Telenor 74.4K/s 98% 1:51 pm

MTH621 Real Analysis I

Question No : 1 of 41 Marks: 1 (Budgeted

The sum of a rational number and an irrational number is

page 16

Answer (Please select your correct option)

- may be a rational depends on the numbers.
- always an irrational number.
- always a rational number.

Telenor
177.4K/s 98% 1:52 pm

Identify the identical sequences.

page 38

Answer (Please select your correct option)

- $\left\{\frac{1}{n^2-2}\right\}_3^{\infty}$ and $\left\{\frac{1}{n}\right\}_1^{\infty}$
- $\left\{\frac{1}{n}\right\}_3^{\infty}$ and $\left\{\frac{1}{n}\right\}_1^{\infty}$
- $\left\{\frac{1}{n-2}\right\}_3^{\infty}$ and $\left\{\frac{1}{n^2}\right\}_1^{\infty}$
- $\left\{\frac{1}{n-2}\right\}_3^{\infty}$ and $\left\{\frac{1}{n}\right\}_1^{\infty}$

Telenor

MTH621 Real Analysis I

188.4K/s 98% 1:52 pm

Question No : 3 of 41

$\liminf_{n\to\infty} s_n = -\infty$ if

page 49

Answer (Please select your correct option)

- (s_n) is not bounded above.
- $\lim_{n\to\infty} s_n = 0$.
- (s_n) is not bounded below.
- (s_n) is convergent.

Telenor

111.9K/s 98% 1:52 pm

Question No : 4 of 41 Marks: 1

If (s_n) is bounded and nonincreasing, then

page 41

Answer (Please select your correct option)

- $\lim_{n\to\infty} s_n = \sup(s_n)$.
- $\lim_{n\to\infty} s_n = \inf(s_n)$.
- (s_n) is divergent.

Telenor
268.3K/s 98% 1:52 pm

Question No : 5 of 41

Marks: 1 (Budgeted Time

Which of the following given sets is compact subset of ℝ

Answer (Please select your correct option)

- (1,5]
- {1,2,4}
- [0,1]
- (2,3)

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MTH621 Real Analysis I

72.3K/s 98% 1:53 pm

Question No : 6 of 41

Marks: 1 (Budge

$S = (2,4) \cup (5,6)$ is

page 34

Answer (Please select your correct option)

- compact.
- closed.
- open.
- semi infinite.

Telenor
180.3K/s 98% 1:53 pm

Question No : 7 of 41

For the function $f(x) = x \sin \frac{1}{x}, \quad x \neq 0$, which statement is true

page 87

Answer (Please select your correct option)

- $\lim_{x \to 1/x} f(x) = 0$
- The function is not defined at $x = 0$.
- $\lim_{x \to 0} f(x) = 1$.
- $\lim_{x \to 0} f(x) = 0$.

Telenor
30K/s 98% 1:53 pm

Question No : 0 of 41

The value of $x + \lim_{x \to 0+} \frac{|x|}{x} =$

page 90

Answer (Please select your correct option)

- $x - 1$.
- $x + 1$.
- x.
- $x + 1/x$

Telenor 17.6K/s 98% 1:53 pm

Question No : 9 of 41 Marks: 1 (Budgeted Time 1 Min)

For the piecewise defined function $f(x) = \begin{cases} x^3, & x \le 0, \\ x^2 \sin\frac{1}{x}, & x > 0, \end{cases}$ the value of $\lim_{x \to 0^+} f(x)$ is

page 117

Answer (Please select your correct option)

- 0
- 2
- 1
- -1

Telenor
90.7K/s 98% 1:54 pm

Question No : 11 of 41 Marks: 1 (Budgeted Tim

If $f(x) = \log x$ and $g(x) = \frac{1}{1-x^2}$, then $g \circ f$ is

page 99

Answer (Please select your correct option)

- $\log \frac{1}{1-x^2}$
- $\frac{1}{1-(\log x)^2}$
- $\log \frac{1}{x^2}$
- $\log \frac{x}{1-x^2}$

Telenor

MTH621 Real Analysis I

154.8K/s 97% 1:54 pm

Question No : 12 of 41

Marks: 1 (Budgeted Tim

The function $g(x) = x^2$ is

page 106

Answer (Please select your correct option)

- decreasing on $[0,\infty)$.
- increasing on $[0,\infty)$.
- having negative values on $[0,\infty)$.
- undefined on $[0,\infty)$.

