lim_{t \rightarrow a} r(t) = (\lim_{t \rightarrow a} x(t))i + (\lim_{t \rightarrow a} y(t))j$

3-space we define $\lim_{t \rightarrow a} r(t) = (\lim_{t \rightarrow a} x(t))i + (\lim_{t \rightarrow a} y(t))j + (\lim_{t \rightarrow a} z(t))k$

CONTINUITY OF VECTOR-VALUED FUNCTIONS

The definition of continuity for vector-valued functions is similar to that for real-valued functions. We shall say that r is continuous at t_0 if

1. $r(t_0)$ is defined
2. $\lim_{t \rightarrow t_0} r(t)$ exists;
3. $\lim_{t \rightarrow t_0} r(t) = r(t_0)$

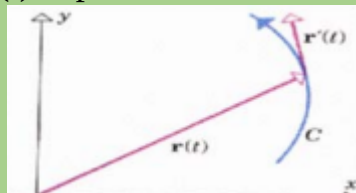
DERIVATIVES OF VECTOR-VALUED FUNCTIONS

The derivative $r'(t)$ of a vector-valued function $r(t)$ is defined by

$$r'(t) = \lim_{h \rightarrow 0} \frac{r(t+h) - r(t)}{h} \text{ Provided this limit exists.}$$

TANGENT VECTORS AND TANGENT LINES GEOMETRIC

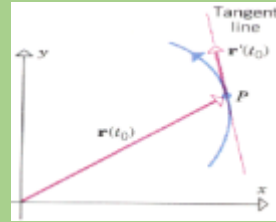
INTERPRETATIONS OF THE DERIVATIVE. Suppose that C is the graph of a vector-valued function $r(t)$ and that $r'(t)$ exists and is nonzero for a given value of t . If the vector $r'(t)$ is positioned with its initial point at the terminal



point of the radius vector.

DEFINITION

Let P be a point on the graph of a vector-valued function $\mathbf{r}(t)$, and let $\mathbf{r}(t_0)$ be the radius vector from the origin to P . If $\mathbf{r}'(t_0)$ exists and $\mathbf{r}'(t_0) \neq \mathbf{0}$, then we call $\mathbf{r}'(t_0)$



$\mathbf{r}'(t_0)$ the tangent vector to the graph of $\mathbf{r}$ at $\mathbf{r}(t_0)$

REMARKS

quiz and mcq

Observe that the graph of a vector-valued function can fail to have a tangent vector at a point either because the derivative in (4) does not exist or because the derivative is zero at the point. If a vector-valued function $\mathbf{r}(t)$ has a tangent vector $\mathbf{r}'(t_0)$ at a point on its graph, then the line that is parallel to $\mathbf{r}'(t_0)$ and passes through the tip of the radius vector $\mathbf{r}(t_0)$ is called the tangent line of the graph of $\mathbf{r}(t)$ at $\mathbf{r}(t_0)$. Vector equation of the tangent line is $\mathbf{r} = \mathbf{r}(t_0) + t \mathbf{r}'(t_0)$.

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Lecture No -29 Change of parameter

When making such a change of parameter, it is important to ensure that the new vector valued function of u is smooth if the original vector-valued function of t is smooth. It can be proved that this will be so if g satisfies the following conditions:

- g is differentiable
- g' is continuous.
- $g'(u) \neq \mathbf{0}$ for any u in the domain of g .
- The range of g is the domain of $\mathbf{r}$.

If g satisfies these conditions, then we call $t = g(u)$ a smooth change of parameter. Henceforth, we shall assume that all changes of parameter are smooth, even if it is not stated explicitly.

ARC LENGTH

Because derivatives of vector-valued functions are calculated by differentiating components, it is natural to define integrals of vector-functions in terms of components.

2-spaces exactly as one would expect:
$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

3-spaces exactly as one would expect:
$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

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Lecture No -30

Exact Differentials

$$Z = f(x, y), \text{ then } dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

The result can be extended to functions of more than two independent variables.

$$Z = f(x, y, w), \text{ then } dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy + \frac{\partial z}{\partial w} dw$$

Make a note of these results in differential form as shown.

