

MTH603

Solved Subjective

FOR FINAL TERM (SPRING , 2021)

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MSC MATHEMATICS



Spring 2012

Question No: 50 (Marks: 5)

$$\sin x = 1 + x^3$$

Solve by using the Newton Raphson method up to one iteration
(Note: Accuracy up to four decimal places is required)

Answer:

$$f(x) = \sin x = 1 + x^3$$

$$f(x) = \sin x - 1 - x^3$$

$$f(1) = \sin 1 - 1 - 1^3 = 0.0174 - 1 - 1 = -1.9826$$

$$f(2) = \sin 2 - 1 - 2^3 = 0.0348 - 1 - 8 = -8.9652$$

$$f'(1) = -\cos x - 3x^2 = -\cos 1 - 3(1)^2 = -0.9998 - 3 = -3.9998$$

$$f'(2) = -\cos x - 3x^2 = -\cos 2 - 3(2)^2 = -0.9993 - 12 = -12.9993$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = -3.9998 - \frac{-8.9652}{-12.9993} = -3.9998 - 0.6896 = -4.6894$$

Q: Evaluate the integral $\int_0^1 (x+1) dx$ By Trapezoidal Rule, h=0.2

Answer:

X	0	0.2	0.4	0.6	0.8	1
Y= (x+1)	1.0	1.2	1.4	1.6	1.8	2

Using trapezoidal rule, taking h=0.2

$$I = \frac{h}{2} (y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4))$$

$$I = \frac{0.2}{2} (1.0 + 2 + 2(1.2 + 1.4 + 1.6 + 1.8))$$

$$I = \frac{0.2}{2} (3 + 12)$$

$$I = \frac{3}{2} = 1.5$$

Q: Evaluate the integral $\int_3^5 (\sin x + \cos x) dx$ By Simpson's 3/8 rule. Take the interval $h=1$.

Answer:

X	3	4	5
Y=(sin x + cos x)	1.0509	1.0673	1.0833

$$\int_3^5 (\sin x + \cos x) dx = \frac{3h}{8} (y_0 + y_2 + 3(y_1))$$

$$\int_3^5 (\sin x + \cos x) dx = \frac{3(1)}{8} (1.0509 + 1.0833 + 3(1.0673))$$

$$\int_3^5 (\sin x + \cos x) dx = \frac{3}{8} (2.1342 + 3.2019)$$

$$\int_3^5 (\sin x + \cos x) dx = \frac{3}{8} (5.3361) = \frac{16.0083}{8} = 2.0010375$$

Q: Using Milne's method find $y(0.1)$ $dy/dx = x - y/x + y$, $Y(0)=1$.

Answer:

$$\frac{dy}{dx} = \frac{x-y}{x+y}, \quad y(0)=1,$$

find $y(0.1)$

$$y'_0 = \frac{x-y}{x+y} = \frac{0-1}{0+1} = -\frac{1}{1} = -1$$

now, using Predictor formula

$$y_1 = y_0 + \frac{4h}{3} (2y'_0)$$

Q: $y' = 2t + 3y$ then find next two derivatives w.r.t t or y .

Answer:

w.r.t t 2nd derivative $Y'' = 2$ 3rd derivative $Y''' = 0$

w.r.t y. 2nd derivative $Y'' = 3$ 3rd derivative $Y''' = 0$

Q: Make backward difference table from;

X	0	3	4
Y	4	7	9

Q:Solve by Gauss-elimination method:

1 0 8/3 x 8

0 -1 1/3 y = 0

0 0 -1/3 z 1

Q: Find y by Euler's method when $x=1$, $dy/dx=1/x+y$, $y(0)=1$, $h=1$

Answer:

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 1 + 1 \left(\frac{1}{x_0 + y_0} \right) = 1 + 1 \left(\frac{1}{1+1} \right) = 1 + 0.5 = 1.5$$

Q: Evaluate the integral by Simpson's 1/3 rule , $\int_0^{\frac{\pi}{4}} (\sin x + 1) dx$ taking interval of $\frac{\pi}{4}$.

Answer:

X	0	$\frac{\pi}{4}$
Y=(sin x + 1)	1	1.7071

$$\int_3^5 (\sin x + 1) dx = \frac{h}{3} (y_0 + y_1)$$

$$\int_3^5 (\sin x + 1) dx = \frac{\pi}{3} (1 + 1.7071)$$

$$\int_3^5 (\sin x + 1) dx = \frac{3.14}{12} (2.7071) = \frac{8.500294}{12} = 0.7083$$

2012 paper1:

Q.1. State the sufficient condition of convergence of the iterative solution to the exact solution? 2m

Answer: page 72

A sufficient condition for convergence of the iterative solution to the exact solution is

$|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|, i=1, 2, \dots, n$ When this condition (diagonal dominance) is true, Jacobi's method

converges

Q.2. Evaluate the integral $\int_3^5 (\log x + 1) dx$ Using trapezoidal rule

Take h=1.....2m

Answer:

X	3	4	5
Y= logx+1	0.6020	0.6989	0.7781

Using trapezoidal rule, taking h=1

$$I = \frac{h}{2} (y_0 + y_3 + 2y_1)$$

$$I = \frac{1}{2} (0.6020 + 0.7781 + 2(0.6989))$$

$$I = \frac{1}{2} (1.3801 + 1.3978)$$

$$I = \frac{2.7779}{2} = 1.38895$$

Q3. Write the formula for finding the value of k₁ in the fourth order R-K Method. 2m

Answer: page 205

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

Q4. Write Adam-Moulton's Predictor formula for finding the solution of a differential equation. 2m

Answer: page 223

$$P: y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Q5. Obtain numerically the solution of $y' = x^2 + 2x + y^2$, $y(0) = 1$ Using Euler's method to find y at $x=1$, $h=1$ 3m

Answer:

$$Y_1 = y_0 + hf(x_0, y_0)$$

$$Y_1 = 1 + (1)(1+1)$$

$$Y_1 = 1 + 2 = 3$$

Q6. If $f(2) = -2.6146$ and $f(3) = 4.7610$, then find the first approximation using the Regula-Falsi method? 3m

Answer:

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$x_3 = 3 - \frac{3 - 2}{4.7610 - (-2.6146)} (4.7610)$$

$$x_3 = 3 - \frac{4.7610}{7.3756}$$

$$x_3 = 3 - \frac{4.7610}{7.3756} = 3 - 0.6455 = 2.3545$$

Q7. If in a solving a given differential equations

$$y_0 = 1, y_1' = 1.1, y_2' = 1.2, y_3' = 1.3, h = 1$$

then find y_4 by Milne Predictor formula

.....3m

Answer:

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$P: y_4 = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3)$$

$$P: y_4 = 1 + \frac{4(1)}{3}(2(1.1) - 1.2 + 2(1.3))$$

$$P: y_4 = 1 + \frac{4}{3}(2.2 - 1.2 + 2.6)$$

$$P: y_4 = 1 + \frac{4}{3}(3.6)$$

$$P: y_4 = 1 + \frac{14.4}{3}$$

$$P: y_4 = \frac{3+14.4}{3} = \frac{17.4}{3} = 5.8$$

Q8. Find F(h), using Richardson's extrapolation limit, while finding

$y'(0.01)$ to the function $y = \frac{1}{x}$ with $h = 0.005$ 3m

Answer:

$$F(h) = \frac{y(x+h) - y(x-h)}{2h}$$

$$F(h) = \frac{\frac{1}{0.01+0.005} - \frac{1}{0.01-0.005}}{2(0.005)}$$

$$F(h) = \frac{\frac{1}{0.015} - \frac{1}{0.005}}{0.01}$$

$$F(h) = \frac{66.66 - 200}{0.01}$$

$$F(h) = \frac{-133.34}{0.01} = -13334$$

Q.9

Evaluate the integral

$$\int_0^3 (x^2 + x) dx$$

using Simpson's $\frac{3}{8}$ rule take $h=1$ 5m

Answer:

