MTH632 Mega File Final Term

Lec (23-45)

Topic (41-121) page# (99-288)

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- MTH632 Quiz#2 All in 1 File pg47
- Paper Final MTH632 (by HSZS-legends) pg30
- Mth632-FINAL-2020.docx pg26
- Mth632 2 pg41
- Mth632 current paper khan pg33 unsolve
- Mth632 objective file final term by Hussnain Ali pg473
- Mth632 final term 2020 wajid pg33 solve
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Mth632 Quiz#2

- Quiz mth632 2-3-1-1-1 8aug2019 pg5
- Mth632 Quizz2(Zaiba akram) 9aug2019 pg10
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MTH632:Quiz#2

Question # 1 of 10 (Start time: 03:55:32 PM, 09 August 2019)

Harmonic conjugate of $u(x, y) = e^y \cos x$ is _____.

Select the correct option

○	$e^y \sin x + c$
●	$-e^y \sin x + c$
○	$e^x \sin y + c$
○	$e^x \cos y + c$

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Question # 2 of 10 (Start time: 03:56:05 PM, 09 August 2019)

The sequence $z_n = \frac{i}{n^3}$ $(n = 1, 2, 3, \ldots)$ for $n \to \infty$ converges to _____.

Select the correct option

●	0
○	1/2
○	1
○	infinity

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Question # 4 of 10 (Start time: 03:57:08 PM, 09 August 2019)

Total Marks: 1

Each simple closed curve C divides the plane into two domains (connected open sets). One of which is unbounded and is called the _____ of C.

Select the correct option

- ○ interior
- ● exterior ✓

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Question # 3 of 10 (Start time: 03:56:36 PM, 09 August 2019)

Total Ma

Any polynomial $p(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n$ $(a_n \neq 0)$, of degree $n (n \geq 1)$ has least one zero, i.e. there exists at least _____ z_0, such that $P(z_0) = 0$.

Select the correct option

Reload Math Equa

- ○ n points
- ○ 10 points
- ● one point ✓
- ○ no points

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MTH632:Quiz#2

Time Left sec(s)

Quiz Start Time: 03:55 PM, 09 August 2019

Question # 5 of 10 (Start time: 03:57:21 PM, 09 August 2019)

Total Marks: 1

A curve C is said to be a contour if it is constructed by joining finitely many smooth curves end to end.

Select the correct option

- ○ False
- ● True

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Question # 7 of 10 (Start time: 03:57:53 PM, 09 August 2019)

The sequence $z_n = -3 + i\frac{1}{n^3}$ $(n = 1, 2, 3, \ldots)$ for $n \to \infty$ converges to _____.

Select the correct option

○	i
○	-1
○	1
●	-3

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Question # 8 of 10 (Start time: 03:58:11 PM, 09 August 2019)

Total Ma

Each simple closed curve C divides the plane into two domains (connected open sets). One of which is bounded and is called _____ of C.

Select the correct option

- (●) interior ✓
- ( ) exterior

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MTH632:Quiz#2

Question # 9 of 10 (Start time: 03:58:23 PM, 09 August 2019)

If $e^{ax}\cos y$ is harmonic then 'a' is _____.

Select the correct option

●	i
○	-1
○	0
○	2

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Question # 10 of 10 (Start time: 03:58:41 PM, 09 August 2019)

Total Marks:

A domain D (connected open set) is called _____ connected if interior of every simple closed contour C is contained in D.

Select the correct option

- ○ multiply
- ● simply ✓

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MTH632 Quiz # 2 — Quiz Start Time: 02:41 PM, 07 June 202

Question # 5 of 10 (Start time: 02:44:53 PM, 07 June 2021) — Total Marks

The polar form of the complex function $f(z) = z^3$ *is*

Select the correct option

Reload Math Equation

- ○ $r^3(\cos 3\theta + i \sin 3\theta)$
- ○ $r^2(\cos 3\theta + i \sin 3\theta)$

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Question # 8 of 10 (Start time: 02:12:44 PM, 07 June 2021)

Total Marks: 1

If $f(z) = x + ay + i(bx + cy)$ is analytic, then a, b, c equals to _____.

Select the correct option

O	b = 1 and a = -c
O	c=1 and a= -b
O	a= 1 and c= -b
O	a = b = c = 1

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The Cauchy - Riemann equations on a pair of real - valued functions of two real variables $u(x, y)$ and $v(x, y)$ are $U_x = V_y$ and _____.

t the correct option

Reload

) $U_y = -V_y$

) $U_y = -V_x$



) $U_x = -V_x$

) $U_x = -V_y$

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ıne 2021)

$$\textit{The value of } w = \frac{1}{z} \textit{ when } z = 2 - i \textit{ is}$$



$\frac{1}{5}(2+i)$

$\frac{1}{5}(2-i)$

Cl

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Quiz # 2

7 of 10 (Start time: 02:11:48 PM, 07 June 2021)

If $f(z) = \dfrac{4z^4 - 1}{(z^2 - 1)(z - 2)}$, then f(z) is not analytic at z =

: correct option

5

4

3

1

$z^2 - 1 = 0$

$z^2 = 1$

$z = +1$ and -1

Cli

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:Quiz # 2

6 of 10 (Start time: 02:11:04 PM, 07 June 2021)

If $f(z) = \frac{z^4 + 1}{(z^2 + 1)(z - 3)}$, then f(z) is not analytic at z =

e correct option

4
3 ✓
1
-1

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Question # 5 of 10 (Start time: 02:10:24 PM, 07 June 2021)

Total Marks: 1

A function f(z) is analytic function if _____ part f(z) is analytic.

Select the correct option

O	(c) both (a) and (b).
O	(b) imaginary
O	(a) real
O	(d) none of these.

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Question # 4 of 10 (Start time: 02:09:45 PM, 07 June 2021)

Total Marks: 1

A point at which a function ceases to be analytic is called a _____ point.

Select the correct option

- O singular
- O regular
- O non-regular
- O non-singular

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MTH632:Quiz # 2

Question # 3 of 10 (Start time: 02:08:40 PM, 07 June 2021)

A function V is said to be harmonic if and only if _____.

Select the correct option

O	$V_x + V_y = 0$
O	$V_{xy} + V_{yx} = 0$
O	$V_{xx} + V_{yy} = 0$ ✓
O	$V_x^2 + V_y^2 = 0$

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7 June 2021)

The polar form of the complex function $f(z) = z^2$ *is*

$r^2(cos\theta + isin\theta)$



$r^2(cos2\theta + isin2\theta)$

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7 June 2021)

The polar form of the complex function $f(z) = \frac{1}{z}$ *is*



$\frac{1}{r}(\cos\theta - i\sin\theta)$

$r(\cos\theta + i\sin\theta)$

$r(\cos\theta - i\sin\theta)$

$\frac{1}{r}(\cos\theta + i\sin\theta)$

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uestion # 5 of 10 (Start time: 02:01:45 PM. 07 June 2021)

In the complex valued function z = x - iy. the value of $U_x =$ _____.

Select the correct option

○	0
○	2
○	1
○	-1

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If we write the function $w = z^2 - z$ *as* $w = u(x, y) + iv(x, y)$, *then* $u(x, y) =$

Select the correct option

O	$x^2 - x - y^2$
O	none of these
O	$x^2-y^2+2xy(i)-(x+iy)$
O	$x^2-y^2-x.\quad u(x'y)$
O	$2xy-y.\quad v(x'y)$
O	$x^2 + y^2$
O	$x^2 - y^2$

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MTH632:Quiz # 2

Question # 10 of 10 (Start time: 12:35:43 PM, 07 June 2021)

In the complex valued function z = x - iy, the value of $V_z =$ _____.

Select the correct option

O	2
O	1
O	-1
O	0

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Question # 4 of 10 (Start time: 12:30:13 PM, 07 June 2021)

If $f(z) = \frac{z^4 + 1}{(z^2 + 1)(z - 3)}$, then f(z) is not analytic at z =

Select the correct option

- O i
- O 1
- O 2
- O -1

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MTH632 Quiz # 2

Quiz Start Time: 12:27

Question # 3 of 10 (Start time: 12:29:09 PM, 07 June 2021)

If we write the function $w = z^2 - z$ as $w = u(x, y) + v(x, y)$, then $v(x, y) =$

Select the correct option

- $2xy - y$
- $x^2 - x - y^2$

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Question # 2 of 10 (Start time: 12:28:26 PM, 07 June 2021)

Total Marks: 1

If a function f(z) = u(x,y) + iv(x,y) is analytic in a domain D, then its component functions u and v are _____ in D.

Select the correct option

o	equal
o	steady
o	harmonic
o	homogeneous

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MTH632:Quiz # 2

Question # 1 of 10 (Start time: 11:09:13 AM, 07 June 2021)

For which values of z is $w = \frac{1}{z}$ defined?

Select the correct option

- ○ w is defined for all $z = 0$.
- ○ w is defined for all $z = 0$. ✓

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Mth632 Quiz#3

Module (-)

- Mth632 quiz 3 by AS 13aug2021 pg37
- MTH632 Quiz 3(11 files merged) 13aug2021 pg11
- MTH632 – Pen Pals Solved quiz 3 13aug2021 pg56
- Math632 current file#3 08-13-2021 16.06 pg47
- Math632 current file#3 08-13-2021 16.06 -1 pg55
- Pen Pals.MTH632.#3 pg82

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Question # 5 of 10 (Start time: 05:10:23 PM, 13 August 2021) Total Marks: 1

Let z_o and z_1 be two points in simply connected domain D
and f be an analytic complex valued function in D and C be contour by joining z_o and z_1 then

Select the correct option

Reload Math Equations

- $\int_{z_o}^{z_1} f(s)ds = F(z_1) + F(z_o)$
- $\int_{z_o}^{z_1} f(s)ds = F(z_o) + F(z_o)$
- $\int_{z_o}^{z_1} f(s)ds = F(z_o) - F(z_1)$
- $\int_{z_o}^{z_1} f(s)ds = F(z_1) - F(z_o)$

Click to Save Answer & Move to Next Question

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uestion # 10 of 10 (Start time: 05:00:12 PM, 13 August 2021)

Let C_1 and C_2 be two simple closed contours with positively oriented s. t C_1 is exterior and C_2 is an interior. If $f(z)$ is analytic in domain

lect the correct option

- $\int_{C_1} f(z)dz = \int_{C_2} f(z)dz$
- $\int_{C_1} f(z)dz > \int_{C_2} f(z)dz$
- $\int_{C_1} f(z)dz \neq \int_{C_2} f(z)dz$
- $\int_{C_1} f(z)dz < \int_{C_2} f(z)dz$

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Question # 7 of 10 (Start time: 04:58:14 PM, 13 August 2021) Tot

Each simple closed curve C divides the plane into interio and exterior domains where one is bounded and other is unbounded. This statement is of .....

Select the correct option

- Green's Theorem
- Cauchy Theorem
- Cauchy Goursat Theorem
- Jordan Curve Theorem



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MTH632:Quiz # 3

Quiz Start Time: 04:50 PM, 13 August 2021

Question # 9 of 10 (Start time: 04:56:23 PM, 13 August 2021)

Total Marks: 1

Each simple closed curve C divides the plane into two domains (connected open sets). One of which is bounded and is called _____ of C.

Select the correct option

- interior
- exterior

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Question # 7 of 10 (Start time: 04:54:55 PM, 13 August 2021) Total Marks: 1

Mathematically, the functions in Green's theorem will be

Select the correct option

- Continuous derivatives
- Discrete derivatives
- Discrete partial derivatives
- Continuous partial derivatives



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Question # 8 of 10 (Start time: 04:55:37 PM, 13 August 2021) Total Marks: 1

A curve is said to be .... if it is constructed by joining finitely many smooth curves end to end

Select the correct option

- disconnected
- contour
- continuous
- discontinuous

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Question # 4 of 10 (Start time: 04:52:17 PM, 13 August 2021)

Total Marks: 1

$$\int_C \frac{f(z)}{(z+1)^4} dz \text{ where } z = e^{2\pi} \text{ then this integral is equal to}$$

Select the correct option

Reload Math Equations

- $\frac{\pi i}{3!} f^{(3)}(1)$
- $\frac{2\pi i}{3!} f^{(3)}(1)$
- $\frac{\pi i}{3!} f^{(3)}(-1)$
- $\frac{2\pi i}{3!} f^{(3)}(-1)$

Click to Save Answer & Move to Next Question

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Question # 3 of 10 (Start time: 04:51:30 PM, 13 August 2021) Total Marks: 1

If the interior of every simple closed contour C is contained in the domain D , then domain is said to be....

Select the correct option

- None of above
- Unbounded
- Disconnected
- Simple connected



Click to Save Answer & Move to Next Question

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MTH632:Quiz # 3

Quiz Start Time: 04:50 PM, 13 August 2021

Question # 2 of 10 (Start time: 04:50:53 PM, 13 August 2021)

Total Marks: 1

If C is simple closed contour with positively oriented such that fixed complex value z_0 lies interior to C, then for integer $n \neq 1$ we have,

$$\int_C \frac{1}{(z - z_0)^n} dz =$$

Select the correct option

Reload Math Equations

- 0
- πi
- $-2\pi i$
- $2\pi i$

Click to Save Answer & Move to Next Question

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sec(s)

MTH632:Quiz # 3

Quiz Start Time: 04:03 PM, 13 August 2021

Question # 2 of 10 (Start time: 04:04:16 PM, 13 August 2021)

Total Marks: 1

$$Evaluate \int_C f(z)dz \quad where\ C : z(t) = e^{it}\ for\ 0 \leqslant t \leqslant \pi\ and f(z) = 1/z$$

Select the correct option

Reload Math Equations

- $-2\pi i$
- $-\pi i$
- $2\pi i$
- $i\pi$

Click to Save Answer & Move to Next Question

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Question # 4 of 10 (Start time: 04:06:54 PM, 13 August 2021) Total Marks: 1

$$Evaluate\ the\ integral \int_C f(z)dz \quad where\ C : z(t) = e^{it}\ for\ 0 \leqslant t \leqslant \pi\ and f(z) = 1/z^2$$

Select the correct option

Reload Math Equations

- $1 + e^{i\pi}$
- $1 - e^{-i\pi}$ ✓
- $1 - e^{i\pi}$
- $1 + e^{-i\pi}$

Click to Save Answer & Move to Next Question

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Question # 7 of 10 (Start time: 04:11:48 PM, 13 August 2021)

Total Marks: 1

$$Consider\ contour\ C : z(t) = e^{it} = Cos\, t + i\, Sint\ be\ counter\ clockwise\ where\ -\pi/2 \leqslant t \leqslant \pi/2\ then\ the\ integral$$
$$\int_C \bar{z}(t)dz =$$

Select the correct option

Reload Math Equations

- $-\pi$
- π
- $-i\pi$
- $i\pi$

Click to Save Answer & Move to Next Question

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Question # 6 of 10 (Start time: 11:05:38 AM, 13 August 2021)

If C is positively oriented unit circle i.e. {C: |z|=1} then

$$\int_C \frac{e^{2z}}{z^3} dz$$

is

Select the correct option

○	$3\pi i$
○ ✓	$2\pi i$
○	$6\pi i$
○	$4\pi i$

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$$Evaluate \int_0^{2\pi} \sin(\frac{\pi}{4} + 3e^{i\theta})d\theta \; if \; f(z) = \sin \; z$$

2π

$\sqrt{2\pi}$

$\frac{\pi}{\sqrt{2}}$

$\frac{\pi}{2}$

Click to Save Answe

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MTH632:Quiz # 3

Quiz Start Time: 04:05 PM, 13 August 2021

Question # 3 of 10 (Start time: 04:07:55 PM, 13 August 2021)

Total Marks: 1

$$Evaluate \int_C f(z)dz \text{ where } C : z(t) = (x + iy)\,t \text{ for } a \leqslant t \leqslant b \text{ and } f(z) = \frac{1}{z}$$

Select the correct option

Reload Math Equations

- $\ln b - \ln a$
- $\frac{(\ln a)}{(\ln b)}$
- $\ln b + \ln a$
- $(\ln a)(\ln b)$

Click to Save Answer & Move to Next Question

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MTH632:Quiz # 3

Quiz Start Time: 04:05 PM, 13 August 2021

Question # 9 of 10 (Start time: 04:14:00 PM, 13 August 2021)

Total Marks: 1

If function f is continuous in simple connected domain D and $\int_C f(z)dz$ for every closed contour C in D, then f is analytic in D.

This statement is of:

Select the correct option

Reload Math Equations

- Cauchy Goursat Theorem
- Cauchy theorem
- Morera's Theorem
- Green's Theorem

Click to Save Answer & Move to Next Question

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MTH632:Quiz # 3

Quiz Start Time: 04:03 PM, 13 August 2021

Question # 8 of 10 (Start time: 04:13:18 PM, 13 August 2021)

Total Marks: 1

Morera's Theorem is the converse of ...

Select the correct option

- Guass Mean value Theorem
- Cauchy Goursat Theorem
- De Movire's theorem
- Green's Theorem

Click to Save Answer & Move to Next Question

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Question # 2 of 10 (Start time: 04:06:45 PM, 13 August 2021) T

Let $f(t)$ be continuous function on contour C and L be length of C. For all t in contour C, $|f(t)| \leqslant M$ then

Select the correct option

Reload Ma

- $\left| \int_C f(t)\, dt \right| > ML$
- $\left| \int_C f(t)\, dt \right| < ML$
- $\left| \int_C f(t)\, dt \right| \leqslant ML$
- $\left| \int_C f(t)\, dt \right| \geqslant ML$

Click to Save Answer & Move to Next Que

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MTH632:Quiz # 3

Quiz Start Time: 04:03 PM, 13 August 2021

Question # 6 of 10 (Start time: 04:11:44 PM, 13 August 2021)

Total Marks: 1

If $f(z)$ is an analytic function in domain D then for $n \geqslant 0$ where n is a positive integer,

Select the correct option

Reload Math Equations

- None of above
- $f^{(n)}(z)$ is analytic in disconnected D
- $f^{(n)}(z)$ is not analytic in simple connected D
- $f^{(n)}(z)$ is an analytic in simple connected D



Click to Save Answer & Move to Next Question

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Question # 1 of 10 (Start time: 04:03:16 PM, 13 August 2021)

Total Marks: 1

Let α lies in the interior of contour C and

$$P_n(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \ldots + a_1 z + a_o z^0$$

then the value of $\int_C \frac{P_n(z)}{z-\alpha} dz$ *is ... when* $n = 3$

Select the correct option

Reload Math Equations

- $2\pi i P_n(\alpha)$
- $2\pi P_n(\alpha)$
- $2\pi P_3(\alpha)$
- $2\pi i P_3(\alpha)$ ✓

Click to Save Answer & Move to Next Question

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Question # 10 of 10 (Start time: 04:03:08 PM, 13 August 2021) Total Marks: 1

A domain D that is not simply connected is...

Select the correct option

- none of above
- doubly connected
- triply connected
- multiply connected



Click to Save Answer & Move to Next Question

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MTH632:Quiz # 3

Quiz Start Time: 03:55 PM, 13 August 2021

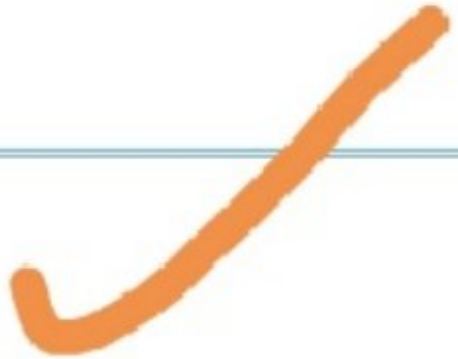
Question # 9 of 10 (Start time: 04:02:31 PM, 13 August 2021)

Total Marks: 1

A curve C is said to be a contour if it is constructed by joining finitely many smooth curves end to end.

Select the correct option

- False
- True



Click to Save Answer & Move to Next Question

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MTH632:Quiz # 3

Quiz Start Time: 10:51 AM, 13 August 2021

Question # 2 of 10 (Start time: 10:53:24 AM, 13 August 2021)

Total Marks: 1

If $f(z)$ is complex valued function and C and $-C$ be the contours with same geometrical representation but opposite in direction. Then $\int_{-C} f(z)dz =$

Select the correct option

Reload Math Equations

- $\int_C f(-z)dz$
- $\int_C f(z)dz$
- $-\int_C f(z)dz$ ✓
- $-\int_{-C} f(z)dz$

Click to Save Answer & Move to Next Question

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Question # 3 of 10 (Start time: 10:55:08 AM, 13 August 2021)

Using Cauchy integral formula to compute the value of integral

$$\int_C \frac{z-2}{z+i} dz \text{ where } f(z) = z - 2 \text{ and } z_o = -i$$

Select the correct option

○	$2\pi(-1-2i)$
○	$2\pi(1-2i)$
○	$2\pi(1+2i)$
○	$2\pi(-1+2i)$

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MTH632:Quiz # 3

Question # 1 of 10 (Start time: 10:51:36 AM, 13 August 2021)

If the function f(z) is analytic in domain D and simple connected contour C lies in D then

Select the correct option

- ✓ $\int_C f(z)dz = 0$
- $\int_C f(-z)dz = 0$
- $\int_C f(\bar{z})dz = 0$
- $\int_C f(z')dz = 0$

Click

Scanned with CamScanner

Evaluate $\int_C f(z)dz$ where $C : z(t) = (x + iy)\,t$ for $a \leqslant t \leqslant b$ and $f(z) = z$

option

Reload Math Equations

$$\frac{(b^2 - a^2)(x + iy)^2}{2}$$

$$\frac{(b^2 + a^2)(x + iy)}{2}$$

$$\frac{(b^2 - a^2)(x + iy)}{2}$$

$$\frac{(b^2 + a^2)(x + iy)^2}{2}$$

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Total Marks: 1

Evaluate the integral $\int_C \frac{e^z}{z^4} dz$ where the contour $C: |z| = 1$

the correct option

Reload Math Equations

$\frac{2\pi i}{3}$

$\frac{4\pi i}{3}$

$\frac{8\pi i}{3}$

$\frac{\pi i}{3}$

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MTH632:Quiz # 3

Quiz Start Time: 08:30 PM, 12 Aug

Question # 6 of 10 (Start time: 12:34:11 PM, 13 August 2021)

Total

Consider the integral $\int_C f(z)dz$ where contour $C: z(t) = e^{it}, L = 2\pi$ and $f(z) = \bar{z}$. The upper bound is

Select the correct option

Reload Math E

- $-\pi$
- -2π
- π
- 2π ✓

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MTH632:Quiz # 3

Quiz Start Time: 08:30 PM, 12 August 2021

Question # 8 of 10 (Start time: 12:37:37 PM, 13 August 2021)

Total Marks: 1

If function f is an analytic function in simple connected domain where $C_R(z_o) = \{z(t) : |z - z_o| = R\}, 0 \leqslant t \leqslant 2\pi$ and R be the radius of circle, then

Select the correct option

Reload Math Equations

- $f(z_o) = \frac{1}{\pi}\int_0^{2\pi} f(z_o + \mathrm{Re}^{it})dt$
- $f(z_o) = \frac{1}{2\pi}\int_0^{2\pi} f(z_o + \mathrm{Re}^{it})dt$
- $f(z_o) = \frac{1}{4\pi}\int_0^{2\pi} f(z_o + \mathrm{Re}^{it})dt$
- $f(z_o) = \frac{1}{2\pi i}\int_0^{2\pi} f(z_o + \mathrm{Re}^{it})dt$

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PM, 13 August 2021)

Consider function $f(z) = \exp(z^2 + 1)$ be differentiable and its derivatives are continuous in domain D then $\int_C f(z)dz = \ldots\ldots$ wh

Select the correct option

O	1
O	0 ✓
O	-1
O	2

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Find value of Green's theorem if P=x2 and Q=y2 for region x=1 and y=2 from origin.

Select the correct option

○	0
○	2
○	1
○	-2

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Question # 4 of 10 (Start time: 01:19:45 PM, 13 August 2021)

A curve C is said to be a contour if it is constructed by joining finitely many smooth curves end to end.

Select the correct option

- ○ False
- ○ True ✓

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Question # 2 of 10 (Start time: 04:34:31 PM, 13 August 2021)

Consider contour $C : z(t) = e^{it} = Cos\, t + i\, Sin\, t$ be counter clockwise where $-\pi/2 \leq t \leq \pi/2$ then the integral

$$\int_C z(t)dz =$$

Select the correct option

- 1
- -1
- 0
- 2

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Let $f(t) = u(t) + iv(t)$ be a piece − wise continuous complex valued function of real variable t, $a \leqslant t \leqslant b$ then

ct the correct option

- $\left|\int_a^b f(t)dt\right| \leqslant \int_a^b |f(t)|dt$
- $\left|\int_a^b f(t)dt\right| \geqslant \int_a^b |f(t)|dt$
- $\left|\int_a^b f(t)dt\right| > \int_a^b |f(t)|dt$
- $\left|\int_a^b f(t)dt\right| < \int_a^b |f(t)|dt$

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Evaluate $\frac{1}{2\pi}\int_0^{2\pi} \exp(\frac{i\pi}{3} + 5e^{i\theta})d\theta$ if $f(z) = \exp(z)$ then we get,

$\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$\cos\frac{\pi}{3} + \sin\frac{\pi}{3}$

$\sin\frac{\pi}{3} + i\cos\frac{\pi}{3}$

$\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

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MTH632 Quiz # 3

Question # 7 of 10 (Start time: 05:10:35 PM 13 August 2021)

If C is senple closed contour and D be the domain that forms interior of C P and Q are continuous function and their partial derivatives are also continuous then

Select the correct option

- $\int_C |P(x,y)dx - Q(x,y)dy| = \iint_D (Q_x(x,y) - P_y(x,y))dxdy$
- $\int_C |P(x,y)dx + Q(x,y)dy| = \iint_D (Q_x(x,y) + P_y(x,y))dxdy$
- $\int_C |P(x,y)dx + Q(x,y)dy| = \iint_D (Q_x(x,y) - P_y(x,y))dxd$ ✓
- $\int_C |P(x,y)dx - Q(x,y)dy| = \iint_D (Q_x(x,y) + P_y(x,y))dxdy$

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MTH632:Quiz # 3

Quiz Start Time: 08:50 PM, 13 August 2021

Question # 2 of 10 (Start time: 08:51:36 PM, 13 August 2021)

Total Marks: 1

Each simple closed curve C divides the plane into two domains (connected open sets). One of which is unbounded and is called the _____ of C.