Area enclosed by the closed curve

Proved that $A = -\oint y dx$

imp

One of the earliest applications of integration is finding the area of a plane figure bounded by the x-axis, the curve $y = f(x)$ and ordinates at $x = x_1$ and $x = x_2$.

$$A_1 = \int_{x_1}^{x_2} y dx \Rightarrow \int_{x_1}^{x_2} f(x) dx$$

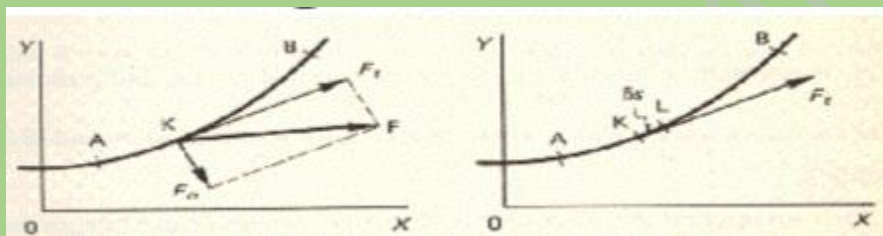
$$A_2 = \int_{x_1}^{x_2} y dx \Rightarrow \int_{x_1}^{x_2} f(x) dx$$

$$A = A_1 - A_2$$

$$A = \int_{x_1}^{x_2} f(x) dx - \int_{x_1}^{x_2} f(x) dx$$

$$A = -\int y dx$$

Line integrals



The work done in moving the particle through small distance δs from K to L along the curve is then approximately $F_t \delta s$. So the total work done in moving a particle along the curve from A to B is given by

$$\lim_{\delta s \rightarrow 0} \sum F_t \delta s = \int F_t ds \text{ From A to B.}$$

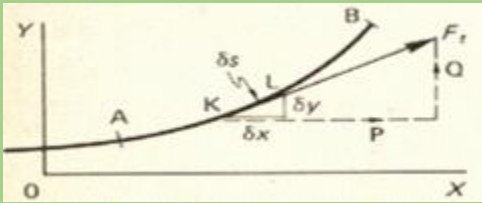
This is normally written $\int_{AB} F_t ds$ where A and B are the end points of the curve, or as $\int_c F_t ds$ where the curve c connecting A and B is defined.

Alternative form of a line integral

It is often more convenient to integrate with respect to x or y than to take arc length as the Variable.

- If F_t has a component.
- P in the x-direction
- Q in the y-direction

Then the work done from K to L can be stated as $P\delta x + Q\delta y$.



$$\therefore \int_{AB} F_t ds = \int_{AB} (P dx + Q dy)$$

Where P and Q are functions of x and y. In general then, the line integral can be expressed as

$$I = \int_c F_t ds = \int_c (P dx + Q dy)$$

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Lecture No -31 Line Integral

ONLY Practices Question

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Lecture No -32 Examples

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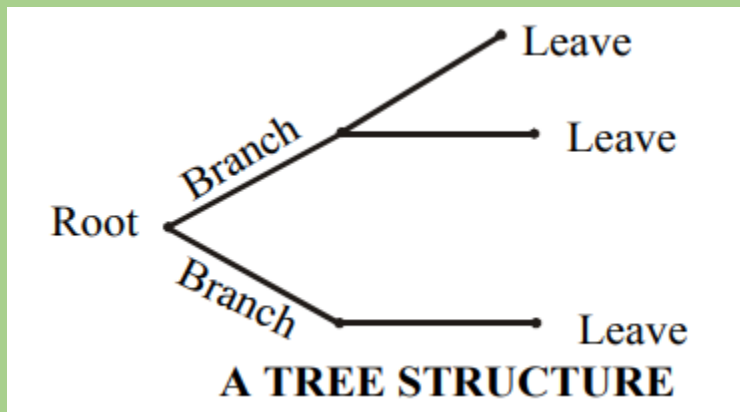
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Lecture No.33 Tree Diagram

TREE DIAGRAM:

A tree diagram is a useful tool to list all the **logical possibilities** of a sequence of events where each event **can occur in a finite number of ways**.