Telenor

100.6K/s 97% 1:54 pm

Question No : 13 of 41

The inverse of the given function $f(x) = 2x + 4, \quad 0 \le x \le 2,$ is

page 110

Answer (Please select your correct option)

- $f^{-1}(y) = \frac{y-4}{2}$
- $f^{-1}(y) = \frac{y-2}{4}$
- $f^{-1}(y) = \frac{x-2}{4}$
- $f^{-1}(y) = \frac{y-4}{4}$

Telenor
121.6K/s 97% 1:55 pm

Question No : 15 of 41

$\lim_{x \to 0^+} x \log x$, has the following indeterminate form

page 126

Answer (Please select your correct option)

- $\infty \times \infty$
- $(0)(\infty)$
- $\frac{0}{0}$,
- 0^0

Telenor

MTH621 Real Analysis I

192.8K/s 97% 1:55 pm

Question No : 16 of 41

If f is differentiable at a local extreme point $x_0 \in D_f^0$, then

page 119

Answer (Please select your correct option)

- $f'(x_0) \neq 0$.
- $f'(x_0) = 0$.
- $x_0 = 0$.
- $|f'(x_0)| < 0$.

Telenor
222.7K/s 97% 1:55 pm

Question No : 17 of 41 Marks: 1 (Budgeted)

If P and P' are partitions of $[a,b]$, then P' is a refinement of P

page 135

Answer (Please select your correct option)

- P has exactly four points.
- if every partition point of P' is also a partition point of P.
- if every partition point of P is also a partition point of P'
- None of the given choices.

Telenor
78.8K/s 97% 1:56 pm

Question No : 19 of 41

If f and g are integrable functions on $[a,b]$ then

Answer (Please select your correct option)

- $\sqrt{f}$ is also integrable on $[a,b]$.
- fg is anti-derivative of f and g on $[a,b]$.
- f^2g is also integrable on $[a,b]$.
- None of the given choices.

Telenor
175.6K/s 97% 1:56 pm

Question No : 20 of 41

If a function f is Riemann integrable on $[a,b]$, then it is

Answer (Please select your correct option)

- differentiable on $[a,b]$.
- Riemann integrable on the any subinterval of $[a,b]$.
- Lipschitz continuous on $[a,b]$.
- None of the given choices.

Telenor 265.7K/s 97% 1:57 pm

Question 1

Define $(f+g)(x), (f-g)(x), (fg)(x)$ and (f/g), for the functions $f(x) = \sqrt{4-x^2}$ and $g(x) = \sqrt{x-1}$.

Solution:

First of all observe that $D_f = [-2, 2]$ and $D_g = [1, \infty)$.
Then $f+g$, $f-g$, and fg are defined on $D_f \cap D_g = [1, 2]$ by

$$\begin{aligned}(f+g)(x) &= \sqrt{4-x^2} + \sqrt{x-1},\\ (f-g)(x) &= \sqrt{4-x^2} - \sqrt{x-1},\end{aligned}$$

and

$$(fg)(x) = (\sqrt{4-x^2})(\sqrt{x-1}) = \sqrt{(4-x^2)(x-1)}.$$

The quotient f/g is defined on $(1, 2]$ by

$$\left(\frac{f}{g}\right)(x) = \sqrt{\frac{4-x^2}{x-1}}.$$

the last expression in is also defined for $-\infty < x < -2$, it does not represent fg for such x, since f and g are not defined on $(-\infty, -2]$.

SSC MTS TARGET 2021

Telenor 96K/s 97% 1:57 pm

Question 2

Find $\lim_{x\to 2}\dfrac{9-x^2}{x+1}$ and $\lim_{x\to 2}(9-x^2)(x+1)$.

