

Mth301 Short Notes Mid term

Lec 1 to22

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MTh lecher #1

What is Integral Calculus?

Deals with the Problem of determining a Function from information about its rates of Change.

Defiance b/w independent or dependent?

The term, function, denotes the dependence of one quantity on other quantity. The quantity x is called the independent variable and the quantity y is called the dependent variable.

Imp for quiz

* The area of a circle depends on its radius r by the equation $A = \pi r^2$ so, we say that A is a function of r .

*The volume of a cube depends on the length of its side x by the equation $V = x^3$ so, we say that V is a function of x .

Define Function of Several Variables?

Many functions depend on more than one independent variable.

Examples quiz

The area of a rectangle depends on its length l and width w by the equation $A = l w$, so we say that A is a function of l and w .

The volume of a rectangular box depends on the length l , width w and height h by the equation $V = l w h$ so, we say that V is a function of l , w and h .

The area of a triangle depends on its base length l and height h by the equation $A = \frac{1}{2} l h$, so we say that A is a function of l and h .

The volume V of a right circular cylinder depends on its radius r and height h by the equation $V = \pi r^2 h$ so, we say that V is a function of r and h .

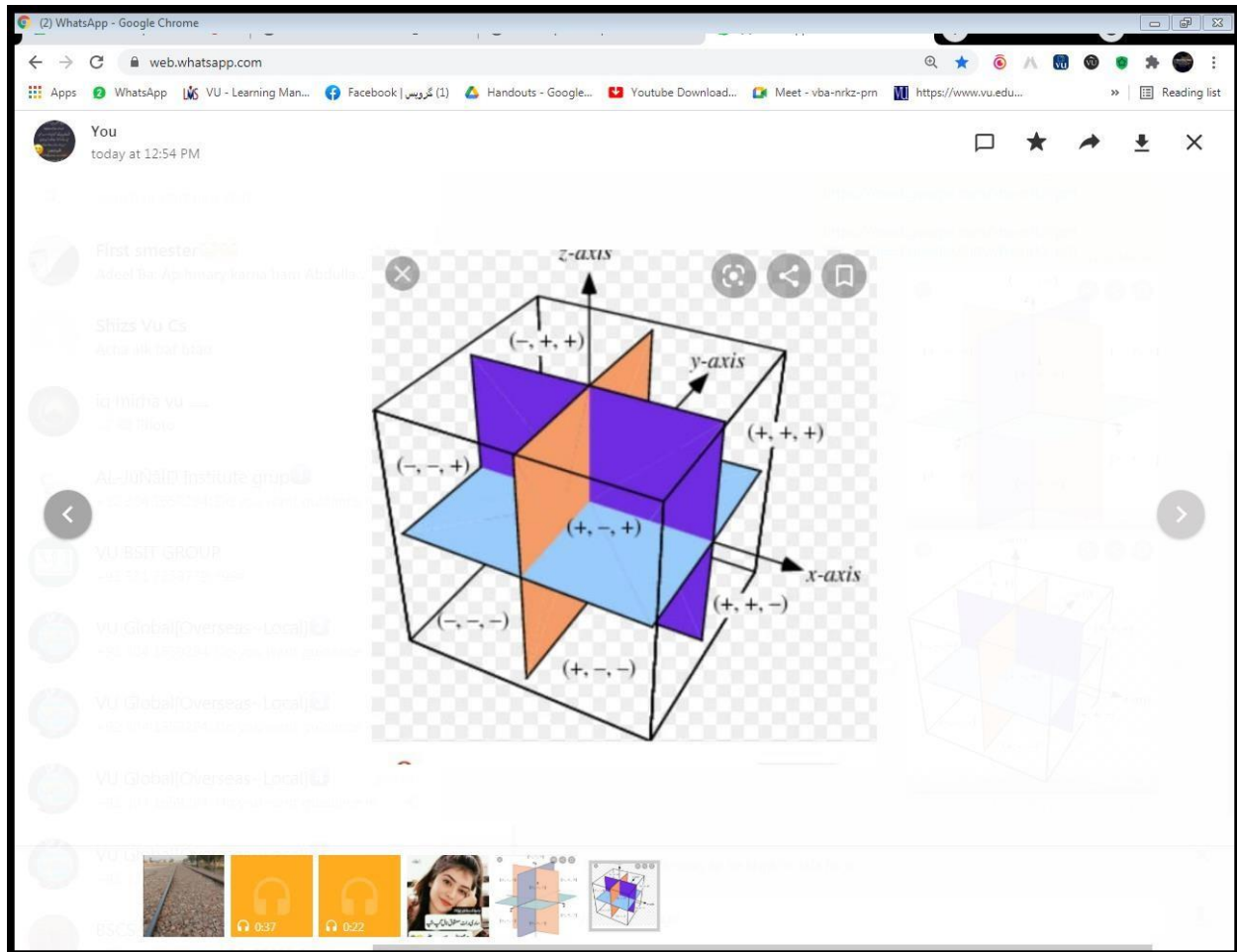
What is quadrant AND OCTANT?

In two dimension plane, say xy - plane, we divide the plane into four parts. Each part is called quadrant. In three dimension space, we divide the whole space in eight parts. Each part is called Octant.

OCTANT

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First octant (+, +, +) Formed by positive sides of the three axis.

Second octant (-, +, +) Formed by -ve x-axis and positive y and z-axis.

Third octant (-, -, +) Formed by -ve x and y axis with positive z-axis.

Fourth octant (+, -, +) Formed by +ve x and z axis and -ve y-axis.

Fifth octant (+, +, -) Formed by +ve x and y axis with -ve z-axis.

Sixth octant (-, +, -) Formed by -ve x and z axis with positive y-axis.

Seventh octant (-, -, -) Formed by -ve sides of three axis.

Eighth octant (+, -, -) Formed by -ve y and z-axis with +ve x-axis.

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Lecture No-2 Values of functions:

Function of One Variable

A function f of one real variable x is a rule that assigns a unique real number $f(x)$ to each point x in some set D of the real line.

Function of two Variables

A function f in two real variables x and y , is a rule that assigns unique real number $f(x,y)$ to each point (x,y) in some set D of the xy -plane.

Function of three variables

A function f in three real variables x , y and z , is a rule that assigns a unique real number $f(x,y,z)$ to each point (x,y,z) in some set D of three dimensional space.

Function of n variables:

A function f in n variable real variables $x_1, x_2, x_3, \dots, x_n$, is a rule that assigns a unique real number $w = f(x_1, x_2, x_3, \dots, x_n)$ to each point $(x_1, x_2, x_3, \dots, x_n)$ in some set D of n dimensional space.

Lecture No-3 Elements of three dimensional geometry.

Distance formula in three dimensions

The distance formula states that the distance between two points in xyz -space is the square root of the sum of the squares of the differences between corresponding coordinates.