X	0	1	2	3
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$Y=(x^2+x)$	0	2	6	12
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$$\int_0^3 (x^2 + x)dx = \frac{3}{8} h [y_0 + y_3 + 3(y_1 + y_2)]$$

$$\int_0^3 (x^2 + x)dx = \frac{3(1)}{8} [0 + 12 + 3(2 + 6)]$$

$$\int_0^3 (x^2 + x)dx = \frac{3}{8} [12 + 24] = \frac{3}{8} (36) = \frac{108}{8} = 13.5$$

Q10. Construct a backward difference table from the following values of x and y.....5m

x	-1	0	1	2	3
Y=f(x)	10	2	10	62	80

Answer:

x	Y=f(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
-1	10				
0	2	-8			
1	10	8	16		
2	62	52	44	28	
3	80	18	-34	-78	-106

Q11. From the following table of values, construct forward difference table.....5m

X	1.00	1.05	1.10	1.15	1.20
y	1.000	1.0247	1.0488	1.0724	1.0954

Answer:

x	Y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.00	1.000				
1.05	1.0247	0.0247			
1.10	1.0488	0.0241	0.0006		
1.15	1.0724	0.0236	-0.0005	-0.0011	
1.20	1.0954	0.023	-0.0006	-0.0001	0.0000

Q12. Use Runge-Kutta Method of order four to find the values of $k_1 k_2 k_3 k_4$ for the initial value problem

$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$ taking $h = 0.1$ 5m

Answer:

$$\text{here } f(x, y) = \frac{1}{2}(2x^3 + y), \quad h = 0.1, \quad x_1 = 1, \quad y_1 = 2$$

$$K_1 = hf(x_1, y_1) = 0.1 \left(\frac{1}{2} (2(1^3 + 2)) \right)$$

$$K_1 = hf(x_1, y_1) = 0.1 \left(\frac{6}{2} \right) = \frac{0.6}{2} = 0.3$$

$$K_2 = hf \left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2} \right) = 0.1 \left(1 + \frac{0.1}{2}, 2 + \frac{0.3}{2} \right)$$

$$K_2 = hf \left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2} \right) = 0.1(1.05, 2 + 0.15)$$

$$K_2 = hf \left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2} \right) = 0.1(1.05, 2.15) = 0.1(1.05 + 2.15) = 0.32$$

$$K_3 = hf \left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2} \right) = 0.1 \left(1 + \frac{0.1}{2}, 2 + \frac{0.32}{2} \right)$$

$$K_3 = hf \left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2} \right) = 0.1(1.05, 2 + 0.16)$$

$$K_3 = hf \left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2} \right) = 0.1(1.05, 2.16) = 0.1(1.05 + 2.16) = 0.321$$

$$K_4 = hf(x_1 + h, y_1 + k_3) = 0.1(1 + 0.1, 2 + 0.321)$$

$$K_4 = hf(x_1 + h, y_1 + k_3) = 0.1(1.1, 2.321) = 0.1(0.1 + 2.21575) = 0.3421$$

Now,

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_2 = 2 + \frac{1}{6}(0.3 + 2(0.32) + 2(0.321) + 0.3421)$$

$$y_2 = 2 + \frac{1}{6}(0.3 + 0.64 + 0.642 + 0.3421) = 2 + \frac{1.9241}{6} = 2 + 0.320683 = 2.320683$$

2012 Paper2

Q1. Obtain numerically the solution of $y' = x + y^2, y(0) = 2$ Using Euler's method to find y at $x=1, h=1....2m$

Answer:

$$h = 1, x = 1 \text{ and } y(0) = 2$$

$$y = y_0 + hf(x_0, y_0)$$

$$y = 2 + 1(1 + 2)$$

$$y = 2 + 3 = 5$$

Q2. If $f(0)=3$ and $f(1)=9$, then the next approximate value of the function using secant method....2m

Answer:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(0)9 - (1)3}{9 - 3}$$

$$x_2 = \frac{0 - 3}{9 - 3} = \frac{-3}{6} = -0.5$$

Q3. Write a backward difference formula of $D^2 f(x)$ 2m

Answer:

$$y_n'' = D^2 y_n = \frac{1}{h^2} \left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \frac{5}{6} \nabla^5 y_n + \dots \right)$$

Q4. Write a formula for Simpson's 1/3 rule.....2m

Answer:

$$\int_{x_0}^{x_{2N}} f(x) dx = \frac{h}{3} [y_0 + 4(y_1 + y_3 + \dots + y_{2N-1}) + 2(y_2 + y_4 + \dots + y_{2N-2}) + y_{2N}] + \text{Error term}$$

Q5.

Evaluate the integral

$$\int_3^5 (\log x + x) dx$$

using Simpson's $\frac{3}{8}$ rule

take $h = 1$

.....3m

Answer:

X	3	4	5
Y=(log x + x)	3.4771	4.6020	5.6989

$$\int_3^5 (\log x + x) dx = \frac{3}{8} h (y_0 + y_2 + 3y_1)$$

$$\int_3^5 (\log x + x) dx = \frac{3}{8} (1) (3.4771 + 5.6989 + 3(4.6020))$$

$$\int_3^5 (\log x + x) dx = \frac{3}{8} (9.176 + 13.806)$$

$$\int_3^5 (\log x + x) dx = \frac{3}{8} (22.982) = \frac{68.946}{8} = 8.61825$$

Q6. Write the two steps of solving the linear equations using Gaussian Elimination method....3m

Answer:

In this method, the solution to the system of equations is obtained in two stages.

i) the given system of equations is reduced to an equivalent upper triangular form using elementary transformations

ii) The upper triangular system is solved using back substitution procedure

Q.7

Use Runge – Kutta Method of order four to find the values of

k_1 and k_2 for the initial value problem

$$y' = yx^2, y(0) = 1, h = 0.1$$

....3m

Answer:

$$f(x, y) = yx^2, y(0) = 1 \text{ and } h = 0.1, x_0 = 0$$

$$k_1 = hf(x_0, y_0) = 0.1(1(0^2)) = 0$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.1\left(0 + \frac{0.1}{2}, 1 + \frac{0}{2}\right) = 0.1(0.05, 1) = 0.1(0.05 + 1) = 0.105$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2)$$

$$y_1 = 1 + \frac{1}{6}(0 + 2(0.105)) = 1 + \frac{0.21}{6} = 1.035$$

Q.8

Evaluate the integral

$$\int_0^3 (x^2 + x) dx$$

Using Trapezoidal rule

take $h = 1$

.....3m

Answer:

x	0	1	2	3
Y=(x ² + x)	0	2	6	12

$$\int_0^3 (x^2 + x) dx = \frac{h}{2}(y_0 + y_3 + 2(y_1 + y_2))$$

$$\int_0^3 (x^2 + x) dx = \frac{1}{2}(0 + 12 + 2(2 + 6))$$

$$\int_0^3 (x^2 + x) dx = \frac{1}{2}(28)$$

$$\int_0^3 (x^2 + x) dx = \frac{28}{2} = 14$$

Q.9

Evaluate the integral

$$\int_0^3 (x+1)dx$$

Using Simpson's $\frac{3}{8}$ rule 5m

Answer:

x	0	1	2	3
Y=(x+1)	1	2	3	4

$$\int_0^3 (x+1)dx = \frac{3}{8}h(y_0 + y_3 + 3(y_1 + y_2))$$

$$h = \frac{b-a}{3n} = \frac{3-0}{3(3)} = \frac{3}{9} = 0.3$$

$$\int_0^3 (x^2 + x)dx = \frac{3}{8}(0.3)(1 + 4 + 3(2 + 3))$$

$$\int_0^3 (x^2 + x)dx = \frac{0.9}{8}(5 + 15)$$

$$\int_0^3 (x^2 + x)dx = \frac{0.9}{8}(20) = \frac{18}{8} = 2.25$$

Q10. Find the dominant Eigen-value and the corresponding eigenvector of the matrix

$$\begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \text{ by Power Method with vector } v^0 = (1, 1, 1)^T \text{ as the initial vector}$$