A tree is normally constructed **from left to right**.



THE INCLUSION-EXCLUSION PRINCIPLE:

1. If A and B are disjoint finite sets, then
 $n(A \cup B) = n(A) + n(B)$ the is sum rule
2. If A and B are finite sets, then
 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ Thus we subtract

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Lecture -34.....Examples

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Lecture No -35 Definite Integrals

Wallis Sine Formula

When n is even:

$$\int_0^{\frac{\pi}{2}} \sin^n x dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \frac{n-7}{n-6} \cdots \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

When n is odd:

$$\int_0^{\frac{\pi}{2}} \cos^n x dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdot \frac{n-7}{n-6} \cdots \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$$

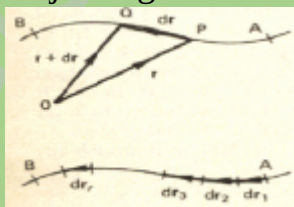
Integration by Parts

$$\int UV dx = U \int V dx - \int \left[\int v dx \cdot \frac{du}{dx} \right] dx$$

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Line Integrals

Let a point p on the curve c joining A and B be denoted by the position vector r

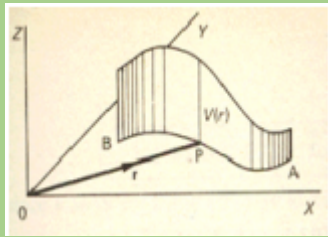


with respect to origin O. If q is a neighboring point on the curve with position vector $r+dr$, then $\overline{PQ} = r$

$$\overline{AB} \sum_{p=1}^n dr_p \text{ If } dr \rightarrow 0, \text{ then the length of the curve } AB = \int_c dr$$

Scalar Field

If a scalar field $V(r)$ exists for all points on the curve, the $\sum_{p=1}^n V(r) dr_p$ with $dr \rightarrow 0$,



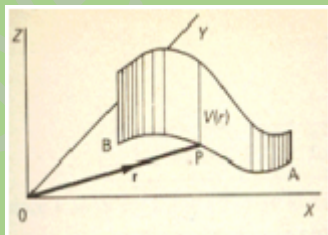
defines the line integral of V i.e. Line integral $= \int_c V(r) dr$

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Lecture No -36 Scalar Field

Scalar Field

If a scalar field $V(r)$ exists for all points on the curve, the $\sum_{p=1}^n V(r) dr_p$ with $dr \rightarrow 0$,



defines the line integral of V i.e. Line integral $= \int_c V(r) dr$

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Lecture No -37 Examples

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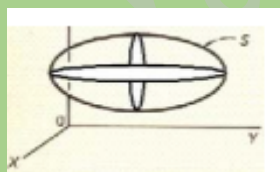
Lecture No -38 Vector Field

Vector Field

In general, the value of the integral $\int_c F \cdot dr$ between two stated points A and B depends on the particular path of integration followed. If, however, the line integral between A and B is independent of the path of integration between the two end points, then the vector field F is said to be conservative. It follows that, for a closed path in a conservative field, $\oint_c F \cdot dr = 0$.

Divergence Theorem (Gauss' theorem)

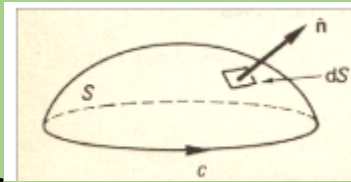
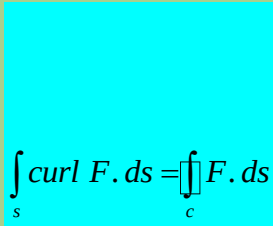
For a closed surface S, enclosing a region V in a vector field F $\int_V \text{div } F \, dv = \int_S F \cdot ds$



In general, this means that the volume integral (triple integral) on the left-hand side can be expressed as a surface integral (double integral) on the right-hand side.