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Midpoint of two points

Measure the distance between the two end points, and divide the result by 2.

$$X = \frac{X_1 + X_2}{2}$$

Direction cosines:

$$\cos \alpha = \frac{x}{op} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

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$$\cos\beta = \frac{y}{op} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\gamma = \frac{z}{op} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

Direction cosines and direction ratios of a line joining two points

The direction ratios of line PQ joining P(x₁,y₁,z₁) and Q(x₂,y₂,z₂) are x₂-x₁=a(say), y₂-y₁=b(say) and z₂-z₁=c(say).

$$\frac{x_2 - x_1}{|pq|} \quad \frac{y_2 - y_1}{|pq|} \quad \frac{z_2 - z_1}{|pq|}$$

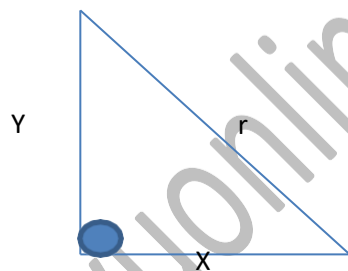
Lecture No-4

Define Cartesian coordinates for plane

Position of any point in the plane can be obtained by the two perpendicular lines known as x and y axis and together we call it as Cartesian coordinates for plane

$$x^2 + y^2 = r^2$$

Conversion formulas between rectangular and cylindrical co-ordinates



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

Conversion formulas between rectangular and spherical co-ordinates

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Proved that $\rho = \sqrt{x^2 + y^2 + z^2}$

$$x^2 + y^2 + z^2 = (\rho \sin \Phi \cos \theta)^2 + (\rho \sin \Phi \sin \theta)^2 + (\rho \cos \Phi)^2$$

$$x^2 + y^2 + z^2 = \rho^2 \{ \sin^2 \Phi (\cos^2 \theta + \sin^2 \theta) + \cos^2 \Phi \}$$

$$= \rho^2 (\sin^2 \Phi + \cos^2 \Phi) = \rho^2 (1)$$

$$\rho^2 = x^2 + y^2 + z^2$$

$$\sqrt{\rho^2} = \sqrt{x^2 + y^2 + z^2}$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$(P, \theta, \Phi) \rightarrow (x, y, z)$$

Lecture No-5 Limit of Multivariable Function

Definition of a Limit of Multivariable Function

Given a function of two variables $f: D \rightarrow \mathbb{R}$, $D \subseteq \mathbb{R}^2$ such that D contains points arbitrarily close to a point (a, b) , we say that the Limit of $f(x, y)$ as (x, y) approaches (a, b) exists and has value L if.

e.g

$$f(x, y) = \sin^{-1}(x + y)$$

Let $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist and let c be a constant.

- 1) $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
- 2) $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$
- 3) $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$
- 4) $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ if $\lim_{x \rightarrow a} g(x) \neq 0$
- 5) $\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$ if n is a positive integer.
- 6) $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$ if $\sqrt[n]{\lim_{x \rightarrow a} f(x)}$ exists.

The following properties are special limit laws :

- 7) $\lim_{x \rightarrow a} c = c$
- 8) $\lim_{x \rightarrow a} x = a$

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Lecture No-6 Geometry of continuous functions

Continuity of functions of two variables A function f of two variables is called continuous at the point (x_0, y_0) if

A $f(x_0, y_0)$ is defined

B $\lim_{(x,y) \rightarrow (x_0, y_0)} f(x,y)$ exists $\rightarrow$ exists.

C $\lim_{(x,y) \rightarrow (x_0, y_0)} f(x,y) = f(x_0, y_0)$

Quiz

(1) $f(x,y) = \ln(2x - y + 1)$

The function f is continuous in the whole region where $2x > y - 1$, $y < 2x + 1$. And its region is shown in figure below.

(2) $f(x,y) = e^{1-xy}$

The function f is continuous in the whole region of xy -plane.

(3) $f(x,y) = \tan^{-1}(y-x)$

The function f is continuous in the whole region of xy -plane.

(4) $f(x,y) = \sqrt{y-x}$

The function is continuous where $x \geq y$

Lecture No -7 Geometric meaning of partial derivative

Partial Derivatives

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Let $z = f(x, y)$ be a function of two variable defined on a certain domain D . For a given change Δx in x , keeping y as it is, the change Δz in z , is given by

$$\Delta z = f(x + \Delta x, y) - f(x, y)$$

Respect of x

$$\frac{\Delta z}{\Delta x} = \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

Respect of y

$$\frac{\Delta z}{\Delta y} = \frac{f(x + \Delta x, y) - f(x, y)}{\Delta y}$$

Partial Derivatives of Higher Orders

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial x} (f_x) = (f_x)_x = f_{xx}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} (f_y) = (f_y)_y = f_{yy}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial y} (f_x) = (f_x)_y = f_{xy}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial x} (f_y) = (f_y)_x = f_{yx}$$

Laplace's Equation

The **definition** of the fundamental solution thus implies that, if the Laplacian of u is integrated over any volume that encloses the source point, then.

Formula

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$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} + \frac{\partial^2 R}{\partial z^2} = 0.$$

Lecture No- 8 More About Euler Theorem Chain Rule

Chain rule in function of one variable

both $f(x)$ and $g(x)$ are functions of one variable.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

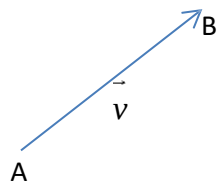
Lecture No- 9

Practice Question

Lecture No -10 Introduction to vectors

Vector

A vector in the plane is a directed line segment.



Formula $\vec{v} = \overrightarrow{AB}$

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Magnitude or Length of a Vector:

Magnitude of the vector $\vec{v}$ is denoted by $|\vec{v}| = |\overline{AB}|$ Is the length of the line segment AB.

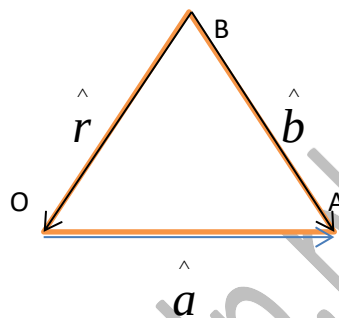
Formula $|\vec{v}| = |\overline{AB}|$

Unit vector

Any Vector whose Magnitude or length is 1 is a unit vector.

Formula $\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

Addition of Vectors



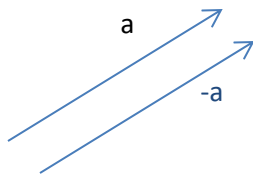
This diagram shows three vectors, in two vectors one vector $\vec{OA}$ is connected with tail of vector $\vec{AB}$. The tail of third vector $\vec{OB}$ is connected with the tail of $\vec{OA}$ and head is connected with the head of vector $\vec{AB}$. This third vector is called Resultant vector.

The resultant vector can be written as

$$\vec{r} = \vec{a} + \vec{b}$$

Equal vectors

Two vectors are equal or same vectors if they have same magnitude and direction.

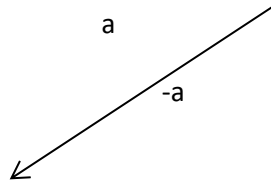


Opposite vectors

Two vectors are opposite vectors if they have same magnitude and opposite direction.

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Parallel vectors

Two vectors are parallel if one vector is scalar multiple of the other.

$$\underline{b} = \lambda \underline{a}$$

Where λ is a non-zero scalar

Scalar product

Scalar product (dot product) ("a dot b") of vector a and b is the

Number

$$a.b = |a||b| \cos\theta.$$

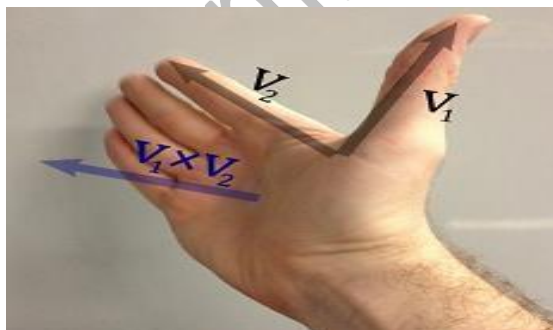
Where θ is the angle between a and b

The angle between two vectors

$$\theta = \cos^{-1} \left(\frac{a.b}{|a||b|} \right)$$

The Cross Product of Two Vectors in Space

Consider two nonzero vectors "a" and "b" in space. The vector product $a \times b$ ("a cross b") to be the vector $a \times b = (|a| |b| \sin \theta) \mathbf{n}$ where n is a vector determined by right hand rule.