.....5m

Answer:

vector $v^0 = (1,1,1)^T$ as the initial vector. Then, compute first iteration

$$u^1 = [A]v^0 = \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 8+1+2 \\ 0+10-1 \\ 6+2+15 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ 23 \end{pmatrix}$$

$$u^1 = 23 \begin{pmatrix} \frac{11}{23} \\ \frac{9}{23} \\ 1 \end{pmatrix} = q_1 v^1$$

second iteration

$$u^2 = [A]v^1 = \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{pmatrix} \frac{11}{23} \\ \frac{9}{23} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{88}{23} + \frac{9}{23} + 2 = \frac{143}{23} \\ 0 + \frac{90}{23} - 1 = \frac{67}{23} \\ \frac{66}{23} + \frac{18}{23} + 15 = \frac{429}{23} \end{pmatrix}$$

$$u^2 = 23 \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = q_2 v^2$$

third iteration

$$u^3 = [A]v^2 = \begin{bmatrix} 8 & 1 & 2 \\ 0 & 10 & -1 \\ 6 & 2 & 15 \end{bmatrix} \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = \begin{bmatrix} 8+1+2 \\ 0+10-1 \\ 6+2+15 \end{bmatrix} \begin{bmatrix} 143 \\ 67 \\ 429 \end{bmatrix} = \begin{bmatrix} 1573 \\ 603 \\ 9867 \end{bmatrix}$$

$$u^3 = 9867 \begin{bmatrix} \frac{1573}{9867} = 0.0159 \\ \frac{603}{9867} = 0.0611 \\ 1.0 \end{bmatrix} = 9867 \begin{bmatrix} 0.0159 \\ 0.0611 \\ 1.0 \end{bmatrix} = q_3 v^2$$

The largest eigen-value and the corresponding eigenvector as accurate to two decimals are

$$\lambda = 9867 \text{ and } (X) = \begin{bmatrix} 0.01 \\ 0.06 \\ 1.0 \end{bmatrix}$$

Q11. From the following table of values, construct backward difference table....5m

X	0.1	0.2	0.3	0.4
F(x)	1.10517	1.22140	1.34986	1.49182

Answer:

x	F(x)	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
0.1	1.10517			
0.2	1.22140	0.11623		
0.3	1.34986	0.12846	0.01223	
0.4	1.49182	0.14196	0.0135	0.00127

Q12. Evaluate the integral

$$\int_0^4 (x^2 + 1) dx \text{ Using Simpson's } \frac{1}{3} \text{ rule}$$

take $h = 1$

....5m

Answer:

x	0	1	2	3	4
$Y = (x^2 + 1)$	1	2	5	10	17

$$\int_0^4 (x^2 + 1) dx = \frac{h}{3} (y_0 + y_4 + 4(y_1 + y_3) + 2(y_2))$$

$$\int_0^4 (x^2 + 1) dx = \frac{1}{3} (1 + 17 + 4(2 + 10) + 2(5))$$

$$\int_0^4 (x^2 + 1) dx = \frac{1}{3} (18 + 48 + 10)$$

$$\int_0^4 (x^2 + 1) dx = \frac{1}{3} (76) = \frac{76}{3} = 25.33$$

2 Marks Questions:

Q. Evaluate the integral

$$\int_0^{\pi/2} \cos 2x dx \text{ Using Simpson's } \frac{1}{3} \text{ rule. Where } h = \pi/4$$

Answer:

x	0	1	2	3	4
$Y = \cos 2x$	0	0.9993	0.9975	0.9945	0.9902

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{h}{3} (y_0 + y_4 + 4(y_1 + y_3) + 2(y_2))$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{\frac{\pi}{2}}{3} (0 + 0.9902 + 4(0.9993 + 0.9945) + 2(0.9975))$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{3.14}{12} (0.9902 + 7.9752 + 1.995)$$

$$\int_0^{\frac{\pi}{2}} \cos 2x dx = \frac{3.14}{12} (10.9604) = \frac{34.415656}{12} = 2.8679$$

Q. Difference between Jacobi's Method and Gauss Seidel Method.

Answer:

The **Jacobi method** is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal. Each diagonal element is solved for, and an approximate value plugged in. The process is then iterated until it converges. This algorithm is a stripped-down version of the Jacobi transformation method of matrix diagonalization.

The Gauss-Seidel Method

With the Jacobi method, the values of obtained in the t th iteration remain unchanged until the entire t th iteration has been calculated. With the Gauss-Seidel method, we use the new values as soon as they are known. For example, once we have computed from the first equation, its value is then used in the second equation to obtain the new and so on.

Q. if $y' = t + 2y$, then find next two derivation in term of t and/or y .

Answer:

w.r.t t 2nd dravative $t'' = 1$ 3rd dravative $t''' = 0$

w.r.t y . 2nd dravative $Y'' = 2$ 3rd dravative $Y''' = 0$

3 Marks Questions:

Q. Evaluate the integral

$$\int_{\pi/2}^{\pi} \sin x dx \text{ Using Simpson's 3/8 rule. Where } h = \pi/4$$

Answer:

X	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Y=sinx	1.0	0.7071	0.0

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3h}{8} (y_0 + y_2 + 3(y_1))$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\left(\frac{\pi}{4}\right)}{8} (1.0 + 0.0 + 3(0.7071))$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\pi}{8} (1.0 + 2.1213)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{2.355}{8} (3.1213) = \frac{7.3506615}{8} = 0.9188$$

Q. Use Ruge-Kutta method of order four to find value of k_1 and k_2 for initial value problem. $y' = 1 + xy$, $y(0) = 2$, $h = 0.2$

Answer:

here $f(x, y) = 1 + xy$, $h = 0.2$, $x_0 = 0$, $y_0 = 2$

$$K_1 = hf(x_0, y_0) = 0.2(1 + (0)(2))$$

$$K_1 = hf(x_0, y_0) = 0.2(1 + 0) = 0.2(1) = 0.2$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2\left(0 + \frac{0.2}{2}, 2 + \frac{0.2}{2}\right)$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2(0.1, 2 + 0.1)$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2(0.1, 2.1) = 0.2(0.1 + 2.1) = 0.44$$

Now,

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2)$$

$$y_1 = 2 + \frac{1}{6}(0.2 + 2(0.44))$$

$$y_1 = 2 + \frac{1}{6}(0.2 + 0.88) = 2 + \frac{1.08}{6} = 2 + 0.18 = 2.18$$

Q. Find the residuals by relation method

$$6x_1 - 3x_2 + x_3 = 11$$

$$2x_1 + x_2 - 8x_3 = -15$$

$$x_1 - 7x_2 + x_3 = 10$$

With starting vector (0,0,0)

Answer:

Re-arrange the equation:

$$6x_1 - 3x_2 + x_3 = 11$$

$$x_1 - 7x_2 + x_3 = 10$$

$$2x_1 + x_2 - 8x_3 = -15$$

Thus, we get

$$R_1 = 11 - 6x_1 + 3x_2 - x_3 = 11 - 6(0) + 3(0) - (0) = 11$$

$$R_2 = 10 - x_1 + 7x_2 - x_3 = 10 - (0) + 7(0) - (0) = 10$$

$$R_3 = -15 - 2x_1 - 3x_2 + 8x_3 = -15 - 2(0) - 3(0) + 8(0) = -15$$

The largest residual in magnitude is R_3

$$Dx_3 = -\frac{R_3}{a_{33}} = -\frac{-15}{8} = 1.875$$

Similarly, we find the new residuals of large magnitude and relax it to zero, and so on.