Select the correct option

- exterior
- interior

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Question # 7 of 10 (Start time: 08:13:38 PM, 13 August 2021)

Let C be simple closed contour in simple connected domain D and f be an analytic function in D and z_o be a point that lies in C then $f(z_o)$ is

Select the correct option

- $\int_C \frac{f(z)}{(z-z_o)}dz$
- $\frac{1}{\pi i}\int_C \frac{f(z)}{(z-z_o)}dz$
- $\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_o)}dz$ ✓
- $2\pi i\int_C \frac{f(z)}{(z-z_o)}dz$

Click to Save An

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MTH632:Quiz # 3

Question # 1 of 10 (Start time: 07:51:31 PM, 13 August 2021)

Let C_1 and C_2 be two simple closed contours with positively oriented such that $C_1 - C_2$ is positively oriented as boundary of the region R then

Select the correct option

- $\int_{C_1 - C_2} f(z)dz \geqslant 0$
- $\int_{C_1 - C_2} f(z)dz = 0$
- $\int_{C_1 - C_2} f(z)dz \neq 0$
- $\int_{C_1 - C_2} f(z)dz \leqslant 0$

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MTH632:Quiz # 3

Quiz Start Time: 08:25 PM, 13 August 2021

Question # 4 of 10 (Start time: 08:27:54 PM, 13 August 2021)

Total Marks: 1

Let $f(z)$ be an analytic in a domain D and D be the interior and boundary of th circle 'r' defined as $|z-a|=r$ and $|f(z)| \leqslant M$, then $|f^{(n)}(a)| \leqslant \frac{n!M}{r^n}$

Select the correct option

Reload Math Equations

- ML-Inequality
- None of above
- Cauchy Inequality
- Cauchy Sichwarz Inequality

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Question # 8 of 10 (Start time: 07:58:14 PM, 13 August 2021)

Evaluate $\int_C f(z)dz$ where C:$f(z) = e^{it}$ for $0 \leqslant t \leqslant 2\pi$ and $f(z) = 1/z$

Select the correct option

- $2\pi i$
- πi
- $-2\pi i$
- $-\pi i$

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Question # 4 of 10 (Start time: 07:54:01 PM, 13 August 2021)

A continuous complex function f(z) defined on a set containing contour C and z(t) be any parameterization of C for a ≤ t ≤ b then

$$\int_C f(z)dz =$$

Select the correct option

○	$\int_a^b f(z)z(t)dz$
○	\int\limits_a^b {{f^'}(z)z(t)dz}
○	\int\limits_a^b {{f^'}(z){z^'}(t)dz}
○ ✓	\int\limits_a^b {f(z){z^'}(t)dz}

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Question # 5 of 10 (Start time: 07:55:33 PM, 13 August 2021)

If domain D is simply connected, then any contour of integral around simple closed curve is

Select the correct option

- -1
- 0
- 2
- 1

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MTH632:Quiz # 3

Question # 6 of 10 (Start time: 07:40:49 PM, 13 August 2021)

Evaluate the integral $\int_C \frac{e^z}{z^4} dz$ where the contour $C: \ |z| = 1$

Select the correct option

- $\frac{\pi i}{3}$
- $\frac{2\pi i}{3}$
- $\frac{4\pi i}{3}$
- $\frac{8\pi i}{3}$

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Question # 3 of 10 (Start time: 07:38:23 PM, 13 August 2021)

A domain D (connected open set) is called _____ connected if interior of every simple closed contour C is contained in D.

Select the correct option

- ○ simply
- ○ multiply

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Question # 5 of 10 (Start time: 08:10:59 PM, 13 August 2021)

If a function $f(z) = e^{iz}$ *and its derivatives are analytic,*

and C *is simple closed contour that bounds some domain which contain* z_o, *then* $\int_C \frac{f(z)}{(z - z_o)^4} dz$

Select the correct option

○	$\frac{2\pi}{3} e^{iz_o}$
○	$\frac{2\pi i}{3} e^{iz_o}$
○	$\frac{\pi}{3} e^{iz_o}$
○	$\frac{\pi i}{3} e^{iz_o}$

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MTH632 Mega File Final Term

Lec (23-45)

Topic (41-121) page# (99-288)

Objective Files (15)

- MTH632 Quiz#2 All in 1 File pg28
- MTH632 Quiz#2 All in 1 File pg47
- Paper Final MTH632 (by HSZS-legends) pg30
- Mth632-FINAL-2020.docx pg26
- Mth632 2 pg41
- Mth632 current paper khan pg33 unsolve
- Mth632 objective file final term by Hussnain Ali pg473
- Mth632 final term 2020 wajid pg33 solve
- Mth632 current Ppr Solution pg43
- MTH632 Final Term correct File pg288
- ZE+BA Group official File(MTH632) pg312 solve
- Mth632 current 6 march2021 pg29
- Mth632 Checked Pen Pals pg178
- MTH632 by Mudassar Iqbal.Lec(38-48) quiz-1 pg1
- MTH632-FINAL-TERM-MCQ's DATA-ABU-SULTAN-1-1 pg19

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MTH632 Complex Analysis and Differential Geometry

Question No :

If $\vec{x}(t)$ be a smooth space curve. Then which of the following is (are) known as the the Frenet formula (formulas)?

Answer (Please select your correct option)

- (a) $\vec{t}' = k\vec{n}$
- (b) $\vec{n}' = \tau\vec{b} - k\vec{t}$
- (c) $\vec{b}' = -\tau\vec{n}$
- (d) All (a), (b) and (c)

Start Time: 11:48 AM

20:00

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

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MTH632 Complex Analysis and Differential Geometry

Question No : 18 of 33

A triplet $\vec{q} = (q_1, q_2, q_3)$ is called a _____ of Euclidean 3-space E^3.

Answer (Please select your correct option)

- line
- point

Start Time 11:48 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13

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Question No : 9 of 20

The points at which $f(z) = \frac{1}{z^2+1}$ is not analytic are _____.

Answer (Please select your correct option)

- 1 and -1
- i and $-i$
- 1 and i
- -1 and $-i$

Start Time: 11:48 AM

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Question No : 5 of 35

The function $f(z) = |z|$ is non-constant _____.

Answer (Please select your correct option)

- analytic function
- nowhere analytic function
- non-analytic function
- entire function

Start Time: 11:48 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12

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MTH632 Complex Analysis and Differential Geometry

Question No : 11 of 20

The principal value of 1^{2i} is _____.

Answer (Please select your correct option)

- 0
- 1
- i
- $2i$

Start Time: 11:48 AM

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Question Summary : (Attempted Question)

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MTH632 Complex Analysis and Differential Geometry

Question No : 22 of 33

Let ϕ and ψ be two 1-forms then their sum is defined as _____ for all tangent vectors v.

Answer (Please select your correct option)

- $(\phi+\psi)(v)$
- $(\phi+\psi)(v)=(\psi)(v)$
- $(\phi+\psi)(v)=(\phi)(v)+(\psi)(v)$
- $(\phi+\psi)(v)=(\phi)(v)$

Start Time: 11:48 AM

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22

Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

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MTH632 Complex Analysis and Differential Geometry

Question No : 13 of 33

The circle $z = e^{i\theta}$, $0 \le \theta \le 2\pi$ is centered at _____.

Answer (Please select your correct option)

- (1,1)
- (1,0)
- (0,1)
- (0,0)

Start Time: 11:48 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12

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MTH632 Complex Analysis and Differential Geometry

Question No : 15 of 33

Let $f(z)=z$ be a continuous complex valued function, then $\int_0^{1+i} f(z)dz =$ _____

Answer (Please select your correct option)

- 1-i
- 0
- 1+i
- i

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14

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MTH632 Complex Analysis and Differential Geometry

Question No : 19 of 33

The poles of $\frac{1}{(z^2-1)^2}$ are _____.

Answer (Please select your correct option)

- $\pm i$
- ± 1
- ± 2
- $\pm 2i$

Start Time: 11:48 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11

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MTH632 Complex Analysis and Differential Geometry

Question No:

If p and q are points in R^3 then _____ is the symmetry property of dot product.

Answer (Please select your correct option)

- $p.q$
- $q.p$
- $p.q=q.p$
- $p.p=q.q$

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13

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MTH632 Complex Analysis and Differential Geometry

Question No : 16 of 23

Using the Cauchy–Goursat theorem when contour C is unit circle: $\int_C ze^{-z}dz =$ _____

Answer (Please select your correct option)

- i
- -1
- 1
- 0

Start Time: 11:48 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14

Conduct Instructions

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MTH632 Complex Analysis and Differential Geometry

Question No : 24 of 33

The arc length of α from $t=a$ to $t=b$ is equal to _____

Answer (Please select your correct option)

- $\int_a^b v(t)dt$
- $\int_a^b \|\alpha'\|dt = \int_a^b v(t)dt$
- $\int_a^b \|\alpha'\|dt$
- $\int_a^b \alpha' dt = \int_a^b v(t)dt$

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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MTH632 Complex Analysis and Differential Geometry

Question No : 17 of 20

By using the Extension of Cauchy Integral formula $\int_c \frac{f(z)}{(z-z_0)^{n+1}} dz =$ _____

Answer (Please select your correct option)

- $f^{(n)}(z_0)$
- $2\pi i f^{(n)}(z_0)$
- $\frac{f^{(n)}(z_0)}{n!}$
- $\frac{2\pi i}{n!} f^{(n)}(z_0)$

Start Time: 11:48 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14

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MTH632 Complex Analysis and Differential Geometry MC

Question No : 11 of 33 **Marks: 1 (Budgeted Time Min)**

The definite integral of a complex valued function $w(t) = u(t) + i\,v(t)$ over an interval $a \le t \le b$ is _____ , where u , v are real valued functions of a real variable t

Answer (Please select your correct option)

- $\int_a^b w(t)dt = \int_a^b u(t)\,v(t)dt$
- $\int_a^b w(t)dt = \int_a^b u(t)dt + i\int_a^b v(t)dt$
- $\int_a^b w(t)dt = \int_a^b u(t)dt - i\int_a^b v(t)dt$
- $\int_a^b w(t)dt = \int_a^b u(t)\,v(t)dt$

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MTH632 Complex Analysis and Differential Geometry

Question No : 1 of 33

In the complex valued function $U(x, y)$ is harmonic in some domain , if _____.

Answer (Please select your correct option)

- $u_{xx} - u_{yy} = 0$
- $u_{xx} + u_{yy} = 0$
- $u_{xx} = u_{yy} = 0$
- $u_{xx} = u_{yy}$

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

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MTH632 Complex Analysis and Differential Geometry

Question No :

The logarithmic function of $e^w = z$, where $z = re^{i\theta}$ and $w = u + iv$, then _____.

Answer (Please select your correct option)

- $w = \ln r + i(\theta + 2n\pi)$
- $w = re^{i\theta}$
- $w = r + i(\theta + 2n\pi)$
- $w = -\ln r + i\,\theta$

Start Time: 11:39 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21

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MTH632 Complex Analysis and Differential Geometry

Question No : 6 of 41

The limit of $f(z) = 2z^2 - 4z + 2$ at $z = i$ is _____

Answer (Please select your correct option)

- 5i
- - 4i ✓
- 2i
- -3i

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Question No : 8 of 41 **Marks: 1 (Budgeted**

If a function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D, then its component functions u and v are _____

Answer (Please select your correct option)

- harmonic in D.
- equal in D.
- homogeneous in D.
- steady in D.

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Question No : 11 of 41

If $f(z) = x + ay + i(bx + cy)$ is analytic, then a, b, c equals to _____.

Answer (Please select your correct option)

- $c=1$ and $a=-b$
- $a=1$ and $c=-b$
- $b=1$ and $a=-c$
- $a=b=c=1$

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Question No : 14 of 41

If $e^{ax}\cos y$ is harmonic then 'a' is _____.

Answer (Please select your correct option)

- 0
- -1
- 2
- i ✓

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MTH632 Complex Analysis and Differential Geometry

Question No : 16 of 41

If $f(z) = z(2-z)$, then $f(1+i) =$ _____.

Answer (Please select your correct option)

- 0
- 2
- i
- $-i$

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Question No : 21 of 41 Marks: 1 (Budgeted Time min)

The derivative of a complex valued function $w(t) = u(t) + i\,v(t)$ is _____ , where u , v are real valued functions of a real variable t.

Answer: (Please select your correct option)

- $w'(t) = u'(t) + iv'(t)$
- $w'(t) = u(t) + iv(t)$
- $w'(t) = u'(t)v'(t)$
- $w'(t) = u'(t) + v'(t)$

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Marks: 1 (Budgeted Time Min)

If the three vectors $\vec{t}$, $\vec{n}$ and $\vec{b}$ are perpendicular to each other and form the Frenet frame, then $\vec{b} \times \vec{t} =$ _____ .

Answer (Please select your correct option)

- $\vec{b}$
- $\vec{t}$
- $\vec{n}$ ✓
- $-\vec{n}$

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Question No : 22 of 41

If $w(t)$ is a piecewise continuous complex-valued function defined on an interval $a \le t \le b$, then _____.

Answer (Please select your correct option)

- (•) $\left|\int_a^b w(t)dt\right| \le \int_a^b |w(t)|dt$
- ( ) $\left|\int_a^b w(t)dt\right| \ge \int_a^b |w(t)|dt$
- ( ) $\left|\int_a^b w(t)dt\right| - \int_a^b |w(t)|dt$
- ( ) $\int_a^b w(t)dt \ge \left|\int_a^b |w(t)|dt\right|$

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MTH632 Complex Analysis and Differential Geome

Question No : 18 of 41

The principal value of i^i is _____.

Answer (Please select your correct option)

- $\exp(\frac{\pi}{2})$
- $\exp(-\frac{\pi}{2})$
- $\exp(i)$
- $\exp(-i)$

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MTH632 Complex Analysis and Differential Geometry

Question No : 30 of 41

Let $\beta : I \to R^1$ be a unit speed curve. The velocity $T = \beta'$, is said to be the ___

Answer (Please select your correct option)

- unit
- unit tangent vector field
- vector
- tangent

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MTH632 Complex Analysis and Differential Geometry

Question No : 19 of 41

Let Z be a complex valued function then $\tanh Z =$ _____

Answer (Please select your correct option)

- $\frac{e^{-z}}{e^{z}+e^{-z}}$
- $\frac{e^{z}+e^{-z}}{e^{z}-e^{-z}}$ coth z
- ✓ $\frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$ Tanh z
- $\frac{e^{-z}}{e^{z}-e^{-z}}$

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Marks: 1 (Budgeted Time Min)

Question No : 28 of 41

The differential of f is, $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$, where df calculates the small changes in the value of f when there is small change in _____.

Answer (Please select your correct option)

(a) x

(b) y

(c) z

(d) All (a), (b) and (c)

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Question No : 29 of 41

A differentiable function from R^n to R^m is known as a____

Answer (Please select your correct option)

- injective
- mapping
- surjective
- bijective

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Question No : 26 of 41 **Marks: 1 (Budgeted Time Min)**

The scalar multiple of a vector $\vec{x} = (x_1, x_2, x_3)$ by a real number k is the point $k\vec{x} = (kx_1, kx_2, kx_3)$, clearly _____ for al k and $\vec{x}$.

Answer (Please select your correct option)

- (a) $k\vec{x} = \vec{0}$
- (b) $0\vec{x} = \vec{0}$
- (c) $k\vec{0} = \vec{0}$
- (d) both (b) and (c)

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MTH632 Complex Analysis and Differential Geometry

Question No : 25 of 41

If a function f is entire and bounded in the complex plane, then $f(z)$ is ___

Answer (Please select your correct option)

- imaginary
- real
- variable
- constant

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Question No : 24 of 41

Let C be the positively oriented circle $|z| = 2$. Since the function $f(z) = \dfrac{z}{9 - z^2}$ is analytic within and on C and since the point $z_0 = -i$ is interior to C, then using the Cauchy Integral formula $\displaystyle\int_C \frac{z}{(9 - z^2)(z + i)} dz =$ _____

Answer (Please select your correct option)

- $\dfrac{\pi}{9}$
- $\dfrac{\pi}{5}$
- 2π
- $2\pi i$

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MTH632 Complex Analysis and Differential Geometry

Question No : 20 of 41

The circle $z = e^{i\theta}$, $0 \le \theta \le 2\pi$ is centered at _____.

Answer (Please select your correct option)

- (1,1)
- (1,0)
- (0,1)
- (0,0)

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MTH632 Complex Analysis and Differential Geometry

Question No : 17 of 41

In the trigonometric functions of complex numbers $Sin\ x =$ _

Answer (Please select your correct option)

- $\frac{e^{ix} - e^{-ix}}{2i}$
- $\frac{e^{ix} - e^{-ix}}{2}$
- $\frac{e^{-ix} - e^{ix}}{2i}$
- $\frac{e^{-ix} - e^{ix}}{2}$

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Question No : 15 of 41 Marks: 1 (Budgeted Tim

A series converges to a number S if and only if the sequence of remainders tends to _____.

Answer (Please select your correct option)

- infinity
- one
- zero

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Question No : 13 of 41

If U and V are harmonic functions then $f(Z) = U + i\,V$ is _____.

Answer (Please select your correct option)

- analytic function.
- need not be analytic function.
- analytic function only at Z=0.
- none of the others.

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Question No : 12 of 41 **Marks: 1**

If a series of complex numbers converges, the n^{th} term converges to zero as n tends to _____.

Answer (Please select your correct option)

- infinity
- finite numbers

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MTH632 Complex Analysis and Differential Geometry

Question No : 9 of 41

If Z_1 and Z_2 are two nonzero complex numbers then $Log(z_1 z_2)$ = _____.

Answer (Please select your correct option)

- $Log\, z_1\, Log\, z_2$
- $Log\, z_1 / Log\, z_2$
- $Log\, z_1 - Log\, z_2$
- $Log\, z_1 + Log\, z_2$

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Question No : 7 of 41

Let $f(z) = z^3 - 3z + 12$, then $f'(z)$ is ____

Answer (Please select your correct option)

- $z^2 - 3$
- $3z^2 - 3$
- $z^2 - 1$
- $z^3 - 3$

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MTH632 Complex Analysis and Differential Geometry

Question No : 3 of 41

The complex numbers $f(z) = 3iz/5$ then $\lim_{z \to 10} f(z) =$ ____

Answer (Please select your correct option)

- $6i$
- $2i$
- $3i$
- $10i$

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MTH632 Complex Analysis and Differential Geometry

Question No : 5 of 41

In the complex valued function $z = x - iy$, the value of $U_x =$ ___

Answer (Please select your correct option)

- -1
- 1
- 0
- 2

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Question No : 23 of 41 **Marks: 1 (Budgeted**

Using the Cauchy–Goursat theorem when contour C is unit circle $\int_C \frac{z^2}{z-3} dz =$ _____

Answer (Please select your correct option)

- 3
- 0 ✓
- 1
- -3

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MTH632 Complex Analysis and Differential Geometry

Question No : 1 of 41

The sum of two complex numbers, $z = 3 + i$, $w = -1 + 2i$ is, $z + w =$ ____

Answer (Please select your correct option)

- $2 + 3i$
- $3 + 2i$
- $-1 + i$
- $-3 + 2i$

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MTH632 Complex Analysis and Differential Geometry

Question No : 10 of 41

The sequence $z_n = \frac{i}{n^2}$ $(n = 1, 2, 3, \ldots)$ for $n \to \infty$ converges to _____ .

Answer (Please select your correct option)

- 1
- 0
- infinity
- 1/2

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MTH632 Complex Analysis and Differential

Question No : 4 of 41

The reciprocal of the number, i is _____

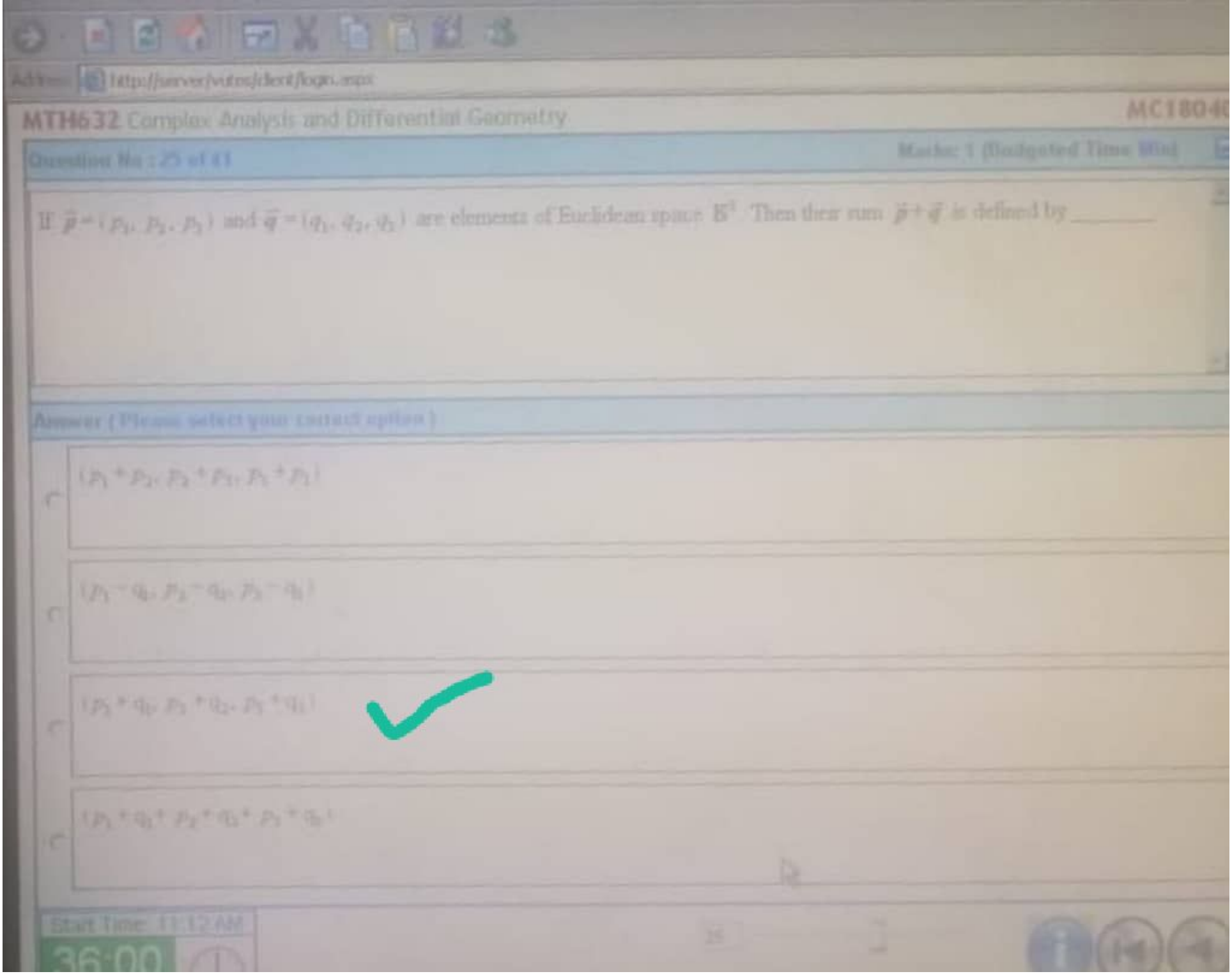
Answer (Please select your correct option)

- ○ -1
- ○ 1
- ○ i
- ◉ $-i$

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Question No : 26 of 41 **Marks: 1 (Budgeted Time Min**

Let $f(z)$ be analytic function and C is contour enclosing point z_0, then Cauchy Integral formula states $f(z_0)=$ -

_____.

Answer (Please select your correct option)

- $\frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_0}dz$ ✓
- $\int_C \frac{f(z)}{z-z_0}dz$
- $\frac{1}{2\pi i}\int_C \frac{f(z)}{z+z_0}dz$
- $\int_C \frac{f(z)}{z+z_0}dz$

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Question No : 21 of 41 **Marks: 1 (Budgeted Time Min)**

The answer of definite integral of a complex valued function $w(t) = \int_{1}^{3} t\,dt$ is _____

Answer (Please select your correct option)

- 1
- 2
- 4 ✓
- 0

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EXAMPLE 2. The unit circle

$$z = e^{i\theta} \qquad (0 \le \theta \le 2\pi) \tag{5}$$

about the origin is a simple closed curve, oriented in the counterclockwise direction. So is the circle

$$z = z_0 + Re^{i\theta} \qquad (0 \le \theta \le 2\pi), \tag{6}$$

centered at the point z_0 and with radius R (see Sec. 6).

The same set of points can make up different arcs.

EXAMPLE 3. The arc

$$z = e^{-i\theta} \qquad (0 \le \theta \le 2\pi) \tag{7}$$

is not the same as the arc described by equation (5). The set of points is the same, but now the circle is traversed in the *clockwise* direction.