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Lecture No -11 the Triple Scalar or Box Product

Triple scalar product

The product $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ is called the triple scalar product of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ (in that order).

$$AS \quad |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = |\mathbf{a} \times \mathbf{b}| |\mathbf{c}| |\cos \theta|$$

$$\text{Volume} = (\text{area of base}) \cdot (\text{high})$$

$$\text{Volume} = |\mathbf{a} \times \mathbf{b}| |\mathbf{c}| |\cos \theta|$$

$$\text{Volume} = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|$$

Directional Derivative

If $f(x, y)$ is differentiable at (x_0, y_0) and if $\mathbf{u} = (u_1, u_2)$ is a unit vector, then the directional derivative of f at (x_0, y_0) in the direction of $\mathbf{u}$ is defined by

$$D_{\mathbf{u}} f(x_0, y_0) = f_x(x_0, y_0) u_1 + f_y(x_0, y_0) u_2$$

Formula for the directional

$$D_{\mathbf{u}} f(x, y) = \nabla f(x, y) \cdot \hat{\mathbf{u}}$$

Properties of Directional Derivatives

1 The function f increases most rapidly when $\cos(0)$ or when $\hat{\mathbf{u}}$ is the direction of ∇f . f increases most rapidly in the direction of the gradient vector ∇f at P . The derivative in this direction is

$$D_{\mathbf{u}} f = |\nabla f| \cos(0) = |\nabla f|.$$

2 Similarly, f decreases most rapidly in the direction of $-\nabla f$. The derivative in this direction is

$$D_{\mathbf{u}} f = |\nabla f| \cos(\pi) = -|\nabla f|.$$

Rapidly in the direction of $-\nabla f$

3 Any direction $\hat{\mathbf{u}}$ orthogonal to the gradient is a direction of zero change in f because θ then equals $\pi/2$ and

$$D_{\mathbf{u}} f = |\nabla f| \cos(\pi/2) = |\nabla f| \cdot 0 = 0$$

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Lecture No -12 Tangent planes to the surfaces

Normal line to the surfaces

The Normal Line at is the line that passes through and is perpendicular to the tangent plane at and perpendicular to the surface at.

Tangent plane

The plane through a point of a surface that contains the tangent lines to all the curves on the surface through the same point. Since the tangent plane and the surface touch at (x_0, y_0) (x_0, y_0) the following point will be on both the surface and the plane.

Surface normal

Its normal (the straight line through P_0 and perpendicular to the tangent) is called the surface normal of S at P_0 .

Different forms of equation of straight line in two dimensional spaces

Slope intercepts form of the Equation of a line.

$$y = mx + c$$

Where m is the slope and c is y intercept.

2. Point Slope Form

Let m be the slope and $p_0 (x_0, y_0)$ be the point of required line, then

$$Y - y_0 = m (x - x_0)$$

3. General Equation of straight line

$$Ax + By + c = 0$$

$$M = \text{slope of line} = \text{Rise/Run} = b/c$$

$$Y - y_0 = b/c (x - x_0)$$

Parametric equation of a line

Parametric equation of a line in two dimensional spaces passing through the point (x_0, y_0) and parallel to the vector $a_i + b_j$ is given by

$$x = x_0 + at, y = y_0 + bt$$

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Eliminating t from both equation

We get $(x - x_0)/a = (y - y_0)/b$

$$y - y_0 = b/a (x - x_0)$$

Parametric vector form:

$$r(t) = (x_0 + at) i + (y_0 + bt) j,$$

Equation of line in three dimensional

Parametric equation of a line in three dimensional space passing through the point (x_0, y_0, z_0) and parallel to the vector $ai + bj + ck$ is given by

$$(x - x_0)/a = (y - y_0)/b = (z - z_0)/c$$

Different form of equation of curve

Curves in the plane are defined in different ways

Explicit form:

$$y = f(x)$$

Implicit form:

$$F(x, y) = 0$$

Parametric form:

$$x = f(t) \text{ and } y = g(t)$$

Parametric vector form:

$$r(t) = f(t) i + g(t) j,$$

Equation of a plane

An **equation** of the **plane** containing the point (x_0, y_0, z_0) with normal vector $N = \langle A, B, C \rangle$ is $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$. Note: The **equation** of any **plane** can be expressed as $Ax + By + Cz = D$. This is called the standard form of the **equation of a plane**.

Normal vector

$$n = ai + bj + ck$$

($\perp$ means perpendicular to)

$$\rightarrow \text{POP} = (x - x_0) i + (y - y_0) j + (z - z_0) k$$

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Gradients and Tangents to Surfaces

$$f(x, y) = c$$

$$z = f(x, y), z = c$$

If a differentiable function $f(x, y)$ has curve having parametric equation

$$x = g(t), y = h(t), r = g(t)i + h(t)j$$

Differentiating both sides of this equation with respect to t leads to the equation a constant value c along a smooth

$$\frac{d}{dx} f(g(t), h(t)) = \frac{d}{dt} (c)$$

$$\frac{\partial f}{\partial x} * \frac{\partial g}{\partial t} + \frac{\partial f}{\partial y} * \frac{\partial h}{\partial t} = 0$$

$$\left(\frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j \right) * \left(\frac{\partial g}{\partial t} i + \frac{\partial h}{\partial t} j \right) = 0$$

$$\nabla f * \frac{dr}{dt} = 0$$

∇f is normal to the tangent vector dr/dt , so it is normal to the curve through (x_0, y_0)

Tangent Plane and Normal Line

Well **tangent planes** to a **surface** are **planes** that just touch the **surface at the point** and are “parallel” to the **surface at the point**. Note that this gives us a **point** that is on the **plane**. Since the **tangent plane** and the **surface** touch at (x_0, y_0) (x_0, y_0) the following **point** will be on both the **surface** and the **plane**.

Normal line to the surface

The **normal vector**, often simply called the “**normal**,” to a **surface** is a **vector** which is **perpendicular to the surface** at a given point. When normal are considered on closed **surfaces**, the inward-pointing **normal** (pointing towards the interior of the **surface**) and outward-pointing **normal** are usually distinguished.

ORTHOGONAL SURFACES

Two **surfaces** are called **orthogonal** at a point of intersection P if their normal is perpendicular at that point. Show that **surfaces** with equations $F(x; y; z) = 0$ and $G(x; y; z) = 0$ are **orthogonal**

let (x, y, z) be any point of intersection

$$f(x, y, z) = 0 \dots\dots (i)$$

$$g(x, y, z) = 0 \dots\dots (ii)$$

Similarly, direction ratios of a line normal to (2)

are g_x, g_y, g_z

$$f_x g_x + f_y g_y + f_z g_z$$

For a function $y = f(x)$ $dy = f'(x) dx$

Is called the differential of functions $f(x)$

dx the differential of x is the same as the actual change in x i.e.

$dx = \Delta x$ whereas dy the Distinction between the increment Δy and the differential dy

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

TOTAL DIFFERENTIAL

If we move from (x_0, y_0) to a point $(x_0 + dx, y_0 + dy)$ nearby, the resulting differential in f is

$$df = f_x(x_0, y_0) dx + f_y(x_0, y_0) dy$$

This change in the linearization of

f is called the total differential of f .

Absolute maximum

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An **absolute maximum** point is a point where the function obtains its greatest possible value. Similarly, an **absolute minimum** point is a point where the function obtains its least possible value.

$$f(x_0, y_0) \geq f(x, y) \text{ for all } (x, y) \in D$$

Absolute minimum

The smallest value that a mathematical function can have over its entire curve (see curve entry 3 sense 5a) .the function defined by $y = 3 - x$ has an **absolute maximum** $M = 2$ and an **absolute minimum** $m = 0$ on the interval $1 < x < 3$.

$$f(x_0, y_0) \leq f(x, y) \text{ for all } (x, y) \in D$$

Relative (local) maximum

A **relative maximum** point is a point where the function changes direction from increasing to decreasing (making that point a "peak" in the graph). Similarly, a **relative minimum** point is a point where the function changes direction from decreasing to increasing (making that point a "bottom" in the graph).

$$f(x_0, y_0) \geq f(x, y) \text{ for all } (x, y) \in D$$

Extreme Value Theorem

If $f(x, y)$ is continuous on a closed and bounded set R , then f has both an absolute maximum and an absolute minimum on R .