5 Marks Questions:

Q. Evaluate the integral

$$\int_0^3 (x^2 + 1)dx \text{ Using Simpson's 3/8 rule. Where } h = 1$$

Answer:

x	0	1	2	3
$Y = (x^2 + 1)$	1	2	5	10

$$\int_0^3 (x^2 + 1)dx = \frac{3}{8}h(y_0 + y_3 + 3(y_1 + y_2))$$

$$\int_0^3 (x^2 + 1)dx = \frac{3(1)}{8}(1 + 10 + 3(2 + 5))$$

$$\int_0^3 (x^2 + 1)dx = \frac{3}{8}(11 + 21)$$

$$\int_0^3 (x^2 + 1)dx = \frac{3}{8}(32) = \frac{96}{8} = 12$$

Q. Solve by Gauss Seidel iterative method to 3 decimal place up to Two iterations

$$8x - y - 2 = 8$$

$$x - 7y + 2z = -4$$

$$2x + y + 9z = 12$$

Answer:

The system is diagonally dominant

$$x = \frac{1}{8}[8 + y + 2z]$$

$$y = -\frac{1}{7}[-4 - x - 2z]$$

$$z = \frac{1}{9}[12 - 2x - y]$$

initial approximantion is (0,0,0)

first iteration

$$x_1 = \frac{1}{8}[8 + 0 + 2(0)] = \frac{8}{8} = 1$$

$$y_1 = -\frac{1}{7}[-4 - 1 - 2(0)] = -\frac{-5}{7} = 0.71$$

$$z_1 = \frac{1}{9}[12 - 2(1) - 0.71] = \frac{9.29}{9} = 1.032$$

second iteration

$$x_2 = \frac{1}{8}[8 + 0.71 + 2(1.032)] = \frac{1}{8}[8 + 0.71 + 2.064] = \frac{10.774}{8} = 1.34675$$

$$y_2 = -\frac{1}{7}[-4 - 1.34675 - 2(1.032)] = -\frac{1}{7}[-4 - 1.34675 - 2.064] = -\frac{-7.41075}{7} = 1.0586$$

$$z_2 = \frac{1}{9}[12 - 2(1.34675) - 1.0586] = \frac{1}{9}[12 - 2.6935 - 1.0586] = \frac{8.2479}{9} = 0.91643$$

Q. Evaluate the integral

$$\int_0^4 (x^2 + x + 2)dx \text{ using Simpson's 1/3 rule. Where } h=1$$

Answer:

x	0	1	2	3	4
Y=(x ² + x+2)	2	4	8	14	22

$$\int_0^4 (x^2 + x + 2)dx = \frac{h}{3} (y_0 + y_4 + 4(y_1 + y_3) + 2(y_2))$$

$$\int_0^4 (x^2 + x + 2)dx = \frac{1}{3} (2 + 22 + 4(4 + 14) + 2(8))$$

$$\int_0^4 (x^2 + x + 2)dx = \frac{1}{3} (24 + 72 + 16)$$

$$\int_0^4 (x^2 + x + 2)dx = \frac{1}{3} (112) = \frac{112}{3} = 37.33$$

Q. Find the 2nd derivative of f(x) at x=0.3 using three point equation

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	0.125	0.352	0.652	0.756	0.812	0.924

Answer:

$$f''(0.3) = \frac{f(x_3) - 2f(x_2) + f(x_1)}{h^2 = 0.01}$$

$$f''(0.3) = \frac{f(0.4) - 2f(0.3) + f(0.2)}{h^2 = 0.01}$$

$$f''(0.3) = \frac{0.756 - 2(0.652) + 0.352}{h^2 = 0.01}$$

$$f''(0.3) = \frac{0.756 - 1.304 + 0.352}{h^2 = 0.01} = -\frac{0.196}{0.01} = -19.6$$

Final 2013:

Q. Find Newton's forward difference table from the following data

X	0.0	0.1	0.2	0.3	0.4
---	-----	-----	-----	-----	-----

F(x)	1	0.9048	0.8187	0.7408	0.6703
-------------	----------	---------------	---------------	---------------	---------------

...5m

Answer:

X	Y=f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.0	1				
0.1	0.9048	-0.0952			
0.2	0.8187	-0.0861	0.091		
0.3	0.7408	-0.0779	0.082	-0.009	
0.4	0.6703	-0.0705	0.074	-0.008	0.001

$$p = \frac{x - x_0}{h} = \frac{x - 0.0}{0.1} = \frac{x}{0.1} - \frac{0.0}{0.1} = \frac{x}{0.1} - 0 = \frac{x}{0.1}$$

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_1 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_2$$

$$x_0 = 0.0, y_0 = 1, \Delta y_0 = -0.0952, \Delta^2 y = 0.091, \Delta^3 y = -0.009, \Delta^4 y = 0.001$$

$$y = 1 + \left(\frac{x}{0.1}\right)(-0.0952) + \frac{\frac{x}{0.1}\left(\frac{x}{0.1} - 1\right)}{2}(0.091) + \frac{\frac{x}{0.1}\left(\frac{x}{0.1} - 1\right)\left(\frac{x}{0.1} - 2\right)}{6}(-0.009) + \frac{\frac{x}{0.1}\left(\frac{x}{0.1} - 1\right)\left(\frac{x}{0.1} - 2\right)\left(\frac{x}{0.1} - 3\right)}{24}(0.001)$$

this is required Newton's interpolating polynomial.

Q5.

Evaluate the integral

$$\int_3^5 (\log x + 2) dx \text{ using Simpson's } \frac{3}{8} \text{ rule. take } h = 1$$

.....3m

Answer:

X	3	4	5
Y=(log x+ 2)	2.4771	2.6020	2.6989

$$\int_3^5 (\log x + 2) dx = \frac{3}{8} h (y_0 + y_2 + 3y_1)$$

$$\int_3^5 (\log x + 2) dx = \frac{3}{8} (1) (2.4771 + 2.6989 + 3(2.6020))$$

$$\int_3^5 (\log x + 2) dx = \frac{3}{8} (5.176 + 7.806)$$

$$\int_3^5 (\log x + 2) dx = \frac{3}{8} (12.982) = \frac{38.946}{8} = 4.86825$$

Q5.

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx \text{ using Simpson's } \frac{3}{8} \text{ rule. take } h = \frac{\pi}{4}$$

.....2m

Answer:

X	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
Y=(cos x + 2)	3	2.7071	2

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx = \frac{3h}{8} (y_0 + y_2 + 3y_1)$$

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx = \frac{3\left(\frac{\pi}{4}\right)}{8} (3 + 2 + 3(2.7071))$$

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx = \frac{3\pi}{8} (5 + 8.1213)$$

$$\int_0^{\frac{\pi}{2}} (\cos x + 2) dx = \frac{9.42}{2} (13.1213) = \frac{123.602646}{2} = 61.801323$$

Q. if $F(h) = 256.2354$ and $F\left(\frac{h}{2}\right) = 257.1379$, then find $F_1\left(\frac{h}{2}\right)$ using Richardson's

extrapolation limit....2m

Answer:

$$F_1\left(\frac{h}{2}\right) = \frac{4f\left(\frac{h}{2}\right) - f\left(\frac{h}{2}\right)}{4^1 - 1}$$

$$F_1\left(\frac{h}{2}\right) = \frac{4(257.1379) - (256.2354)}{3}$$

$$F_1\left(\frac{h}{2}\right) = \frac{1028.5516 - 256.2354}{3}$$

$$F_1\left(\frac{h}{2}\right) = \frac{772.3162}{3} = 257.43873$$

7. if in solving given differential equation, $y' = 0.1$, $y'_1=1.1$, $y'_2=1.2$, $y'_3=1.3$, $h=1$, $y_3=2$ then find the y_4 by Adam–moulton's predictor formula .