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Question No : 20 of 41

The circle $z = e^{i\theta}$, $0 \le \theta \le 2\pi$ is oriented in _____ direction.

Answer (Please select your correct option)

- upward
- downward
- counter-clockwise
- clockwise

$e^{i\theta}$ ✓

$e^{-i\theta}$ ✗

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Question No : 17 of 41

For hyperbolic functions of complex numbers $Cosh\ x$ = _____.

Answer (Please select your correct option)

- ( ) $\frac{e^{x} + e^{-x}}{2i}$
- ( ) $\frac{e^{x} - e^{-x}}{2i}$
- ( ) $\frac{e^{-x} - e^{x}}{2}$
- ( ) $\frac{e^{x} + e^{-x}}{2}$ ✓

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Question No : 15 of 41 **Marks: 1 (Budg**

The absolute convergence of a series of complex numbers implies the convergence of that _____.

Answer (Please select your correct option)

- sequence
- series

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Question No : 14 of 41

Harmonic conjugate of $u(x, y) = e^y \cos x$ is _____.

Answer (Please select your correct option)

- $e^x \cos y + c$
- $e^x \sin y + c$
- $e^y \sin x + c$
- $-e^y \sin x + c$ ✓

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Question No : 11 of 41

A point at which a function ceases to be analytic is called a _____.

Answer (Please select your correct option)

- singular point
- non-singular point
- regular point
- non-regular point

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Question No : 10 of 41 **Marks: 1 (Budgeted Time Min)**

Any polynomial $p(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n$ $(a_n \neq 0)$ of degree n $(n \geq 1)$ has at least one zero. That is, there exists at least _____ z_0, such that $P(z_0) = 0$.

Answer (Please select your correct option)

- one point
- n points
- no points
- 10 points

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MTH632 Complex Analysis and Differential Geometry

Question No : 6 of 41 **Marks: 1 (Bud**

In the complex valued function $f(z) = \frac{1}{re^{i\theta}}$, the value of U_θ = _____.

Answer (Please select your correct option)

- e^{ir} ✔
- $\sin\theta$
- $\frac{\sin\theta}{r}$
- $-\frac{\sin\theta}{r}$

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Question No : 7 of 41

Let $f(z) = 2z^3 + 2z - 7$, then $f'(z)$ is _____

Answer (Please select your correct option)

- $6z^2 + 2$
- $z^2 + 2$
- $2z^2 - 7$
- $2z^3 + 2$

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MTH632 Complex Analysis and Differential Geometry

Question No : 4 of 41

The absolute value of the number, $z = 1-i$ is _____

Answer (Please select your correct option)

- 2
- $\sqrt{2}$
- 0
- 1

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MTH632 Complex Analysis and Differential Geometry

Question No : 5 of 41

The magnitude of $\exp(-2+3i)$ is _____

Answer (Please select your correct option)

- exp(-2*3)
- exp(2*3)
- exp(3)
- exp(-2) ✓

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MTH632 Complex Analysis and Differential Geometry

Question No : 3 of 41

The conjugate of complex number $z = -3 - 3i$ is _____.

Answer (Please select your correct option)

- $3 + 3i$
- $-3 + 6i$
- $3 - 3i$
- $-3 + 3i$ ✓

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MTH632 Complex Analysis and Differential Geometry

Question No : 2 of 41

The modulus of $z = 3+2i$ is _____.

Answer (Please select your correct option)

- $\sqrt{5}$
- $\sqrt{6}$
- $\sqrt{12}$
- $\sqrt{13}$ ✓

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Question No : 3 of 33

The sequence $z_n = -2 + i\frac{1}{n^3}$ $(n = 1, 2, 3, \ldots)$ for $n \to \infty$ converges to _____.

Answer (Please select your correct option)

- C i
- C -1
- C 1
- C -2 ✓

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MTH632 Complex Analysis and Differential Geometry

Question No : 7 of 33

A function U is said to be harmonic if and only if _____.

Answer (Please select your correct option)

- $U_x + U_y = 0$
- $U_x + U_y = 0$
- $U_x^2 + U_y^2 = 0$
- $U_{xx} + U_{yy} = 0$ ✓

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

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Question No : 9 of 33

If $f(z) = |\bar{z}|$, then $f(3-4i) =$ _____.

Answer (Please select your correct option)

- 0
- 5 ✓
- -5
- 12

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Question Summary : (Attempted Question)

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MTH632 Complex Analysis and Differential Geometry

Question No : 12 of 33

Let Z be a complex valued function then $\arcsin(Z) =$ _____.

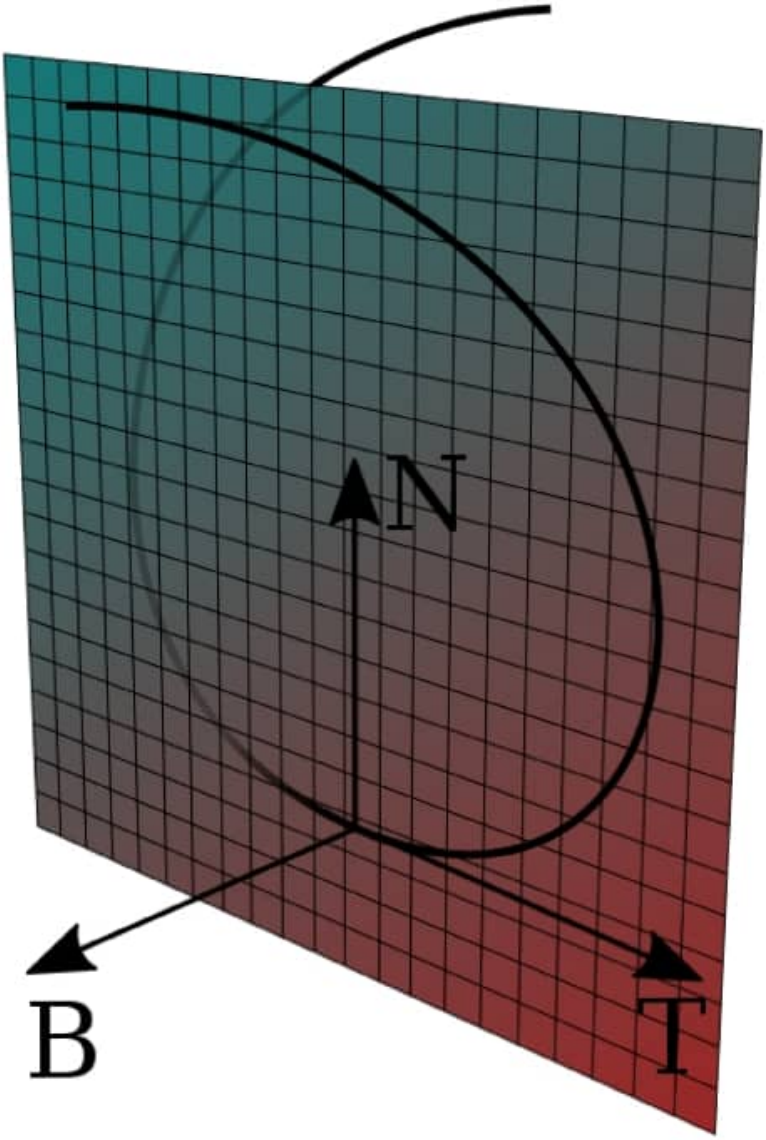
Answer (Please select your correct option)

- $\log(iz + (1-z^2)^{1/2})$
- $-i\log(iz + (1-z^2)^{1/2})$ ✓
- $\log(iz - (1-z^2)^{1/2})$
- $-i\log(iz - (1-z^2))$

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A space curve; the vectors **T**, **N** and **B**; and the osculating plane spanned by **T** and **N**

The tangent, normal, and binormal unit vectors, often called **T**, **N**, and **B**, or collectively the **Frenet–Serret frame** or **TNB frame**, together form an orthonormal basis spanning $\mathbf{R}^3$ and are defined as follows:

- **T** is the unit vector tangent to the curve, pointing in the direction of motion.
- **N** is the normal unit vector, the derivative of

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A space curve; the vectors **T**, **N** and **B**; and the osculating plane spanned by **T** and **N**

The tangent, normal, and binormal unit vectors, often called **T**, **N**, and **B**, or collectively the **Frenet–Serret frame** or **TNB frame**, together form an orthonormal basis spanning $\mathbf{R}^3$ and are defined as follows:

- **T** is the unit vector tangent to the curve, pointing in the direction of motion.
- **N** is the normal unit vector, the derivative of **T** with respect to the arclength parameter of the curve, divided by its length.
- **B** is the binormal unit vector, the cross product of **T** and **N**.

The Frenet–Serret formulas are:

$$\begin{aligned}\frac{d\mathbf{T}}{ds} &= \kappa\mathbf{N},\\ \frac{d\mathbf{N}}{ds} &= -\kappa\mathbf{T} + \tau\mathbf{B},\\ \frac{d\mathbf{B}}{ds} &= -\tau\mathbf{N},\end{aligned}$$

where d/ds is the derivative with respect to

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MTH632 Complex Analysis and Differential Geometry

Question No : 20 of 33 Mar

If the three vectors $\vec{t}$, $\vec{n}$ and $\vec{b}$ are perpendicular to each other and form the Frenet frame, then $\vec{t} \times \vec{n} =$ _____.

Answer (Please select your correct option)

- $-\vec{b}$
- $\vec{b}$ ✓
- $\vec{t}$
- $\vec{n}$

$t \times n = b$, $n \times t = -b$

~~$n \times b = t$, $b \times n = -t$~~

$b \times t = n$, $t \times b = -n$

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Question Summary (Attempted Question)

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MTH632 Complex Analysis and Differential Geometry

Question No : 21 of 33

The restriction of a curve $\alpha : I \to R^1$ to a close interval $[a, b]$ is known as a _____.

Answer (Please select your correct option)

- curve
- segment
- line
- curve segment ✓

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

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MTH632 Complex Analysis and Differential Geometry Marks:

Question No : 22 of 33

An important consequence of the wedge product is that "repeats are zero" due to "alternation rule", that means _____.

Answer (Please select your correct option)

- (a) $dx \wedge dx = 0$
- (b) $dy \wedge dy = 0$
- (c) $dz \wedge dz = 0$
- (d) All (a), (b) and (c)

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MTH632 Complex Analysis and Differential Geometry

Let Z be a complex valued function then $\sinh Z =$ _____

Answer (Please select your correct option)

- ( ) $\frac{e^{z} - e^{-z}}{2}$ ✔
- ( ) $\frac{e^{z} + e^{-z}}{2}$
- ( ) $\frac{e^{z} - e^{-z}}{2i}$
- ( ) $\frac{e^{z} + e^{-z}}{2i}$

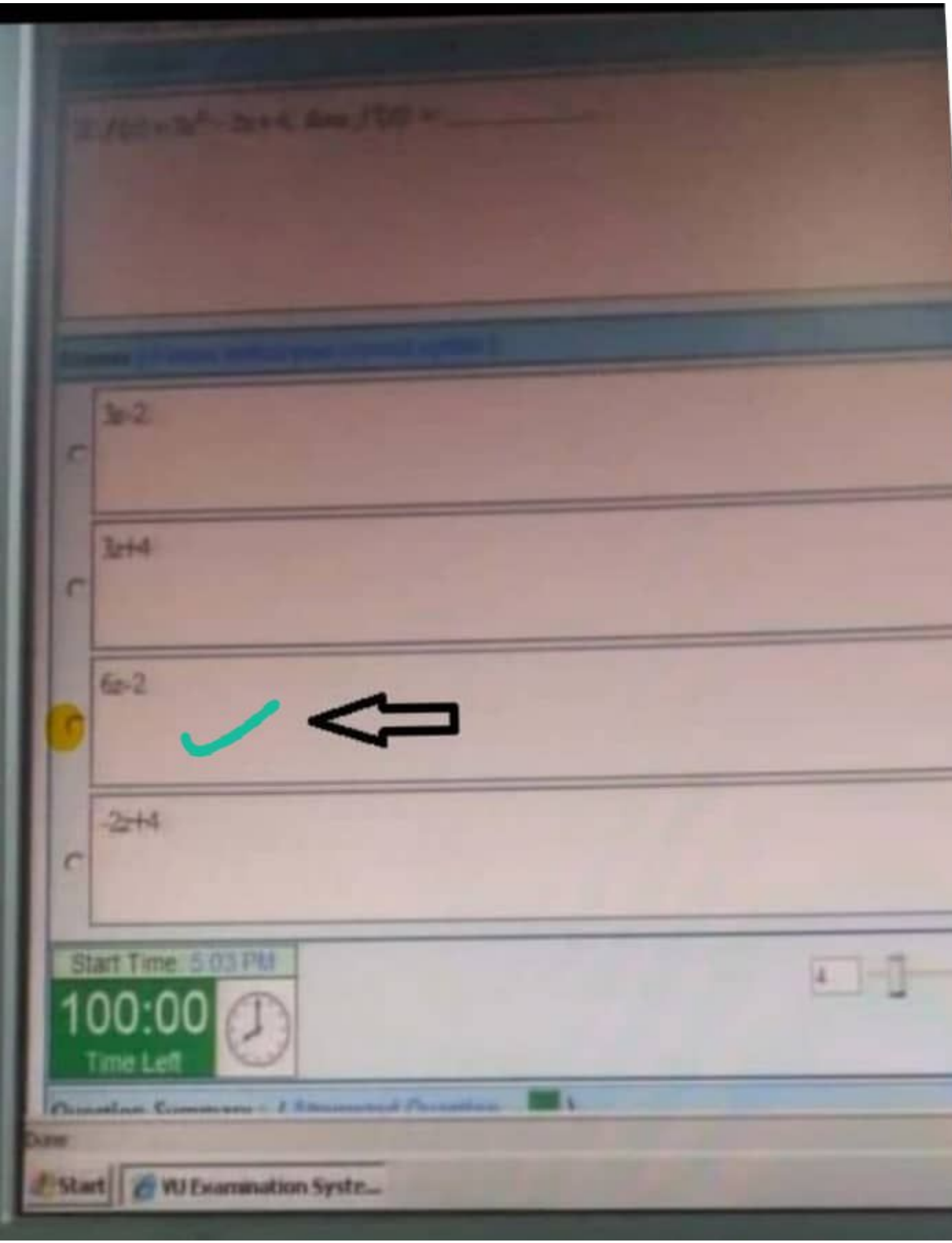
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Question Summary : (Attempted Question)

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Let $f(x) = \frac{1}{x^2+1}$ is discontinuous at $x =$ _____

Answer (Please select your correct option)

- $\pm i$
- ± 1
- ± 2
- ± 3

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MTH632 Complex Analysis and Differential Geometry

In the trigonometric functions of complex numbers $Cos\, x =$ _

Answer (Please select your correct option)

- $\frac{e^{ix}+e^{-ix}}{2i}$
- $\frac{e^{ix}+e^{-ix}}{2}$ ⇦ ✓
- $\frac{e^{-ix}-e^{ix}}{2i}$
- $\frac{e^{-ix}-e^{ix}}{2}$

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Complex Analysis and Differential Geometry

The Cauchy-Goursat theorem shows that _____

Answer | Please select your correct option |

- $\int_c f(z)dz = -1$
- $\int_c f(z)dz = 2\pi i$
- $\int_c f(z)dz = 0$
- $\int_c f(z)dz = 1$

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MTH632 Complex Analysis and Differential Geometry

Question No : 21 of 49 Marks: 1 (Budgeted Time 1 M

Let C denote a contour of length L, and suppose that a function $f(z)$ is piecewise continuous on C. If M is a nonnegative const such that $|f(z)| \leq M$ for all points z on C at which $f(z)$ is defined, then _____

Answer (Please select your correct option)

- $\int_C f(z)dz \geq ML$
- $\left|\int_C f(z)dz\right| \geq ML$
- $\left|\int_C f(z)dz\right| \leq ML$ ✓
- $\int_C f(z)dz \leq ML$

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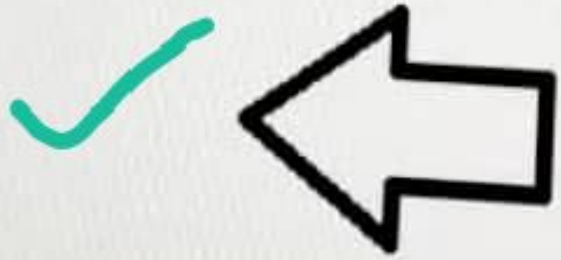
MTH632 Complex Analysis and Differential Geometry

Question No : 8 of 33

Any function which is analytic at a point z_0 must have a _____ series about z_0

Answer (Please select your correct option)

- Maclaurin's
- Taylor's



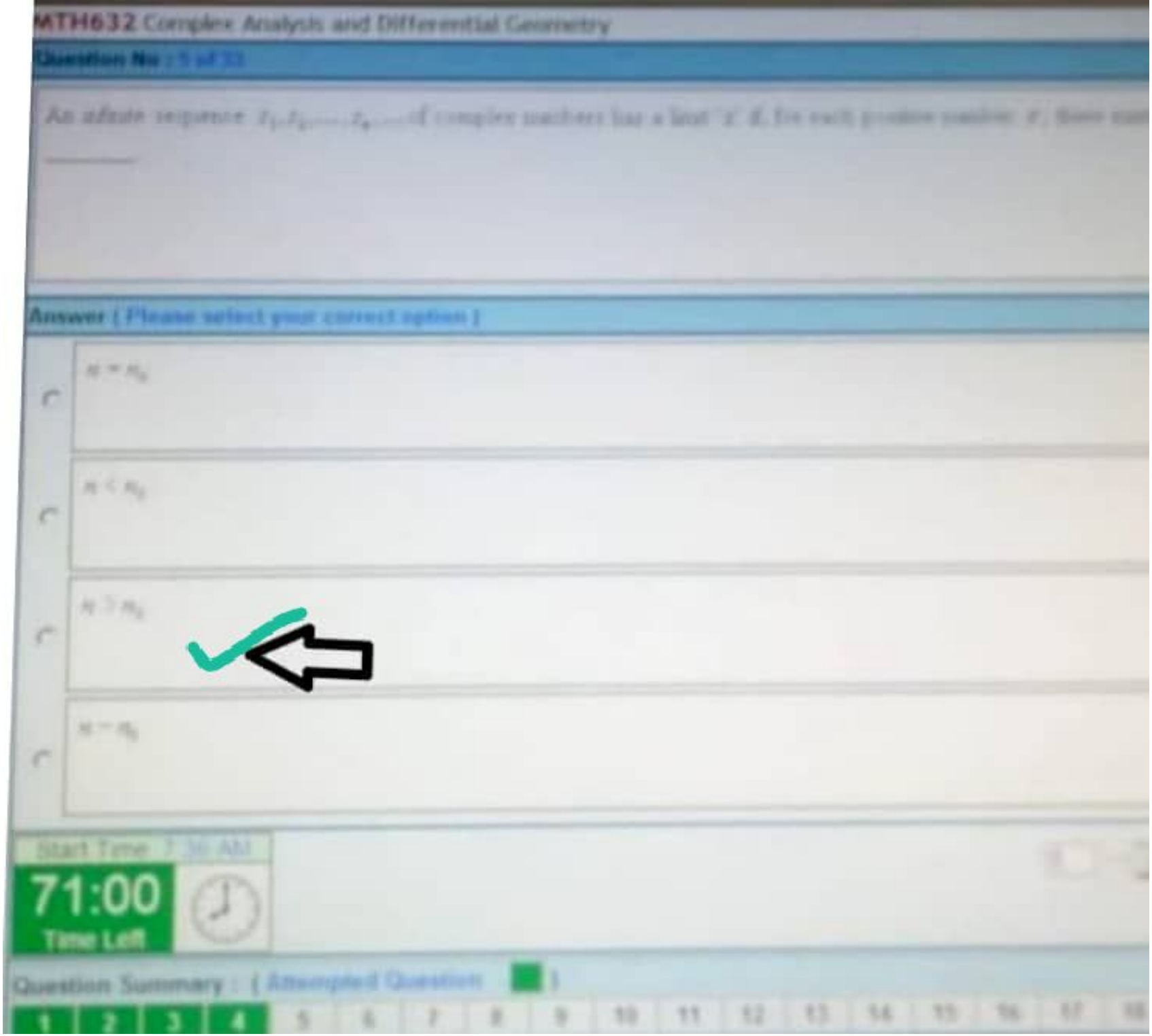
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Question Summary : (Attempted Question)

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MTH632 Complex Analysis and Differential Geometry

Question No : 17 of 33

Let C be the positively oriented circle $|z| = 2$. Since the function $f(z) = \frac{z}{9 - z^2}$ is analytic within and on C and

Cauchy Integral formula $\int_C \frac{z}{(9 - z^2)(z + 1)} dz =$ _____

Answer (Please select your correct option)

- $\frac{\pi}{9}$
- $\frac{\pi}{5}$ ✓
- 2π
- $2\pi i$

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Question Summary : (Attempted Question)

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MTH632 Complex Analysis and Differential Geometry

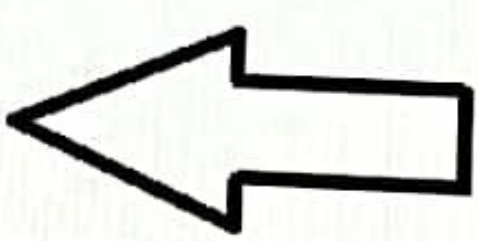
Question No : 23 of 33

A function $F: \boldsymbol{R}^n \to \boldsymbol{R}^m$ is differentiable if all its Euclidean coordinate functions are

Answer (Please select your correct option)

integrable

differentiable



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Question Summary : (Attempted Question)

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MTH632 Complex Analysis and Differential Geometry

Question No : 4 of 33

A function $f(z)$ is analytic function if _____ part $f(z)$ is analytic

Answer (Please select your correct option)

(a) real

(b) imaginary

(c) both (a) and (b)



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Question Summary : (Attempted Question)

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MTH632 Complex Analysis and Differential Geometry

Question No : 15 of 33

Let $f(z)=z$ be a continuous complex valued function, then $\int_0^{1+i} f(z)dz =$ _____

Answer (Please select your correct option)

- 1-i
- 0
- 1+i
- i ✓

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14

Conduct Instructions Click here

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	Module # 001
1.	Complex Analysis related to complex numbers and their functions. We have different types of numbers which help us in different solutions. Natural Number, Integers, Rational Number i.e. $\mathbb{Q} = \{\frac{a}{b} : a, b \epsilon \mathbb{Z}, b \neq 0\}$, Irrational Numbers that cannot be written as $\frac{p}{q}$ where $p, q \epsilon \mathbb{Z}$.
2.	The decimal expression is non-terminating and non-periodic.
3.	Real Numbers: Union of rational and irrational numbers are called real numbers i.e. $\mathbb{R} = \mathbb{Q} U \mathbb{Q}'$
4.	Real numbers are not enough to solve our all problems if we consider an equation $x^2+2x+2=0$ then we get $x = \sqrt{-1}$ so we denote it i i.e. $i = \sqrt{-1}$
5.	By including i we get numbers of the form $a + ib : a, b \epsilon \mathbb{R}$ and such numbers are called complex numbers.
6.	"Cardano" was first to introduce complex numbers $a + \sqrt{-1}\, b$ into algebra but had misgivings about it.
7.	"Rafael Bombelli authored l'Algebra (1572 and 1579) a set of three books "At first , the things seemed to me to be based more on sophism than on truth, but I searched until I found the proof".
8.	L.Euler (1701-1783) introduced the notation $i = \sqrt{-1}$
	Module # 002
9.	A complex number z can be defined as an ordered pair z=(x,y) where x,y are real numbers.
10.	Complex numbers can be interpreted as points in the complex plane.
11.	Real numbers is a subset of complex numbers.
12.	When real numbers x are displayed as points (x,0) on the real axis the set of complex numbers includes the real numbers as a subset.
13.	Complex numbers of the form (0,y) corresponds to y-axis and are pure imaginary numbers when $y \neq 0$. The y-axis is then referred to the imaginary axis.
14.	For complex number z=(x,y) the real numbers x,y are known as the real and imaginary parts of z respectively and we write as x=Re z and y=Im z
15.	Two complex numbers z1 and z2 are equal whenever they have the same real parts and the same imaginary parts.
16.	The statement z1=z2 means that z1 and z2 corresponds to same point in the complex plane or z-plane.
17.	Let z1=(x1,y1), z2=(x2,y2) then z1=z2 iff x1=x2 and y1=y2.
	Module # 003
18.	Two ways to represent complex numbers are Addition of Complex numbers and Product of Complex Numbers.