Extreme values or extrema of f

The maximum and minimum values of f are referred to as extreme values or extrema of f .

Let a function f of two variables be defined on an open disc.

$$K = \left\{ (x, y) : (x - x_0)^2 + (y - y_0)^2 < r^2 \right\}$$

$$f_x(x_0, y_0) = 0 = f_y(x_0, y_0).$$

Saddle Point

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A differentiable function $f(x, y)$ has a saddle point (a, b) if in every open disk centered at (a, b) there are domain points (x, y) where $f(x, y) > f(a, b)$ and domain points (x, y) where $f(x, y) < f(a, b)$. The corresponding point $(a, b, f(a, b))$ on the surface $z = f(x, y)$ is called a saddle point of the surface

Lecture No #15 Examples

$$f(x, y) = \sqrt{x^2 + y^2}$$

$$f_x(x, y) = \frac{x}{\sqrt{x^2 + y^2}}$$

$$f_y(x, y) = \frac{y}{\sqrt{x^2 + y^2}}$$

The partial derivatives exist at all points of the domain of f except at the origin which is f in the domain. Thus $(0, 0)$ is a critical point of f .

$$f_x(x, y) = 0 \text{ only if } x=0$$

and

$$f_y(x, y) = 0 \text{ only if } y=0$$

The only critical point is $(0,0)$ and $f(0,0)=0$. Since $f(x, y) \geq 0$ for all (x, y) , $f(0, 0) = 0$ is the absolute minimum value of f .

practice Question

Lecture No -16 Extreme Valued Theorems

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EXTREME VALUED

THEOREM If the function f is continuous on the closed interval $[a, b]$, then f has an absolute maximum value and an absolute minimum value on $[a, b]$.

practice Question

Lecture No -17

Examples

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Find the absolute maximum and minimum values of

$f(x,y)=xy-x-3y$ on the closed triangular region R with vertices $(0, 0)$, $(0, 4)$, and $(5, 0)$.

$$f(x,y) = xy - x - 3y \dots\dots\dots(1)$$

$$f_x(x,y) = y - 1$$

$$f_y(x,y) = x - 3$$

For critical points

$$f_x(x,y) = 0$$

$$y - 1 = 0$$

$$y = 1 \dots\dots\dots(2)$$

$$f_y(x,y) = 0$$

$$3x - 3 = 0$$

$$x = 1 \dots\dots\dots(3)$$

(i) The line segment between $(0, 0)$ and $(5, 0)$

$y = 0$, $x=5$ putting in (1)

$$u(x) = f(x, 0) = -x, 0 \leq x \leq 5 \dots\dots\dots(4)$$

ii) The line segment between $(0, 0)$ and $(0, 4)$

$x = 0$, $y=4$ putting in (1)

$$v(y) = f(0, y) = -3y, 0 \leq y \leq 4 \dots\dots\dots(5)$$

iii) The line segment between $(5, 0)$ and $(0, 4)$

in the xy - plan, an equation is.

$$y = -\frac{4x}{5} + 4, 0 \leq x \leq 5 \dots\dots\dots(6)$$

$$w(x) = f\left(x, -\frac{4x}{5} + 4\right)$$

$$w(x) = -\frac{4x^2}{5} + \frac{3}{5}x + 12 \quad 0 \leq x \leq 5$$

derivative of w

$$w'(x) = -\frac{8}{5}x + \frac{3}{5}, \quad w'(x) = 0$$

$$x = \frac{3}{8}$$

find integral

$$\int_0^1 (xy + y^2) dx$$

$$\int_0^1 (xy + y^2) dy = x \int_0^1 y dy + \int_0^1 y^2 dy$$

$$\int_0^1 (xy + y^2) dy = x \left[\frac{y^2}{2} \right]_0^1 + \left[\frac{y^3}{3} \right]_0^1 = x \left(\frac{1}{2} \right) + \frac{1}{3}$$

$$\int_0^1 (xy + y^2) dy = \frac{x}{2} + \frac{1}{3}$$

Iterated or Repeated Integral

The expression $\int_c^d \left[\int_a^b f(x, y) dx \right] dy$ is called iterated or repeated integral

Lecture No -19 Use of Integrals

Area as anti-derivatives

The **derivative** of the **area** function $A(x)$ is $f(x)$, so $A(x)$ is an **ant derivative** of $f(x)$. As both $A(x)$ and $F(x)$ are **ant derivatives** of $f(x)$, they must differ by a constant, i.e., $A(x) = F(x) + K$ for some constant K . Since $A(a) = 0$, this implies $F(a) + K = 0$.

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example

4

$$\int_0^4 2x dx$$

$$\int_0^4 2x dx = \left| x^2 \right|_0^4$$

$$\left| x^2 \right|_0^4 = 16$$

Area of triangle = $\frac{1}{2}$ base $\times$ altitude

$$\frac{1}{2}(4)(8) = 16$$

Volume as anti-derivative

$$\iint_R cf(x, y) dx dy$$

$$= c \iint_R f(x, y) dx dy \dots\dots\dots c \text{ is constant}$$

$$\iint_R [f(x, y) + g(x, y)] dx dy$$

$$= \iint_R f(x, y) dx dy + \iint_R g(x, y) dx dy$$

$$\iint_R [f(x, y) - g(x, y)] dx dy$$

$$= \iint_R f(x, y) dx dy - \iint_R g(x, y) dx dy$$

Lecture No -20

Double integral for non-rectangular region

Double integral for non-rectangular region

Type I regions bounded the left and right by vertical lines $x=a$ and $x=b$ and is bounded below and above by curves $y=g_1(x)$ and $y=g_2(x)$ where $g_1(x) \leq g_2(x)$ for $a \leq x \leq b$.

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$$\int_R f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx$$

Type II region is bounded below and above by the horizontal lines $y=c$ and $y=d$ and is $h_1(y) \leq h_2(y)$ for $c \leq y \leq d$

$$\int_R f(x, y) dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) dx dy$$

Lecture No -21 Examples

Question

Use a double integral to find the volume of the solid that is bounded

above by the plane $Z = 4 - x - y$ and below by the rectangle $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 2\}$

solution

$$V = \iint_R (4 - x - y) dA$$

$$V = \int_0^2 \int_0^1 (4 - x - y) dx dy$$

$$V = \int_0^2 \left[4x - \frac{x^2}{2} - xy \right]_{x=0}^{x=1} dy$$

$$V = \int_0^2 \left(\frac{7}{2} - y \right) dy = \left[\frac{7}{2}y - \frac{y^2}{2} \right]_0^2 = 5$$

Lecture No -22 Examples

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$$\iint_R xy dA$$

R is the region bounded by the Trapezium with the vertices $(1, 3)$ $(5, 3)$ $(2, 1)$ $(4, 1)$

$$\text{Slope of } AD = \frac{3-1}{1-2} = -2$$

Equation of line AD

$$y - 1 = -2(x - 2)$$

$$y - 1 = -2x + 4$$

$$-2x = y - 5$$

$$x = -\frac{y-5}{2}$$

$$\text{Slope of } BC = \frac{3-1}{5-4} = 2$$

$$y - 1 = 2(x - 4)$$

$$y - 1 = 2x - 8$$

$$2x = y + 7$$

$$x = \frac{y + 7}{2}$$

$$= \int_1^3 \int_{-(y-5)/2}^{(y+7)/2} xy$$

$$= \int_1^3 \left| dx \right|_{-(y-5)/2}^{(y+7)/2} y dy$$

$$= \left| y^3 + \frac{3y^2}{2} \right|_1^3$$

$$= (3^3) + \frac{3(3)^2}{2} - 1 - \frac{3}{2}$$

$$= 38 \text{ Answer}$$

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MTh lecher #1

What is Integral Calculus?