Answer:

$$P: y_{n+1} = y_n + \frac{h}{24} (25y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

$$P: y_{3+1=4} = y_3 + \frac{1}{24} (25y'_3 - 59y'_{3-1=2} + 37y'_{3-2=1} - 9y'_{3-3=0})$$

$$P: y_4 = y_3 + \frac{1}{24} (25y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$

$$P: y_4 = 2 + \frac{1}{24} (25(1.3) - 59(1.2) + 37(1.1) - 9(0.1))$$

$$P: y_4 = 2 + \frac{1}{24} (32.5 - 70.8 + 40.7 - 0.9)$$

$$P: y_4 = 2 + \frac{1}{24} (1.5) = 2 + \frac{1.5}{24} = 2 + 0.0625 = 2.0625$$

12. using Adam-moulton's predictor formula find the predicted value of $f(0.4)$ from ordinary differential equation $y'=1+2xy$; $y(0)=0$; $h=0.1$

x	0	0.1	0.2	0.3
y	0	.1007	.2056	.3199

Answer:

$$h = 0.1$$

$$f(x, y) = 1 + 2xy$$

$$y_0' = 1 + 2x_0y_0 = 1 + 2(0)(0) = 1$$

$$y_1' = 1 + 2x_1y_1 = 1 + 2(0.1)(0.1007) = 1.02014$$

$$y_2' = 1 + 2x_2y_2 = 1 + 2(0.2)(0.2056) = 1.08224$$

$$y_3' = 1 + 2x_3y_3 = 1 + 2(0.3)(0.3199) = 1.19194$$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24}(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(55(1.19194) - 59(1.08224) + 37(1.02014) - 9(1))$$

$$y_4 = 0.446773833$$

Computing y'_4 for the Corrector Formula;

$$y'_4 = 1 + 2x_4y_4 = 1 + 2(0.4)(0.446773833)$$

$$y'_4 = 1.3574190664$$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$

$$y_4 = y_3 + \frac{h}{24}(9y'_4 + 19y'_3 - 5y'_2 + y'_1)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(9(1.3574190664) + 19(1.19194) - 5(1.08224) + 1.02014)$$

$$y_4 = 0.446869048$$

Q. using Adam-moulton's predictor formula find the predicted value of $f(0.4)$ from ordinary differential equation $y'=1+xy$; $y(0)=0$; $h=0.1$

x	0	0.1	0.2	0.3
y	0	.1007	.2056	.3199

Answer:

$$h = 0.1$$

$$f(x, y) = 1 + xy$$

$$y_0' = 1 + x_0 y_0 = 1 + (0)(0) = 1$$

$$y_1' = 1 + x_1 y_1 = 1 + (0.1)(0.1007) = 1.2007$$

$$y_2' = 1 + x_2 y_2 = 1 + (0.2)(0.2056) = 1.04112$$

$$y_3' = 1 + x_3 y_3 = 1 + (0.3)(0.3199) = 1.09597$$

Now, Using Adam's P-C Pair Formula:-

$$y_{n+1} = y_n + \frac{h}{24} (55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Putting the values;

$$y_4 = y_3 + \frac{h}{24} (55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$

$$y_4 = 0.3199 + \frac{0.1}{24} (55(1.09597) - 59(1.04112) + 37(1.2007) - 9(1))$$

$$y_4 = 0.3199 + 0.00416(60.27835 - 61.42608 + 44.4259 - 9) = 0.3199 + 34.27817 = 34.59807$$

Computing y'_4 for the Corrector Formula;

$$y'_4 = 1 + x_4 y_4 = 1 + (0.4)(34.59807)$$

$$y'_4 = 14.839228$$

Now Applying the Corrector Formula;

$$y_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$

$$y_4 = y_3 + \frac{h}{24}(9y'_4 + 19y'_3 - 5y'_2 + y'_1)$$

$$y_4 = 0.3199 + \frac{0.1}{24}(9(14.839228) + 19(1.09597) - 5(1.04112) + 1.2007)$$

$$y_4 = 0.3199 + 0.00416(133.553052 + 20.82343 - 5.2056 + 1.2007)$$

$$y_4 = 0.3199 + 0.625545781 = 0.945445781$$

Q.2. Evaluate the integral $\int_0^{\frac{\pi}{2}} \cos x dx$ Using trapezoidal rule

Take $h = \frac{\pi}{4} \dots\dots 2m$

Answer:

X	0	$\frac{\pi}{4}$
Y= cosx	1	0.7071

Using trapezoidal rule, taking $h = \frac{\pi}{4}$

$$I = \frac{h}{2}(y_0 + y_1)$$

$$I = \frac{\frac{\pi}{4}}{2}(1 + 0.7071)$$

$$I = \frac{3.14}{2}(1.7071)$$

$$I = \frac{5.360294}{2} = 2.680147$$

Q. obtain numerically the solution of

$y' = x + y^2$, $y(0) = 2$ using Euler's method to find y at $x = 1$, $h = 1$.

Answer:

Here: $y(0) = 2$, $x = 1$, $h = 1$

$$y = y_0 + hf(x_0, y_0)$$

$$y = 2 + 1(1 + 4) = 2 + 5 = 7$$

Q5.

Evaluate the integral $\int_{\frac{\pi}{2}}^{\pi} \sin x dx$ using Simpson's $\frac{3}{8}$ rule. take $h = \frac{\pi}{4}$

.....2m

Answer:

X	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Y=sinx	1	0.7071	0

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3h}{8}(y_0 + y_2 + 3y_1)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\left(\frac{\pi}{4}\right)}{8}(1 + 0 + 3(0.7071))$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3\pi}{8}(1 + 2.1213)$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{9.42}{2}(3.1213) = \frac{29.402646}{2} = 14.701323$$

Q. Prove that $E^{\frac{1}{2}} = \mu + \frac{\delta}{2}$

Answer:

$$E^{\frac{1}{2}} = \mu + \frac{\delta}{2}$$

$$E^{\frac{1}{2}} = \frac{1}{2} \left(E^{\frac{1}{2}} + E^{-\frac{1}{2}} \right) + \frac{E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2}$$

$$E^{\frac{1}{2}} = \frac{E^{\frac{1}{2}} + E^{-\frac{1}{2}}}{2} + \frac{E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2}$$

$$E^{\frac{1}{2}} = \frac{E^{\frac{1}{2}} + E^{-\frac{1}{2}} + E^{\frac{1}{2}} - E^{-\frac{1}{2}}}{2} = \frac{2E^{\frac{1}{2}}}{2} = E^{\frac{1}{2}}$$

Q.2. Evaluate the integral $\int_0^2 (x^2 - 3x + 4) dx$ **Using trapezoidal rule**

Take h=1.....2m

Answer:

X	0	1	2
Y= (x ² -3x+4)	4	2	2

Using trapezoidal rule, taking h=1

$$I = \frac{h}{2} (y_0 + y_2 + 2(y_1))$$

$$I = \frac{1}{2} (4 + 2 + 2(2))$$

$$I = \frac{1}{2} (6 + 4)$$

$$I = \frac{10}{2} = 5$$

Q12. Evaluate the integral

$$\int_0^4 (x^2 + 2) dx \text{ Using Simpson's } \frac{1}{3} \text{ rule}$$

take h=1

....5m

Answer:

x	0	1	2	3	4
Y=(x ² + 2)	2	3	6	11	18

$$\int_0^4 (x^2 + 2)dx = \frac{h}{3} (y_0 + y_4 + 4(y_1 + y_3) + 2(y_2))$$

$$\int_0^4 (x^2 + 2)dx = \frac{1}{3} (2 + 18 + 4(3 + 11) + 2(6))$$

$$\int_0^4 (x^2 + 2)dx = \frac{1}{3} (20 + 56 + 12)$$

$$\int_0^4 (x^2 + 2)dx = \frac{1}{3} (88) = \frac{88}{3} = 29.33$$

Q. Evaluate the integral $\int_0^3 x dx$ Using Simpson's 3/8 rule. Take h=1 (5)

Answer:

x	0	1	2	3
Y=x	0	1	2	3

$$\int_0^3 x dx = \frac{3h}{8} (y_0 + y_3 + 3(y_1 + y_2))$$

$$\int_0^3 x dx = \frac{3}{8} (0 + 3 + 3(1 + 2))$$

$$\int_0^3 x dx = \frac{3}{8} (3 + 9)$$

$$\int_0^3 x dx = \frac{3}{8} (12) = \frac{36}{8} = 4.5$$

Final 2014

1- Write three equations for the following system of linear equation for Gauss Seidal Approximation Method (2)

$$83x + 11y - 4z = 95$$

$$7x + 52y + 13z = 104$$

$$3x + 8y + 29z = 71$$

2- Evaluate the integral

$$\int_3^5 (\log x + 2) dx$$

Using trapezoidal rule.