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19.	Commutative Law w.r.t Addition and Associative Law w.r.t Addition holds in Complex Numbers.
20.	**Addition of two complex numbers:** For z1=(x1,y1), z2=(x2,y2) the addition is defined as z1+z2=(x1,y1,)+(x2,y2,)=(x1+x2, y1+y2)
21.	**Product of two complex numbers:** For z1=(x1,y1), z2=(x2,y2) the product is defined as z1.z2=(x1,y1,).(x2,y2,)=(x1x2-y1y2, x1y2+x2y1)
22.	Any complex number z=(x,y) can be written as z=(x,y)=(x,0)+(0,y)
23.	Any complex number z=(x,y) can be written as z=(x,y)=(x,0)+(0,y), Also we have (0,1)(y,0)=(0,y) so we can write z=(x,0)+(0,1)(y,0). The numbers of the form (x,0) are real numbers so can simply denoted by x, numbers of the form (y,0) are real numbers so can simply denoted by y and (0,1) is the number denoted by "i" then z=(x,0)+(0,1)(y,0)=x+iy
24.	(0,1) is such a complex number when it is multiplied with itself then it gives -1. i.e (0,1)(0,1)=(0-1,0+0)=(-1,0)=-1or i^2=-1
25.	Z=x+iy is used to represent the complex numbers and this representation is due to the mathematician Gauss.
	Module # 004
26.	Properties of Addition of Complex Numbers: i. The addition of complex numbers follows all the properties of real numbers. ii. **Additive Identity**: There is a unique complex number 'w' such that z+w=w+z=z such number w is 0+i0 or (0,0). iii. **Additive Inverse**: For every complex number z=x+iy there is a unique ξ such that z+$\xi = 0$, obviously $\xi = -x - iy$ and is known as additive inverse.
27.	Complex numbers can be represented by two ways. i. Ordered pair representation ii. z=x+iy form. Ordered Pair Representation of complex numbers is the real foundational definition on which complex number system is built while z=x+iy is used or preferred due to simplicity.
	Module # 005
28.	Properties of Multiplication of Complex Numbers: i. The product of complex numbers follows all the properties of real numbers. ii. The commutative law z1z2=z2z1. The commutative implies iy=yi so z=x+iy and z=x+yi are same. iii. The associative law z1(z2z3)=(z1z2)z3. We can write z1z2z3 without parenthesis. iv. **Multiplicative Identity**: There is a unique complex number 'w' such that zw=wz=z such number w is 1+i0 or (1,0). iv. **Multiplicative Inverse**: For every complex number z=x+iy other than (0,0) there is a unique z^{-1} such that $zz^{-1} = 1$, In fact $z^{-1} = \frac{1}{z} = \frac{1}{x+iy}$

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29.	Division of Complex numbers can be calculated using $\frac{z1}{z2} = \left(\frac{x1x2+y1y2}{x2^2+y2^2}, \frac{y1x2-x1y2}{x2^2+y2^2}\right)$
30.	**Distributive Law**: The Operation of multiplication is distributive over addition that is z1(z2+z3)=z1z2+z1z3
31.	**Binomial Formula**: For any two complex numbers z1, z2 we have $(z1+z2)^n=\sum_{k=0}^{n}\binom{n}{k} z_1^k z_2^{n-k}$ $for\ n = 1,2,3, \ldots \ldots Here\ \binom{n}{k} = \frac{n!}{k!(n-k)!}$
	Module # 006
32.	A complex number 'z' can be associated with the directed line segment or vector from origin to the point of z.
33.	Multiplication of two complex number is neither dot product nor vector product.
34.	The vector interpretation helps us defining modulus or absolute value of complex number.
35.	The modulus or absolute value of a complex number z=(x,y)=x+iy is defined and denoted as $\lvert Z\rvert = \sqrt{x^2+y^2}$
	Module # 007-008
36.	Properties of modulus of a complex number: An important consequence of triangular inequality is $\lvert Z_1 + Z_2\rvert \geq \big\lvert\lvert Z_1\rvert - \lvert Z_2\rvert\big\rvert$
37.	The complex conjugate or simple conjugate of a complex number z=x+iy is denoted by$\bar{z}$ and is given by; $\bar{z} = (x,-y) = \overline{(x+iy)} = x - iy$
38.	For two complex numbers z1 and z2 we have $\overline{z1+z2} = \overline{z1} + \overline{z2}, \overline{z1-z2} = \overline{z1} - \overline{z2}, \overline{z1z2} = \overline{z1}\,\overline{z2}$ and $\overline{\left(\frac{z1}{z2}\right)} = \frac{\overline{z1}}{\overline{z2}}$ if $\overline{z2} \neq 0$
39.	Complex conjugate of z=x+iy are Re(z)=x and Im(z)=y
40.	As z=x+iy and $\bar{z}$=x-iy so $z + \bar{z} = 2x\ or\ x = \frac{z+\bar{z}}{2}\ or\ Re(z) = \frac{z+\bar{z}}{2}\ and\ z - \bar{z}$=2iy or y=$\frac{z-\bar{z}}{2i}\ or\ Im(z) = \frac{z-\bar{z}}{2i}$
41.	By following above point(in 40) we can prove that $Re(iz) = -Im(z)\ and\ Im(iz) = Re(z)$
42.	Complex conjugate helps us in proving properties of modulus . If z=x+iy then $\lvert z\rvert = \sqrt{x^2+y^2}$
43.	$\lvert z\rvert^2 = z\bar{z}$
44.	Complex conjugate fulfill triangular inequality property i.e. $\lvert z_1 + z_2\rvert \leq \lvert z_1\rvert + \lvert z_2\rvert$
	Module # 009
45.	The number z=x+iy can be written in polar form as z=(r$Cos\theta, rSin\theta$) $or\ z = r(Cos\theta + iSin\theta)$ and the values of r and θ are known as polar coordinates of Z.
46.	If z=0 then coordinates of θ are undefined.
47.	The length of $r = \lvert z\rvert$ is not allowed to be negative since it is length of radius vector for z.

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48.	The real number θ represents the angle measured in radians that z makes with positive x-axis.
49.	Polar representation of z=1+i is $z = \sqrt{2}\left(Cos\left(\frac{\pi}{4}\right) + Sin\left(\frac{\pi}{4}\right)\right)$**(PREPARE EXAMPLE P#25)**
50.	For a complex number z=(x,y) the coordinates θ has infinite number of values. All these values differ by integral multiples of 2π
51.	**Define Arguments of Z=(x,y)** : If $z \neq 0$, arg(z)={ θ :z=r(Cos θ +i Sin θ)}
52.	If $z \neq 0$ be a complex number then principal value of argument of z is denoted by Arg z is a unique value of θ such that z=r(Cos θ +i Sin θ) and $-\pi < \theta \leq \pi$
53.	Principal value of Arg z is also referred as "The argument of z" so we can say that arg z=Arg z +2 $n\pi$ where n is any integer.
54.	If x=0 then we can not use $tan\theta = \left(\frac{y}{x}\right)$ to find arg z.
55.	If $z \neq 0$ and x=0 then we use the following rule Arg z=$\frac{\pi}{2}$ if Im z>0 and Arg z=-$\frac{\pi}{2}$ if Im z<0
	Module # 010
56.	The expression $e^{i\theta}$ *or* exp$(i\theta)$ is defined by means of Euler formula as $e^{i\theta} = Cos\,\theta + i\,Sin\theta$
57.	The exponential of complex number in Euler formula is $z = re^{i\theta}$
58.	Euler's Identity: For $\theta = \pi$ we get z=$e^{i\pi} = Cos\,\pi + iSin\pi = -1$ so we have $e^{i\pi} + 1 = 0$ this is known as Euler's Identity.
	Module # 011
59.	The expression $(Cos\,\theta + i\,Sin\theta)^n = Cos\,n\theta + iSinn\theta$ where n=0,$\pm$1, $\pm$2, $\pm$3, $\pm$4, ... is known as De-Moivre's Formula.
	Module # 012-013
60.	A number u is said to be an nth root of complex number z if u^n=z
61.	If p(z) is a polynomial of degree n (n>0) with complex coefficient then equation p(z)=0 has precisely n solutions.
62.	There are precisely n number of nth roots of a complex number z_0.
63.	We denote the set of all nth roots of a complex number z0 by $z_0^{\frac{1}{2}}$
64.	For positive real number r_0 the symbol $\sqrt[n]{r_0}$ is reserved for one positive root.
65.	Equality of complex number in exponential form is $z = re^{i\theta}$

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66.	Consider a complex number $z_0 = r_0 e^{i\theta_0}$ then the nth root of z is given by $z = \sqrt[n]{r_0}\, exp\left[i\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right], k = 9, \pm1, \pm2, \pm3, \pm4, - - - -$
Module # 014-016	
67.	Equation of a circle of radius ε and center z_0 is given by $\lvert z - z_0\rvert = \varepsilon$
68.	A closed disk with center z_0 and radius ε is defined by $\lvert z - z_0\rvert \leq \varepsilon$
69.	A open disk with center z_0 and radius ε is defined by $\lvert z - z_0\rvert < \varepsilon$
70.	A point z_0 is **interior point** of a set S whenever there is a neighborhood of z_0 contained in S.
71.	A point z_0 is **exterior point** of a set S whenever there is a neighborhood of z_0 not contained in S.
72.	A point z_0 is **boundary point** of a set S if it is neither an interior point nor an exterior point.
73.	A set S is called an **Open Set** if every point of S in an interior point of S.
74.	A set S is called a **Closed Set** if it contains all its boundary points.
75.	A point z_0 is called an **accumulation Point** of a set S is each deleted neighborhood of z_0 contains at least one point of S.
76.	If a set is closed then it contains all its accumulation points.
77.	**Closure of a set** S is the closed set consisting of all points in S together with boundary points.
78.	Some sets are neither open nor closed.
79.	Some sets are open as well as closed.
80.	A curve is piece of string placed on a flat surface.
81.	A curve C is the range of a function given by z(t)=(x(t),y(t))=x(t)+iy(t) where t∈ $[a, b]$
82.	A curve is called **simple** if it does not cross itself.
83.	A set S is said to be **connected set** if for every pair of points $z1, z2 \in S$ there is a curve C joining them which is entirely contained in S.
84.	A connected open set is called a domain.
85.	A set is said to be **bounded set** if it can be completely contained in some closed disk.
86.	A set which is not bounded is called unbounded.
Module # 29	

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87.	Continuity means without any break or sudden jump or holes at any point.
88.	If the small change in the input of a function brings to a small change in the output of that function then function is continuous
89.	A function f(x) is continuous at a point X_0 if • $\lim_{x\to x_0} f(x)$ Exists. • $f(x_0)$ Exists. • $\lim_{x\to x_0} f(x) = f(x_0)$
90.	A function is said to be continuous on a set A if it is continuous at each point of A.
91.	The mean value theorem for complex numbers does not hold.
92.	In order to calculate limit at a point x_0 function f(x) must be defined in neighborhood of x_0.
93.	The function $f(x) = \{^{x^2+1}_{x} \; ^{x\leq 0}_{x>0}$ is continuous at x=1 but not continuous at x=0
94.	Let U(x,y) be a real valued function of the two real variables x and y then U is continuous at a point (x_0, y_0) if the following three conditions are satisfied; • $\lim_{(x,y)\to(x_0,y_0)} U(x,y)$ Exists. • $U(x_0, y_0)$ Exists. • $\lim_{(x,y)\to(x_0,y_0)} U(x,y) = U(x_0, y_0)$
95.	The function $U(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} = 0 & (x,y) \neq (0,0) \\ 0 & (x,y) \neq (0,0) \end{cases}$ is continuous at (0,0)
96.	A complex function f(z) of complex variable z for all values of z in some neighborhood of z_0 is continuous at z_0 if it satisfied the following three conditions; • $\lim_{z\to z_0} f(z)$ Exists. • $f(z_0)$ Exists. • $\lim_{z\to z_0} f(z) = f(z_0)$
97.	For every $\varepsilon > 0$ *there exist a* $\delta > 0$ such that $\lvert f(z) - L\rvert < \varepsilon$ whenever $0 < \lvert z - z_0\rvert < \delta$ Where $\lim_{z\to z_0} f(z) = f(z_0) \implies \lim_{z\to z_0} f(z) = L$
98.	A complex function f(z) of complex variable z for all values of z in some neighborhood of z_0 is continuous at z_0 if for each $\varepsilon > 0$ *there exist a* $\delta > 0$ such that $\lvert f(z) - f(z_0)\rvert < \varepsilon$ whenever $0 < \lvert z - z_0\rvert < \delta$
Module # 30	
99.	Let a function f(z) = u(x, y) + i v(x, y) be defined in some neighborhood of z_0 then f is continuous at $z_0 = x_0 + iy_0$ iff u and v are continuous at (x_0, y_0)

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100.	If f and g are continuous at the point z_0 then the following functions are continuous at z_0. • The sum f+g, where (f+g)(z)=f(z)+g(z) • The difference f-g, where (f-g)(z)=f(z)-g(z) • The Product fg, where (fg)(z)=f(z)g(z) • The quotient $\frac{f}{g}$, where $\left(\frac{f}{g}\right)(z) = \frac{f(z)}{g(z)}$ provided $g(z) \neq 0$
101.	Any polynomial $p(z) = a_0 + a_1 z + a_2 z^2 + \cdots \ldots \ldots + a_n z^n (a_n \neq 0)$ is continuous at each point Z_0 in the complex plane.
102.	$\lim_{z \to i} \frac{z^2+2}{z^2-2z+1} = \frac{1}{-2i}$
	Module # 31
103.	Consider two complex valued functions f(z) and g(z) then its composition is denoted by '**g of (z)**' and is defined as '**gof(z)=g(f(z))**'
104.	If f and g are continuous at the point z_0 then 'gof' is continuous at z_0.
	Module # 32
105.	If a function f(z) is continuous and non zero at a point z_0 then $f(z) \neq 0$ throughout some neighborhood of that point.
106.	If a function f is continuous at throughout a region R that is both closed and bounded then there exist a non real number M such that $\lvert f(z) \leq M \rvert$ for all points Z in R where equality holds for at least one such Z.
	Module # 33
107.	The mapping $\frac{1}{Z}$ is called the reciprocal function. In exponential notation it can be expressed as $z = re^{i\theta}$ then we obtain $w = \frac{1}{Z} = \frac{1}{re^{i\theta}} = \frac{1}{r}e^{-i\theta}$ Case 1: If z lies on a unit circle. Case 2: If z lies outside the unit circle. Case 3: If z lies inside the unit circle.
108.	Image of the half plane $\{z:Re(z) \geq \frac{1}{2}$ under the mapping $w = \frac{1}{Z}$ is the closed disk i.e. $\{w:\lvert w-1 \rvert \leq 1\}$
109.	For transformation $\frac{1}{Z}$ image of portion of the half plane $Re(z) \geq \frac{1}{2}$ is inside the closed disk i.e. $\{z:\lvert z-\frac{1}{2} \rvert \leq 1\}$
	Module # 34
110.	The derivative of 'f' at x_0 is written as $f'(x_0)$ and is defined as $f'(x_0) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$ provided the limit exists or $f'(x_0) = \lim_{\Delta_x \to 0} \frac{f(x_0+\Delta_x)-f(x_0)}{\Delta_x}$
111.	The derivative of f at z_0 is written as $f'(z_0) = \lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ provide the limit exists. The function f must be defined is a neighborhood of z_0

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112.	If we write $\Delta_z = z - z_0$, *then the funciton* $f'(z_0) = \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ can be expressed as $f'(z_0) = \lim_{\Delta_z\to 0} \frac{f(z_0+\Delta_z)-f(z_0)}{\Delta_z}$
113.	If we let w=f(z) and $\Delta w = f(z) - f(z_0)$ then we can use the Leibniz notation $\frac{dw}{dz}$ for the derivative $f'(z_0) = \frac{dw}{dz} = \lim_{\Delta_z\to 0} \frac{\Delta w}{\Delta z}$ Hence $f'(z_0) = \lim_{\Delta_z\to 0} \frac{\Delta w}{\Delta z} = \lim_{\Delta_z\to 0} \frac{f(z_0+\Delta_z)-f(z_0)}{\Delta_z}$ and often we drop the subscript z_0 and write the definition as $f'(z) = \lim_{\Delta_z\to 0} \frac{\Delta w}{\Delta z} = \lim_{\Delta_z\to 0} \frac{f(z+\Delta_z)-f(z)}{\Delta_z}$
114.	If $f(z) = z^2$ then $f'(z) = 2z$. **(PROVE IT-PAGE 66)**
115.	The function f(z)=Re(z) is differentiable nowhere. **(PROVE IT-PAGE 67)**
	Module # 35
116.	Consider the real valued function $f(z)=\|z\|^2$. Find the points where f(z) is differentiable. **(PROVE IT-PAGE 69)**
117.	If f(z) is differentiable at z_0 then f(z) is continuous at z_0. **(PROVE IT-PAGE 73)**
	Module # 36
118.	Chain rule is useful when we want to differentiate composition of two functions.
119.	Given two functions f(z) and g(z) composition of f and g is given by; fog(z)=f(g(z)) or $\frac{d}{dz}(f(g(z)) = f'(g(z)).g'(z))$ **(PREPARE EXAMPLE P#78)**

	Module # 37 – The Cauchy – Riemann Equation
120.	Augustin-Lonis Cauchy (French Mathematician, Engineer and Physicist – 21 March 1789 to 23 May 1857) and Bernhard Riemann.
121.	Cauchy – Riemann equations are a pair of equations that first order partial derivative of the component function u and v of a function f(z)=u(x,y)+iv(x,y) must satisfy at a point $(x_0,y_0)$when derivative of f exist there.
122.	If Cauchy – Riemann equations are not satisfied at z_0 then function is not differentiable at z_0.
123.	A Cauchy – Riemann equations are satisfied at a point z_0 then function may or may not be differentiable at z_0. **(PROVE IT-PAGE 81)**
	Module # 38
124.	If f(z)=f(x+iy)=u(x,y)+iv(x,y) differentiable at a point $z_0=x_0+iy_0$ then u and v satisfy Cauchy-Riemann equations $Ux(x_0,y_0)=Vy(x_0,y_0)$ and $Uy(x_0,y_0)=-Vx(x_0,y_0)$
	Module # 39
125.	Let (x_0,y_0) be an arbitrary point defined in some neighborhood of the point $Z_0=x_0+iy_0$ and suppose that f(z)=f(x+iy)=u(x,y)+iy(x,y) then $U(x,y)=e^{-y}Cosx$ ------- $Ux(x,y)=-e^{-y}Sinx$ -------$Uy(x,y)= -e^{-y}Cosx$ and $V(x,y)=e^{-y}Sinx$ ------- $Vx(x,y)=-e^{-y}Cosx$ -------$Vy(x,y)= -e^{-y}Sinx$

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126.	The above partial derivatives are continuous at (X0,Y0) and satisfy Cauchy – Riemann equations Ux=Vy and Uy=-Vx i.e. Ux=)=-e^{-y}Sinx =Vy and Uy=-e^{-y}Cosx =-Vx
	Module # 41
127.	A complex function f is analytic at the point Z_0 provided there is some $\varepsilon > 0$ such that $f'(z)$ exist in ε neighborhood of Z_0.
	Module # 45
128.	Harmonic Functions are real valued functions but they are very strongly related to complex valued functions.
129.	**Harmonic Function Definition**: A real valued function H of two variables x and y is said to be harmonic in a given domain D of xy-plane if following two conditions are satisfied by function; 1. Throughout the domain it has continuous partial derivatives of the first order and second order. 2. It satisfied following partial differential equation Hxx(x,y)+Hyy(x,y)=0 or $\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}=0$
130.	Notations: The sum $\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}$ of two partial derivatives is denoted by $\nabla^2\varphi$ and is called the Laplacian of φ.
131.	Laplacian equation or sometimes known as Potential equation is $\nabla^2\varphi = 0$ **(PREPARE EXAMPLE - PAGE 27)**
	Module # 46
132.	If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then its component functions u and v are harmonic in D. **(PREPARE EXAMPLE - PAGE 28)**
133.	**Harmonic Conjugate**: Let u(x, y) and v(x, y) are two functions such that u(x, y) and v(x, y) are harmonic in D and first partial derivatives of U and V satisfy Cauchy-Riemann equations throughout D then V is said to be harmonic conjugate of U in D.
134.	A function f(z)=U(x,y)+iv(x,y) is analytic in a domain D if and only iff V is harmonic conjugate of U.
135.	The function $f(z)=z^2$ i.e $f(x+iy)=(x+iy)^2=(x^2-y^2)+i(2xy)$ is analytic everywhere. It means $U(x,y)= x^2-y^2$ and V(x,y)= 2xy
136.	The function 2xy and x^2-y^2 are harmonic functions but $f(x+iy)=(x+iy)^2=(2xy)+i(x^2-y^2)$ is not harmonic.
	Module # 47
137.	Find Harmonic Conjugate of $U(x,y)=x^3-3xy^2-5y$. **(PREPARE EXAMPLE - PAGE 32)**
	Module # 48
138.	Harmonic Functions are very important in applied mathematics for solving real life problems e.g. for fluid flow problems.
139.	Incompressible means density w.r.t times remains constant or density does not change with time and Frictionless means when fluid flows then its friction with the walls is zero.
140.	An analytic function has analytic anti-derivative.

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141.	The curves given by {(x,y):$\varphi(x,y) = constant$} are called equi-potentials or stream lines.
142.	For F(x,y)= $\varphi(x,y) + i\Psi(x,y)$ the component $\varphi(x,y)$ is harmonic also $\varphi_y = -\Psi_x$
	Module # 49
143.	**Sequence:** A sequence is any collection of objects, events or numbers that has some pattern that we can find e.g. 1,2,3,5,8,13,……
144.	A sequence is a function whose domain is a positive integer and whose range is a subset of complex numbers.
145.	$f: z^+ \to \mathbb{C}$ or $f: \mathbb{N} \to \mathbb{C}$ is a sequence
146.	Examples of complex valued sequence: $f_1(n) = n + i2n, \quad n \in \mathbb{N}$
147.	A sequence is usually denoted as $\{Z_n\}_{n=1}^{\infty}$ OR $\{Z_n\}_1^{\infty}$
	Module # 50
148.	A sequence is function from natural number to complex number.
149.	**Limit of a sequence**: A sequence {Xn} has P as its limit as n approaches to infinity provided the terms Xn can be made as close as we want to P by making n large enough.
150.	A sequence of real numbers {Xn} converges of P if for every $\varepsilon > 0$ there exist Nε such that |Xn-P|< ε whenever n>N
151.	A sequence of complex numbers {Zn} converges to ζ if for each $\varepsilon > 0$ there exist Nε scuh that |Zn- ζ|< ε whenever n>Nε denotes as $\lim_{n\to\infty} Zn = \zeta$

	Module # 51
152.	A complex sequence {Zn} is bounded provided that there exist a positive real number R and an integer N such that |Zn|<R for all n>N.
153.	If {Zn} is a convergent sequence then {Zn} is bounded.
154.	**Cauchy Sequence**: The sequence {Zn} is a Cauchy Sequence if for every $\varepsilon > 0$ there is a positive integer Nε such that if n,m>Nε then |Zn-Zm|<ε
155.	{Zn} is a Cauchy Sequence if and only if {Zn} converges.