Solution:

If c is a constant, then $\lim_{x\to x_0} c = c$, and $\lim_{x\to x_0} x = x_0$. Therefore

$$\begin{aligned}\lim_{x\to 2}(9-x^2) &= \lim_{x\to 2} 9 - \lim_{x\to 2} x^2 \\ &= \lim_{x\to 2} 9 - (\lim_{x\to 2} x)^2 \\ &= 9 - 2^2 = 5,\end{aligned}$$

and

$$\lim_{x\to 2}(x+1) = \lim_{x\to 2} x + \lim_{x\to 2} 1 = 2 + 1 = 3.$$

Therefore,

$$\lim_{x\to 2}\frac{9-x^2}{x+1} = \frac{\lim_{x\to 2}(9-x^2)}{\lim_{x\to 2}(x+1)} = \frac{5}{3}$$

and

$$\lim_{x\to 2}(9-x^2)(x+1) = \lim_{x\to 2}(9-x^2)\lim_{x\to 2}(x+1) = 5\cdot 3 = 15.$$

Telenor 91.6K/s 97% 1:57 pm

Question 3

Check the continuity of $g(x) = \begin{cases} x+1, & x > 0, \\ x-1, & x < 0; \end{cases}$ at x=0

Solution:

every open interval containing $x_0 = 0$ also contains points x_1 and x_2 such that $|g(x_1) - g(x_2)|$ is as close to 2 · Therefore, $\lim_{x \to x_0} g(x)$ does not exist.

Telenor 14.5K/s 97% 1:57 pm

Question 4

Discuss the continuity of f defined on $[0,2]$ by

$$f(x) = \begin{cases} x^2, & 0 \le x < 1, \\ x+1, & 1 \le x \le 2 \end{cases}$$

Solution: : If $0 < x, x_0 < 1$, then

$$\begin{aligned} |f(x) - f(x_0)| &= |x^2 - x_0^2| = |x - x_0|\,|x + x_0| \\ &\le 2|x - x_0| < \varepsilon \quad \text{if} \quad |x - x_0| < \varepsilon/2. \end{aligned}$$

Hence, f is continuous at each x_0 in $(0,1)$. If $1 < x, x_0 < 2$, then

$$\begin{aligned} |f(x) - f(x_0)| &= |(x+1) - (x_0+1) = |x - x_0| \\ &< \varepsilon \quad \text{if} \quad |x - x_0| < \varepsilon. \end{aligned}$$

Hence, f is continuous at each x_0 in $(1,2)$.

Telenor 93.8K/s 97% 1:57 pm

Question 5

Discuss the continuity of $f(x) = \sqrt{x}, \quad 0 \leq x < \infty.$

Solution: Consider

$$|f(x) - f(0)| = \sqrt{x} < \varepsilon \quad \text{if} \quad 0 \leq x < \varepsilon^2,$$

so $\lim_{x \to 0_+} f(x) = f(0)$. If $x_0 > 0$ and $x \geq 0$, then

$$|f(x) - f(x_0)| = |\sqrt{x} - \sqrt{x_0}| = \frac{|x - x_0|}{\sqrt{x} + \sqrt{x_0}} \leq \frac{|x - x_0|}{\sqrt{x_0}} < \varepsilon \quad \text{if} \quad |x - x_0| < \varepsilon\sqrt{x_0}$$

so $\lim_{x \to x_0} f(x) = f(x_0)$. Hence, f is continuous on $[0, \infty)$.

Telenor 70K/s 96% 1:57 pm

Question 7

the function $f(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0, \\ 0, & x = 0, \end{cases}$

Solution:

The function is continuous at all x_0 except $x_0 = 0$.

As x approaches 0 from either side, $f(x)$ oscillates between -1 and 1 with ever-increasing frequency, so neither $\lim_{x\to 0+} f(x)$ nor $\lim_{x\to 0-} f(x)$ exists.

Therefore, the discontinuity of f at 0 is not a jump discontinuity, and if $\rho > 0$, then f is not piecewise continuous on any interval of the form $[-\rho, 0]$, $[-\rho, \rho]$, or $[0, \rho]$.

Telenor 88.6K/s 96% 1:58 pm

Question 8

Removable discontinuity: Let f be defined on a deleted neighborhood of x_0 and discontinuous (perhaps even undefined) at x_0. We say that f has a removable discontinuity at x_0 if $\lim_{x \to x_0} f(x)$ exists.

In this case, the function

$$g(x) = \begin{cases} f(x) & \text{if } x \in D_f \text{ and } x \neq x_0, \\ \lim_{x \to x_0} f(x) & \text{if } x = x_0, \end{cases}$$

is continuous at x_0.

Telenor 86.7K/s 96% 1:58 pm

Question 9

If $f(x) = \log x$ and $g(x) = \dfrac{1}{1-x^2}$, we define $f \circ g$? and $g \circ f$?