Take h=1 (2)

Answer:

X	3	4	5
Y=(log x+2)	2.4771	2.6020	2.6989

$$\int_3^5 (\log x + 2) dx = \frac{h}{2} (y_0 + y_2 + 2(y_1))$$

$$\int_3^5 (\log x + 2) dx = \frac{1}{2} (2.4771 + 2.6989 + 2(2.6020))$$

$$\int_3^5 (\log x + 2) dx = \frac{1}{2} (2.4771 + 2.6989 + 5.204)$$

$$\int_3^5 (\log x + 2) dx = \frac{1}{2} (10.38) = \frac{10.38}{2} = 5.19$$

3- Write a formula for finding the value of k_2 in fourth order R-K method?(2)

Answer:

$$k_2 = hf \left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2} \right)$$

4- If $y' = t^2 - y$, then find the next two derivatives in terms of t and y . (2)

Answer:

w.r.t t 2nd derivative $t'' = 2t$ 3rd derivative $t''' = 2$

w.r.t y 2nd derivative $Y'' = 1$ 3rd derivative $Y''' = 0$

5- Define intermediate value property? (3)

Answer:

If $f(x)$ is a real valued continuous function in the closed interval $a \leq x \leq b$ if $f(a)$ and $f(b)$ have opposite signs once; that is $f(x)=0$ has at least one root β such that $a \leq \beta \leq b$

Simply

If $f(x) = 0$ is a polynomial equation and if $f(a)$ and $f(b)$ are of different signs, then $f(x)=0$ must have at least one real root between a and b .

Numerical methods for solving either algebraic or transcendental equation are classified into two groups.

6- If in solving a given differential equation,

$y_0 = 1, y'_1 = 1.1, y'_2 = 1.2, y'_3 = 1.3, h = 1$ Then find y_4 by Milne's Predictor formula. (3)

Answer:

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$P: y_4 = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3)$$

$$P: y_4 = 1 + \frac{4(1)}{3}(2(1.1) - 1.2 + 2(1.3))$$

$$P: y_4 = 1 + \frac{4}{3}(2.2 - 1.2 + 2.6)$$

$$P: y_4 = 1 + \frac{4}{3}(3.6)$$

$$P: y_4 = 1 + \frac{14.4}{3}$$

$$P: y_4 = \frac{3+14.4}{3} = \frac{17.4}{3} = 5.8$$

7- Evaluate the integral

$$\int_0^4 x^2 dx$$

Using trapezoidal rule. Take h=1. (3)

Answer:

X	0	1	2	3	4
Y=x ²	0	1	4	9	16

$$\int_3^5 (\log x + 2) dx = \frac{h}{2}(y_0 + y_4 + 2(y_1 + y_2 + y_3))$$

$$\int_3^5 (\log x + 2) dx = \frac{1}{2}(0 + 16 + 2(1 + 4 + 9))$$

$$\int_3^5 (\log x + 2) dx = \frac{1}{2}(0 + 16 + 28)$$

$$\int_3^5 (\log x + 2) dx = \frac{1}{2}(44) = \frac{44}{2} = 22$$

8- Obtain numerically the solution of

$$Y' = x^2 + 2x + y^2, y(6) = 1$$

Using Euler's method to find y at x=1, h=1. (3)

Answer:

$$y(6) = 1, x_6 = 1, h = 1$$

$$y_7 = y_6 + hf(x_6, y_6)$$

$$y_7 = 1 + 1(x^2 + 2x + y^2)$$

$$y_7 = 1 + 1(1 + 2 + 1) = 1 + 4 = 5$$

9- Using predictor formula find the predicted value of $y(0.4)$ from ordinary differential equation

$$Y' = x + y; y(0) = 1, h = 0.1$$

With the help of the following table (5)

X	0	0.1	0.2	0.3
Y	1	0.9097	0.8375	0.7816

Answer:

Here,

$$x_0 = 0, \quad x_1 = 0.1, \quad x_2 = 0.2, \quad x_3 = 0.3, \quad x_4 = 0.4$$

$$y_1' = x_1 - y_1 = 0.1 - 0.9097 = -0.8097$$

$$y_2' = x_2 - y_2 = 0.2 - 0.8375 = -0.6375$$

$$y_3' = x_3 - y_3 = 0.3 - 0.7816 = -0.4816$$

Now, using Predictor Formula

$$y_4 = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4 = 1 + \frac{4 \cdot 0.1}{3}(-1.9451)$$

$$y_4 = 0.740653333$$

Using the predicted value, we shall now compute the corrected value as;

$$y_4 = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$$

Now,

$$y_4' = x_4 - y_4 = 0.4 - 0.740653333 = -0.340653333$$

10- Use the regula-falsi to solve the equation :

$F(x) = e^x - 3x = 0$. (Note only 1 iteration is required upto 4 decimal places is required) (5)

Answer:

$$f(x) = e^x - 3x = 0$$

$$f(1) = e^1 - 3(1) = 2.7182 - 3 = -0.2818$$

$$f(2) = e^2 - 3(2) = 7.3890 - 6 = 1.389$$

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$$

$$x_3 = 2 - \frac{2 - 1}{1.389 - (-0.2818)} (1.389)$$

$$x_3 = 2 - \frac{1.389}{1.6708} = 2 - 0.8313 = 1.1687$$

11- Evaluate the integral

$$\int_0^4 x dx \text{ Using Simpson's 1/3 rule (5)}$$

Answer:

x	0	0.5	1	1.5	2	2.5	3	3.5	4
Y=x	0	0.5	1	1.5	2	2.5	3	3.5	4

$$\int_0^4 x dx = \frac{h}{3} (y_0 + y_8 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6))$$

$$h = \frac{b-a}{2n} = \frac{4-0}{2(4)} = \frac{4}{8} = 0.5$$

$$\int_0^4 x dx = \frac{0.5}{3} (0 + 4 + 4(0.5 + 1.5 + 2.5 + 3.5) + 2(1 + 2 + 3))$$

$$\int_0^4 x dx = \frac{0.5}{3} (4 + 32 + 12)$$

$$\int_0^4 x dx = \frac{0.5}{3} (48) = \frac{24}{3} = 8$$

12- Find $y(1)$ for the following initial value problem $y' = x$, $y(0) = 2$

Using simple Euler's method, taking $h=1$ (5)

Answer:

$$y(0) = 2, x_0 = 0, h = 1$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 2 + 1(0 + 2)$$

$$y_1 = 2 + 2 = 4$$

what is Simpson's 3/8th rule.

Answer: The general formula for Simpson's 3/8th rule is $3h/8[f_0 + 3(f_1 + f_2) + 2f_3 + (3f_4 + f_5) + 2f_6 + \dots + 3(f_{n-2} + f_{n-1}) + f_n]$ Now if we have to calculate the integral by using this rule then we can simply proceed just write first and last value and distribute all the remaining values with prefix 3 and 2. Like you have f_0, f_1, f_2, f_3, f_4 . Then the integral can be calculated as $3h/8[f_0 + f_4 + 3(f_1 + f_2) + 2f_3]$. If we have values like $f_0, f_1, f_2, f_3, f_4, f_5$. Then integral can be calculated as $3h/8[f_0 + 3(f_1 + f_2) + 2f_3 + 3f_4 + f_5]$. Similarly proceeding in this fashion we can calculate the integral in this fashion.

Question: what is classic Runge-Kutta method

Answer: The fourth order Runge-Kutta method is known as the classic formula of classic Runge-Kutta method. $y_{n+1} = y_n + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$ Where $k_1 = hf(x_n, y_n)$ $k_2 = hf(x_n + h/2, y_n + k_1/2)$ $k_3 = hf(x_n + h/2, y_n + k_2/2)$ $k_4 = hf(x_n + h, y_n + k_3)$

Question: What is meant by TOL?

Answer: The TOL means the extent of accuracy which is needed for the solution. If you need the accuracy to two places of decimal then the TOL will be 10^{-2} . Similarly the 10^{-3} means that the accuracy needed to three places of decimal. Suppose you have the root from last iteration 0.8324516 and 0.8324525. If we subtract both and

consider absolute value of the difference 0.0000009 now it can be written as 0.09×10^{-5} so the TOL in this case is 10^{-5} . similarly if we have been provided that you have to for the TOL 10^{-2} you will check in the same way. In the given equation you will solve the equation by any method and will consider some specific TOL and try to go to that TOL. Some time no TOL is provided and you are asked to perform to some specific no of iterations.