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S#	Statement	Answer
156.	The derivative of f at z_0 i.e. $f' = \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ provided the limit exists. The function must be defined is a neighborhood of ____	z_0
157.	If Cauchy – Riemann equations are not satisfied at a point z_0 then function is not _____ at z_0.	Differentiable
158.	In the complex valued function z=x-iy, the value of Ux=_____	1
159.	In the complex valued function z=x-iy, the value of Vx=_____	0
160.	In the complex valued function z=x+iy, the value of Ux=_____	1
161.	In the complex valued function z=x+iy, the value of Vx= _____	0
162.	Consider $\lim_{z\to z_0} f(z) = A$ and $\lim_{z\to z_0} g(z) = B$ then the $\lim_{z\to z_0}(f(z) \pm g(z)) =$ _____	$A \pm B$
163.	Consider $\lim_{z\to z_0} f(z) = A$ and $\lim_{z\to z_0} g(z) = B$ then the $\lim_{z\to z_1}(f(z)g(z)) =$ _____	A.B
164.	If Cauchy – Riemann on a pair of real – valued functions of two real variables u(x,y) and v(x,y) are Ux=Vy and ___.	Uy= -Vx
165.	Each simple closed curve C divides the plane into two domains (Connected Open Sets). One of which is bounded and is called the ___ of C.	Interior
166.	Each simple closed curve C divides the plane into two domains (Connected Open Sets). One of which is unbounded and is called the _____ of C.	Exterior
167.	If we write $\Delta_z = z - z_0$, *then the funciton* $f'(z_0) = \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ can be expressed as $f'(z_0) =$ ___	$\lim_{\Delta_z\to 0} \frac{f(z_0+\Delta_z)-f(z_0)}{\Delta_z}$
168.	A Cauchy – Riemann equations are satisfied at a point z_0 then function _____ differentiable at z_0.	C. Both (a) and (b)
169.	Consider $\lim_{z\to z_0} f(z) = A$ and $\lim_{z\to z_0} g(z) = B$ then the $\lim_{z\to z_1} \frac{f(z)}{g(z)} =$ _____	$\frac{A}{B}, B \neq 0$
170.	If a function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D, then its component functions u and v are ____ in D.	Harmonic
171.	A curve C is said to be a contour if it is constructed by joining finitely many smooth curves end to end	True
172.	The sequence $z_n = -3 + i\frac{1}{n^3}$ (n = 1,2,3,4, … …) converges to _____-	-3
173.	Any polynomial $p(z) = a_0 + a_1 z + a_2 z^2 + \cdots \ldots \ldots + a_n z^n (a_n \neq 0)$ of degree $n(n \geq 1)$ has least one zero, i.e. there exists at least ---------- Z_0 such that $P(Z_0) = 0$	One point
174.	The sequence $z_n = \frac{1}{n^3}$ (n = 1,2,3,4, … …)for $n \to \infty$ converges to _____.	0

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175.	A domain D (Connected Open Set) is called _____ connected if interior of every simple closed contour C is contained in D.	Simply
176.	If $f(z) = x + ay + i(bx + cy)$ is analytic, then a, b, c equals to_____.	c=1 and a=-b
177.	A function V is said to be harmonic if and only if _____	Vxx+Vyy=0
178.	A function f(z) is analytic function if _____ part of f(z) is analytic.	C. Both (a) and (b)
179.	Harmonic conjugate of $u(x,y)=e^y \cos x$ is _____.	$-e^y \sin x + C$
180.	If $e^{ax}\cos y$ is harmonic then 'a' is _____.	i
181.	If U and V are harmonic functions then f(Z)=U+iV is_____.	Analytic Function
182.	A point at which a function ceases to be analytic is called a _____ point.	Singular
183.	Cauchy-Riemann equations are	Ux=Vy and Uy=-Vx
184.	If f(z)=u+iv in polar form is analytic then $\frac{\partial u}{\partial r}$ is	$\frac{1}{r}\frac{\partial v}{\partial \theta}$
185.	If f(z)=u+iv in polar form is analytic then $\frac{\partial u}{\partial \theta}$ is	$-r\frac{\partial v}{\partial r}$
186.	A function v is called a conjugate harmonic function for a harmonic function u in Ω whenever	f=u+iv is analytic
187.	The function $f(z) = \lvert z \rvert$ is non-constant.	nowhere analytic function
188.	The function $f(x+iy)=x^3+ax^2y+bxy^2+cy^3$ is analytic if	a=3i, b= -3 and c=i
189.	There exist no analytic function f such that	Re $f(z)=y^2-2x$
190.	The harmonic conjugate of $2x-x^3+3xy^2$ is	$2y-3x^2y+y^3$
191.	The harmonic conjugate of u(x,y)=2x(1-y) is	x^2-y^2+2y+C
192.	If the real part of an analytic function f(z) is x^2-y^2-y, then imaginary part is	2xy+x
193.	If the imaginary part of an analytic function f(z) is 2xy+y, then real part is	x^2-y^2+x
194.	$f(z) = \bar{z}$ is differentiable	Nowhere
195.	$f(z) = \lvert\bar{z}\rvert^2$ is differentiable	Only at z=0
196.	$f(z) = \lvert\bar{z}\rvert^2$ is	Differentiable at z=0 but not

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		analytic at z=0
197.	$if\ f(z) = \begin{cases} \frac{xy}{(x^2+y^2)}, & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$ then f(z) is	Not differentiable at z=0
198.	$f(z) = e^z$ is analytic	Everywhere
199.	$e^x(\cos y - i \sin y)$ *is*	Not analytic
200.	If f(z) is analytic then $\overline{f(\bar{z})}$ is	Analytic
201.	The point at which $f(z) = \frac{z^2-z}{(z^2-3z+2)}$ is not analytic are;	*i and -i*
202.	The harmonic conjugate of u=log$\sqrt{x^2+y^2}$ is	$tan^{-1}\left(\frac{y}{x}\right)$
203.	If $f(z) = z(2-z)$ then f(1+i)=	i
204.	If $f(z) = \lvert\bar{z}\rvert$ then f(3-4i)=	5
205.	Critical point of bilinear transformation $w = \frac{a+bz}{c+dz}$ are	$-\frac{c}{d}, \infty$
206.	The point coincide with their transformation are known as	Fixed points
207.	$w = \frac{a+bz}{c+dz}$ is a bilinear transformation when	$ad - bc \neq 0$
208.	$w = \frac{1}{z}$ is known as _____--	Inversion
209.	$w = z + \alpha$ is known as _____-	Translation
210.	A translation of the type $w = \alpha z + \beta$ where α *and* β are the complex constants, known as a ___	Linear transformation
211.	A mapping that preserves angles between oriented curves both in magnitude and in sense is called a/an _____ mapping.	Conformal
212.	The mapping defined by an analytic function $f(z)$ is conformal at all points z except at points where ____	$f'(z) = 0$
213.	The fixed points of the transformation $w = z^2$ are _____	0,1
214.	The invariant points of the mapping $w = \frac{z}{2-z}$ are _____	0,1
215.	The fixed points of $w = \frac{z-1}{z+1}$ are _____	$\pm i$
216.	The mapping $w = z + \frac{1}{z}$ transforms circles of constant radius into_____.	confocal ellipses

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217.	Under the transformations $w = \frac{1}{z}$,the image of the line $y = \frac{1}{4}$ in z-plane is ___	Circle $u^2 + v^2 + 4v = 0$
218.	The bilinear transformation that maps the points $0, i, \infty$ respectively into $0,1, \infty$ *is* $w =$ _____	$-iz$
219.	The bilinear transformation which maps the points $z = 1, z = 0, z = -1$ of z-plane into $w = i, w = 0, w = -1$ of w-plane respectively is ___	$w = iz$

MODULE#055 to85	
220.	**Geometric Series**: A finite series of the form $\sum_{n=0}^{\infty} Z^n$ is called geometric series.
221.	For what value of Z the geometric series $\sum_{n=0}^{\infty} Z^n$ converges? If $\|z\| < 1$ the series $\sum_{n=0}^{\infty} Z^n$ converges to $f(z) = \frac{1}{(1-z)}$. If If $\|z\| \geq 1$ the series $\sum_{n=0}^{\infty} Z^n$ diverges. **PROOF M#55-P#2**
222.	If $\|z\| > 1$ the series $\sum_{n=1}^{\infty} Z^{-n}$ converges to $f(z) = \frac{1}{(1-z)}$. If If $\|z\| \geq 1$ the series $\sum_{n=0}^{\infty} Z^n$ diverges.
223.	If $\sum_{n=0}^{\infty} \zeta_n$ is a complex series such that $\lim_{n\to\infty} \frac{\|\zeta_{n+1}\|}{\|\zeta_n\|} = L$ then the series is absolutely convergent if $L < 1$ and divergent if $L > 1$
224.	**PREPARE EXAMPLE….M#56-P#5**
225.	**Accumulation Point:** A point $Z_0$is called an accumulation point of a set S is each deleted neighborhood of Z_0 contains at least one point of S.
226.	**Limit Supermum**: Let $\{t_n\}$ be a sequence of positive real numbers. The limit supremum of the sequence is denoted by $\lim_{n\to\infty} Sup\ t_n$ is the smallest real number L having the property that for any $\varepsilon > 0$ there are at most finitely many terms in a sequence that are larger than $L + \varepsilon$. If there is no such L then $\lim_{n\to\infty} Sup\ t_n = \infty$
227.	The series $\sum_{n=0}^{\infty} \zeta_n$ has $\lim_{n\to\infty} Sup\ \|\zeta_n\|^{\frac{1}{n}} = L$ then the series absolutely convergent if L<1 and divergent if L>1.
228.	**PREPARE EXAMPLE….M#58-P#14-15**
229.	Define a function: An infinite series or an expression of the form $\sum_{n=1}^{\infty} a_n$ is called function.
230.	The series $\sum_{n=0}^{\infty} C_n(z-\alpha)^n$ defines a function only if it converges.
231.	$f(z) = \sum_{n=0}^{\infty} C_n(z-\alpha)^n$ is called power series function.
232.	Domain: Collection of all points z for which the series converges.

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233.	Suppose $f(z) = \sum_{n=0}^{\infty} C_n(z-\alpha)^n$ then the set of points z for which the series converges is one of the following : 1. The single point $z = \alpha$ 2. The disk $D_s(\alpha) = \{z: \lvert z-\alpha\rvert < p\}$ 3. The entire complex plane
234.	**Radius of convergence**: The number p such that the series $\sum_{n=0}^{\infty} C_n(z-\alpha)^n$ converges for all z satisfying $\lvert z-\alpha\rvert < p$ and diverges for all z satisfying $\lvert z-\alpha\rvert > p$ is called radius of convergence.
235.	If the series $\sum_{n=0}^{\infty} C_n(z-\alpha)^n$ converges for the single point $z = \alpha^n$ then radius of convergence=0.
236.	If the series $\sum_{n=0}^{\infty} C_n(z-\alpha)^n$ converges for the disk $Dp = \alpha^c = \{z: \lvert z-\alpha\rvert < p\}^n$ then radius of convergence=p.
237.	If the series $\sum_{n=0}^{\infty} C_n(z-\alpha)^n$ converges for the entire plane then radius of convergence=∞.
238.	**PREPARE EXAMPLE....M#61-P#25-26-27-28**
239.	$\sum_{n=0}^{\infty}(n+1)z^n = \frac{1}{(1-z)^2}$ for all $z \in D_1(0)$
240.	The function e^z is an entire function satisfying the following conditions: $\frac{d}{dz}e^z = e^z$, $e^{z1+z2} = e^{z1}.e^{z2}$
241.	Euler Formula: If θ is a real number then $e^{i\theta} = cos\theta + i\ sin\theta$.
242.	The exponential function $\exp(z) = e^x(Cosy + i\ siny)$ is not one-to-one mapping but onto mapping exist.
243.	Log(z) is a single value function from non zero complex numbers to the fundamental period strip.
244.	**PREPARE EXAMPLE....M#__P#56**
245.	$Log(z) = ln\lvert z\rvert + i(Arg(z) + 2n\pi)$ is multi valued function but $Log(z) = ln\lvert z\rvert + iArg(z)$ is a single valued function.
246.	$Log(z)$ has two properties: Continuity and differentiability .
247.	$Log(z)$ has become ln(x) when restricted to real numbers.
248.	The image of $Log(z)$ is one sheet of complex numbers.
249.	The graph of $Log(z)$ looks like many sheets one for each 'n'
250.	The function $Log(z)$ is discontinuous at all the points of negative real axis.

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251.	Let $z_1 and\ z_2$ be non-complex numbers. The multi valued function log obeys the following properties. i. $\text{Log}(z_1 z_2) = \log(z_1) + \log(z_2)$ ii. $\text{Log}(\frac{z_1}{z_2}) = \log(z_1) - \log(z_2)$
252.	The identity $log(z_1 z_2) = \log(z_1) + \log(z_2)$ hold true if and only iff $-\pi < Arg(z_1) < Arg(z_2) \leq \pi$
253.	**PREPARE EXAMPLE....M#__P#69**
254.	**PREPARE EXAMPLE....M#__P#74**
255.	Find the derivative of b^2. **PREPARE EXAMPLE....M#__P#77**
256.	If z is a complex number then $e^{iz} = cosz + i\ sinz$.
257.	For $z = x + iy$ we have $Cos\ z = \text{Cos}\, x\ Coshy - i\ Sinx\ Sinhy$, $Sin\ (z + 2\pi) = Sin\ z$, $Cos\ (z + 2\pi) = Cos\ z$, $Sin\ (z + \pi) = -Sin\ z, Cos\ (z + \pi) = -Cos\ z$, $Tan\ (z + \pi) = Tan\ z, Cot\ (z + \pi) = Cot\ z$,
258.	$Sin\ z = 0$ iff $z = n\pi, n \in z$
259.	$Cos\ z = 0$ iff $z = \left(\frac{n+1}{2}\right)\pi, n \in z$
260.	**PREPARE EXAMPLE....M#__P#92**
261.	The periodic nature of $Sinz$ makes it a multiple one-to-one function.

MODULE#086 to100

262.	The parametric representation $z(t) = cost + isint$ for $0 \leq t \leq 2\pi$ represents a _____ in the complex plane.	Circle
263.	The parametric representation $z(t) = t^2 + it$ for $-1 \leq t \leq 1$ represents a _____ in the complex plane.	Parabola
264.	A curve $c: z(t) = x(t) + iy(t)$ for $a \leq t \leq b$, is simple if it does not cross it self, that is _____	$z(t_1) \neq z(t_2)$ whenever $t_1 \neq t_2$
265.	A curve $c: z(t) = x(t) + iy(t)$ for $a \leq t \leq b$, is closed if _____	$z(a) = z(b)$
266.	The complex valued function $f(t) = u(t) + iv(t)$ is said to be differentiable $[a, b]$ if both $x(t) and\ y(t)$ are _____ on $[a, b]$	Differentiable
267.	A curve $c: z(t) = x(t) + iy(t)$ for $a \leq t \leq b$, is said to be smooth curve if _____is continuous and nonzero on the interval.	$z'(t)$
268.	A curve C is said to _____ if it is constructed by joining finitely many smooth curves end to end.	Contour

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269.	If $f(t) = u(t) + iv(t)$, where $u(t)$ and $v(t)$ are real valued functions of the real variable t for $a \le t \le b$, then $\int_a^b f(t)dt =$	$\int_a^b u(t)dt + i\int_a^b v(t)dt$
270.	For $f(t) = u(t) + iv(t)$, and $g(t) = p(t) + iq(t)$,we have $\int_a^b (f(t) + g(t))dt =$	$\int_a^b f(t)dt + \int_a^b g(t)dt$
271.	For $f(t) = u(t) + iv(t)$, and a partition of the interval $[a, b]$ given as $a \le t \le d$ and $d \le t \le b$,we have $\int_a^b (f(t) + g(t))dt =$	$\int_a^d f(t)dt + \int_d^b f(t)dt$
272.	Mean value theorem for real integrals: If $h(x)$ is continuous of $[a, b]$ then there exist $c \in [a, b]$ such that $\int_a^b h(x)dx =$	$h(x)(b-a)$
273.	Mean value theorem for real integrals does not hold in case of integrals of complex functions over _____ intervals.	Real
274.	Consider $f(t) = e^{it}$ for $0 \le t \le 2\pi$ then evaluate $\int_0^{2\pi} e^{it}\, dt =$	0
275.	For any $c \in [0,2\pi]$, we have $\|f(c)(2\pi - 0)\| = \|e^{it}\|\|2\pi\| =$	2π
276.	A curve in the plane is defined as $c: z(t) = x(t) + iy(t)$ for $a \le t \le b$, where t is a _____	Parameter
277.	Evaluate the integral $\int_c \exp(z)\, dz =$	$\exp(a+ib) - 1$
278.	Evaluate the integral $\int_{c1} z\, dz$ and $\int_{c2} z\, dz =$	0
279.	Evaluate the integral $\int_c z\, dz =$ _____ where the contour C is circle from $-i$ to i. $c: z(t) = e^{it}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$ =____	0, 0
280.	Evaluate the integral $\int_c z^a\, dz =$ _____. Where the contour C is semicircle from 2 to -2.	$\frac{2^{a+1}}{a+1}[(-1)^{a+1} - 1]$
281.	For a piece wise continuous complex valued function of real variable t , $f(t) = u(t) + iv(t); a \le t \le b$ we have $\left\|\int_a^b f(t)\, dt\right\| \le \int_a^b \|f(t)\|dt$	
282.	Each simple closed curve C divides the plane into two domains (Connected open set). One is bounded and is called _____ of C the other is unbounded and is called the exterior of C.	Interior
283.	Each simple closed curve C divides the plane into two domains (Connected open set). One is bounded and is called interior of C the other is unbounded and is called _____ the of C.	Exterior
284.	Simply connected domain: A domain D (Connected open set) is called _____connected domain if interior of every simple closed contour C is contained in D.	Simply
285.	A domain D that is not simply connected is called _____connected.	Multiply
286.	Positive Direction: _____direction of a contour C is the direction corresponding to increasing values of a parameter t.	Positive

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287.	Negative Direction: _____direction of a contour C is the direction opposite to the positive direction.	Negative
288.	The line integral $\int_c ydx + xdy =$ _____	0
289.	Jordon Curve Theorem: Each simple closed curve C divides the plane into two domains (Connected open set). One is bounded and is called interior of C the other is unbounded and is called the exterior of C.	
290.	In case of simple closed contour positive orientation (Contour Clockwise direction) and negative direction corresponds to clock wise direction, if C is positively oriented then –C is negatively oriented.	
291.	**Green's theorem:** Let C be a simple closed contour with positive orientation and Let R be the domain that forms the interior of C. If P and Q are continuous and have continuous partial derivatives then $\int_c P(x,y)dx + Q(x,y)dx = \int_R \int Q_x(x,y) - P_y(x,y)dxdy$ is called green's theorem.	
292.	**Cauchy Theorem:** Let f be analytic in simply connected domain D and f is continuous in domain D. If C is simple closed contour that lies in D then $\int_c f(z)dz = 0$	
293.	**Cauchy-Goursat Theorem:** Let f be analytic in simply connected domain D and f is continuous in domain D. If C is simple closed contour that lies in D then $\int_c f(z)dz = 0$	
294.	Let Z_0 *and* Z_1 be points in a domain D. A contour integral $\int_c f(z)dz$ is said to be independent of the path if its value is the same for all contour C in D with initial point Z_0 and terminal point Z_1	
295.	Generalization of Cauchy-Goursat Theorem: Let C_1 *and* C_2 be two simple closed positively oriented contours such that C_2 lies interior to C_1. If f(z) is analytic in a domain D that contains both C_1 and C_2 then $\int_{c1} f(z)dz = \int_{c2} f(z)dz$	
296.	Remember: $\int_{c1} \frac{dz}{(z-z0)^n} = 2ni \; if \; n = 1$ and $\int_{c1} \frac{dz}{(z-z0)^n} = 0 \; if \; n \neq 1$	
	MODULE#101 to121	
297.		
	FROM PAST PAPERS	
298.	The scalar multiple of a vector $\vec{x} = (x_1, x_2, x_3)$ by a real number k is the point $k\vec{x} = (kx_1, kx_2, kx_3)$, clearly _____ for all k and $\vec{x}$.	Both b and c
299.	Let $f(z)$ be analytic function and C is contour enclosing point x_0, then Cauchy Integral Formula states f(z0)=	$\frac{1}{2\pi i}\int_c \frac{f(z)}{z - z_0}dz$

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300.	If $\vec{x}(t)$ be a smooth space curve. Then which of the following is (are) known as Frenet formula (formulas)?	All a,b,c
301.	An important consequences of the wedge product is that "repeats are zero" due to "alternation rule", that means___	All a,b,c
302.	If the three vectors $\vec{t}, \vec{n}$ *and* $\vec{b}$ are perpendicular to each other and form the Frenet frame , then $\vec{b} \times \vec{t} =$ ___	$\vec{n}$
303.	If p *and* q are points in R^3 then _____ is the symmetric property of dot product.	$p.q = q.p$
304.	The argument of complex number $(1+i)^{\pi}$ is _____	90^0 (Not confirm)
305.	If $f(x) = 3x^2 - 2x + 4$ then $f'(x) =$_____	$6x - 2$
306.	If $f(z) = z^3 - 3z + 12$ then $f'(z) =$_____	$3z^2 - 3$
307.	Let $f(z) = \frac{1}{z^2+1}$ is discontinuous at z=_____	$\pm i$
308.	If $u(x,y) = 5x(4-2y)$ is harmonic in some domain then harmonic conjugate v(x,y)=	$20y - 5y^2 + f(x)$
309.	Any polynomial $p(z) = a_0 + a_1 z + a_2 z^2 + \cdots \ldots \ldots + a_n z^n (a_n \neq 0)$ of degree $n(n \geq 1)$ has least one zero, i.e. there exists at least ----------Z_0 such that $P(Z_0) = 0$	One point
310.	Any function which is analytic at a point x_0 must have a _____ series about x_0.	Taylor's
311.	Harmonic conjugate of $u(x,y)=e^y \cos x$ is _____.	$-e^y \sin x + C$
312.	The point at which $f(z) = \frac{1}{z^2+1}$ is not analytic are;	*i and -i*
313.	In the trigonometric functions of complex numbers Cos x=	$\frac{e^{ix} + e^{-ix}}{2}$
314.	Let Z be a complex valued function then arcsin(Z)=	$-ilog(iz + (1-z^2)^{\frac{1}{2}})$
315.	Let C denote a contour of length L, and suppose that a function f(z) is piece wise continuous on C. If M is non negative constant such that $\lvert f(z) \rvert \leq M$ for all points z on C at which f(z) is defined, then _____	$\left\lvert \int_c f(z)dz \right\rvert \leq ML$
316.	The Cauchy-Goursat theorem shows that _____	$\int_c f(z)dz = 0$
317.	The derivative of a complex valued function $w(t) = u(t) + iv(t)$ *is* _____, where u and v are real valued functions of a real variable t.	$w'(t) = u'(t) + iv'(t)$ *is*
318.	The circle $z = e^{i\theta}, 0 \leq \theta \leq \pi$ is centered at _____	(0,0)
319.	The principal value of $i^i =$	$exp(-\frac{\pi}{2})$

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MTH632 Mega File Final Term

Lec (23-45)

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Subjective Files (6)

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- Mth632 ex_2 solution mage by jathol pg6

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Mth632 Imp Formulas

⇒ CRE in cartesian:

$u_x = v_y$, $-v_x = u_y$

⇒ in polar:

$r u_r = v_\theta$, $-r v_r = u_\theta$

⇒ CRE for velocity vector:-

$$P_x(x,y) = -q_y(x,y)$$

$$P_y(x,y) = q_x(x,y)$$

⇒ equipotential:

$$\{(x,y) : \phi(x,y) = \text{constant}\}$$

⇒ Streamline:

$$\{(x,y) : \psi(x,y) = \text{constant}\}$$

⇒ Comparison test:

$$|z_n| \le M_n \quad \forall n$$

$$\sum_{n=1}^{\infty} z_n = \sum_{n=1}^{\infty} x_n + i y_n$$

⇒ d'Alembert's Ratio test:

$$\lim_{n\to\infty} \frac{|z_{n+1}|}{|z_n|} = L$$

$L<1 \to$ cgt, $L>1 \to$ divergent.

⇒ Root Test:

$$\lim_{n\to\infty} \text{Sup} \, |z_n|^{\frac{1}{n}} = L$$

$L<1 \to$ cgt, $L>1 \to$ dvg.

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→ infinite series in this form:-

∵ Power series function:-

$$f(z) = \sum_{n=0}^{\infty} c_n (z-\alpha)^n$$

c_n and α fixed complex no.

→ Cauchy's root test:-

$$\rho = \frac{1}{\lim_{n\to\infty} |c_n|^{1/n}}$$

→ Cauchy - Hadamard root test:-

$$\rho = \frac{1}{\lim_{n\to\infty} \sup |c_n|^{1/n}}$$

→ d' Alembert's ratio test:-

$$\rho = \frac{1}{\lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right|}$$

→ $c_k = \frac{f^{(k)}(\alpha)}{k!}$

→ Bessel Function:-

$$J_0(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{z}{2}\right)^{2n}$$

$$J_0'(z) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!(n+1)!} \left(\frac{z}{2}\right)^{2n+1}$$

$$\because J_1(z) = -J_0'(z)$$

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⇒ e^x has pave series:

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

⇒ $w = \exp\{\ln|w| + i(\text{Arg}(w) + 2n\pi)\}$

$\log w = \ln|w| + i(\text{Arg}(w) + 2n\pi)$

multi valued ← $\log(z) = \ln|z| + i\{\text{Arg}(z) + 2n\pi\}$

principal ← $\text{Log}(z) = \ln|z| + i\,\text{Arg}(z)$.

Single valued.

⇒ $\sinh z = \frac{e^z - e^{-z}}{2}$, $\cosh z = \frac{e^z + e^{-z}}{2}$.

⇒ $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$, $\cos z = \frac{e^{iz} + e^{-iz}}{2}$.

imp prop:

$\cosh(iz) = \cos z$, $\cos(iz) = \cosh z$

$\sinh(iz) = i\sin z$, $\sin(iz) = i\sinh z$.

$\cosh^2 z - \sinh^2 z = 1$

⇒ $\text{arc}\sin(z) = -i\log\left(iz + (1-z^2)^{1/2}\right)$

⇒ $\text{arc}\cos(z) = -i\log\left(z + i(1-z^2)^{1/2}\right)$.

⇒ $\text{arc}\tan(z) = \frac{i}{2}\log\left(\frac{i+z}{i-z}\right)$

⇒ $\frac{d}{dz}\text{arc}\sin z = \frac{1}{\sqrt{1-z^2}}$

⇒ $\text{arc}\sinh(z) = \log\left[z + (z^2+1)^{1/2}\right]$

⇒ $\text{arc}\cosh(z) = \log\left[z + (z^2-1)^{1/2}\right]$.

⇒ $\text{arc}\tanh(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$.

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$\frac{d}{dz}\operatorname{arcsinh} z = \frac{1}{\sqrt{1+z^2}}$, $\frac{d}{dz}\operatorname{arccosh} z = \frac{1}{\sqrt{z^2-1}}$

$\frac{d}{dz}\operatorname{arctanh} z = \frac{1}{1-z^2}$

⇒ Arc length ⇒

$$L(c) = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

⇒ integral of $f(t)$:

$$\int_a^b f(t)\,dt = \int_a^b u(t)\,dt + i\int_a^b v(t)\,dt.$$

⇒ Fundamental theorem of calculus:

$$F'(t) = f(t)$$

$$\int_a^b f(t)\,dt = F(t)\Big|_a^b = F(b) - F(a).$$

⇒ Mean value theorem for real integrals:

$$\int_a^b h(x)\,dx = h(x)(b-a).$$

⇒ X (Cauchy - Goursat theorem:.) X

⇒ Green's Theorem:

$$\int_c P(x,y)\,dx + Q(x,y)\,dy = \iint_R Q_x(x,y) - P_y(x,y)\,dx\,dy.$$

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⇒ Cauchy Integral Formula(I):-

$$f(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z - z_0} dz$$

⇒ Cauchy Integral Formula (II):- for derivatives⇒:-

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z - z_0)^{n+1}} dz$$

for $n = 0$

we get Cauchy's Integral formula (I)

⇒ Gauss's Mean Value Theorem:-

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + Re^{it}) dt$$

⇒ Cauchy's Inequality:-

if $f(z) \leq M$ + points then;

$$|f^n(z_0)| \leq \frac{n! M}{R^n}$$

⇒ Liouville's theorem:-

$$|f'(z_0)| \leq \frac{M}{R}$$

⇒ Directional derivative:-

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

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Imp Loo) .. MTH632.

1- laplace eq:
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

2- $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$, $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$.