Solution:

The domain of the given functions are

$$D_f = (0, \infty) \quad \text{and} \quad D_g = x \in \mathbb{R} / \; x \neq \pm 1.$$

Since $g(x) > 0$ if $x \in T = (-1, 1)$, the composite function $f \circ g$ is defined on $(-1, 1)$ by

$$(f \circ g)(x) = \log \frac{1}{1-x^2}.$$

$g \circ f$

The function $g \circ f$ is defined on $(0, 1/e) \cup (1/e, e) \cup (e, \infty)$ by

$$(g \circ f)(x) = \frac{1}{1 - (\log x)^2}.$$

Telenor 163.9K/s 96% 1:58 pm

Question 10

the functions $f(x) = \sqrt{x}$, $g(x) = \dfrac{9 - x^2}{x + 1}$. Check the continuity of $f \circ g$.

Solution:

The function f is continuous for $x > 0$, and the function g is continuous for $x \neq -1$.

Since $g(x) > 0$ if $x < -3$ or $-1 < x < 3$, the composite function

$$(f \circ g)(x) = \sqrt{\frac{9 - x^2}{x + 1}}$$

is continuous on $(-\infty, -3) \cup (-1, 3)$. It is also continuous from the left at -3 and 3.

Telenor 88.7K/s 96% 1:58 pm

Question 11

Define Intermediate theorem?

Suppose that f is continuous on $[a,b]$, $f(a) \neq f(b)$, and μ is between $f(a)$ and $f(b)$. Then $f(c) = \mu$ for some c in (a,b).

Define Uniform Continuity?

A function f is *uniformly continuous* on a subset S of its domain if, for every $\varepsilon > 0$, there is a $\delta > 0$ such that

$$|f(x) - f(x')| < \varepsilon \text{ whenever } |x - x'| < \delta \text{ and } x, x' \in S.$$

Telenor 83.7K/s 96% 1:58 pm

Question 12

Check the uniform continuity of the function $f(x) = 2x$.

Solution:

For the function $f(x)$, we have

$$|f(x) - f(x')| = 2|x - x'| < \varepsilon \quad \text{if} \quad |x - x'| < \varepsilon/2.$$

The function $f(x)$ is uniformly continuous on $(-\infty, \infty)$.

Telenor 332.6K/s 96% 1:58 pm

Question 14

Show that the function $g(x) = x^2$ is uniformly continuous on $[-r, r]$ for any finite r.

Solution:

$$|g(x) - g(x') = |x^2 - (x')^2|$$
$$= |x - x'|\,|x + x'| \leq 2r|x - x'|$$

so

$$|g(x) - g(x')| < \varepsilon \quad \text{if} \quad |x - x'| < \delta = \frac{\varepsilon}{2r} \quad \text{and} \quad -r \leq x, x' \leq r.$$

Telenor 90.9K/s 96% 1:59 pm

Question 16

Monotonic functions: A function f is *nondecreasing* on an interval I if

$$f(x_1) \leq f(x_2) \quad \text{whenever} \quad x_1, x_2 \in I, \quad x_1 < x_2.$$

or *nonincreasing* on I if

$$f(x_1) \geq f(x_2) \quad \text{whenever } x_1 \text{ and } x_2 \text{ are in } I \text{ and } x_1 < x_2.$$

Telenor 186.8K/s 96% 1:59 pm

Question 17

If f is nondecreasing, then $\lim_{x \to a+} f(x) = \alpha$

Proof:

We first show that $\lim_{x \to a+} f(x) = \alpha$.

If $M > \alpha$, there is an x_0 in (a, b) such that $f(x_0) < M$. Since f is nondecreasing, $f(x) < M$ if $a < x < x_0$. Therefore, if $\alpha = -\infty$, then $\lim_{x \to a+} f(x) = -\infty$.

If $\alpha > -\infty$, let $M = \alpha + \varepsilon$, where $\varepsilon > 0$.

Then $\alpha \leq f(x) < \alpha + \varepsilon$, so

$$|f(x) - \alpha| < \varepsilon \quad \text{if} \quad a < x < x_0.$$

If $a = -\infty$, this implies that $f(-\infty) = \alpha$. If $a > -\infty$, let $\delta = x_0 - a$. Then is equivalent to

$$|f(x) - \alpha| < \varepsilon \quad \text{if} \quad a < x < a + \delta,$$

which implies that $f(a+) = \alpha$.

Telenor 93.5K/s 96% 2:00 pm

Question 19

the function $f(x) = |x|$.