Question: what is meant by uniqueness of LU method.

Answer: An invertible (whose inverse exists) matrix can have LU factorization if and only if all its principal minors (the determinant of a smaller matrix in a matrix) are non zero. The factorization is unique if we require that the diagonal of L or U must have 1's. the matrix has a unique LDU factorization under these condition. If the matrix is singular (inverse does not exists) then an LU factorization may still exist, a square matrix of rank (the rank of a matrix in a field is the maximal no of rows or column) k has an LU factorization if the first k principal minors are non zero. These are the conditions for the uniqueness of the LU decomposition.

Question: how the value of h is calculated from equally spaced data.

Answer: Consider the following data x y 1 1.6543 2 1.6984 3 2.4546 4 2.9732 5 3.2564 6 3.8765 Here for $h = 2 - 1 = 3 - 2 = 1$ x y 0.1 1.6543 0.2 1.6984 0.3 2.4546 0.4 2.9732 0.5 3.2564 0.6 3.8765 Here for the calculation of $h = 0.2 - 0.1 = 0.3 - 0.2 = 0.1$ I think so that you may be able to understand.

what is the relation ship between $p=0$ and non zero p in interpolation.
 Answer: $f_p = f_0 + pDf_0 + \frac{1}{2}(p^2 - p)D^2f_0 + \frac{1}{6}(p^3 - 3p^2 + 2p)D^3f_0 + \frac{1}{24}(p^4 - 6p^3 + 11p^2 - 6p)D^4f_0 + \dots$ This is the interpolation formula For the derivative you will have to take the derivative of this formula w.r.t p and you will get $f'_p = \frac{1}{h} \{ Df_0 + \frac{1}{2}(2p - 1)D^2f_0 + \frac{1}{6}(3p^2 - 6p + 2)D^3f_0 + \dots \}$ Now put here $p=0$ $f'_p = \frac{1}{h} \{ Df_0 + \frac{1}{2}(0 - 1)D^2f_0 + \frac{1}{6}(0 - 0 + 2)D^3f_0 + \dots \}$ So the formula becomes $f'_p = \frac{1}{h} \{ Df_0 + \frac{D^2f_0}{2} + \frac{D^3f_0}{3} + \dots \}$ so this is the relation ship between non zero and zero p no when you have to calculate p the you use formula $p = \frac{x - x_0}{h}$ so this is the impact of x.

Question: What is chopping and Rounding off?

Answer: Chopping and rounding are two different techniques used to truncate the terms needed according to your accuracy needs. In chopping you simply use the mentioned number of digits after the decimal and discard all the remaining terms. Explanation $(\frac{1}{3} - \frac{3}{11}) + \frac{3}{20} = (0.333333... - 0.27272727...) + 0.15 = (0.333 - 0.272) + 0.15$ This is the three digit chopping.

Question: When the forward and backward interpolation formulae are used?

Answer: In interpolation if we have at the start then we use the forward difference formula and the formula to calculate p is $\frac{x - x_0}{h}$. If the value of x lies at the end then we use Newton's backward formula and formula to calculate the value of p is $\frac{x - x_n}{h}$. Now I come to your question as in this case the value lies at the end so 6 will be used as

the x_n . This procedure has been followed by the teacher in the lectures. But some authors also use another technique that is if you calculate the value of p and that is negative then the origin is shifted to that value for which the value of p becomes positive. And then according to that origin the values of differences are used and you need not follow this procedure.

Question: What is forward and backward difference operator and the construction of their table.

Answer: For forward $D_{fr} = f_{r+1} - f_r$ $D_{f0} = f_1 - f_0$ In terms of y $D_{y,r+1} = y_{r+1} - y_r$ D stands for the forward difference operator For backward $D_{br} = f_r - f_{r-1}$ $D_{f1} = f_1 - f_0$ In terms of y $D_{y1} = y_1 - y_0$ Here D stands for backwards operator Now the construction of the difference table is based on X Y 1st forward 2nd forward 3rd forward x_1 Y_1 $Y_2 - Y_1 = Dy_0$ x_2 Y_2 $Y_3 - Y_2 = Dy_1$ x_3 Y_3 $Y_4 - Y_3 = Dy_2$ x_4 Y_4 Now consider the construction of table for the backward table X Y 1st forward 2nd forward 3rd forward x_1 Y_1 $Y_2 - Y_1 = Dy_1$ x_2 Y_2 $Y_3 - Y_2 = Dy_2$ x_3 Y_3 $Y_4 - Y_3 = Dy_3$ x_4 Y_4 Dear student this is the main difference in the construction of the forward and backwards difference table when you proceed for forward difference table you get in first difference the value Dy_0 but in the construction of backwards difference table in the first difference you get Dy_1 and in the second difference in the forward difference table you get $D^2 y_0$ and in the backward difference table the first value in the second difference is $D^2 y_1$. I think so you have made it clear.

Question: What is Jacobi's method?

Answer: Jacobi's Method It is an iterative method and in this method we first of all check either the system is diagonally dominant and, if the system is diagonally dominant then we will calculate the value of first variable from first equation in the form of other variables and from the second equation the value of second variable in the form of other variables and so on. We are provided with the initial approximations and these approximations are used in the first iteration to calculate first approximation of all the variables. The approximations calculated in the first iteration are used in the second iteration to calculate the second approximations and so on.

What is the condition that a root will lie in an interval.

Answer: Suppose that you have a function $f(x)$ and an interval $[a,b]$ and you calculate both $f(a)$ and $f(b)$ if $f(a)f(b) < 0$ then there must exist a root for this function in this interval. In simple words $f(a)$ and $f(b)$ must have opposite signs.

Question: How the divided difference table is constructed?

Answer: x y First difference Second difference Third difference 1 0.2 y_0 $y_1 - y_0 / x_1 - x_0 = sy_0$ $sy_1 - sy_0 / x_2 - x_0 = s_2y_0$ $s_2y_0 - s_2y_1 / x_3 - x_0 = s_3y_0$ $s_3y_1 - s_3y_0 / x_4 - x_0 = s_4y_0$ 2 0.69 y_1 $y_2 - y_1 / x_2 - x_1 = sy_1$ $sy_2 - sy_1 / x_3 - x_1 = s_2y_1$ $s_2y_2 - s_2y_1 / x_4 - x_1 = s_3y_1$ 3 0.985 y_2 $y_3 - y_2 / x_3 - x_2 = sy_2$ $sy_3 - sy_2 / x_4 - x_2 = s_2y_2$ 4 1.2365 y_3 $y_4 - y_3 / x_4 - x_3 = sy_3$ 5 2.3651 y_4 This is the complete table showing the central differences . Here s stands for difference

Question: What is Gauss-seidel Method

Answer: It is also an iterative method and in this we also check either the system is diagonally dominant and, if so then we proceed on the same lines as in the Jacobi's method. The difference in both is when we calculate a value of any variable that is instantly replaced by the previous value. If we have initial values like $x_1=0, x_2=0$ and $x_3=0$ in the case of three variables in the first iteration we will use all the three values to calculate x_1 but to calculate the value of x_2 we will use the recent value of x_1 which is calculated in the first iteration.

Question: What is partial and full pivoting?

Answer: Partial and full pivoting, In gauss elimination method when you have any of diagonal element a_{ii} zero it means the solution does not exist to avoid this we change the equation so that a non zero pivot is achieved. Now you have an argument matrix in which the elements in the first column are 1,3,4 respectively here in case of partial pivoting we will replace first element with the last element it is done by replacing the first equation with last equation it is known as partial pivoting. In full pivoting we change rows and columns but that is not implemented manually it is used in computers.

Question: How the initial vector is chosen in power method?

Answer: Choose the initial vector such that the largest element is unity. But no problem with this you will proceed with this vector as it is and will not change it. In the given question the initial vector is provided so you have to proceed with the given vector and will normalize this vector and normalizing this vector you will keep in mind that the greatest value should be unity. An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to unity. Actually this condition is for the normalization of the vectors when you have to normalize the vector you keep in mind that the largest entry in the vector must be 1 and so you take the largest element and divide the remaining values by the greatest value

What is meant by diagonally dominant system?