3- Chain rule:.
$$d(h(f)) = h'(f)\,df$$
product rule.
$$d(f(g)) = g\,df + f\,dg.$$
By tangent vector:.
$$df(v_p) = [2P_1P_2 + P_2^2 - 1]v_1 + [P_1^2 + 2P_2P_1 - 1]v_2$$

4- If $u(x,y)$ given then how we will find $v(x,y)$?
- 1st shout it is harmonic. for this
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$
- Then; $u_x = v_y \Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
integrating $\Rightarrow$ and get $v(x,y)$.

5- $\cos z = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$, $\sin z = \sum \frac{(-1)^{n-1} z^{(2n+1)}}{(2n+1)!}$

6) Fernet - Serret Formula.
$$\vec{n} = \tau\vec{B} - k\vec{t}$$

7- How we show that function is analytic:
→ CRE satisfied.

8- For Harmonic conjugate:.
Satisfied this eq $\Rightarrow$ $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

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→ nowhere differentiable means:-
check CRE. if it satisfied
then diffe then not →.

→ Cauchy - Integral Formula:-

i)- $\int_C \frac{f(z)}{z-a} dz = 2\pi i f(a)$.

ii) - $\int_C \frac{f(z)}{(z-a)^{n+1}} = \frac{2\pi i}{n!} f^n(a)$.

→ diff at $z=0$

$$\frac{f(z+h)-f(z)}{h} \to \frac{f(h)-0}{h}$$

→ Analytic region:-
For this use CRE:
$u_x = v_y$, $-v_x = u_y$.

→ $\log(z) = \ln z + i\,\mathrm{Arg}(z)$

→ CRE:- $u_x = v_y$, $-v_x = u_y$.
$u_\theta = -r v_r$, $r u_r = v_\theta$.

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2022/03/10 16:01

Frenet Serret Formulas:

1) $\frac{d\vec{T}}{ds} = k\vec{N}$

$\vec{T}\cdot\vec{T} = 1 \Rightarrow \vec{T}\cdot\frac{d\vec{T}}{ds} + \frac{d\vec{T}}{ds}\cdot\vec{T} = 0$

$2\left(\vec{T}\cdot\frac{d\vec{T}}{ds}\right) = 0$

$\therefore$ Hence $\vec{T}$ and $\frac{d\vec{T}}{ds}$ are perpendicular

and $\frac{d\vec{T}}{ds} \perp \vec{N}$

and $\frac{d\vec{T}}{ds} \parallel \vec{N}$

So; $\frac{d\vec{T}}{ds} = k\vec{N}$

2) $\frac{d\vec{B}}{ds} = -\tau\vec{N}$

$\because \vec{B} = \vec{T}\times\vec{N}$

$\frac{d\vec{B}}{ds} = \frac{d\vec{T}}{ds}\times\vec{N} + \vec{T}\times\frac{d\vec{N}}{ds}$

$\frac{d\vec{B}}{ds} = k\vec{N}\times\vec{N} + \vec{T}\times\frac{d\vec{N}}{ds}$ —①

$\Rightarrow \frac{d\vec{B}}{ds} = \vec{T}\times\frac{d\vec{N}}{ds}$ —(i).

$\therefore \vec{T}\cdot\frac{d\vec{B}}{ds} = \vec{T}\cdot\vec{T}\times\frac{d\vec{N}}{ds}$ by ①

$\Rightarrow \vec{T}\cdot\frac{d\vec{B}}{ds} = 0$

$\frac{d\vec{B}}{ds}$ is $\perp$ to $\vec{T} \Rightarrow \frac{d\vec{B}}{ds} \parallel \vec{N}$

$\frac{d\vec{B}}{ds} = -\tau\vec{N}$ → Torsion

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3) $\frac{d\vec{N}}{ds} = \tau\vec{B} - k\vec{T}$

$\vec{N} = \vec{B} \times \vec{T}$

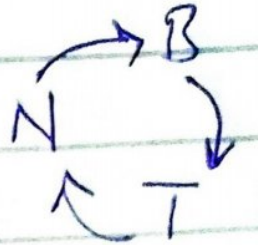
$\frac{d\vec{N}}{ds} = \frac{d\vec{B}}{ds} \times \vec{T} + \vec{B} \times \frac{d\vec{T}}{ds}$

$= -\tau\vec{N} \times \vec{T} + \vec{B} \times k\vec{N}$

$= -\tau(-\vec{B}) + \vec{B} \times k\vec{N}$

$= -\tau(-\vec{B}) + k(-\vec{T})$

$\frac{d\vec{N}}{ds} = \tau\vec{B} - k\vec{T}$



$N \times B = T$

$B \times T = N$

$T \times N = B.$

but

$T \times B = -N$

$B \times N = -T$

$N \times T = -B.$

OPPO A31
2022/03/10 16:01

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10:44 4G 0.03 KB/S ··· 88

$$\sin(ix) = i\sinh x \qquad \sinh(ix) = i\sin x$$

$$\cos(ix) = \cosh x \qquad \cosh(ix) = \cos x$$

$$\tan(ix) = i\tanh x \qquad \tanh(ix) = i\tan x$$

$$\sec(ix) = \operatorname{sech} x \qquad \operatorname{sech}(ix) = \sec x$$

$$\csc(ix) = -i\operatorname{csch} x \qquad \operatorname{csch}(ix) = -i\csc x$$

$$\cot(ix) = -i\coth x \qquad \coth(ix) = -i\cot x$$

eFunda

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10:45 4G 0.03 KB/S ... 88

(1) $\sinh x = \dfrac{e^x - e^{-x}}{2}$,[Read as 'hyperbolic sine x']

(2) $\cosh x = \dfrac{e^x + e^{-x}}{2}$,[Read as 'hyperbolic cosine x']

(3) $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(4) $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

(5) $\operatorname{cosech} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

(6) $\operatorname{sec} hx = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

- $\sinh 0 = 0, \quad \cosh 0 = 1, \quad \tanh 0 = 0$

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The Complex Logarithm

Example: Find the values of $\log(2+i2)$ and $\mathrm{Log}(2+i2)$.

Solution: we know that

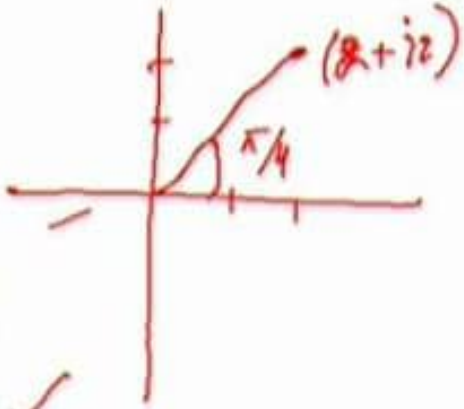
$$\log(z) = \ln|z| + i(Arg(z) + 2n\pi)$$

$$\mathrm{Log}(z) = \ln|z| + iArg(z)$$

$z = 2+i2$

$|z| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$Arg(z) = \pi/4$



$\mathrm{Log}(z) = \mathrm{Log}(2+i2) = \ln 2\sqrt{2} + i\,\pi/4$

$\log(2+i2) = \ln 2\sqrt{2} + i(\pi/4 + 2n\pi)$

VU

MTH632_Topic068

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11:01 0.00 KB/S 73

The Complex Logarithm

Proof ($\log(z_1 z_2) = \log(z_1) + \log(z_2)$):

We know that

$$\log(z) = \ln|z| + i(\arg(z))$$ ✓

$\log(z_1 z_2) = \ln|z_1 z_2| + i(\arg(z_1 z_2))$

$= \ln|z_1||z_2| + i(\arg(z_1) + \arg(z_2))$ ∵ $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

$= \ln|z_1| + \ln|z_2| + i\arg(z_1) + i\arg(z_2)$

$= [\ln|z_2| + i\arg(z_2)] + [\ln|z_1| + i\arg(z_1)]$

$= \log(z_2) + \log(z_1) = \log(z_1) + \log(z_2)$

VU

MTH632_Topic071

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11:03 195 KB/S 73

The Complex Logarithm

Example: Let $z_1 = -1$ and $z_2 = -2$. Check whether the following holds

$$\text{Log}(z_1 z_2) = \text{Log}(z_1) + \text{Log}(z_2) .$$

Solution: We know that

$$\text{Log}(z) = \ln|z| + iArg(z)$$

$\text{Log}(-1) = \ln|-1| + i\,Arg(-1)$

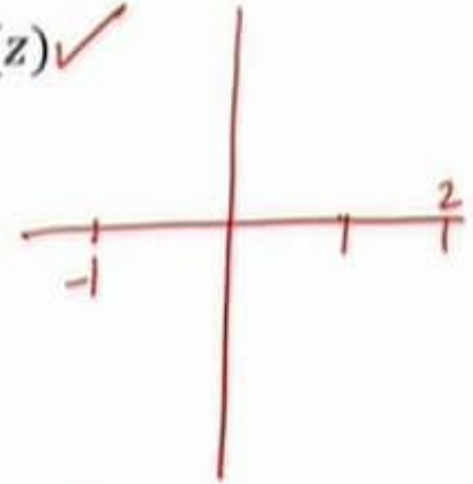
$\text{Log}(-1) = i\pi$

$\text{Log}(-2) = \ln|-2| + i\,Arg(-2)$

$\text{Log}(-2) = \ln|2| + i\pi$

$z_2 = 2$,

$\text{Log}(z_1 z_2) = \ln|2|$



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11:09 279 KB/S 71

The Complex Logarithm

Show that

$$\frac{d}{dz}\log_\alpha z = \frac{1}{z}, \text{ for } z = re^{i\theta}, \quad \alpha < \theta < \alpha + 2\pi$$

$$1 = \frac{d}{dz} z = \frac{d}{dz} \exp\left[\log_\alpha(z)\right]$$

$$= \exp\left[\log_\alpha(z)\right] \frac{d}{dz} \log_\alpha(z)$$

$$1 = z \cdot \frac{d}{dz} \log_\alpha(z) \Rightarrow \boxed{\frac{d}{dz} \log_\alpha(z) = \frac{1}{z}}$$

VU

MTH632_Topic072

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11:15 2.00 KB/S ... 70

Complex Exponents

Example: Evaluate P. V. i^i

$$i^i = \exp[i \operatorname{Log}(i)]$$
$$= \exp[i(\ln|1| + i \operatorname{Arg}(i))]$$
$$= \exp[i(i\tfrac{\pi}{2})]$$
$$= \exp(-\pi/2) = 0.20788 \checkmark$$

VU

MTH632_Topic073

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Complex Exponents

Definition (Complex exponent)

Let $z \neq 0$ and c be a complex number. We define z^c as

$$z^c = \exp[\, c \log(z)]$$ ✓

Example: Evaluate $4^{\frac{1}{2}}$. , $c = \frac{1}{2}$, $z = 4$

$$4^{1/2} = \exp\left[\tfrac{1}{2}\log(4)\right] = \exp\left[\tfrac{1}{2}\left(\ln|4| + i\,(\mathrm{Arg}(4) + 2n\pi)\right)\right]$$

$$= \exp\left[\tfrac{1}{2}\ln|4| + \tfrac{i}{2}\,2n\pi\right]$$

$$= \exp[\ln 2 + in\pi]$$

$n=0$ $4^{1/2}$: $\exp[\ln 2] = 2$, $n=1$ $4^{1/2}$: $\exp[\ln 2 + i\pi]$

$= 2(-1) = -2$

$\Rightarrow 4^{1/2} = \{2, -2\}$

VU

MTH632_Topic073

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11:25 251 KB/S ... 68

Complex Exponents

Example: Evaluate and compare the values of

$(i^2)^i$ and i^{2i}

Solution:

$$(i^2)^i = (-1)^i = \exp[i \log(-1)] = \exp[i(\ln|1| + i(\pi + 2n\pi))]$$

$$= \exp[i(i(\pi + 2n\pi))]$$

$$(i^2)^i = \exp(-\pi(1 + 2n)) \checkmark$$

$$i^{2i} = \exp(2i \log(i)) = \exp[2i(\ln 1 + i(\tfrac{\pi}{2} + 2n\pi))]$$

$$i^{2i} = \exp[-2(\tfrac{\pi}{2} + 2n\pi)] = \exp[-\pi(1 + 4n\pi)] \checkmark$$

VU

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Theorem: The identity

$$\text{Log}(z_1 z_2) = \text{Log}(z_1) + \text{Log}(z_2)$$

holds true iff

$$-\pi < Arg(z_1) + Arg(z_2) \leq \pi$$

Proof: Given $\text{Log}(z_1 z_2) = \text{Log}(z_1) + \text{Log}(z_2)$

To show: $-\pi < Arg(z_1) + Arg(z_2) \leq \pi$

$$\text{Log}(z_1 z_2) = \ln|z_1||z_2| + i\,Arg(z_1 z_2)$$

$$\text{Log}(z_1) + \text{Log}(z_2) = [\ln|z_1| + i\,Arg(z_1)] + [\ln|z_2| + i\,Arg(z_2)]$$

$$\rightarrow \quad Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$$

VU

Theorem: The identity

$$\text{Log}(z_1 z_2) = \text{Log}(z_1) + \text{Log}(z_2)$$

holds true iff

$$-\pi < Arg(z_1) + Arg(z_2) \leq \pi$$

Proof: Conversely assume: $-\pi < Arg(z_1) + Arg(z_2) \leq \pi$

To show $\text{Log}(z_1 z_2) = \text{Log}(z_1) + \text{Log}(z_2)$

$Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$ (exercise)

$$\text{Log}(z_1 z_2) = \ln|z_1||z_2| + i\,Arg(z_1 z_2)$$

$$= \ln|z_1| + \ln|z_2| + i\,Arg(z_1) + i\,Arg(z_2) = \text{Log}(z_1) + \text{Log}(z_2)$$

VU

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Jathal

23-8-2020

MTH-632

Final term Solution

Made by Jathal

Q#01 prove that

IMP

$$\frac{d}{dz}\sin z = \cos z$$

proof:

L.H.S

$$\frac{d}{dz}\sin z = \frac{d}{dz}\left(\frac{e^{iz}-e^{-iz}}{2i}\right)$$

$$= \frac{1}{2i}\left(\frac{d}{dz}e^{iz} - \frac{d}{dz}e^{-iz}\right)$$

$$= \frac{1}{2i}\left(ie^{iz} - (-ie^{-iz})\right)$$

$$= \frac{1}{2i}\left(ie^{iz} + ie^{-iz}\right)$$

$$= \frac{i}{2i}\left(e^{iz} + e^{-iz}\right)$$

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$$= \frac{e^{iz} + e^{-iz}}{2} \qquad \because \frac{e^{iz} + e^{-iz}}{2} = \cos z$$

$$= \cos z \qquad \because \frac{e^{iz} - e^{-iz}}{2i} = \sin z$$

L.H.S = R.H.S

Proved

Q#02 Show That

Imp

$$\frac{d}{dz} \cos z = -\sin z$$

Proof:

$$\frac{d}{dz} \cos z = \frac{d}{dz}\left(\frac{e^{iz} + e^{-iz}}{2}\right)$$

$$= \frac{1}{2}\left(\frac{d}{dz} e^{iz} + \frac{d}{dz} e^{-iz}\right)$$

$$= \frac{1}{2}\left(ie^{iz} + (-ie^{-iz})\right)$$

$$= \frac{i}{2}\left(e^{iz} - e^{-iz}\right)$$

Multiply and div by i

$$= \frac{i}{i} \cdot \frac{i}{2}\left(e^{iz} - e^{-iz}\right)$$

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$$= \frac{i^2}{2i}\left(e^{iz} - e^{-iz}\right)$$

$$= (-1)\left(\frac{e^{iz} - e^{-iz}}{2i}\right) \qquad \because i^2 = -1$$

$$= -\left(\frac{e^{iz} - e^{-iz}}{2i}\right)$$

$$= -\sin z$$

Proved

Show that

$$\cos z + i\sin z = e^{iz}$$

Proof:

We know that

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \qquad \text{---(i)}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \qquad \text{---(ii)}$$

By adding eq (i) and (ii)

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(4)

$$\cos z + i\sin z = \frac{e^{iz}+e^{-iz}}{2} + i\frac{e^{iz}-e^{-iz}}{2i}$$

$$= \frac{e^{iz}+e^{-iz}}{2} + \frac{e^{iz}-e^{-iz}}{2}$$

$$= e^{iz}$$

Show That

$$\sin(z_1+z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$$

Proof:

We know That

$$2\sin z_1 \cos z_2 = \sin(z_1+z_2) + \sin(z_1-z_2) \quad (1)$$

Interchaning z_1 by z_2

$$2\cos z_1 \sin z_2 = \sin(z_2+z_1) + \sin(z_2-z_1)$$

$$2\cos z_1 \sin z_2 = \sin(z_1+z_2) + \sin(-z_1+z_2)$$

$$2\cos z_1 \sin z_2 = \sin(z_1+z_2) - \sin(z_1-z_2) \quad (2)$$

By adding eq (1) and (11)

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$$2\sin z_1\cos z_2 = \sin(z_1+z_2)+\sin(z_1-z_2)$$
$$2\cos z_1\sin z_2 = \sin(z_1+z_2)-\sin(z_1-z_2)$$
$$+$$
$$2(\sin z_1\cos z_2+\cos z_1\sin z_1) = 2\sin(z_1+z_2)$$

OR

$$\sin(z_1+z_2) = \sin z_1\cos z_2+\cos z_1\sin z_2$$

Proved

Show That

$$\cos(z_1+z_2) = \cos z_1\cos z_2-\sin z_1\sin z_2$$

proof:

As we know That

$$\sin(z_1+z_2) = \sin z_1\cos z_2+\cos z_1\sin z_2$$

By diff it w.r.t z_1 we get

$$\cos(z_1+z_2) = \cos z_1\cos z_2+(-\sin z_1)\sin z_2$$
$$\cos(z_1+z_2) = \cos z_1\cos z_2-\sin z_1\sin z_2$$

Proved

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Show That

$$\sin^2 z + \cos^2 z = 1$$

Proof:

We know That

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \quad \text{---} (1)$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \quad \text{---} (11)$$

So L.H.S

$$\sin^2 z + \cos^2 z = \left(\frac{e^{iz} - e^{-iz}}{2i}\right)^2 + \left(\frac{e^{iz} + e^{-iz}}{2}\right)^2$$

$$= \frac{e^{2iz} + e^{-2iz} - 2e^{iz}e^{-iz}}{-4} + \frac{e^{2iz} + e^{-2iz} + 2e^{iz}e^{-iz}}{4}$$

$$= \frac{-e^{2iz} - e^{-2iz} + 2e^{iz}e^{-iz} + e^{2iz} + e^{-2iz} + 2e^{iz}e^{-iz}}{4}$$

$$= \frac{4e^{iz}e^{-iz}}{4}$$

$$= e^{iz}e^{-iz}$$

$$= e^{iz-iz}$$

$$= e^0$$

$$= 1$$

R.H.S proved

Show that

$1 + \tan^2 z = \sec^2 z$

Proof:

From the identity

$\sin^2 z + \cos^2 z = 1$

We have

$$\frac{\sin^2 z}{\cos^2 z} + \frac{\cos^2 z}{\cos^2 z} = \frac{1}{\cos^2 z}$$

$$\tan^2 z + 1 = \sec^2 z$$

Proved

Show that

$1 + \cot^2 z = \operatorname{cosec}^2 z$

proof:

from the identity

$\sin^2 z + \cos^2 z = 1$

We have

$$\frac{\sin^2 z}{\sin^2 z} + \frac{\cos^2 z}{\sin^2 z} = \frac{1}{\sin^2 z}$$

$$1 + \cot^2 z = \operatorname{cosec}^2 z$$

Proved

Since $\frac{\sin^2 z}{\cos^2 z} = \tan^2 z$, $\frac{\cos^2 z}{\sin^2 z} = \cot^2 z$, $\frac{1}{\cos^2 z} = \sec^2 z$, $\frac{1}{\sin^2 z} = \operatorname{cosec}^2 z$

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(8)

Show That

$$|\sin z|^2 = \sin^2 x + \sinh^2 y$$

Solution:

As We know That

$$\sin z = \sin x \cosh y + i \cos x \sinh y$$

We find That

$$|\sin z|^2 = \sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y$$

$$= \sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y$$

$$= \sin^2 x (1 + \sinh^2 y) + (1 - \sin^2 x) \sinh^2 y$$

$$= \sin^2 x + \sin^2 x \sinh^2 y + \sinh^2 y - \sin^2 x \sinh^2 y$$

$$= \sin^2 x + \sinh^2 y$$

Proved

Jathol

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(9)

Show That

$$|\cos z|^2 = \cos^2 x + \sinh^2 y$$

Solution:

As we know That

$$\cos z = \cos x \cosh y + i \sin x \sinh y$$

from this we have

$$|\cos z|^2 = \cos^2 x \cosh^2 y + \sin^2 x \sinh^2 y$$

$$= \cos^2 x (1 + \sinh^2 y) + (1 - \cos^2 x) \sinh^2 y$$

$$= \cos^2 x + \cos^2 x \sinh^2 y + \sinh^2 y - \cos^2 x \sinh^2 y$$

$$= \cos^2 x + \sinh^2 y$$

proved

Show that

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R.H.S

$\cos(\overline{iz}) = \cos(-ix/-y)$

$= \cos(-(y+ix))$

$= \cos(y+ix)$

$= \cos y \cosh x -$

Show that v. Imp

$$\overline{\cos(iz)} = \cos(\overline{iz})$$

Sol:

L.H.S

$\overline{\cos(iz)} = \overline{\cos(-y+ix)}$

$= \overline{\cos(-y)\cosh x - i\sin(-y)\sinh x}$

$= \overline{\cos y \cosh x + i \sin y \sinh x}$

$= \cos y \cosh x - i \sin y \sinh x$

$\because z = x+iy$

$iz = ix + i^2 y$

$= ix - y$

$\Rightarrow -y + ix$

R.H.S

$\cos(\overline{iz}) = \cos(\overline{-y+ix})$ $\quad \therefore iz = -y+ix$

$= \cos(-y-ix)$ $\quad = \cos(-y)$

$= \cos(-(y+ix))$ $\quad = \cos y$

$= \cos(y+ix)$

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$= \cos y \cosh u - \sin y \sinh u$

L.H.S = R.H.S

Show that

$$\frac{d}{dz} \arcsin z = \frac{1}{\sqrt{1-z^2}}$$

Sol:

let $w = \arcsin z$

$\therefore \arcsin z = \sin^{-1} w$

$z = \sin w$

diff b.s w.r.t 'z'

$$\frac{d}{dz} z = \frac{d}{dz} \sin w$$

$$1 = \cos w \cdot \frac{dw}{dz}$$

$$\frac{1}{\cos w} = \frac{dw}{dz}$$

$$\frac{dw}{dz} = \frac{1}{\sqrt{1-\sin^2 w}}$$

$$\frac{dw}{dz} = \frac{1}{\sqrt{1-z^2}}$$

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$$\frac{d}{dz} w = \frac{1}{\sqrt{1-z^2}}$$

$$\Rightarrow \frac{d}{dz} \arcsin z = \frac{1}{\sqrt{1-z^2}}$$

Proved

Prove Jhaj $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz = e + 1/e$

Solution:

L.H.S

$$\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$$

$$= \left| \frac{\sin(z/2)}{1/2} \right|_0^{\pi+2i}$$

$$= 2 \left| \sin(z/2) \right|_0^{\pi+2i}$$

$$= 2 \left[\sin\left(\frac{\pi+2i}{2}\right) - \sin(0) \right]$$

$$= 2 \left(\sin\left(\frac{\pi}{2} + i\right) - 0 \right)$$

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Since

formula: $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$

$$= \frac{1}{2i}\left(e^{i(\pi/2+i)} - e^{-i(\frac{\pi}{2}+i)}\right)$$

$$= \frac{1}{i}\left(e^{i\frac{\pi}{2}+i^2} - e^{(-i\frac{\pi}{2}-i^2)}\right)$$

$$= \frac{1}{i}\left(e^{i\pi/2-1} - e^{-i\frac{\pi}{2}+1}\right)$$

$$= \frac{1}{i}\left(e^{i\pi/2}\cdot e^{-1} - e^{-i\frac{\pi}{2}}\cdot e^{1}\right)$$

Since

formula: $e^{iz} = \cos z + i\sin z$

$$= \frac{1}{i}\left(\left(\cos \pi/2 + i\sin \pi/2\right)e^{-1}\right.$$

$$\left. - \left(\cos(-\pi/2) + i\sin(-\pi/2)\right)e\right)$$

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$$= \frac{1}{i}\left((0+i)e^{-1} - (0-i)e\right)$$

$$= \frac{1}{i}\left(ie^{-1} + ie\right)$$

$$= \frac{i}{i}\left(e + e^{-1}\right)$$

$$= \left(e + \tfrac{1}{e}\right)$$

$$= R.H.S$$

proved

Imp

find $\int_c f(z)\,dz$ if

$$f(z) = \frac{z+2}{z}, \quad z = e^{i\theta}$$

Sol:

$$\int_c f(z)\,dz$$

$$= \int_0^{\pi} \left(\frac{z+2}{z}\right) dz$$

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$$= \int_0^{\pi} \left(\frac{z+2}{z}\right)(i2e^{i\theta})\, d\theta$$

$$= \int_0^{\pi} \left(\frac{z}{z} + \frac{2}{z}\right)(i2e^{i\theta})\, d\theta$$

$$= \int_0^{\pi} \left(1 + \frac{2}{2e^{i\theta}}\right)(i2e^{i\theta})\, d\theta$$

$$= \int_0^{\pi} \left(1 + \frac{1}{e^{i\theta}}\right)(i2e^{i\theta})\, d\theta$$

$$= \int_0^{\pi} \left(\frac{e^{i\theta}+1}{e^{i\theta}}\right)(i2e^{i\theta})\, d\theta$$

$$= 2i \int_0^{\pi} \frac{e^{i\theta}+1}{e^{i\theta}} \cdot e^{i\theta}\, d\theta$$

$$= 2i \int_0^{\pi} (e^{i\theta}+1)\, d\theta$$

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$$= 2i\left[\int_0^{\pi} e^{i\theta}\,d\theta + \int_0^{\pi} 1\,d\theta\right]$$

$$= 2i\left[\left|\frac{e^{i\theta}}{i}\right|_0^{\pi} + |\theta|_0^{\pi}\right]$$

$$= 2i\left[\left(\frac{e^{i\pi}}{i} - \frac{e^{0}}{i}\right) + (\pi - 0)\right]$$

$$= 2i\left(\frac{e^{i\pi}}{i} - \frac{1}{i}\right) + 2i(\pi)$$

$$= \frac{2i}{i}\left(e^{i\pi} - 1\right) + 2i\pi$$

$$= 2\left(\left(\cos\pi + i\sin\pi\right) - 1\right) + 2i\pi$$

$$= 2\left(\left(-1 + 0\right) - 1\right) + 2i\pi$$

$$= 2(-1 - 1) + 2i\pi$$

$$= -4 + 2i\pi$$

$\because e^{iz} = \cos z + i\sin$

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Imp

Find antiderivative of

$$\int_{i}^{i/2} e^{\pi z}\, dz$$

Solution:

$$\int_{i}^{1/2} e^{\pi z}\, dz$$

$$= \left| \frac{e^{\pi z}}{\pi} \right|_{i}^{i/2}$$

$$= \frac{1}{\pi}\left(e^{\pi i/2} - e^{i\pi} \right)$$

$$= \frac{1}{\pi}\left(e^{i\pi/2} - e^{i\pi} \right)$$

$$= \frac{1}{\pi}\left(\cos(\pi/2) + i\sin(\pi/2) - (\cos\pi + i\sin\pi) \right)$$

$$= \frac{1}{\pi}\left((0 + i) - (-1 + 0) \right)$$

$$= \frac{1}{\pi}\left(i + 1 \right)$$

$$= \frac{i+1}{\pi}$$

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Imp

Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies laplaces equation.