Deals with the Problem of determining a Function from information about its rates of Change.

Defiance b/w independent or dependent?

The term, function, denotes the dependence of one quantity on other quantity. The quantity x is called the independent variable and the quantity y is called the dependent variable.

Imp for quiz

* The area of a circle depends on its radius r by the equation $A = \pi r^2$ so, we say that A is a function of r .

*The volume of a cube depends on the length of its side x by the equation $V = x^3$ so, we say that V is a function of x .

Define Function of Several Variables?

Many functions depend on more than one independent variable.

Examples quiz

The area of a rectangle depends on its length l and width w by the equation $A = l w$, so we say that A is a function of l and w .

The volume of a rectangular box depends on the length l , width w and height h by the equation $V = l w h$ so, we say that V is a function of l , w and h .

The area of a triangle depends on its base length l and height h by the equation $A = \frac{1}{2} l h$, so we say that A is a function of l and h .

The volume V of a right circular cylinder depends on its radius r and height h by the equation $V = \pi r^2 h$ so, we say that V is a function of r and h .

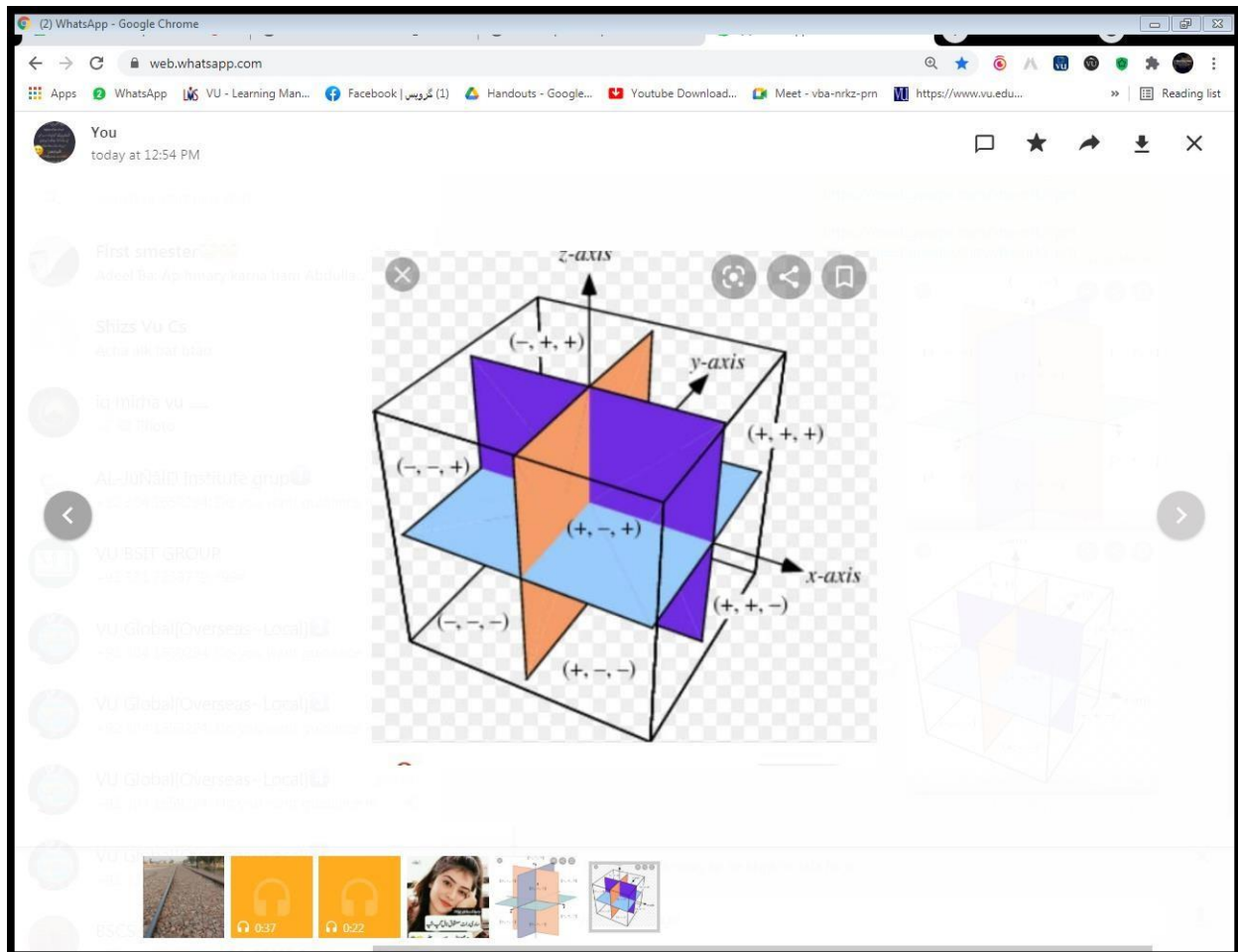
What is quadrant AND OCTANT?

In two dimension plane, say xy - plane, we divide the plane into four parts. Each part is called quadrant. In three dimension space, we divide the whole space in eight parts. Each part is called Octant.

OCTANT

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First octant (+, +, +) Formed by positive sides of the three axis.

Second octant (-, +, +) Formed by -ve x-axis and positive y and z-axis.

Third octant (-, -, +) Formed by -ve x and y axis with positive z-axis.

Fourth octant (+, -, +) Formed by +ve x and z axis and -ve y-axis.

Fifth octant (+, +, -) Formed by +ve x and y axis with -ve z-axis.

Sixth octant (-, +, -) Formed by -ve x and z axis with positive y-axis.

Seventh octant (-, -, -) Formed by -ve sides of three axis.

Eighth octant (+, -, -) Formed by -ve y and z-axis with +ve x-axis.

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Lecture No-2 Values of functions:

Function of One Variable

A function f of one real variable x is a rule that assigns a unique real number $f(x)$ to each point x in some set D of the real line.

Function of two Variables

A function f in two real variables x and y , is a rule that assigns unique real number $f(x,y)$ to each point (x,y) in some set D of the xy -plane.

Function of three variables

A function f in three real variables x , y and z , is a rule that assigns a unique real number $f(x,y,z)$ to each point (x,y,z) in some set D of three dimensional space.

Function of n variables:

A function f in n variable real variables $x_1, x_2, x_3, \dots, x_n$, is a rule that assigns a unique real number $w = f(x_1, x_2, x_3, \dots, x_n)$ to each point $(x_1, x_2, x_3, \dots, x_n)$ in some set D of n dimensional space.

Lecture No-3 Elements of three dimensional geometry.

Distance formula in three dimensions

The distance formula states that the distance between two points in xyz -space is the square root of the sum of the squares of the differences between corresponding coordinates.

