

Date: \_\_\_\_\_

### \* Christoffel Symbol of first kind

$$\Gamma_{ijk} = \frac{1}{2} \left[ \frac{\partial}{\partial x^i} (g_{jk}) + \frac{\partial}{\partial x^j} (g_{ik}) - \frac{\partial}{\partial x^k} (g_{ij}) \right]$$

$$= \frac{1}{2} [g_{ik,j} + g_{ji,k} - g_{jk,i}]$$

### \* Transformation law for Christoffel Symbol of first kind.

$$\bar{\Gamma}_{ijk} = \Gamma_{rst} \frac{\partial x^r}{\partial \bar{x}^i} \frac{\partial x^s}{\partial \bar{x}^j} \frac{\partial x^t}{\partial \bar{x}^k} + \frac{\partial^2 x^r}{\partial \bar{x}^i \partial \bar{x}^j} \frac{\partial x^s}{\partial \bar{x}^k}$$

### \* Christoffel Symbol of Type II

$$\Gamma_{jk}^i = g^{ir} \Gamma_{jkr}$$

$$\Gamma_{jkr}^i = \frac{g^{ir}}{2} [-g_{jkr,i} + g_{kir,j} + g_{rki,j}]$$

### \* Transformation law for Christoffel Symbol of first kind.

$$\Gamma_{jk}^i = \Gamma_{st}^r \frac{\partial x^i}{\partial \bar{x}^j} \frac{\partial x^s}{\partial \bar{x}^k} \frac{\partial x^t}{\partial \bar{x}^r} + \frac{\partial^2 x^r}{\partial \bar{x}^j \partial \bar{x}^k} \frac{\partial x^i}{\partial \bar{x}^r}$$

### \* Covariant:

$$T_{i;jk} = \frac{\partial T_i}{\partial x^k} - \Gamma_{ik}^t T_t$$

### \* Contravariant:

$$T^i_{;jk} = \frac{\partial T^i}{\partial x^k} - \Gamma_{tk}^i T^t$$

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## \* Gradient

$$\nabla \phi = \text{grad } \phi = \frac{\partial \phi}{\partial x^i} \vec{e}_i$$

## \* divergence

$$\text{div}(A^i) = A^i_{;i}$$

$$= A^i_{;i} + \Gamma^i_{ik} A^k$$

## \* In covariant form

$$\text{div}(A_i) = A_i{}^{;i} = A_i{}^{;i} - \Gamma^i_{ii} A_i$$

## \* Curl

$$(\nabla \times A)_i = \epsilon_{ijk} A_{k;j}$$

In covariant derivative.

$$(\nabla \times A)_i = \epsilon_{ijk} A_{k;j}$$

$$A_{k;j} = A_{k;j} - \Gamma^i_{kj} A_i$$

## \* Laplacian:-

$$\nabla^2 \phi = \nabla \cdot \nabla \phi = \nabla \cdot \text{grad } \phi$$

$$\nabla \phi = \frac{\partial \phi}{\partial x^i} \vec{e}_i$$

$$\text{div}(\text{grad } \phi)$$

$$\nabla \cdot \nabla \phi = \left( \frac{\partial \phi}{\partial x^i} \right)_{;i}$$

## Two events

$$x_1^0, x_1^1, x_2^0, x_2^1$$

$$x_2^0, x_2^1, x_2^2, x_2^3$$

$$x_1^1 = -\frac{\beta x_2^0 - x_1^0}{\sqrt{1-\beta^2}}$$

$$x_2^1 = -\frac{\beta x_2^0 - x_1^0}{\sqrt{1-\beta^2}}$$

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$$x_2^1 = -\frac{\beta x_2^0 - x_1^0}{\sqrt{1-\beta^2}}$$

$$x_2^1 = -\frac{\beta x_2^0 - x_1^0}{\sqrt{1-\beta^2}}$$

## Time Dilation (increase)

$$\Delta x^0 = \frac{1}{\sqrt{1-\beta^2}} \Delta x^1$$

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proper time velocity

$$u^i = \frac{dx^i}{d\tau}$$

proper time acceleration

$$b^i = \frac{du^i}{d\tau} = \frac{d^2x^i}{d\tau^2}$$

Rest Mass

$$\hat{m} = \frac{m}{\sqrt{1-v^2/c^2}}$$

4-momentum:

$$p^i = (p^0, \vec{p}) = \hat{m} \vec{v}_i$$

Lorentz force ( $\vec{F}_0, \vec{F}$ )

$$\vec{F} = \frac{d}{dt} (\hat{m} \vec{v}_0) = \frac{m \vec{v}}{\sqrt{1-v^2/c^2}}$$

$$= \frac{d}{dt} \left( \frac{m \vec{v}}{\sqrt{1-v^2/c^2}} \right)$$

Minowski force

$$K = \frac{F_i}{\sqrt{1-v^2/c^2}}$$

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### \* Generalization of $\frac{d}{dt} \frac{dx^i}{dt}$

$$\vec{a} = (a^i) = \frac{d^2 x^i}{dt^2} + \Gamma_{rs}^i \frac{dx^r}{dt} \frac{dx^s}{dt}$$

### \* Curvature :-

$$K(s) = \sqrt{g_{ij} \frac{dx^i}{ds} \frac{dx^j}{ds}}$$

$$S(t) = \int_a^t \sqrt{g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt}} dt$$

### \* Generalized formula for Curvature

$$K(s) = \sqrt{g_{ij} b^i b^j}$$

$$b^i = \frac{d}{ds} \left( \frac{dx^i}{ds} \right) = \frac{d^2 x^i}{ds^2} + \Gamma_{rs}^i \frac{dx^r}{ds} \frac{dx^s}{ds}$$

$$S(t) = \int_a^t \sqrt{g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt}} dt$$

### \* Minkowski Spacetime:

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$

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## \* Absolute Tensor

$$T_{ij}^k \frac{dx^i}{dt}$$

$$\frac{dT^i}{dt} = \frac{dT^i}{dt} + \Gamma_{rs}^i T^s \frac{dx^r}{dt}$$

## \* Torsion Tensor:

$$T_{\alpha\beta}^\gamma = T_{\beta\alpha}^\gamma - T_{\alpha\beta}^\gamma$$

## \* Acceleration in General Coordinates

$$\vec{r} = \vec{r}(x^i(t))$$

$$\vec{a} = (a^i) = \frac{d}{dt} \frac{dx^i}{dt} = \frac{d^2 x^i}{dt^2}$$

## \* Instantaneous acceleration

$$a = \sqrt{\delta_{ij} a^i a^j}$$

## \* Formula for acceleration

$$= \sqrt{|g_{ij} a^i a^j|}$$

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### Type B

Range bcd

$$R_0 = 3^1 C_3 = \frac{n(n-1)(n-2)}{2}$$

### Type C

Range abcde

$$R_0 = 2^1 C_4 = \frac{n(n-1)(n-2)(n-3)}{12}$$

### Corollary:

$$R_{1112} = R_{2121} = -R_{1221} = -R_{2112}$$

### Rici Tensors

$$R_{ab} = R^c_{acb}$$

$$R_{ab} = R_{ba}$$

$$R_{aa} = g^{ab} R_{ab} = R^a_a$$

### Riemannian Curvature

$$K = K(X, Y, Z, V) = \frac{R_{abcd} X^a Y^b Z^c V^d}{g_{pq} X^p Y^q Z^r V^r}$$

$$C_{pqrs} = \frac{1}{2} (R_{pqrs} + R_{rspq} - R_{rpqs} - R_{sqrp})$$

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## Riemann tensor

$$(V_{i;j})_{;k} = V_{i;jk}$$

$$R^i_{jkl} = \frac{\partial \Gamma^i_{jk}}{\partial x^l} - \frac{\partial \Gamma^i_{jl}}{\partial x^k} + \Gamma^i_{jk} \Gamma^l_{ml} - \Gamma^i_{jl} \Gamma^l_{mk}$$