Solution:

The functions can be written as

$$f(x) = \begin{cases} x, & x > 0, \\ -x, & x \leq 0, \end{cases} \quad \Rightarrow \quad f'(x) = \begin{cases} 1, & x > 0, \\ -1, & x \leq 0, \end{cases}$$

$$\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0+} \frac{x}{x}$$

$$\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0-} \frac{-x}{x} = -1$$

are different,

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

does not exist thus, f is not differentiable at 0, even though it is continuous at 0.

Telenor 74.5K/s 96% 2:00 pm

Question 20

Calculate the derivative of $h(x) = \sin\frac{1}{x}, \quad x \neq 0.$

Solution:

If

$$f(x) = \sin x \quad \text{and} \quad g(x) = \frac{1}{x}, \quad x \neq 0,$$

then

$$h(x) = f(g(x))$$

$$= \sin\frac{1}{x}, \quad x \neq 0,$$

and

$$h'(x) = f'(g(x))g(x)$$

$$= \left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right), \quad x \neq 0.$$

Telenor 78.1K/s 96% 2:00 pm

Question 21

For the piecewise defined function

$$f(x) = \begin{cases} x^3, & x \le 0, \\ x^2 \sin \frac{1}{x}, & x > 0, \end{cases}$$

Investigate the one sided derivatives

Solution:

Solution: For the given function, we have

$$f'(x) = \begin{cases} 3x^2, & x < 0, \\ 2x \sin \frac{1}{x} - \cos \frac{1}{x}, & x > 0. \end{cases}$$

We will calculate the one sided derivatives

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0+} \frac{x^2 \sin \frac{1}{x} - 0}{x - 0} = \lim_{x \to 0+} x \sin \frac{1}{x} = 0,$$

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} \frac{x^3 - 0}{x - 0} = \lim_{x \to 0-} x^2 = 0;$$

hence, $f'(0) = f'_+(0) = f'_-(0) = 0$.

Telenor 131.4K/s 96% 2:00 pm

Question 22

Differentiable function: A function f is *differentiable on the closed interval* $[a, b]$ if f is differentiable on the open interval (a, b) and $f'_+(a)$ and $f'_-(b)$ both exist.

Continuously differentiable We say that f is *continuously differentiable on* $[a, b]$ if f is differentiable on $[a, b]$, f' is continuous on (a, b),

$$f'_+(a) = \lim_{x \to a+} f'(x)$$

, and

$$f'_-(b) = \lim_{x \to b-} f'(x).$$

Telenor 61.7K/s 96% 2:00 pm

Question 23

Rolle's Theorem

Theorem: Suppose that f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), and $f(a) = f(b)$. Then $f'(c) = 0$ for some c in the open interval (a, b).

Proof:

Since f is continuous on $[a, b]$, f attains a maximum and a minimum value on $[a, b]$. If these two extreme values are the same, then f is constant on (a, b), so $f'(x) = 0$ for all x in (a, b).

If the extreme values differ, then at least one must be attained at some point c in the open interval (a, b), and $f'(c) = 0$.

Telenor 23.9K/s 96% 2:01 pm

Question 24

Define

The Mean Value Theorem

Theorem: Suppose that f is differentiable on $[a, b]$, $f'(a) \neq f'(b)$, and μ is between $f'(a)$ and $f'(b)$. Then $f'(c) = \mu$ for some c in (a, b).

Generalized Mean Value Theorem

Theorem: If f and g are continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then

$$[g(b) - g(a)]f'(c) = [f(b) - f(a)]g'(c) \qquad \text{for some } c \text{ in } (a, b).$$

Telenor 79.4K/s 96% 2:01 pm

Question 25

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then

$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad \text{for some } c \text{ in } (a, b).$$

Proof:

The function

$$h(x) = [b - a]f(x) - [f(b) - f(a)]x$$

is continuous on $[a, b]$ and differentiable on (a, b).

Furthermore

$$h(a) = h(b) = bf(a) - f(b)a.$$

Therefore, Rolle's theorem implies that $h'(c) = 0$ for some c in (a, b).

Since $h'(c) = [b - a]f'(c) - [f(b) - f(a)]$.

Telenor 7.1K/s 96% 2:01 pm

Question26

Define

Lipschitz Continuity

A function that satisfies inequality

$$|f(x) - f(x')| \leq M|x - x'|, \quad x, x' \in (a, b),$$

for all x and x' in an interval, where $M > 0$ is some real number is said to satisfy a *Lipschitz condition* on the interval.