Sol:

Given that

$$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$

diff w.r.t 'x'

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x$$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 6 \quad \text{--- (1)}$$

Now diff w.r.t 'y'

$$\frac{\partial u}{\partial y} = -6xy - 6y$$

$$\frac{\partial^2 u}{\partial y^2} = -6x - 6 \quad \text{--- (1)}$$

By adding eq (1) and (1)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 6 - 6x - 6$$

$(\cos\pi + i\sin\pi)$

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$= 0$

$\therefore$ U satisfies the laplace equation.

Imp

Question: $z_n = \frac{1}{n^3} - i$ solve it for convergence also find value of ε.

Solution:

$$z_n = \frac{1}{n^3} - i$$

$$\lim_{n \to \infty} z_n = \lim_{n \to \infty} \left(\frac{1}{n^3} - i \right)$$

$$= \lim_{n \to \infty} \left(\frac{1}{n^3} \right) - \lim_{n \to \infty} (i)$$

$$= \frac{1}{\infty} - i$$

$$= -i < 0 \text{ (convergenes)}$$

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Imp (20)

Evaluate $\bar{z}_1 - \bar{z}_2$ when $z_1 = 5+i$ and $z_2 = -8+3i$

Sol:

$$z_1 = 5+i \quad , \quad z_2 = -8+3i$$

$$\bar{z}_1 = 5-i \quad , \quad \bar{z}_2 = -8-3i$$

$$\bar{z}_1 - \bar{z}_2 = (5-i) - (-8-3i)$$

$$= 5 - i + 8 + 3i$$

$$= 2i + 13$$

if $x(t) = a\sin t\hat{e}_1 + a\cos t\hat{e}_2 +$

Solve $x'(t)$

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Imp

if $x(t) = a\cos t\hat{e}_1 + a\sin t\hat{e}_2 + bt\hat{e}_3$

then solve $\dfrac{x'(t)}{|x'(t)|}$

Solution:

$$x(t) = a\cos t\hat{e}_1 + a\sin t\hat{e}_2 + bt\hat{e}_3$$

diff w.r.t "t"

$$x'(t) = -a\sin t\hat{e}_1 + a\cos t\hat{e}_2 + b\hat{e}_3$$

$$|x'(t)| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + b^2}$$

$$= \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}$$

$$= \sqrt{a^2(\sin^2 t + \cos^2 t) + b^2}$$

$$= \sqrt{a^2 + b^2}$$

So

$$\frac{x'(t)}{|x'(t)|} = \frac{-a\sin t\hat{e}_1 + a\cos t\hat{e}_2 + b\hat{e}_3}{\sqrt{a^2+b^2}}$$

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Find $V_p[f]$ if $A(1,1,0)$ $v = (1,0,-3)$ and $f = x^2yz$

Solution:

$$f = x^2yz$$

$$\frac{\partial f}{\partial x} = 2xyz \quad , \quad \frac{\partial f}{\partial y} = x^2z$$

$$\frac{\partial f}{\partial z} = x^2y$$

at point $P = (1,1,0)$

$$\frac{\partial f}{\partial x} = 0 \quad , \quad \frac{\partial f}{\partial y} = 0 \quad , \quad \frac{\partial f}{\partial z} = 1$$

Here

$$V_p[f] = v\frac{\partial f}{\partial x}(P) + v\frac{\partial f}{\partial y}(P) + \frac{\partial f}{\partial z}(P)$$

$$= 1(0) + 0(0) + (-3)(1)$$

$$= -3$$

-3 is directional derivative.

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Find Singularity if

$$f(z) = \frac{1}{z^2+2z+2}$$

Solution:

Here $z^2+2z+2=0$

by quadratic formula

$$z = \frac{-2 \pm \sqrt{4-4(1)(2)}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

$z = -1 \pm i$ is Singuler point

Show that $\sum_{n=1}^{\infty} \bar{z}_n = \bar{S}$

Solution:

As we know That

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$$z_n = x_n + iy_n$$

$$\bar{z}_n = x_n - iy_n$$

and

$$S = X + iY$$

$$\bar{S} = X - iY$$

$$\sum_{n=1}^{\infty} x_n = X \quad ; \quad \sum_{n=1}^{\infty} y_n = Y$$

So

$$\sum_{n=1}^{\infty} \bar{z}_n = \sum_{n=1}^{\infty} (x_n - iy_n)$$

$$= \sum_{n=1}^{\infty} x_n - \sum_{n=1}^{\infty} y_n$$

$$= X - iY$$

$$\sum_{n=1}^{\infty} \bar{z}_n = \bar{S}$$

Proved

Show that $\overline{(-1)^{1/\pi}} = \exp(1+2n)i$

Solution:

L.H.S

$$= (-1)^{1/\pi}$$

$$= \exp\left[\tfrac{1}{\pi}\log(-1)\right]$$

$$= \exp\left[\tfrac{1}{\pi}\log(1)\right]$$

$$= \exp\left[\tfrac{1}{\pi}\left(\ln|1| + i[\arg(1) + 2n\pi]\right)\right]$$

$$= \exp\left[\tfrac{1}{\pi}(0 + (\pi + 2n\pi)i\right]$$

$$= \exp\left[\tfrac{1}{\pi} \cdot \pi(1+2n)i\right]$$

$$= \exp[(1+2n)i]$$

$$= \exp(1+2n)i$$

$$= R.H.S$$

Prove That

$$d(fg) = gfd + fdg$$

Solution:

$$d(fg) = \sum \frac{\partial}{\partial x_i}(fg)\,dx_i$$

$$= \sum \left(\frac{\partial f}{\partial x_i} g + \frac{\partial g}{\partial x_i} f\right) dx_i$$

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$$= \sum \frac{\partial f}{\partial x_i} g\, dx_i + \sum \frac{\partial g}{\partial x_i} f\, dx_i$$

$$= g \sum \frac{\partial f}{\partial x_i} dx_i + f \sum \frac{\partial g}{\partial x_i} dx_i$$

$$= g\,df + f\,dg \quad \cdots (1)$$

$$= R.H.S$$

Show that the function $f(z) = \bar{z}$ nowhere diff.

Solution:

Given that

$$u + iv = x - iy$$

$$u = x, \quad v = -y$$

$$u_x = 1, \quad v_x = 0$$

$$u_y = 0, \quad v_y = -1$$

here $u_x \neq v_y$

so $f(z) = \bar{z}$ nowhere differentiable.

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Test the analyticity of function $w = \sin z$

Sol:

Let $w = f(z)$

$f(z) = \sin z$

$u + iv = \sin(x + iy)$

$= \sin x \cosh y + i \cos x \sinh y$

Now

$u = \sin x \cosh y$; $v = \cos x \sinh y$

$u_x = \cos x \cosh y$; $v_x = -\sin x \sinh y$

$u_y = \sin x \sinh y$; $v_y = \cos x \cosh y$

$u_x = v_y$ and $u_y = -v_x$

$\therefore$ function is analytic.

Show that $f(z) = e^z$ is analytic

Sol:

$f(z) = e^z$

$u + iv = e^{x+iy}$

$= e^x \cdot e^{iy}$

$= e^x \cdot (\cos y + i \sin y)$

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$u+iv = e^x \cos y + ie^x \sin y$

$u = e^x \cos y$, $v = e^x \sin y$

$u_x = e^x \cos y$, $v_x = e^x \sin y$

$u_y = -e^x \sin y$, $v_y = e^x \cos y$

here

$u_x = v_y$ and $u_y = -v_x$

So function is analytic.

Given an example such that u and v are harmonic but u+iv is not analytic.

Solution.

consider the function

$f(z) = \bar{z}$

$u+iv = x - iy$

$u = x$, $v = -y$

$u_x = 1$, $v_x = 0$

$u_{xx} = 0$, $v_{xx} = 0$

$u_y = 0$, $v_y = -1$

$u_{yy} = 0$, $v_{yy} = 0$

here cauchy-Rieman equation not satisfied.

$u_x \neq v_y$

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$\therefore f(z)$ is not analytic

But $U_{xx} + U_{yy} = 0$ and $V_{xx} + V_{yy} = 0$ gives U and V are harmonic.

Write $-3i^4$ in the form of $a+ib$

Soli

$$\text{let } z = -3i^4$$
$$z = -3(i^2)^2$$
$$z = -3(-1)^2$$
$$= -3$$
$$z = -3 + 0i$$

Find the arc length of

$$S = \int_a^b \left| \frac{\overrightarrow{dr}}{dt} \right| dt$$

$\vec{r}(t) = (\cos t, \sin t, t)$

$0 \leq t \leq 2\pi$

Solution:

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We start from the
are length

$$s(t) = \int_a^b \| x'(t) \| \, dt$$

here

$$x(t) = (\cos t, \sin t, t)$$

$$x'(t) = (-\sin t, \cos t, 1)$$

$$\| x'(t) \| = \sqrt{\sin^2 t + \cos^2 t + 1}$$

$$= \sqrt{1+1}$$

$$= \sqrt{2}$$

So

$$\int_0^{2\pi} \sqrt{2} \, dt$$

$$\sqrt{2} \int_0^{2\pi} dt$$

$$\sqrt{2} \Big| t \Big|_0^{2\pi}$$

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$$= \sqrt{2}\,(2\pi - 0)$$

$$s(t) = 2\pi\sqrt{2}$$

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$$\int_0^{\pi} \frac{\cosh}{z^4}\, dz$$ by cauchy integral formula.

Solution:

We know the cauch Integral formula:

$$\int_c \frac{f(z)\, dz}{(z-z_0)^{n+1}} = \frac{2\pi}{n!} \left| f^{(n)}(z_0) \right|$$

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$$\int_C \frac{\cosh z}{z^4}\, dz$$

$$= \int_C \frac{\cosh z}{(z-z_0)^{3+1}}\, dz$$

$$= \frac{2\pi}{3!} \left| \frac{d^3}{dz^3} \cosh z \right|_{z_0=0}$$

$$= \frac{2\pi}{3!}(0)$$

$$= 0$$

$u(x,y) = 2x(1-y)$ harmonic in some domain and find a harmonic conjugate $v(x,y)$ when

$$u(x,y) = 2x(1-y)$$

Solution:

It is straightforward to show that $u_{xx} + u_{yy} = 0$ When $u(x,y) = 2x(1-y)$ to find harmonic conjugation

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$v(x,y)$. We start with

$u_x = 2 - 2y$ Now

$u_x = v_y \Rightarrow v_y = 2 - 2y$

$\Rightarrow v(x,y) = \underbrace{2y - y^2}_{v_y} + \underbrace{\phi(x)}_{v_x}$

then

$u_y = -v_x \Rightarrow -2x = -\phi'(x) \Rightarrow \phi'(x) = 2x$

$\Rightarrow \phi(x) = x^2 + c$

consequently

$$v(x,y) = 2y - y^2 + (x^2 + c)$$
$$= x^2 - y^2 + 2y + c$$

$$\int_c \frac{e^{-z}}{(z - \pi i/2)} dz$$ by cauch integral.

solution:

$$\int_c \frac{e^{-z}}{(z - \pi i/2)^{0+1}}$$

since $n = 1$

$\therefore f(z) = e^{-z}$

$f'(z) = e^{-z}$

$f'(i\pi/2) = e^{-i\pi/2}$

$$\frac{2\pi i}{1!} \left| f'(\pi i/2) \right|$$

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$= 2\pi i\left(e^{-i\frac{\pi}{2}}\right)$

$= 2\pi i\left(\cos\left(-\frac{\pi}{2}\right) + i\sin(-\pi/2)\right)$

$= 2\pi i(0 - i)$

$= -2\pi i^2$

$= 2\pi$

Find analytical region

$f(z) = (x-y)^2 + 2i(x+y)$

Sol:

$u + iv = (x-y)^2 + 2i(x+y)$

$u = (x-y)^2$, $v = 2(x+y)$

$u_x = 2(x-y)$, $v_x = 2$

$u_y = 2(x-y)(-1)$, $v_y = 2$

$\Rightarrow -2(x-y)$,

$\because f(z)$ is analytic, So u and v should satisfy cauchy-equation.

$u_x = v_y \Rightarrow 2(x-y) = 2$

$\Rightarrow x - y = 1$

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also

$$v_y = -v_x \Rightarrow -2(x-y) = -2$$
$$\Rightarrow x - y = 1$$

$\Rightarrow$ $f(z)$ is analytic at all points satisfying $x + y - 1$.

hence the analytic region is the line.

if $f(z) = z(2 - z^*)$ then find $f(1+i)$

Solution: $f(z) = 2z - z^2$

$$f(1+i) = 2(1+i) - (1+i)^2$$
$$= 2 + 2i - 1 - i^2 - 2i$$
$$= 2 - 1 + 1$$
$$= 2$$

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Serret frenet formulas

$\hat{t}, \hat{n}, \hat{b}$

i) $\frac{d\hat{t}}{ds} = k\hat{n} \Rightarrow \hat{t}' = k\hat{n}$

ii) $\frac{d\hat{r}}{ds} = \tau\hat{b} \Rightarrow \hat{n}' = \tau\hat{b} - k\hat{t}$

iii) $\frac{d\hat{b}}{ds} = -\tau\hat{n} \Rightarrow \hat{b}' = -\tau\hat{n}$

Prof of (i)

$\hat{t}' = k\hat{n}$

Proof:

$\hat{t}$ is unit vector

$\hat{t} \cdot \hat{t} = 1$ ——— (i)

diff w.r.t 's'

$\frac{d\hat{t}}{ds} \cdot \hat{t} + \hat{t} \frac{d\hat{t}}{ds} = 0$

$2\hat{t} \cdot \hat{t}' = 0$

$\hat{t} \cdot \hat{t}' = 0$ ——— (ii)

$\hat{t}'$ is $\perp$ to $\hat{t}$

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$\frac{d\hat{t}}{ds} = \hat{t}'$ along $\hat{n}$

$\frac{d\hat{t}}{ds} = \pm \left| \frac{d\hat{t}}{ds} \right| \hat{n}$

$\hat{t}' = k\hat{n}$

proved

Made by
jathol
MSc 4th Semester

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MTH632

Final term solution file:

Sabin Saeed

Q1: Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies Laplace's equation.

Sol: Given $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x \quad ; \quad \frac{\partial u}{\partial y} = -6xy - 6y$$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 6 \qquad \frac{\partial^2 u}{\partial y^2} = -6x - 6$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 6 - 6x - 6 = 0$$

$\therefore$ u satisfies Laplace's equation.

Q2: Show that

(a) $\text{Log}(1+i)^2 = 2\text{Log}(1+i)$

(b) $\text{Log}(-1+i)^2 \neq 2\,\text{Log}(-1+i)$

Sol:

(a) $(1+i) = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$$= \sqrt{2}\, e^{i\pi/4}$$

$$(1+i)^2 = \left(\sqrt{2}e^{i\pi/4}\right)^2 = 2e^{i\pi/2}$$

therefore $\text{Arg}(1+i) = \frac{\pi}{4}$ and $\text{Arg}(1+i)^2 = \frac{\pi}{2}$

so that

$$\text{Log}(1+i)^2 = \ln|(1+i)^2| + i\,\text{Arg}(1+i)^2$$

$$= \ln|2e^{i\pi/2}| + i\frac{\pi}{2}$$

$$= \ln\left|2\left(\cos\frac{\pi}{2} + i\sin\frac{2\pi}{2}\right)\right| + \frac{i\pi}{2}$$

$$= \ln|2| + \frac{i\pi}{2}$$

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"x" and "÷" by 2

$$= 2\left(\frac{1}{2}\ln|2| + i\frac{\pi}{4}\right) = 2\left(\ln\sqrt{2} + i\frac{\pi}{4}\right) = 2\cdot Log(1+i)$$

(b) $$-1+i = \sqrt{2}\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \sqrt{2}e^{i3\pi/4}$$

$$(-1+i)^2 = -2\cdot i = 2e^{-i\pi/2}$$

therefor

$$Arg(-1+i) = \frac{3\pi}{4} \quad ; \quad Arg(-1+i)^2 = \frac{\pi}{2}$$

so that

$$Log(-1+i)^2 = \ln|(-1+i)^2| + iArg(-1+i)^2$$

$$= \ln|2e^{-i\pi/2}| + i\frac{\pi}{2}$$

$$= \ln|2| + i\frac{\pi}{2} \quad \text{---(i)}$$

$$2Log(-1+i) = \ln 2 + i\frac{3\pi}{2} \quad \text{---(ii)}$$

from (i) and (ii)

$$\Rightarrow Log(-1+i)^2 \neq 2Log(-1+i)$$

Q3: $$\frac{d}{dz}\arcsin z = \frac{1}{\sqrt{1-z^2}}$$

Sol: Let $w = \arcsin z$

$$z = \sin w$$

Take derivative of b/s

$$\frac{d(z)}{dz} = \frac{d(\sin w)}{dz}$$

$$1 = \frac{d}{dw}\sin w \cdot \frac{dw}{dz}$$

$$1 = \cos w \cdot \frac{dw}{dz}$$

$$\frac{1}{\cos w} = \frac{dw}{dz}$$

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$$\frac{dw}{dz} = \frac{1}{\sqrt{1-\sin^2 w}}$$

$$\frac{dw}{dz} = \frac{1}{\sqrt{1-z^2}}$$

$$\Rightarrow \frac{d}{dz}\arcsin z = \frac{1}{\sqrt{1-z^2}} \quad \text{(proved)}$$

Q4: Prove that $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz = e + 1/e$

Sol: $L.H.S = \int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$

$$= \left| \frac{\sin(z/2)}{1/2} \right|_0^{\pi+2i} = 2\left| \sin\left(\frac{z}{2}\right) \right|_0^{\pi+2i}$$

$$= 2\left[\sin\left(\frac{\pi+2i}{2}\right) - \sin(0) \right]$$

$$= 2\left[\sin\left(\frac{\pi}{2}+i\right) \right]$$

$$= 2\left[\frac{e^{i(\frac{\pi}{2}+i)} - e^{-i(\frac{\pi}{2}+i)}}{2i} \right]$$

$$= \frac{1}{i}\left[e^{(i\pi/2+i^2)} - e^{(-i\pi/2-i^2)} \right]$$

$$= \frac{1}{i}\left[e^{i\pi/2-1} - e^{-i\pi/2+1} \right]$$

$$= \frac{1}{i}\left[e^{i\pi/2}\cdot e^{-1} - e^{-i\pi/2}\cdot e^{1} \right]$$

$$= \frac{1}{i}\left[\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)e^{-1} - \left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)e \right]$$

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$$= \frac{1}{i}\left[(0+i)e^{-1} - (0-i)e^{1}\right]$$

$$= \frac{1}{i}\left(i\frac{1}{e} + ie\right)$$

$$= \frac{i}{i}\left(\frac{1}{e} + e\right) = R.H.S$$

proved.

Q5: $z_n = \frac{1}{n^3} - i$ solve it for convergence also find value of ε.

Sol: $z_n = \frac{1}{n^3} - i$

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty}\left(\frac{1}{n^3} - i\right)$$

$$= \lim_{n\to\infty}\frac{1}{n^3} - \lim_{n\to\infty}(i)$$

$$= \frac{1}{\infty} - i = -i \angle 0 \text{ (converges)}$$

Q6: Evaluate $\bar{z}_1 - \bar{z}_2$ when $z_1 = 5+i$ and $z_2 = -8+3i$

Sol: $z_1 = 5+i \quad ; \quad z_2 = -8+3i$

$\bar{z}_1 = 5-i \quad ; \quad \bar{z}_2 = -8-3i$

$\bar{z}_1 - \bar{z}_2 = (5-i) - (-8-3i)$

$= 5 - i + 8 + 3i$

$= 13 + 2i$

$= 2(6+i)$

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Q7: If $\int_C f(z)\,dz$ & $f(z) = \frac{z+2}{z}$

and $z = 2e^{i\theta}$

Sol: $\int_C f(z)\,dz$

$= \int_0^{\pi} \left(\frac{z+2}{z}\right) dz$ $\quad \because z = 2e^{i\theta}$, $dz = 2ie^{i\theta}\,d\theta$

$= \int_0^{\pi} \left(1 + \frac{2}{z}\right) dz = \int_0^{\pi} \left(1 + \frac{2}{2e^{i\theta}}\right)(i2e^{i\theta})\,d\theta$

$= \int_0^{\pi} \left(1 + \frac{1}{e^{i\theta}}\right)(2ie^{i\theta})\,d\theta$

$= 2i \int_0^{\pi} \frac{e^{i\theta}+1}{e^{i\theta}} \cdot e^{i\theta}\,d\theta$

$= 2i \left[\int_0^{\pi} e^{i\theta}\,d\theta + \int_0^{\pi} d\theta\right]$

$= 2i \left[\frac{e^{i\theta}}{i}\Big|_0^{\pi} + 2i\,\theta\Big|_0^{\pi}\right]$

$= 2(e^{i\pi} - e^{0}) + 2i\pi$ $\quad \because e^{i\pi} = \cos\pi + i\sin\pi = -1 + 0$

$= 2e^{i\pi} - 2 + 2i\pi$

$= -2 - 2 + 2i\pi$

$= -4 + 2i\pi$

Q8: If $x(t) = +a\sin t\,\hat{e}_1 + a\cos t\,\hat{e}_2 + bt\,\hat{e}_3$

then solve $\frac{x'(t)}{|x'(t)|}$

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Sol: Given $x(t) = a\cos t\,\hat{e}_1 + a\sin t\,\hat{e}_2 + bt\,\hat{e}_3$

$$x'(t) = -a\sin t\,\hat{e}_1 + a\cos t\,\hat{e}_2 + b\hat{e}_3$$

$$|x'(t)| = \sqrt{a^2\sin^2(t) + a^2\cos^2 t) + b^2}$$

$$= \sqrt{a^2(\sin^2 t + \cos^2 t) + b^2}$$

$$= \sqrt{a^2 + b^2}$$

we know that

$$\hat{t} = x = \frac{x'(t)}{|x'(t)|} = \frac{-a\sin(t)\hat{e}_1 + a\cos(t)\hat{e}_2 + b\hat{e}_3}{\sqrt{a^2+b^2}}$$

Q9: If $p = (1,1,0)$ and $v = (1,0,3)$,

$f = 3x^2yz$

Then find $V_p[f]$.

Sol: 1st method:

$$(P+tv) = (1,1,0) + t(1,0,3)$$

$$= (1+t, 1, -3t)$$

$$f(P+tv) = f(1+t, 1, -3t)$$

$$= (1+t)^2(1)(-3t)$$

$$= -3t^3 - 6t^2 - 3t$$

$$\frac{d}{dt}(f(P+tv)) = \frac{d}{dt}(-3t^3 - 6t^2 - 3t)$$

$$= -9t^2 - 12t - 3$$

Applying definition of $V_p[f]$

$$V_p[f] = \frac{d}{dt}(f(P+tv))\Big|_{t=0}$$

$$= -9t^2 - 12t - 3\Big|_{t=0}$$

$$= -3$$

So -3 is directional derivative.

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2nd method:

$$f = x^2yz$$

$$\frac{\partial f}{\partial x} = 2xyz \;,\quad \frac{\partial f}{\partial y} = x^2z \;,\quad \frac{\partial f}{\partial z} = x^2y$$

at point $P=(1,1,0)$

$$\frac{\partial f}{\partial x} = 0 \;,\quad \frac{\partial f}{\partial y} = 0 \;,\quad \frac{\partial f}{\partial z} = 1$$

Here

$$V_p[f] = \sum v_i \frac{\partial f}{\partial x_i}(P)$$

$$= 1(0) + 0(0) \neq 3(1)$$

$$= -3$$

-3 is directional derivative.