## properties of Riemann Tensor

$$R^i_{jkl} = -R^i_{jlk}$$

$$R_{ijkl} = -R_{jikl} \quad (\text{second skew symmetric})$$

$$R_{ijkl} = -R_{klij} \quad (\text{first skew symmetric})$$

$$R_{ijkl} = R_{klij}$$

$$R_{ijkl} + R_{iklj} + R_{iljk} = 0 \quad (\text{Bianchi's identity})$$

$$R_{abcd} = R_{cdab} = -R_{acdb} \Rightarrow R_{abcd} = 0$$

Type: A  $R_{abab}$   $a < b$

$$r_A = r_2 = \frac{n(n-1)}{2!}$$

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### \* Lorentz velocity:-

$$\vec{v}_0 = c = v_0$$

$$\tilde{v}_x = \frac{v_x - v}{1 - \frac{v_x v}{c^2}}$$

$$\tilde{v}_y = v_y \frac{\sqrt{1 - \beta^2}}{1 - \frac{v_x v}{c^2}}$$

$$\tilde{v}_z = \frac{v_z \sqrt{1 - \beta^2}}{1 - \frac{v_x v}{c^2}}$$

### \* Lorentz acceleration:-

$$\tilde{a}_x = \frac{a_x (1 - \beta^2)^{3/2}}{(1 - \frac{v_x v}{c^2})^3}$$

$$\tilde{a}_y = \frac{a_y + [a_x v_y - v_x a_y] \frac{v}{c^2} (1 - \beta^2)}{(1 - \frac{v_x v}{c^2})^3}$$

$$\tilde{a}_z = \frac{a_z + [a_x v_z - v_x a_z] \frac{v}{c^2} (1 - \beta^2)}{(1 - \frac{v_x v}{c^2})^3}$$

### Proportions

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\tau = \frac{s}{c} = \frac{1}{c} \int_{t_0}^t \sqrt{g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt}} dt$$

$\vec{v} < c$

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### Simple Lorentz transformation

$$x^0 = \frac{x^0 - \beta x^1}{\sqrt{1 - \beta^2}}, \quad x^1 = \frac{\beta x^0 - x^1}{\sqrt{1 - \beta^2}}$$

$$x^2 = x^2, \quad x^3 = x^3$$

Relativity velocity,  $\beta = v/c$

$$A = \begin{pmatrix} a & b & 0 & 0 \\ b & a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} a & -b & 0 & 0 \\ -b & a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$a^2 - b^2 = 1$

$$A = \begin{pmatrix} \frac{1}{\sqrt{1 - \beta^2}} & -\frac{\beta}{\sqrt{1 - \beta^2}} & 0 & 0 \\ \frac{\beta}{\sqrt{1 - \beta^2}} & \frac{1}{\sqrt{1 - \beta^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

### Length contraction:

$$\Delta x' = \frac{1}{\sqrt{1 - \beta^2}} \Delta x \quad \Rightarrow \quad \Delta x' = \sqrt{1 - \beta^2} \Delta x$$

# Energy momentum tensor of perfect fluid

$$T^{ij} = \left( \rho + \frac{p}{c^2} \right) u^i u^j - p g^{ij}$$

## Co-moving

$$u^i = (c, 0, 0, 0)$$

$$T^{ij} = \begin{bmatrix} \rho + \frac{p}{c^2} & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

$$T^{ij} = \left( \rho + \frac{p}{c^2} \right) u^i u^j - p g^{ij}$$

$$= \left( \rho + \frac{p}{c^2} \right) c^2 - p$$

$$T^{00} = \rho c^2$$

$$T^{11} = p$$

$$T^{22} = p$$

$$T^{33} = p$$

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## Non-Comoving

$$u = (\gamma c, \gamma v^x, \gamma v^y, \gamma v^z)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$T^{ij} = \rho \gamma^2 \begin{pmatrix} c & cv^x & cv^y & cv^z \\ cv^x & (v^x)^2 & v^x v^y & v^x v^z \\ cv^y & v^y v^x & (v^y)^2 & v^y v^z \\ cv^z & v^z v^x & v^z v^y & (v^z)^2 \end{pmatrix}$$