Telenor 4.5K/s 95% 2:02 pm

Question 28

Define Upper Integral

If f is bounded on $[a,b]$ and $P=\{x_0,x_1,\ldots,x_n\}$ is a partition of $[a,b]$, let

$$M_j=\sup_{x_{j-1}\le x\le x_j} f(x)$$

The *upper sum of* f *over* P is

$$S(P)=\sum_{j=1}^{n} M_j(x_j-x_{j-1}),$$

and the *upper integral of* f *over*, $[a,b]$, denoted by

$$\overline{\int_a^b} f(x)\,dx,$$

is the infimum of all upper sums.

Telenor 159.6K/s 95% 2:02 pm

Question 27

Give an example of a function which is continuous but not Lipchitz continuous.

Solution:

Example: The function $f(x) = \sqrt{|x|}$ is continuous at $x = 0$, but not Lipschits continuous at $x = 0$.

Telenor 108.7K/s 95% 2:02 pm

Question 29

Define Lower Integral

If f is bounded on $[a,b]$ and $P=\{x_0,x_1,\ldots,x_n\}$ is a partition of $[a,b]$, let

The *lower sum of* f *over* P is

$$s(P)=\sum_{j=1}^{n} m_j(x_j-x_{j-1}),$$

and the *lower integral of* f *over* $[a,b]$, denoted by

$$\underline{\int_a^b} f(x)\,dx,$$

is the supremum of all lower sums.

Telenor 60.7K/s 95% 2:02 pm

Question 30

Let

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational,} \\ 1 & \text{if } x \text{ is rational,} \end{cases}$$ Find $\overline{\int_a^b} f(x)\,dx$

Solution:

Let $P = \{x_0, x_1, \ldots, x_n\}$ be a partition of $[a, b]$. Since every interval contains both rational and irrational numbers

$$M_j = 1, \quad 1 \le j \le n.$$

Hence,

$$S(P) = \sum_{j=1}^{n} 1 \cdot (x_j - x_{j-1}) = b - a$$

Since all upper sums equal $b - a$ we have

$$\overline{\int_a^b} f(x)\,dx = b - a$$

Telenor 74.9K/s 95% 2:02 pm
MTH621 Past Paper Solved (part 4) Education Information ...

Question 31

Let

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational,} \\ 1 & \text{if } x \text{ is rational,} \end{cases} \qquad \text{Find} \qquad \int_a^b f(x)\,dx$$

Solution:

Let $P = \{x_0, x_1, \ldots, x_n\}$ be a partition of $[a, b]$. Since every interval contains both rational and irrational numbers

$$m_j = 0 \qquad 1 \le j \le n.$$

Hence,

$$s(P) = \sum_{j=1}^{n} 0 \cdot (x_j - x_{j-1}) = 0.$$

Since all lower sums equal 0, from definition we have

$$\int_a^b f(x)\,dx = 0.$$

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MTH621 25TH FEB SHIFT 1 JEE 2021 25th Feb Shift 1 jee JEE

Telenor 97.2K/s 95% 2:02 pm

Question 32

If f is bounded on $[a,b]$, then $\underline{\int_a^b} f(x)\,dx \le \overline{\int_a^b} f(x)\,dx$.

Proof:

Suppose that P_1 and P_2 are partitions of $[a,b]$ and P' is a refinement of both.

We have proved

$$S(P) \ge S(P') \ge S(P) - 2Mr\|P\| \quad (*)$$

and

$$s(P) \le s(P') \le s(P) + 2Mr\|P\| \quad (**).$$

Letting $P = P_1$ in $(**)$ and $P = P_2$ in $(*)$ shows that

$$s(P_1) \le s(P') \quad \text{and} \quad S(P') \le S(P_2).$$

Since $s(P') \le S(P')$, this implies that $s(P_1) \le S(P_2)$. Thus, every lower sum is a lower bound for the set of all upper sums.

Since $\overline{\int_a^b} f(x)\,dx$ is the infimum of this set, it follows that

$$s(P_1) \le \overline{\int_a^b} f(x)\,dx$$

for every partition P_1 of $[a,b]$.

This means that $\overline{\int_a^b} f(x)\,dx$ is an upper bound for the set of all lower sums.