Stokes Theorem

If F is a vector field existing over an open surface S and around its boundary



closed curve C , $\int_S \text{curl } F \cdot d\mathbf{s} = \oint_C F \cdot d\mathbf{s}$

This means that we

can express a surface integral in terms of a line integral round the boundary curve. The proof of this theorem is rather lengthy and is to be found in the Appendix.

Important formula

$$\mathbf{n} = \frac{\nabla s}{|\nabla s|} \quad |\nabla s| = \sqrt{x^2 + y^2 + z^2}$$

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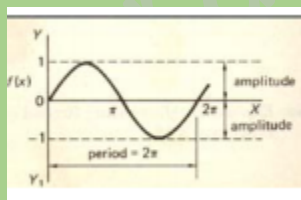
Lecture No -39

Periodic Functions

important quiz

Periodic functions

A function $f(x)$ is said to be periodic if its function values repeat at regular intervals of the independent variable. The regular interval between repetitions is the period of the oscillations. $f(x + p) = f(x)$.



Useful integrals

important quiz

The following integrals appear frequently in our work on Fourier series, so it will help if we obtain the result in readiness. In each case, m and n are integers other than zero.

$$a) \int_{-\pi}^{\pi} \sin nx \, dx = \left[\frac{-\cos nx}{n} \right]_{-\pi}^{\pi} = \frac{1}{n} \{-\cos n\pi + \cos n\pi\} = 0$$

$$b) \int_{-\pi}^{\pi} \cos nx \, dx = \left[\frac{\sin nx}{n} \right]_{-\pi}^{\pi} = \frac{1}{n} \{\sin n\pi + \sin n\pi\} = 0$$

$$c) \int_{-\pi}^{\pi} \sin^2 nx \, dx = \frac{1}{2} \int_{-\pi}^{\pi} (1 - \cos 2nx) \, dx = \frac{1}{2} \left[x - \frac{\sin 2nx}{2n} \right]_{-\pi}^{\pi} = \pi (n \neq 0)$$

$$d) \int_{-\pi}^{\pi} \cos^2 nx \, dx = \frac{1}{2} \int_{-\pi}^{\pi} (1 + \cos 2nx) \, dx = \frac{1}{2} \left[x + \frac{\sin 2nx}{2n} \right]_{-\pi}^{\pi} = \pi (n \neq 0)$$

$$e) \int_{-\pi}^{\pi} \sin nx \cos mx \, dx = \frac{1}{2} \int_{-\pi}^{\pi} \{\sin(n+m)x + \sin(n-m)x\} \, dx = \frac{1}{2} \{0+0\} = 0$$

from result (a) with $n \neq m$

$$f) \int_{-\pi}^{\pi} \cos nx \cos mx \, dx = \frac{1}{2} \int_{-\pi}^{\pi} \{\cos(n+m)x + \cos(n-m)x\} \, dx = \frac{1}{2} \{0+0\} = 0$$

from result (b) with $n \neq m$.

$$g) \int_{-\pi}^{\pi} \sin nx \sin mx \, dx = -\frac{1}{2} \int_{-\pi}^{\pi} (-2) \sin nx \sin mx \, dx = -\frac{1}{2} \int_{-\pi}^{\pi} \cos(n+m)x - \cos(n-m)x \, dx$$

$$= \frac{1}{2} \int_{-\pi}^{\pi} \{0-0\} = 0 \quad \text{from result (b) with } n \neq m$$

What is Fourier series?

An infinite series in which the terms are constants multiplied by sine or cosine functions of integer multiples of the variable and which is used in the analysis of periodic functions.

Periodic functions of period 2π

The basis of a Fourier series is to represent a periodic function by a trigonometrical series of the form

$$F(x) = A_0 + c_1 \sin(x + \alpha_1) + c_2 \sin(2x + \alpha_2) + c_3 \sin(3x + \alpha_3) + \dots + c_n \sin(nx + \alpha_n) + \dots$$

Where A_0 is a constant term

$C_1, C_2, C_3, \dots, C_n$ denote the amplitudes of the compound sine terms $\alpha_1, \alpha_2, \alpha_3, \dots$ are constant auxiliary angles.