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Midpoint of two points

Measure the distance between the two end points, and divide the result by 2.

$$X = \frac{X_1 + X_2}{2}$$

Direction cosines:

$$\cos \alpha = \frac{x}{op} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

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$$\cos\beta = \frac{y}{op} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\gamma = \frac{z}{op} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

Direction cosines and direction ratios of a line joining two points

The direction ratios of line PQ joining P(x₁,y₁,z₁) and Q(x₂,y₂,z₂) are x₂-x₁=a(say), y₂-y₁=b(say) and z₂-z₁=c(say).

$$\frac{x_2 - x_1}{|pq|} \quad \frac{y_2 - y_1}{|pq|} \quad \frac{z_2 - z_1}{|pq|}$$

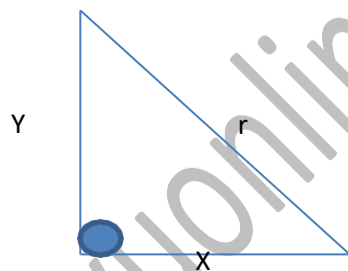
Lecture No-4

Define Cartesian coordinates for plane

Position of any point in the plane can be obtained by the two perpendicular lines known as x and y axis and together we call it as Cartesian coordinates for plane

$$x^2 + y^2 = r^2$$

Conversion formulas between rectangular and cylindrical co-ordinates



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

Conversion formulas between rectangular and spherical co-ordinates

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Proved that $\rho = \sqrt{x^2 + y^2 + z^2}$

$$x^2 + y^2 + z^2 = (\rho \sin \Phi \cos \theta)^2 + (\rho \sin \Phi \sin \theta)^2 + (\rho \cos \Phi)^2$$

$$x^2 + y^2 + z^2 = \rho^2 \{ \sin^2 \Phi (\cos^2 \theta + \sin^2 \theta) + \cos^2 \Phi \}$$

$$= \rho^2 (\sin^2 \Phi + \cos^2 \Phi) = \rho^2 (1)$$

$$\rho^2 = x^2 + y^2 + z^2$$

$$\sqrt{\rho^2} = \sqrt{x^2 + y^2 + z^2}$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$(P, \theta, \Phi) \rightarrow (x, y, z)$$

Lecture No-5 Limit of Multivariable Function

Definition of a Limit of Multivariable Function

Given a function of two variables $f: D \rightarrow \mathbb{R}$, $D \subseteq \mathbb{R}^2$ such that D contains points arbitrarily close to a point (a, b) , we say that the Limit of $f(x, y)$ as (x, y) approaches (a, b) exists and has value L if.

e.g

$$f(x, y) = \sin^{-1}(x + y)$$

Let $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist and let c be a constant.

- 1) $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
- 2) $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$
- 3) $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$
- 4) $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ if $\lim_{x \rightarrow a} g(x) \neq 0$
- 5) $\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$ if n is a positive integer.
- 6) $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$ if $\sqrt[n]{\lim_{x \rightarrow a} f(x)}$ exists.

The following properties are special limit laws :

- 7) $\lim_{x \rightarrow a} c = c$
- 8) $\lim_{x \rightarrow a} x = a$

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Lecture No-6 Geometry of continuous functions

Continuity of functions of two variables A function f of two variables is called continuous at the point (x_0, y_0) if

A $f(x_0, y_0)$ is defined

B $\lim_{(x,y) \rightarrow (x_0, y_0)} f(x,y)$ exists $\rightarrow$ exists.

C $\lim_{(x,y) \rightarrow (x_0, y_0)} f(x,y) = f(x_0, y_0)$

Quiz

(5) $f(x,y) = \ln(2x - y + 1)$

The function f is continuous in the whole region where $2x > y - 1$, $y < 2x + 1$. And its region is shown in figure below.

(6) $f(x,y) = e^{1-xy}$

The function f is continuous in the whole region of xy -plane.

(7) $f(x,y) = \tan^{-1}(y-x)$

The function f is continuous in the whole region of xy -plane.

(8) $f(x,y) = \sqrt{y-x}$

The function is continuous where $x \geq y$

Lecture No -7 Geometric meaning of partial derivative

Partial Derivatives

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Let $z = f(x, y)$ be a function of two variable defined on a certain domain D . For a given change Δx in x , keeping y as it is, the change Δz in z , is given by

$$\Delta z = f(x + \Delta x, y) - f(x, y)$$

Respect of x

$$\frac{\Delta z}{\Delta x} = \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

Respect of y

$$\frac{\Delta z}{\Delta y} = \frac{f(x + \Delta x, y) - f(x, y)}{\Delta y}$$

Partial Derivatives of Higher Orders

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial x} (f_x) = (f_x)_x = f_{xx}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} (f_y) = (f_y)_y = f_{yy}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial}{\partial y} (f_x) = (f_x)_y = f_{xy}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial x} (f_y) = (f_y)_x = f_{yx}$$

Laplace's Equation

The **definition** of the fundamental solution thus implies that, if the Laplacian of u is integrated over any volume that encloses the source point, then.

Formula

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$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} + \frac{\partial^2 R}{\partial z^2} = 0.$$

Lecture No- 8 More About Euler Theorem Chain Rule

Chain rule in function of one variable

both $f(x)$ and $g(x)$ are functions of one variable.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

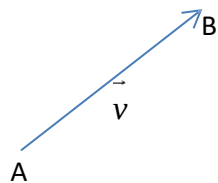
Lecture No- 9

Practice Question

Lecture No -10 Introduction to vectors

Vector

A vector in the plane is a directed line segment.



Formula $\vec{v} = \overrightarrow{AB}$

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Magnitude or Length of a Vector:

Magnitude of the vector $\vec{v}$ is denoted by $|\vec{v}| = |\overline{AB}|$ Is the length of the line segment AB.

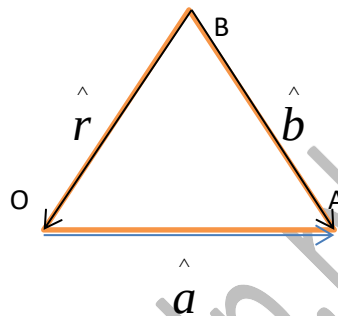
Formula $|\vec{v}| = |\overline{AB}|$

Unit vector

Any Vector whose Magnitude or length is 1 is a unit vector.

Formula $\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

Addition of Vectors



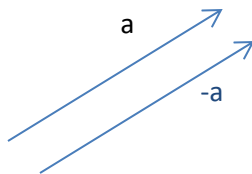
This diagram shows three vectors, in two vectors one vector $\vec{OA}$ is connected with tail of vector $\vec{AB}$. The tail of third vector $\vec{OB}$ is connected with the tail of $\vec{OA}$ and head is connected with the head of vector $\vec{AB}$. This third vector is called Resultant vector.

The resultant vector can be written as

$$\vec{r} = \vec{a} + \vec{b}$$

Equal vectors

Two vectors are equal or same vectors if they have same magnitude and direction.

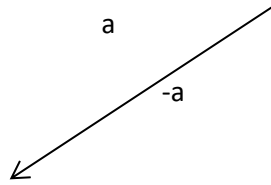


Opposite vectors

Two vectors are opposite vectors if they have same magnitude and opposite direction.

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Parallel vectors

Two vectors are parallel if one vector is scalar multiple of the other.

$$\underline{b} = \lambda \underline{a}$$

Where λ is a non-zero scalar

Scalar product

Scalar product (dot product) ("a dot b") of vector a and b is the

Number

$$a.b = |a||b| \cos\theta.$$

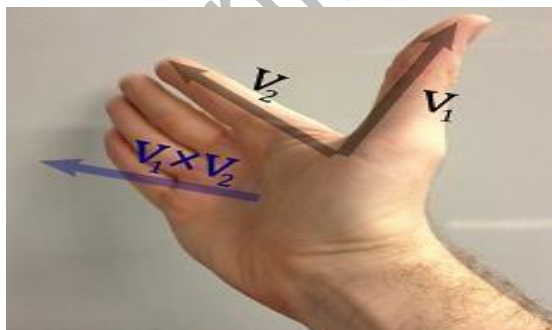
Where θ is the angle between a and b

The angle between two vectors

$$\theta = \cos^{-1} \left(\frac{a.b}{|a||b|} \right)$$

The Cross Product of Two Vectors in Space

Consider two nonzero vectors "a" and "b" in space. The vector product $a \times b$ ("a cross b") to be the vector $a \times b = (|a| |b| \sin \theta) \mathbf{n}$ where n is a vector determined by right hand rule.