Answer: A system $a_1x+b_1y+c_1z=d_1$, $a_2x+b_2y+c_2z=d_2$, $a_3x+b_3y+c_3z=d_3$ is said to be diagonally dominant if the following condition holds. $|a_1| \Rightarrow$ (greater or equal) $|b_1|+|c_1|$ $|b_2| \Rightarrow$ (greater or equal) $|a_2|+|c_2|$ $|c_3| \Rightarrow$ (greater or equal) $|a_3|+|b_3|$

Question: State the sufficient condition for the convergence of the system of equation by iterative methods.

Answer: A sufficient condition for convergence of iterative solution to exact solution is $|a_1| \Rightarrow$ (greater or equal) $|b_1|+|c_1|$ $|b_2| \Rightarrow$ (greater or equal) $|a_2|+|c_2|$ $|c_3| \Rightarrow$ (greater or equal) $|a_3|+|b_3|$ For the system $a_1x+b_1y+c_1z=d_1$, $a_2x+b_2y+c_2z=d_2$, $a_3x+b_3y+c_3z=d_3$ Similarly for the system with more variables we can also construct the same condition

Question: The calculation for numerical analysis should be done in degree or radians.
Answer: All the calculation for numerical analysis should be done in radians not in degrees set your calculator in radians mode and suppose the value of $\pi=3.14$.

Question: How we can identify that Newton forward or backwards interpolation formula is to be used.
Answer: If the value at which we have to interpolate is in the start of the table then we will use Newton's forward interpolation formula if it is at the end of the table then we will use the Newton's backward interpolation formula.

Question: What is meant precision and accuracy?
Answer: Precision and accuracy are two terms which are used in numerical calculations by precision we mean that how the values in different iterations agree to each other or how close are the different values in successive iterations. For example you have performed 3 different iterations and result of all the iteration are 1.32514, 1.32516, 31.32518 these three values are very precise as these values agree with each other. Accuracy means the closeness to the actual value. Suppose that you have calculated an answer after some iteration and the answer is 2.718245 and the actual answer is 2.718254 and the answer calculated is very accurate but if this answer is 2.72125 then it is not accurate.

What is meant by iterative methods?
Answer: The methods which need one or more iterations are known as iterative methods like bisection method, Newton raphson method, and many other methods.

Question: What is graphically meant by the root of the equation?
Answer: If the graph of a function $f(x) = 0$ cuts the x-axis at a point a then a is known as the root of the equation.

Question: Q. What is the difference between open and bracketing method?
Answer: In open methods we need only one initial approximation of the root that may be any where lying and if it is not very close then we have to perform more iteration and the example of open method is Newton raphson method. In bracketing method we bracket the root and find that interval in which root lies means we need two initial approximations for the root finding. Bisection method is an example of the bracketing method.

Question: Condition for the existence of solution of the system of equations.
Answer: If the $|A|$ is not equal to zero then the system will have a unique solution if $|A| \neq 0$

then the system will have no solution

Question: Should the system be diagonally dominant for gauss elimination method?

Answer: The system of equation need not to be diagonally dominant for Gauss elimination method and gauss Jordan method for both the direct method it is not necessary for the system to be diagonally dominant .it should be diagonally dominant for iterative methods like jacobie and gauss seidel method.

What is a transcendental equation?

Answer: An equation is said to be a transcendental equation if it has trigonometric, exponential and logarithmic function or combination of all these functions.

Question: How the value of h is calculated in interpolation?

Answer: There are two types of data in the interpolation one is equally spaced and other is unequally spaced in equally spaced data we need to calculate the value of h that is calculated by subtracting any two consecutive values and taking their absolute value.

Question: What is an algebraic equation?

Answer: An algebraic equation is an equation which is purely polynomial in any variable. Supposed $x^2+3x+2=0$, $x^4+3x^2=0$, $y^3+6y^2=0$ all are algebraic equations as these are purely polynomial in x and y variable.

Question: What is Descartes rule of signs?

Answer: The number of positive roots of an algebraic equation $f(x) = 0$ can not exceed the no of changes in signs. Similarly the no of negative roots of negative roots of and algebraic equation can not exceed the no of changes in sign of equation $f(-x) = 0$.

Question: What are direct methods?

Answer: The numerical methods which need no information about the initial approximation are known as direct methods like Graeffe's root squaring method.

What is direct methods of solving equations?

Answer : Those methods which do not require any information about the initial approximation of root to start the solution are known as direct methods. The examples of direct methods are Graeffe root squaring method, Gauss elimination method and Gauss Jordan method. All these methods do not require any type of initial approximation.

Question: What is Iterative method of solving equations?
Answer: These methods require an initial approximation to start. Bisection method, Newton raphson method, secant method, jacobi method are all examples of iterative methods

Question: If an equation is a transcendental, then in which mode the calculations should be done?
Answer: All the calculations in the transcendental equations should be done in the radian mode.

Question: What is the convergence criterion in method of iteration?
Answer: If x be a root of $f(x) = 0$ which is equivalent to I be any interval containing the point x and will converge to the root provided that the initial approximation is chosen in I

Question: When we stop doing iterations when TOL is given?
Answer: Here if TOL is given then we can simply find the value of TOL by subtracting both the consecutive roots and write it in the exponential notation if the required TOL is obtained then we stop.

Answer: What is inherent error and give its example.
It is that quantity of error which is present in the statement of the problem itself, before finding its solution. It arises due to the simplified assumptions made in the mathematical modeling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the problem. e.g. if writing 8375 instead of 8379 in a statement lies in the category of inherent error.

Question: What is local round-off error?
Answer: At the end of computation of a particular problem, the final results in the computer, which is obviously in binary form, should be converted into decimal form-a form understandable to the user-before their print out. Therefore, an additional error is committed at this stage too. This error is called local round-off error.

Question: What is meant by local truncation error?
Answer: Retaining the first few terms of the series up to some fixed terms and truncating the remaining terms arise to local truncation error.

Question: What is transcendental equation and give two examples.
Answer: An equation is said to be transcendental equation if it has logarithmic, trigonometric and exponential function or combination of all these three. For

example it is a transcendental equation as it has an exponential function These all are the examples of transcendental equation.

Question: What is meant by intermediate value property?

Answer: If $f(x)$ is a real valued continuous function in the closed interval if $f(a)$ and $f(b)$ have opposite signs once; that is $f(x)=0$ has at least one root such that Simply If $f(x)=0$ is a polynomial equation and if $f(a)$ and $f(b)$ are of different signs ,then $f(x)=0$ must have at least one real root between a and b .

What is the order of global error in Simpson's $3/8$ rule?

Answer: The global error in Simpson's $3/8$ rule is of the order of $O(h^4)$.

Question: Which equation models the rate of change of any quantity with respect to another?

Answer: An ordinary differential equation models the rate of change of any quantity with respect to another.

Question: By employing which formula, Adam-Moulton P-C method is derived?

Answer: Adam-Moulton P-C method is derived by employing Newton's backward difference interpolation formula.

Question: What are the commonly used number systems in computers?

Answer: Binary Octal Decimal Hexadecimal

Question: If a system has the base M , then how many different symbols are needed to represent an arbitrary number? Also name those symbols.

Answer: M 0, 1, 2, 3, ... , $M-1$

Is the order of global error in Simpson's $1/3$ rule equal to the order of global error in Simpson's $3/8$ rule?

Answer: Yes. The order of global error in Simpson's $1/3$ rule equal to the order of global error in Simpson's $3/8$ rule.

Question: What is the order of global error in Trapezoidal rule?

Answer: The global error in Trapezoidal rule is of the order of $O(h^2)$.

Question: What is the formula for finding the width of the interval?

Answer: Width of the interval, h , is found by the formula $h=(b-a)/n$

Question: What type of region does the double integration give?

Answer: Double integration gives the area of the rectangular region.

Question: Compare the accuracy of Romberg's integration method to trapezoidal and Simpson's rule.

Answer: Romberg's integration method is more accurate to trapezoidal and Simpson's rule