Each sine term, $C_n \sin (nx + \alpha_n)$ can be expanded thus:

$$C_n \sin (nx + \alpha_n) = C_n \{ \sin nx \cos \alpha_n + \cos nx \sin \alpha_n \} = (C_n \sin \alpha_n) \cos nx + (C_n \cos \alpha_n) \sin nx$$

$$= C_n \cos nx + b_n \sin nx$$

the whole series becomes.

Important quiz

$$f(x) = A_0 + \sum_{n=1}^{\infty} \{ a_n \cos nx + b_n \sin nx \}$$

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Lecture No- 40 Fourier series

$$f(x) = A_0 + \sum_{n=1}^{\infty} \{ a_n \cos nx + b_n \sin nx \}$$

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Lecture No -41 Examples

ONLY Practices Question

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Lecture No -42 Examples

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Lecture No.-43

Functions With Periods Other than 2π

So far, we have considered functions $f(x)$ with period 2π . In practice, we often encounter functions defined over periodic intervals other than 2π , e.g. from 0 to T , $-\frac{T}{2}$, $\frac{T}{2}$. A function f will be periodic with period m , so if we have

$f(a + m) = f(a)$, For every $m > 0$.

It shows that the function $f(a)$ possesses the same values after an interval of “ m ”. One can say that after every interval of “ m ” the function f repeats all its values.

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Lecture No -44

Laplace Transforms

Laplace Transform

The Laplace Transform of a function $F(t)$ is denoted by $L\{F(t)\}$ and is defined as the integral of $F(t)$ e^{-st} between the limits $t=0$ and $t = \infty$.

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt.$$

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt.$$

$$= a \int_0^{\infty} e^{-st} dt$$

$$= a \frac{e^{-st}}{-s} \Big|_0^{\infty} = \frac{-a}{s} \{0 - 1\} = \frac{a}{s}$$

Complex Numbers

$z = 3 + 5i$, is called a complex number where 3 is real part and 5 is imaginary part of the complex number. In general $z = a + bi$, is called a complex number. Where a is real part and b is imaginary part of the complex number.

So,

Complex Number = (Real Part) + i (Imaginary Part)

Conjugate complex

Numbers For a complex number $a + ib$, the complex number $a - ib$ is called the conjugate of $a + ib$. Conjugate complex numbers are identical except the signs in the middle for the brackets.

- $(4 + 5i)$ and $(4 - 5i)$ are conjugate complex numbers
- $(6 + 2i)$ and $(2 + 6i)$ are not conjugate complex numbers
- $(5 - 3i)$ and $(-5 + 3i)$ are not conjugate complex numbers

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Lecture No -45 Theorems

Theorem 1

The First Shift Theorem

The first Shift theorem states that if $L\{F(t)\} = f(s)$ then $L\{e^{(-at)} F(t)\} = f(s+a)$ The transform $L\{e^{(-at)} F(t)\}$ is thus the same as $L\{F(t)\}$ with s everywhere in the result replaced by $(s+a)$.

Theorem 2

Multiplying by t

If $L\{F(t)\} = f(s)$ then $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(s) ds$

Standard Forms

$F(t)$	$\mathcal{L}\{F(t)\} = f(s)$
a	$\frac{a}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
t^n	$\frac{n!}{s^{n+1}}$

(n a positive integer)

Table of inverse transforms

Standard transforms

$f(s)$	$F(t)$
$\frac{a}{s}$	a
$\frac{1}{s-a}$	e^{-at}
$\frac{n!}{s^{n+1}}$	t^n (n a positive integer)
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n a positive integer)
$\frac{a}{s^2 + a^2}$	$\sin at$
$\frac{s}{s^2 + a^2}$	$\cos at$
$\frac{a}{s^2 - a^2}$	$\sinh at$
$\frac{s}{s^2 - a^2}$	$\cosh at$

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PRAY FOR ALL HELPERS

THANK YOU. □□ □□□□

prayers remember me