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Lecture No -11 the Triple Scalar or Box Product

Triple scalar product

The product $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ is called the triple scalar product of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ (in that order).

$$AS \quad |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = |\mathbf{a} \times \mathbf{b}| |\mathbf{c}| |\cos \theta|$$

$$\text{Volume} = (\text{area of base}) \cdot (\text{high})$$

$$\text{Volume} = |\mathbf{a} \times \mathbf{b}| |\mathbf{c}| |\cos \theta|$$

$$\text{Volume} = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|$$

Directional Derivative

If $f(x, y)$ is differentiable at (x_0, y_0) and if $\mathbf{u} = (u_1, u_2)$ is a unit vector, then the directional derivative of f at (x_0, y_0) in the direction of $\mathbf{u}$ is defined by

$$D_{\mathbf{u}} f(x_0, y_0) = f_x(x_0, y_0) u_1 + f_y(x_0, y_0) u_2$$

Formula for the directional

$$D_{\mathbf{u}} f(x, y) = \nabla f(x, y) \cdot \hat{\mathbf{u}}$$

Properties of Directional Derivatives

1 The function f increases most rapidly when $\cos(0)$ or when $\hat{\mathbf{u}}$ is the direction of ∇f . f increases most rapidly in the direction of the gradient vector ∇f at P . The derivative in this direction is

$$D_{\mathbf{u}} f = |\nabla f| \cos(0) = |\nabla f|.$$

2 Similarly, f decreases most rapidly in the direction of $-\nabla f$. The derivative in this direction is

$$D_{\mathbf{u}} f = |\nabla f| \cos(\pi) = -|\nabla f|.$$

Rapidly in the direction of $-\nabla f$

3 Any direction $\hat{\mathbf{u}}$ orthogonal to the gradient is a direction of zero change in f because θ then equals $\pi/2$ and

$$d_{\mathbf{u}} f = |\nabla f| \cos(\pi/2) = |\nabla f| \cdot 0 = 0$$

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Lecture No -12 Tangent planes to the surfaces

Normal line to the surfaces

The Normal Line at is the line that passes through and is perpendicular to the tangent plane at and perpendicular to the surface at.

Tangent plane

The plane through a point of a surface that contains the tangent lines to all the curves on the surface through the same point. Since the tangent plane and the surface touch at (x_0, y_0) (x_0, y_0) the following point will be on both the surface and the plane.

Surface normal

Its normal (the straight line through P_0 and perpendicular to the tangent) is called the surface normal of S at P_0 .

Different forms of equation of straight line in two dimensional spaces

Slope intercepts form of the Equation of a line.

$$y = mx + c$$

Where m is the slope and c is y intercept.

2. Point Slope Form

Let m be the slope and $p_0 (x_0, y_0)$ be the point of required line, then

$$Y - y_0 = m (x - x_0)$$

3. General Equation of straight line

$$Ax + By + c = 0$$

$$M = \text{slope of line} = \text{Rise/Run} = b/c$$

$$Y - y_0 = b/c (x - x_0)$$

Parametric equation of a line

Parametric equation of a line in two dimensional spaces passing through the point (x_0, y_0) and parallel to the vector $ai + bj$ is given by

$$x = x_0 + at, y = y_0 + bt$$

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Eliminating t from both equation

We get $(x - x_0)/a = (y - y_0)/b$

$$y - y_0 = b/a (x - x_0)$$

Parametric vector form:

$$r(t) = (x_0 + at) i + (y_0 + bt) j,$$

Equation of line in three dimensional

Parametric equation of a line in three dimensional space passing through the point (x_0, y_0, z_0) and parallel to the vector $ai + bj + ck$ is given by

$$(x - x_0)/a = (y - y_0)/b = (z - z_0)/c$$

Different form of equation of curve

Curves in the plane are defined in different ways

Explicit form:

$$y = f(x)$$

Implicit form:

$$F(x, y) = 0$$

Parametric form:

$$x = f(t) \text{ and } y = g(t)$$

Parametric vector form:

$$r(t) = f(t) i + g(t) j,$$

Equation of a plane

An **equation** of the **plane** containing the point (x_0, y_0, z_0) with normal vector $N = \langle A, B, C \rangle$ is $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$. Note: The **equation** of any **plane** can be expressed as $Ax + By + Cz = D$. This is called the standard form of the **equation of a plane**.

Normal vector

$$n = ai + bj + ck$$

($\perp$ means perpendicular to)

$$\rightarrow \text{POP} = (x - x_0) i + (y - y_0) j + (z - z_0) k$$

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Gradients and Tangents to Surfaces

$$f(x, y) = c$$

$$z = f(x, y), z = c$$

If a differentiable function $f(x, y)$ has curve having parametric equation

$$x = g(t), y = h(t), r = g(t)i + h(t)j$$

Differentiating both sides of this equation with respect to t leads to the equation a constant value c along a smooth

$$\frac{d}{dx} f(g(t), h(t)) = \frac{d}{dt} (c)$$

$$\frac{\partial f}{\partial x} * \frac{\partial g}{\partial t} + \frac{\partial f}{\partial y} * \frac{\partial h}{\partial t} = 0$$

$$\left(\frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j \right) * \left(\frac{\partial g}{\partial t} i + \frac{\partial h}{\partial t} j \right) = 0$$

$$\nabla f * \frac{dr}{dt} = 0$$

∇f is normal to the tangent vector dr/dt , so it is normal to the curve through (x_0, y_0)

Tangent Plane and Normal Line

Well **tangent planes** to a **surface** are **planes** that just touch the **surface at the point** and are “parallel” to the **surface at the point**. Note that this gives us a **point** that is on the **plane**. Since the **tangent plane** and the **surface** touch at (x_0, y_0) (x_0, y_0) the following **point** will be on both the **surface** and the **plane**.

Normal line to the surface

The **normal vector**, often simply called the “**normal**,” to a **surface** is a **vector** which is **perpendicular to the surface** at a given point. When normal are considered on closed **surfaces**, the inward-pointing **normal** (pointing towards the interior of the **surface**) and outward-pointing **normal** are usually distinguished.

ORTHOGONAL SURFACES

Two **surfaces** are called **orthogonal** at a point of intersection P if their normal is perpendicular at that point. Show that **surfaces** with equations $F(x; y; z) = 0$ and $G(x; y; z) = 0$ are **orthogonal**

let (x, y, z) be any point of intersection

$$f(x, y, z) = 0 \dots\dots (i)$$

$$g(x, y, z) = 0 \dots\dots (ii)$$

Similarly, direction ratios of a line normal to (2)

are g_x, g_y, g_z

$$f_x g_x + f_y g_y + f_z g_z$$

For a function $y = f(x)$ $dy = f'(x) dx$

Is called the differential of functions $f(x)$

dx the differential of x is the same as the actual change in x i.e.

$dx = \Delta x$ whereas dy the Distinction between the increment Δy and the differential dy

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$$

TOTAL DIFFERENTIAL

If we move from (x_0, y_0) to a point $(x_0 + dx, y_0 + dy)$ nearby, the resulting differential in f is

$$df = f_x(x_0, y_0) dx + f_y(x_0, y_0) dy$$

This change in the linearization of

f is called the total differential of f .

Absolute maximum

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An **absolute maximum** point is a point where the function obtains its greatest possible value. Similarly, an **absolute minimum** point is a point where the function obtains its least possible value.

$$f(x_0, y_0) \geq f(x, y) \text{ for all } (x, y) \in D$$

Absolute minimum

The smallest value that a mathematical function can have over its entire curve (see curve entry 3 sense 5a) .the function defined by $y = 3 - x$ has an **absolute maximum** $M = 2$ and an **absolute minimum** $m = 0$ on the interval $1 < x < 3$.

$$f(x_0, y_0) \leq f(x, y) \text{ for all } (x, y) \in D$$

Relative (local) maximum

A **relative maximum** point is a point where the function changes direction from increasing to decreasing (making that point a "peak" in the graph). Similarly, a **relative minimum** point is a point where the function changes direction from decreasing to increasing (making that point a "bottom" in the graph).

$$f(x_0, y_0) \geq f(x, y) \text{ for all } (x, y) \in D$$

Extreme Value Theorem

If $f(x, y)$ is continuous on a closed and bounded set R , then f has both an absolute maximum and on absolute minimum on R .