Since $\underline{\int_a^b} f(x)\,dx$ is the supremum of this set

Telenor 123.5K/s 95% 2:03 pm

Question 32

If f is bounded on $[a,b]$, then $\underline{\int_a^b} f(x)\,dx \le \overline{\int_a^b} f(x)\,dx$.

Proof:

Suppose that P_1 and P_2 are partitions of $[a,b]$ and P' is a refinement of both.

We have proved

$$S(P) \ge S(P') \ge S(P) - 2Mr\|P\| \quad (*)$$

and

$$s(P) \le s(P') \le s(P) + 2Mr\|P\| \quad (**).$$

Letting $P = P_1$ in $(**)$ and $P = P_2$ in $(*)$ shows that

$$s(P_1) \le s(P') \quad \text{and} \quad S(P') \le S(P_2).$$

Since $s(P') \le S(P')$, this implies that $s(P_1) \le S(P_2)$. Thus, every lower sum is a lower bound for the set of all upper sums.

Since $\overline{\int_a^b} f(x)\,dx$ is the infimum of this set, it follows that

$$s(P_1) \le \overline{\int_a^b} f(x)\,dx$$

for every partition P_1 of $[a,b]$.

This means that $\overline{\int_a^b} f(x)\,dx$ is an upper bound for the set of all lower sums.

Since $\underline{\int_a^b} f(x)\,dx$ is the supremum of this set

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Question 33

If f is monotonic on $[a,b]$, then f is integrable on $[a,b]$.

Proof:

Let $P = \{x_0, x_1, \ldots, x_n\}$ be a partition of $[a,b]$. Since f is nondecreasing,

$$\begin{aligned} f(x_j) &= M_j = \sup_{x_{j-1} \le x \le x_j} f(x) \\ f(x_{j-1}) &= m_j = \inf_{x_{j-1} \le x \le x_j} f(x). \end{aligned}$$

Hence,

$$S(P) - s(P) = \sum_{j=1}^{n} (f(x_j) - f(x_{j-1}))(x_j - x_{j-1}).$$

Since $0 < x_j - x_{j-1} \le \|P\|$ and $f(x_j) - f(x_{j-1}) \ge 0$,

$$\begin{aligned} S(P) - s(P) &\le \|P\| \sum_{j=1}^{n} (f(x_j) - f(x_{j-1})) \\ &= \|P\|(f(b) - f(a)). \end{aligned}$$

Therefore,

$$S(P) - s(P) < \varepsilon \quad \text{if} \quad \|P\|(f(b) - f(a)) < \varepsilon,$$

so f is integrable on $[a,b]$.

Telenor 83.1K/s 95% 2:03 pm

Question 34:

If f is continuous on $[a,b]$, then f is integrable on $[a,b]$.

Proof:

Let $P = \{x_0, x_1, \ldots, x_n\}$ be a partition of $[a,b]$.
Since f is continuous on $[a,b]$, there are points c_j and c_j' in $[x_{j-1}, x_j]$ such that

$$f(c_j) = M_j = \sup_{x_{j-1} \le x \le x_j} f(x)$$

and

$$f(c_j') = m_j = \inf_{x_{j-1} \le x \le x_j} f(x).$$

Therefore,

$$S(P) - s(P) = \sum_{j=1}^{n} \left[f(c_j) - f(c_j') \right] (x_j - x_{j-1}).$$

Since f is uniformly continuous on $[a,b]$, there is for each $\varepsilon > 0$ a $\delta > 0$ such that

$$|f(x') - f(x)| < \frac{\varepsilon}{b-a}.$$

If x and x' are in $[a,b]$ and $|x - x'| < \delta$. If $\|P\| < \delta$, then $|c_j - c_j'| < \delta$ and, from

$$S(P) - s(P) < \frac{\varepsilon}{b-a} \sum_{j=1}^{n} (x_j - x_{j-1}) = \varepsilon.$$

Hence, f is integrable on $[a,b]$.

Telenor 72K/s 95% 2:04 pm

Question 35

Determine whether the function $f(x) = 1 + x$ is integrable on $[a, b]$?