Conservation eq. for energy  
momentum tensor

$$\frac{\partial T^{ij}}{\partial x^j} = 0$$

$$i=0, \quad i=t$$

$$\frac{\partial T^{tj}}{\partial x^j} = \frac{\partial T^{tt}}{c \partial t} + \frac{\partial T^{tx}}{\partial x} + \frac{\partial T^{ty}}{\partial y} + \frac{\partial T^{tz}}{\partial z}$$

$$= \frac{1}{c} \frac{\partial (\rho \gamma^2 c^2)}{\partial t} + \frac{\partial (\rho \gamma^2 v^x)}{\partial x}$$

$$+ \frac{\partial (\rho \gamma^2 v^y)}{\partial y} + \frac{\partial (\rho \gamma^2 v^z)}{\partial z}$$

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$$\Rightarrow T^{ij} = \rho u^i u^j = \rho u^0 u^0 - \rho c^2 \delta^{ij}$$

$$T^{00} = \rho c^2$$

$$T^{ij} = \begin{bmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v^i = \frac{dx^i}{dt} = (c, x, y, z)$$

$$= (c, v^x, v^y, v^z)$$

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma = \frac{dt}{d\tau}$$

$$u^i = \frac{dx^i}{d\tau} = (ct, x, y, z)$$

$$= \frac{dt}{d\tau} \frac{dx^i}{dt} = \gamma \frac{dx^i}{dt}$$

$$= \gamma (c, v^x, v^y, v^z)$$

$$v = 0$$

$$u^i = (c, 0, 0, 0)$$

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Greene: / Normal / Riemannian  
 $g_{\mu\nu}$

Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Energy momentum tensor

$$T_{\mu\nu}$$

Covariant derivative of  
energy momentum tensor

$$\nabla_{\mu} T^{\mu}_{\nu} = 0$$

Stress energy tensor

$$T^{\mu}_{\nu} = \rho u^{\mu} u_{\nu}$$

$u^{\mu}$  (proper  
velocity)

Energy-momentum tensor of  
Dust in Minkowski  
space-time (flat space)

Co-moving:

$$u^i = (1, 0, 0, 0)$$

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Mass: in  $S' = m_0$

In  $S$  frame

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$p = \frac{p_0}{\sqrt{1 - v^2/c^2}}$$

$$\frac{p}{m} = \frac{p_0}{m_0} \Rightarrow p = \frac{p_0 m}{m_0}$$

$$\vec{j} = p \vec{u}$$

$$= \frac{p_0}{\sqrt{1 - v^2/c^2}} p$$

$$\vec{j} = \frac{p_0 m \vec{u}}{m_0}$$

$$\vec{j} = \frac{p_0}{m_0} p$$

Four current:

$$J^\mu = (p_0, \vec{j})$$

$$\vec{j} = j_x \hat{i} + j_y \hat{j} + j_z \hat{k}$$

$$J_\mu = (p_0, -\vec{j})$$

Conserved  
current

$$\nabla_\mu J^\mu = 0$$

In special

$$J^\mu_{;\mu} = 0$$

$$\frac{dL}{dt}$$

E/e

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Charge density:

$$\rho_0 = \frac{Ne}{L_0} \quad \text{in } S'$$

In  $S$

$$\rho = \frac{Ne}{L_0 \sqrt{1-v^2/c^2}}$$

$$\rho = \frac{\rho_0}{\sqrt{1-v^2/c^2}}$$

Current density:

in  $S'$  (the current frame)

In  $S$

$$\vec{j} = \rho \vec{v}$$

$$\vec{j} = \frac{\rho_0 \vec{v}}{\sqrt{1-v^2/c^2}}$$

$$\rho = \frac{\rho_0}{\sqrt{1-v^2/c^2}}$$

In component form

$$j_x = \frac{\rho_0 v_x}{\sqrt{1-v^2/c^2}} \quad j_y = \frac{\rho_0 v_y}{\sqrt{1-v^2/c^2}} \quad j_z = \frac{\rho_0 v_z}{\sqrt{1-v^2/c^2}}$$

## Einstein field Eq.

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Poisson law:-

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 4\pi G \rho$$

General relativistic analogue of Poisson eq.

$$(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{; \alpha} = 0$$

$\Downarrow$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}$$

$$\kappa = \frac{8\pi G}{c^4}$$

Maxwell Field Eq.'s

$$\nabla \cdot \vec{E} = \rho$$

$$\epsilon_0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$

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## Navier-Stokes

$$u^i = v^i$$

$$\tau^{ij} = \left( p + \frac{p}{c_2} \right) v^i v^j - p g^{ij}$$

$$\tau^{00} = \left( p + \frac{p}{c_2} \right) v^0 v^0 - p$$

$$\tau^{00} = p c_2^2 + p v^0 v^0 - p$$

(In Non-relativistic pressure is zero.)