Extreme values or extrema of f

The maximum and minimum values of f are referred to as extreme values of extrema of f .

Let a function f of two variables be defined on an open disc.

$$K = \left\{ (x, y) : (x - x_0)^2 + (y - y_0)^2 < r^2 \right\}$$

$$f_x(x_0, y_0) = 0 = f_y(x_0, y_0).$$

Saddle Point

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A differentiable function $f(x, y)$ has a saddle point (a, b) if in every open disk centered at (a, b) there are domain points (x, y) where $f(x, y) > f(a, b)$ and domain points (x, y) where $f(x, y) < f(a, b)$. The corresponding point $(a, b, f(a, b))$ on the surface $z = f(x, y)$ is called a saddle point of the surface

Lecture No #15 Examples

$$f(x, y) = \sqrt{x^2 + y^2}$$

$$f_x(x, y) = \frac{x}{\sqrt{x^2 + y^2}}$$

$$f_y(x, y) = \frac{y}{\sqrt{x^2 + y^2}}$$

The partial derivatives exist at all points of the domain of f except at the origin which is f in the domain. Thus $(0, 0)$ is a critical point of f .

$$f_x(x, y) = 0 \text{ only if } x=0$$

and

$$f_y(x, y) = 0 \text{ only if } y=0$$

The only critical point is $(0,0)$ and $f(0,0)=0$. Since $f(x, y) \geq 0$ for all (x, y) , $f(0, 0) = 0$ is the absolute minimum value of f .

practice Question

Lecture No -16 Extreme Valued Theorems

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EXTREME VALUED

THEOREM If the function f is continuous on the closed interval $[a, b]$, then f has an absolute maximum value and an absolute minimum value on $[a, b]$.

prectice Qustion

Lecture No -17

Examples

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Find the absolute maximum and minimum values of

$f(x,y)=xy-x-3y$ on the closed triangular region R with vertices $(0, 0)$, $(0, 4)$, and $(5, 0)$.

$$f(x,y) = xy - x - 3y \dots\dots\dots(1)$$

$$f_x(x,y) = y - 1$$

$$f_y(x,y) = x - 3$$

For critical points

$$f_x(x,y) = 0$$

$$y - 1 = 0$$

$$y = 1 \dots\dots\dots(2)$$

$$f_y(x,y) = 0$$

$$3x - 3 = 0$$

$$x = 1 \dots\dots\dots(3)$$

(i) The line segment between $(0, 0)$ and $(5, 0)$

$y = 0$, $x=5$ putting in (1)

$$u(x) = f(x, 0) = -x, 0 \leq x \leq 5 \dots\dots\dots(4)$$

ii) The line segment between $(0, 0)$ and $(0, 4)$

$x = 0$, $y=4$ putting in (1)

$$v(y) = f(0, y) = -3y, 0 \leq y \leq 4 \dots\dots\dots(5)$$

iii) The line segment between $(5, 0)$ and $(0, 4)$

in the xy - plan, an equation is.

$$y = -\frac{4x}{5} + 4, 0 \leq x \leq 5 \dots\dots\dots(6)$$

$$w(x) = f\left(x, -\frac{4x}{5} + 4\right)$$

$$w(x) = -\frac{4x^2}{5} + \frac{3}{5}x + 12 \quad 0 \leq x \leq 5$$

derivative of w

$$w'(x) = -\frac{8}{5}x + \frac{3}{5}, \quad w'(x) = 0$$

$$x = \frac{3}{8}$$

find integral

$$\int_0^1 (xy + y^2) dx$$

$$\int_0^1 (xy + y^2) dy = x \int_0^1 y dy + \int_0^1 y^2 dy$$

$$\int_0^1 (xy + y^2) dy = x \left[\frac{y^2}{2} \right]_0^1 + \left[\frac{y^3}{3} \right]_0^1 = x \left(\frac{1}{2} \right) + \frac{1}{3}$$

$$\int_0^1 (xy + y^2) dy = \frac{x}{2} + \frac{1}{3}$$

Iterated or Repeated Integral

The expression $\int_c^d \left[\int_a^b f(x, y) dx \right] dy$ is called iterated or repeated integral

Lecture No -19 Use of Integrals

Area as anti-derivatives

The **derivative** of the **area** function $A(x)$ is $f(x)$, so $A(x)$ is an **ant derivative** of $f(x)$. As both $A(x)$ and $F(x)$ are **ant derivatives** of $f(x)$, they must differ by a constant, i.e., $A(x) = F(x) + K$ for some constant K . Since $A(a) = 0$, this implies $F(a) + K = 0$.

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example

4

$$\int_0^4 2x dx$$

$$\int_0^4 2x dx = \left| x^2 \right|_0^4$$

$$\left| x^2 \right|_0^4 = 16$$

Area of triangle = $\frac{1}{2}$ base $\times$ altitude

$$\frac{1}{2}(4)(8) = 16$$

Volume as anti-derivative

$$\iint_R cf(x, y) dx dy$$

$$= c \iint_R f(x, y) dx dy \dots\dots\dots c \text{ is constant}$$

$$\iint_R [f(x, y) + g(x, y)] dx dy$$

$$= \iint_R f(x, y) dx dy + \iint_R g(x, y) dx dy$$

$$\iint_R [f(x, y) - g(x, y)] dx dy$$

$$= \iint_R f(x, y) dx dy - \iint_R g(x, y) dx dy$$

Lecture No -20

Double integral for non-rectangular region

Double integral for non-rectangular region

Type I regions bounded the left and right by vertical lines $x=a$ and $x=b$ and is bounded below and above by curves $y=g_1(x)$ and $y=g_2(x)$ where $g_1(x) \leq g_2(x)$ for $a \leq x \leq b$.

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$$\int_R f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx$$

Type II region is bounded below and above by the horizontal lines $y=c$ and $y=d$ and is $h_1(y) \leq h_2(y)$ for $c \leq y \leq d$

$$\int_R f(x, y) dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) dx dy$$

Lecture No -21 Examples

Question

Use a double integral to find the volume of the solid that is bounded

above by the plane $Z = 4 - x - y$ and below by the rectangle $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 2\}$

solution

$$V = \iint_R (4 - x - y) dA$$

$$V = \int_0^2 \int_0^1 (4 - x - y) dx dy$$

$$V = \int_0^2 \left[4x - \frac{x^2}{2} - xy \right]_{x=0}^{x=1} dy$$

$$V = \int_0^2 \left(\frac{7}{2} - y \right) dy = \left[\frac{7}{2}y - \frac{y^2}{2} \right]_0^2 = 5$$

Lecture No -22 Examples

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$$\iint_R xy dA$$

R is the region bounded by the Trapezium with the vertices $(1, 3)$ $(5, 3)$ $(2, 1)$ $(4, 1)$

$$\text{Slope of } AD = \frac{3-1}{1-2} = -2$$

Equation of line AD

$$y - 1 = -2(x - 2)$$

$$y - 1 = -2x + 4$$

$$-2x = y - 5$$

$$x = -\frac{y-5}{2}$$

$$\text{Slope of } BC = \frac{3-1}{5-4} = 2$$

$$y - 1 = 2(x - 4)$$

$$y - 1 = 2x - 8$$

$$2x = y + 7$$

$$x = \frac{y + 7}{2}$$

$$= \int_1^3 \int_{-(y-5)/2}^{(y+7)/2} xy$$

$$= \int_1^3 \left| dx \right|_{-(y-5)/2}^{(y+7)/2} y dy$$

$$= \left| y^3 + \frac{3y^2}{2} \right|_1^3$$

$$= (3^3) + \frac{3(3)^2}{2} - 1 - \frac{3}{2}$$

$$= 38 \text{ Answer}$$

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