Solution:

If

$$f(x) = \sin x \quad \text{and} \quad g(x) = \frac{1}{x}, \quad x \neq 0,$$

then

$$h(x) = f(g(x)) = \sin \frac{1}{x}, \quad x \neq 0,$$

and

$$h'(x) = f'(g(x))g(x) = \left(\cos \frac{1}{x}\right)\left(-\frac{1}{x^2}\right), \quad x \neq 0.$$

Telenor 96.3K/s 95% 2:04 pm

Question 36

If f is integrable on $[a, b]$ and $a \leq a_1 < b_1 \leq b$, then f is integrable on $[a_1, b_1]$.

Proof:

Thus, $\overline{P} = \{x_r, x_{r+1}, \ldots, x_s\}$ is a partition of $[a_1, b_1]$ over which the upper and lower sums of f satisfy

$$S(\overline{P}) - s(\overline{P}) < \varepsilon.$$

Therefore, f is integrable on $[a_1, b_1]$.

If f is integrable on $[a, b]$ and $a \leq a_1 < b_1 \leq b$, then f is integrable on $[a_1, b_1]$.

Telenor
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Question No : 21 of 41

If f is integrable on $[a,b]$ and $a \le c \le b$, then the function F defined by $F(x) = \int_c^x f(t)dt$, then which statement is true

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Answer (Please select your correct option)

- $F(x)$ is a differentiable function on $[a,b]$.
- $F(x)$ is a constant function on $[a,b]$.
- $F(x)$ is monotone function on $[a,b]$.
- $F(x)$ is Lipschitz continuous on $[a,b]$.

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MTH621 Real Analysis I

Question No : 22 of 41

If $f(x) = e^x$, then $f^{(n)}(x) =$

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Answer (Please select your correct option)

- e^x
- ne^x
- e^{nx}
- e^n

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85.1K/s 92% 2:13 pm

Question No : 23 of 41

$\sin 2t \Big|_{-\pi/4}^{\pi/4} =$

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Answer (Please select your correct option)

- 1
- 2
- 4
- 0

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60.5K/s 92% 2:13 pm

Question No : 24 of 41

By putting $x = \sin t$, in $1-2x^2 =$....

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Answer (Please select your correct option)

- $\cos 2t$
- $\sin 2t$
- $\cos t$
- $-\cos 2t$

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108.3K/s 92% 2:14 pm

Question No : 25 of 41

If $f(x) = 1 - \frac{1}{x^3}$, then $\lim_{x \to \infty} f(x) =$

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Answer (Please select your correct option)

- ∞
- 0
- -1
- 1

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MTH621 Real Analysis I
27.6K/s 92% 2:14 pm

Question No : 26 of 41

If f is a differentiable function at a local extreme point x_0 then $f'(x_0) =$

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Answer (Please select your correct option)

- 0
- positive.
- negative.
- does not exist.

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106.6K/s 92% 2:14 pm

Question No : 27 of 41

Upper sum of f over P

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Answer (Please select your correct option)

- $S(P) = \sum_{j=1}^{n} M_j (x_j - x_{j-1})$ ⇦
- $S(P) = \sum_{j=1}^{n} m_j (x_j - x_{j-1})$
- $\int_a^b f(x)dx$
- $\int_a^b f(x)dx$

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28.4K/s 92% 2:14 pm

Question No : 28 of 41

Identify the anti-derivative of the function $f(x) = x^2 + \sin x$

Answer (Please select your correct option)

- $F(x) = \frac{x^3}{3} + \cos x.$
- $F(x) = \frac{x^2}{2} - \sin x$
- $F(x) = \frac{x^3}{3} - \cos x$
- $F(x) = \frac{x^4}{4} - \cos x.$

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Question No : 29 of 41 Marks: 1 (Budgeted Time 1)

For the function $F(x) = \int_0^x f(t)dt = \frac{x^2}{2}$, $0 < x \leq 1$ Identify the function such that $F(x)$ is anti-derivative of the function

Answer (Please select your correct option)

- $f(x) = x^2 + 1$
- $f(x) = x, \quad 0 \leq x \leq 1$
- $f(x) = 2x, \quad 0 \leq x \leq 1$
- $f(x) = x - 2, \quad 0 \leq x \leq 1$

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Question No : 30 of 41

For the function $f(x) = \begin{cases} x & 0 \le x < 1 \\ 2 & 1 \le x \le 2 \end{cases}$ which statement is true?

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Answer (Please select your correct option)

- Continuous at $x = 1$
- $\lim_{x \to 1^+} = 1$
- $\lim_{x \to 1^-} = 2$
- the function is nondecreasing on the interval [0, 2]

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