$$\tau^{01} = \left( p + \frac{p}{c_2} \right) v^0 v^1 - 0$$

$$= \left( p + \frac{p}{c_2} \right) (v^0 v^1)$$

$$\tau^{01} = v^0 v^1 \left( p + \frac{p}{c_2} \right)$$

$$\tau^{02} = v^0 v^2$$

$$\tau^{03} = v^0 v^3 \quad g^{ij} = -1$$

$$\tau^{11} = \left( p + \frac{p}{c_2} \right) (v^1 v^1) - p (g^{11})$$

$$= \left( p + \frac{p}{c_2} \right) (v^1 v^1) - p$$

$$\tau^{12} = v^1 v^2$$

$$\tau^{13} = v^1 v^3$$

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## Properties of $F^{\mu\nu}$

- Anti Symmetry  $F^{\mu\nu} = -F^{\nu\mu}$
- Trace :  $F = F^{\mu}_{\mu} = 0$
- It has six independent components.
- $F_{\mu\nu} F^{\mu\nu} = 2(B^2 - E^2/c^2)$

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Short expressions for components

$$F_{ij} = \epsilon_{ijk} B^k \quad F_{0i} = -\frac{E_i}{c}$$

$$F^{ij} = \epsilon^{ijk} B_k \quad F^{0i} = \frac{E_i}{c}$$

Maxwell field eq. in tensor form

$$F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} = 0$$

$$F^{\alpha\beta}{}_{;\beta} = \mu_0 J^\alpha$$

In flat space

$$F^{\alpha\beta}{}_{;\beta} = \mu_0 J^\alpha$$

$$F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} = 0$$

Electromagnetic EMT (Energy momentum tensor)

$$T^\alpha_\beta = \frac{1}{4} \delta^\alpha_\beta (F_{\mu\nu} F^{\mu\nu}) - F^\alpha_\gamma F^\gamma_\beta$$

or

$$T^{\alpha\beta} = \frac{1}{4} \delta^{\alpha\beta} (F_{\mu\nu} F^{\mu\nu}) - F^{\alpha\mu} F^\beta_\mu$$

\* Trace (Electromagnetic EMT)

$$T^\alpha_\alpha = T^0_0 + T^1_1 + T^2_2 + T^3_3$$

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$$\frac{\partial (\frac{\phi}{c})}{\partial t} + \nabla \cdot \vec{A} = 0$$

$$\frac{1}{c^2} \frac{\partial \phi}{\partial t} + \nabla \cdot \vec{A} = 0$$

### \* Electromagnetic field tensor

$$F^{\alpha\beta} = \nabla^{\alpha} A^{\beta} - \nabla^{\beta} A^{\alpha}$$

$$A^{\alpha} = (\frac{\phi}{c}, \vec{A})$$

• In flat space

$$F^{\alpha\beta} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & B_x & B_y & 0 \end{bmatrix}$$

$$F_{\alpha\beta} = g_{\alpha\gamma} F^{\gamma\delta} g_{\delta\beta}$$

$$F_{\alpha\beta} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

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Conservation eq. for four vector  
current density

$$\nabla_\alpha J^\alpha = J^\alpha_{;\alpha} = 0$$

In special relativity

$$J^\alpha_{;\alpha} = J^0_{;0} + J^1_{;1} + J^2_{;2} + J^3_{;3}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

Electromagnetic four potential

$$A^\mu = \left( \frac{\phi}{c}, \vec{A} \right)$$

$$V = q\phi$$

$$\vec{W} = q\vec{A}$$

Magnetic and electric field in  
term

$$\vec{B} = \nabla \times \vec{A}, \quad \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

Four potential conservation

$$\nabla_\alpha A^\alpha = 0, \quad A^\alpha_{;\alpha} = 0$$

In flat space

$$A^\alpha_{;\alpha} = A^0_{;0} + A^1_{;1} + A^2_{;2} + A^3_{;3}$$