Q10: Find singularity if $f(z) = \frac{1}{z^2+2z+2}$

Sol: Here $z^2+2z+2=0$

applying quadratic formula

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(2) \pm \sqrt{(2)^2-4(1)(2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2} = -1 \pm i$$

$z = -1 \pm i$ is singular point.

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Q11: Prove that $\frac{d}{dz}(\sin z) = \cos(z)$

Sol: $L.H.S = \frac{d}{dz}(\sin z)$

$$= \frac{d}{dz}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}z^{2n+1}$$

$$= \sum_{n=0}^{\infty}\frac{d}{dz}\frac{(-1)^n}{(2n+1)!}z^{2n+1}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n(2n+1)}{(2n+1)2n!}\cdot z^{2n}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{2n!}\cdot z^{2n}$$

$$= \cos z = R.H.S$$

Q#12: Show that $\sum_{n=1}^{\infty}\bar{z}_n = \bar{S}$

Sol: We know that

$$\bar{z}_n = x_n - iy_n$$

and $S = X + iY$; $\bar{S} = X - iY$

$\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$

$$\sum_{n=1}^{\infty}\bar{z}_n = \sum_{n=1}^{\infty}(x_n - iy_n)$$

$$= \sum_{n=1}^{\infty}x_n - i\sum_{n=1}^{\infty}y_n$$

$$= X - iY$$

$\sum_{n=0}^{\infty}\bar{z}_n = \bar{S}$ proved

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Q 13: Show that $(-1)^{1/\pi} = \exp(1+2n)i$

Sol: $L \cdot H \cdot S = (-1)^{1/\pi}$

$= \exp\left[\frac{1}{\pi}\log(-1)\right]$

$= \exp\left[\frac{1}{\pi}\log(1)\right]$

$= \exp\left[\frac{1}{\pi}(\ln|1| + i[\arg(1) + 2n\pi])\right]$

$= \exp\left[\frac{1}{\pi}(0 + (\pi + 2n\pi)i)\right]$

$= \exp\left[\frac{1}{\pi} \cdot \pi(1+2n)i\right]$

$= \exp(1+2n)i$ R·H·S

Q 14: Show that $\overline{\cos(iz)} = \cos(\bar{i}\bar{z})$

Sol: L·H·S $\overline{\cos(iz)}$ $\qquad z = x + iy$

$= \overline{\cos(-y+ix)}$ $\qquad iz = ix - y$

$= \cos y \cosh x - \sin y \sinh x$ —(a) $\qquad \bar{i}\bar{z} = -ix - y$

R·H·S $= \cos(\bar{i}\bar{z})$

$= \cos(-ix - y)$

$= \cos(-(y+ix))$ $\qquad \because \cos(-x) = \cos x$

$= \cos(y+ix)$

$= \cos y \cosh x - \sin y \sinh x$ —(b)

From (a) and (b)

L·H·S = R·H·S

Q 15: prove that $d(fg) = gfd + f\,dg$

Sol: $d(fg) = \sum \frac{\partial(fg)}{\partial x_i} dx_i$

$= \sum \left(\frac{\partial f}{\partial x_i} \cdot g + \frac{\partial g}{\partial x_i} \cdot f\right) dx_i$

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$$= \sum \frac{\partial f}{\partial x_i} \cdot g \, dx_i + \sum \frac{\partial g}{\partial x_i} \cdot f \, dx_i$$

$$= g \sum \frac{\partial f}{\partial x_i} dx_i + f \sum \frac{\partial g}{\partial x_i} dx_i$$

$$= g\,df + f\,dg \quad \text{(proved)}$$

Q 16: Find df and $df(v_p)$ if
$f = (x^2-1)y + (y^2-1)x$.

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Q17: Show that the function $f(z) = \bar{z}$ nowhere differentiable.

Sol: Given $u+iv = x-iy$

$u = x$; $v = -y$

$u_x = 1$; $v_x = 0$

$u_y = 0$; $v_y = -1$ $\therefore$ $u_x \neq v_y$

$\therefore$ C-R eq. are not satisfied

$f(z) = \bar{z}$ nowhere differentiable.

Q18: Test the analyticity of function $w = \sin z$

Sol: $w = f(z) = \sin z$

$u+iv = \sin(x+iy)$

$(u+iv) = \sin x \cosh y + i \cos x \sinh y$

Now

$u = \sin x \cosh y$; $v = \cos x \sinh y$

$u_x = \cos x \cosh y$; $v_x = -\sin x \sinh y$

$u_y = +\sin x \sinh y$; $v_y = \cos x \cosh y$

$u_x = u_y$ & $-v_x = u_y$

C-R equation satisfied.

$\therefore$ function is analytic.

Q19: Show that $f(z) = e^z$ is analytic

Sol: $f(z) = e^z$

$u+iv = e^{x+iy}$

$u+iv = e^x \cdot e^{iy} = e^x(\cos y + i \sin y) = e^x \cos y + ie^x \sin y$

$u = e^x \cos y$; $v = e^x \sin y$

$u_x = e^x \cos y$; $v_x = e^x \sin y$

$u_y = -e^x \sin y$; $v_y = e^x \cos y$

$u_x = v_x$ & $u_y = -v_x \Rightarrow$ function is analytic.

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Q 20: Give an example such that u and v are harmonic but u+iv is not analytic.

Sol: Consider the function $w = \bar{z} = x - iy$

$u = x$; $v = -y$

$u_x = 1$ $\quad$ $v_y = -1$

$u_{xx} = 0$ $\quad$ $v_{yy} = 0$

$u_y = 0$ $\quad$ $v_x = 0$

$u_{yy} = 0$ $\quad$ $v_{xx} = 0$

$u_x \neq v_x$

$\therefore$ f(z) is not analytic.

But $u_{xx} + u_{yy} = 0$ and $v_{xx} + v_{yy} = 0$ gives u and v are harmonic.

Q #21: Write $-3i^4$ in the form of a+ib.

Sol: Let $z = -3i^4$

$z = -3(i^2)^2$

$z = -3(-1)^2 = -3$

$z = -3 + 0i$

Q 22: Find antiderivative of $\int_i^{i/2} e^{\pi z}\, dz$.

Sol: $\int_i^{i/2} e^{\pi z}\, dz$

$= \frac{e^{\pi z}}{\pi}\Big|_i^{i/2} = \frac{1}{\pi}\left[e^{i\pi/2} - e^{i\pi}\right]$

$= \frac{1}{\pi}\left[i - (-1)\right] = \frac{i+1}{\pi}$

$\because e^{i\pi} = \cos\pi + i\sin\pi$

$= -1 + 0 = -1$

$\because e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$= 0 + i = i$

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Q 23: Prove that $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and find its conjugate harmonic.

Sol: Given $u = \frac{1}{2}\log(x^2+y^2)$

Differentiate partially w.r.t. "x" and "y"

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2} \quad ; \quad \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{y^2-x^2}{(x^2+y)^2} \quad ; \quad \frac{\partial^2 u}{\partial y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{y^2-x^2+x^2-y^2}{(x^2+y^2)^2} = 0$$

$\Rightarrow u$ is harmonic

Let $v(x,y)$ be the conjugate harmonic. Then $w = u+iv$ is harmonic

By C-R equations, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

we have

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

$$dv = -\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

$$dv = \frac{-y}{x^2+y^2}dx$$

Integrating

$$v = \tan^{-1}(y/x) + c$$

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Hijri 1441 SEPTEMBER 2019

MTH 632
Current Papers

Notes

Q: Write $-3i^4$ in the form of $a+ib$.

A: $a+ib = -3i^4$

$= -3(i^2)^2$

$= -3(-1)^2$

$a+ib = -3+0i$

Q: Find antiderivative of $\int_i^{i/2} e^{\pi z}\, dz$

Notes **A:**

$$I = \int_i^{i/2} e^{\pi z}\, dz$$

$$= \frac{e^{\pi z}}{\pi}\Bigg|_i^{i/2}$$

Important

$$= \frac{1}{\pi}\left[e^{i\pi/2} - e^{i\pi}\right]$$

$$= \frac{1}{\pi}\left[i - (-1)\right] = \frac{i+1}{\pi}$$

$\because e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$= 0 + i = i$

$e^{i\pi} = \cos\pi + i\sin\pi$

$= -1 + 0 = -1$

20 FRI 20 Muharram

21 SAT 21 Muharram

22 SUN 22 Muharram

September 2019

M	T	W	T	F	S	S
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16	17	18	19	20	21	22

M	T	W	T	F	S	S
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23	24	25	26	27	28	29

M	T	W	T	F	S	S
9	10	11	12	13	14	15
30						

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23 MON 23 Muharram

Q: $Z_n = \frac{1}{n^3} - i$ Solve it for convergence

A:

$$Z_n = \frac{1}{n^3} - i$$

Notes

24 TUE 24 Muharram

$$\lim_{n\to\infty} Z_n = \lim_{n\to\infty}\left(\frac{1}{n^3} - i\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{n^3}\right) - \lim_{n\to\infty}(i)$$

$$= \frac{1}{\infty} - i = -i < 0$$

(converges)

Notes

25 WED 25 Muharram

Q: Prove that

$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

satisfies Laplace's equation.

A: $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

Notes

26 THU 26 Muharram

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x \quad , \quad \frac{\partial u}{\partial y} = -6xy - 6y$$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 6 \quad , \quad \frac{\partial^2 u}{\partial y^2} = -6x - 6$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 6 - 6x - 6 = 0$$

27 FRI 27 Muharram

Hence, u satisfies Laplace's equation.

Notes

Q: Evaluate $\bar{z}_1 - \bar{z}_2$ when $z_1 = 5+i$ & $z_2 = -8+3i$.

28 SAT 28 Muharram

A:

$z_1 = 5+i$, $z_2 = -8+3i$

Notes

$\bar{z}_1 = 5-i$, $\bar{z}_2 = -8-3i$

$$\bar{z}_1 - \bar{z}_2 = 5 - i + 8 + 3i$$

$$= 13 + 2i$$

29 SUN 29 Muharram

Q: If $\int_C f(z)\,dz$ & $f(z) = \frac{z+2}{z}$

Important

& $z = 2e^{i\theta}$

A: $I = \int_C f(z)\,dz$

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SEP/OCT 2019

30 MON 30 Muharram

$$I = \int_0^{\pi} \left(\frac{z+2}{z}\right) dz \qquad \because z = 2e^{i\theta}$$

$$dz = 2ie^{i\theta} d\theta$$

$$= \int_0^{\pi} \left(\frac{z}{z} + \frac{2}{z}\right) dz$$

1 TUE 1 Safar

$$= \int_0^{\pi} \left(1 + \frac{2}{z}\right) dz$$

$$= \int_0^{\pi} \left(1 + \frac{2}{2e^{i\theta}}\right) 2ie^{i\theta} d\theta$$

2 WED 2 Safar

$$= \int_0^{\pi} \left(1 + \frac{1}{e^{i\theta}}\right) 2ie^{i\theta} d\theta$$

$$= 2i \int_0^{\pi} \left(\frac{e^{i\theta}+1}{e^{i\theta}}\right) \cdot e^{i\theta} d\theta$$

3 THU 3 Safar

$$= 2i \int_0^{\pi} (e^{i\theta} + 1)\, d\theta$$

$$= 2i \left[\int_0^{\pi} e^{i\theta} d\theta + \int_0^{\pi} d\theta\right]$$

Hijri 1441 OCTOBER 2019

$$I = 2i \left[\frac{e^{i\theta}}{i}\Big|_0^{\pi} + \theta\Big|_0^{\pi}\right]$$

4 FRI 4 Safar

$$= 2i \left[\frac{e^{i\pi} - e^{0}}{i}\right] + 2i(\pi - 0)$$

$$= 2e^{i\pi} - 2 + 2i\pi \qquad \because e^{i\pi} = \cos\pi + i\sin\pi$$

$$= 2(-1) - 2 + 2i\pi \qquad = -1 + 0 = -1$$

5 SAT 5 Safar

$$= 2i\pi - 4$$

Q: Poove that

$$\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz = e + \frac{1}{e}$$

6 SUN 6 Safar

A: L.H.S

$$I = \int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$$

$$= \frac{\sin\frac{z}{2}}{1/2}\Big|_0^{\pi+2i}$$

October 2019

M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31											

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7 MON 7 Safar

8 TUE 8 Safar

9 WED 9 Safar

10 THU 10 Safar

Notes

$$I = 2\left|\sin\frac{z}{2}\right|_0^{\pi+2i}$$

$$= 2\left[\sin\left(\frac{\pi+2i}{2}\right) - \sin(0)\right]$$

$$= 2\left[\sin\frac{\pi}{2} + i\right]$$

$$= 2\left[\frac{e^{i(\frac{\pi}{2}+i)} - e^{-i(\frac{\pi}{2}+i)}}{2i}\right]$$

$$= \frac{1}{i}\left[e^{i\frac{\pi}{2}-1} - e^{-i\frac{\pi}{2}+1}\right]$$

$$= \frac{1}{i}\left[e^{i\frac{\pi}{2}}\cdot e^{-1} - e^{-i\frac{\pi}{2}}\cdot e^{1}\right]$$

$$= \frac{1}{i}\left[(i)e^{-1} - (-i)e^{1}\right]$$

$$= \frac{1}{i}\left(i\cdot\frac{1}{e} + ie\right)$$

$$= \frac{1}{e} + e$$

Hence, Proved

$\because e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$= 0 + i = i$

$e^{-i\frac{\pi}{2}} = \cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})$

$= 0 - i = -i$

11 FRI 11 Safar

12 SAT 12 Safar

13 SUN 13 Safar

Notes

Q: Find df & $df(V_p)$ of $f = (x^2-1)y + (y^2-1)x$

A:

As

chain rule

$$d(h(f)) = h'(f)df$$

Product rule

$$d(fg) = g\,df + f\,dg$$

Now,

$$f = (x^2-1)y + (y^2-1)x$$

$$df = (2x\,dx)y + (x^2-1)dy + (2y\,dy)x + (y^2-1)dx$$

$$= 2xy\,dx + (x^2-1)dy + 2xy\,dy + (y^2-1)dx$$

$$df = (2xy + y^2 - 1)dx + (x^2 + 2xy - 1)dy$$

Important

By Tangent vector

$$df(V_p) = [2P_1P_2 + P_2^2 - 1]V_1 + [P_1^2 + 2P_2P_1 - 1]V_2$$

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14 MON 14 Safar

Notes

Q: Find $\hat{t} = \dfrac{x'(t)}{|x'(t)|}$

where

$$x(t) = a\cos(t)\hat{e}_1 + a\sin(t)\hat{e}_2 + bt\hat{e}_3$$

A:

$$x(t) = a\cos(t)\hat{e}_1 + a\sin(t)\hat{e}_2 + bt\hat{e}_3$$

15 TUE 15 Safar

Notes

$$x'(t) = -a\sin(t)\hat{e}_1 + a\cos(t)\hat{e}_2 + b\hat{e}_3$$

$$|x'(t)| = \sqrt{(a\sin t)^2 + (a\cos t)^2 + (b)^2}$$

$$= \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}$$

16 WED 16 Safar

Notes

$$= \sqrt{a^2(\sin^2 t + \cos^2 t) + b^2}$$

$$= \sqrt{a^2(1) + b^2}$$

$$|x'(t)| = \sqrt{a^2 + b^2}$$

17 THU 17 Safar

Notes

we know that

$$\hat{t} = x = \frac{x'(t)}{|x'(t)|} = \frac{-a\sin(t)\hat{e}_1 + a\cos(t)\hat{e}_2 + b\hat{e}_3}{\sqrt{a^2 + b^2}}$$

Notes

Q: If $u(x,y) = e^y\cos x$ then find $v(x,y)$.

A: $u(x,y) = e^y\cos x$

18 FRI 18 Safar

Notes

$$\frac{\partial u}{\partial x} = -e^y\sin x \quad , \quad \frac{\partial u}{\partial y} = e^y\cos x$$

19 SAT 19 Safar

$$\frac{\partial^2 u}{\partial x^2} = -e^y\cos x \quad , \quad \frac{\partial^2 u}{\partial y^2} = e^y\cos x$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -e^y\cos x + e^y\cos x$$

Notes

$$= 0$$

It is harmonic function.

20 SUN 20 Safar

Now, to find v.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = -e^y\sin x$$

Important

By integrating w.r.t. y

$$v(x,y) = -e^y\sin x + g(x)$$

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21 MON 21 Safar

By taking derivative w.r.t. x

$$\frac{\partial v}{\partial x} = -e^y \cos x + g'(x)$$

Put $\frac{\partial v}{\partial x} = -e^y \cos x$

22 TUE 22 Safar

$$-e^y \cos x = -e^y \cos x + g'(x)$$

$$g'(x) = 0$$

$$g(x) = c$$

Hence,

$$v(x, y) = -e^y \sin x + c$$

23 WED 23 Safar

Q: Find Singularity for function

$$f(z) = \frac{1}{z^2 + 2z + 2}$$

24 THU 24 Safar

A: Here

$$z^2 + 2z + 2 = 0$$

By applying quadratic formula

$a = 1$, $b = 2$, $c = 2$

$$z = \frac{(-2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{(-2) \pm \sqrt{4 - 8}}{2}$$

$$z = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

Hence, z is singular Point.

25 FRI 25 Safar

26 SAT 26 Safar

27 SUN 27 Safar

Q: Show that

$$\overline{\cos(iz)} = \cos(\bar{iz})$$

Important

A: L.H.S

$$= \overline{\cos(iz)}$$

$\because z = x + iy$

$\bar{z} = x - iy$

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28 MON 28 Safar

Notes

$= \overline{\cos(-y+ix)}$ ∵ $z = x+iy$

$= \cos(-y+ix)$ $iz = ix-y$

$= \cos y\cosh x - \sin y \sinh x$ — (1) $\bar{iz} = -ix-y$

29 TUE 29 Safar

Notes

R.H.S

$= \cos(\bar{iz})$

$= \cos(-ix-y)$

$= \cos(-(y+ix))$

$= \cos(y+ix)$

$= \cos y\cosh x - \sin y \sinh x$ — (2)

30 WED 1 R. Awwal

from EN (1) & (2)

L.H.S = R.H.S

---

31 THU 2 R. Awwal

Notes

Q: Prove that $\frac{d}{dz}(\cos z) = -\sin z$

A: L.H.S

$= \frac{d}{dz}(\cos z)$

Notes

$= \frac{d}{dz} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$

$= \sum_{n=0}^{\infty} \frac{d}{dz} \frac{(-1)^n z^{2n}}{(2n)!}$

Notes

$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2n\,(z^{2n-1})}{(2n)(2n-1)!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n (z^{2n-1})}{(2n-1)!}$

Notes

$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1} \cdot (-1)^1 (z^{2n-1})}{(2n-1)!}$

$= -\sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^{(2n-1)}}{(2n-1)!}$

$= -\sin z$

1 FRI 3 R. Awwal

2 SAT 4 R. Awwal

3 SUN 5 R. Awwal

Important

Hence, Proved

November 2019

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

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Notes

4 MON 6 R. Awwal

Q: Fesnet – Seoret Formola

Poove that

② $\because \bar{n} = \tau\bar{B} - k\bar{t}$

$n = B \times t$

differentiate w.r.t. s

Notes

5 TU E 7 R. Awwal

$\bar{n} = B\bar{t} + t\bar{B}$

As $t = n \times b$

$n = b \times t$

$B = t \times n$

6 WED 8 R. Awwal

$\bar{n} = K(b \times n) + (-\tau(n \times t))$ $\because \bar{t} = k\bar{n}$

$\bar{b} = -\tau\bar{n}$

$= -k\bar{t} - \tau(-B)$

$= -k\bar{t} + \bar{B}\tau$

$\bar{n} = \bar{B}\tau - k\bar{t}$

Notes

7 THU 9 R. Awwal

① Poove that

$\bar{t} = k\bar{n}$

$K = \frac{d\theta}{ds}$

$K = \frac{d\theta}{|dt|} \cdot \frac{|dt|}{ds}$

Notes

8 FRI 10 R. Awwal

$K = \left|\frac{d\theta}{dt}\right| \cdot \left|\frac{dt}{ds}\right|$ $\because \left|\frac{d\theta}{dt}\right| = 1$

$K = \left|\frac{dt}{ds}\right|$ $\because t = \frac{d\theta}{ds}$

Notes

$= \left|\frac{d\gamma'}{ds}\right| = |\gamma''| = K$

γ'' is collinear with $\hat{n}$

9 SAT 11 R. Awwal

$|\gamma''| = \pm kn$

$\gamma'' = kn$ $\because \bar{t} = \gamma''$

Notes

Hence, $t' = k\bar{n}$

10 SUN 12 R. Awwal

Poove that

③ $\bar{b} = -\tau\bar{n}$

As

$t \cdot b = 0$

Important differentiate with w.r.t b

$t \cdot \bar{b} + \bar{t} \cdot b = 0$ $\because \bar{t} = k\bar{n}$

$t \cdot \bar{b} + k\bar{n} \cdot b = 0$ $\because n \cdot b = 0$

$t \cdot \bar{b} = 0$

November 2019

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18	19	20	21	22	23	24
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11 MON 13 R. Awwal

Notes

$\Rightarrow$ $\bar{b}$ is Proper to t

also $\bar{b}$ is collinear with n

$\bar{b} = \pm \iota n$

$\bar{b} = -\iota n$

b has opposite direction so -ve sign

12 TUE 14 R. Awwal

Notes

Q: If x, y & ϕ, ψ then Prove that

~~$d(fg) =$~~

$d(fg) = gfd + fdg$

13 WED 15 R. Awwal

Notes

A:

$$d(fg) = \sum \frac{\partial}{\partial x_i}(fg)\,dx_i$$

$$= \sum \left(\frac{\partial f}{\partial x_i} g + \frac{\partial g}{\partial x_i} f\right) dx_i$$

14 THU 16 R. Awwal

Notes

$$= \sum \frac{\partial f}{\partial x_i} g\,dx_i + \sum \frac{\partial g}{\partial x_i} f\,dx_i$$

$$= g \sum \frac{\partial f}{\partial x_i} dx_i + f \sum \frac{\partial g}{\partial x_i} dx_i$$

$$= g\,df + f\,dg$$

Notes

Q: If $P = (1, 1, 0)$ & $v = (1, 0, -3)$

$f = x^2yz$, then find $v_p[f]$

15 FRI 17 R. Awwal

A:

$f = x^2yz$

Notes

$$\frac{\partial f}{\partial x} = 2xyz, \quad \frac{\partial f}{\partial y} = x^2z, \quad \frac{\partial f}{\partial z} = x^2y$$

at Point $P = (1, 1, 0)$

16 SAT 18 R. Awwal

$$\left.\frac{\partial f}{\partial x}\right|_{(1,1,0)} = 0, \quad \left.\frac{\partial f}{\partial y}\right|_{(1,1,0)} = 0, \quad \left.\frac{\partial f}{\partial z}\right|_{(1,1,0)} = 1$$

Notes

Now

$$v_p[f] = \sum v_i \frac{\partial f}{\partial x_i}(P)$$

$$= 1(0) + 0(0) - 3(1)$$

$$= -3$$

17 SUN 19 R. Awwal

Important

-3 is directional desivative.

November 2019

M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
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18 MON 20 R. Awwal

19 TUE 21 R. Awwal

20 WED 22 R. Awwal

21 THU 23 R. Awwal

Notes

Q: Prove that

$$\frac{d}{dz}(\sin^{-1}hz) = \frac{1}{\sqrt{1+z^2}}$$

A: Let

$w = \sin^{-1}hz$

$z = \sinh w$

Taking derivative ~~w.r.t z~~

$$\frac{d}{dz}(z) = \frac{d}{dz}(\sinh w)$$

$$1 = \frac{d}{dw}(\sinh w).\frac{dw}{dz}$$

$$1 = -\cosh w.\frac{dw}{dz}$$

$$-\frac{1}{\cosh w} = \frac{dw}{dz}$$

$$\frac{dw}{dz} = \frac{1}{\sqrt{1+(\cosh w)^2}}$$

$$\frac{d}{dz}(\sin^{-1}hz) = \frac{1}{\sqrt{1+z^2}}$$

---

22 FRI 24 R. Awwal

23 SAT 25 R. Awwal

24 SUN 26 R. Awwal

Notes

Q: If $\lim_{n\to\infty} |z_n| = z$ then Prove that $||z_n|-|z|| \le |z_n - z|$ and show that $\lim_{n\to\infty} |z_n| = z$

A:

$\because \quad z_n = (z_n - z) + z$

$|z_n| \le |z_n - z| + |z|$

So, $|z_n| - |z| \le |z_n - z|$

$\because \quad z = (z - z_n) + z_n$

$|z| \le |z - z_n| + |z_n|$

$-|z_n| + |z| \le |z_n - z|$

Hence,

$-|z_n| + |z| \le |z_n - z| \le |z_n| - |z|$

Important

$$||z_n| - |z|| \le |z_n - z|$$

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Week 48

Notes

Q: Prove that

$$\frac{d}{dz}(\sin z) = \cos z$$

A:

Notes

L.H.S

$$= \frac{d}{dz}(\sin z)$$

$$= \frac{d}{dz}\sum_{n=0}^{\infty}\frac{(-1)^n (z)^{2n+1}}{(2n+1)!}$$

Notes

$$= \sum_{n=0}^{\infty}\frac{d}{dz}\frac{(-1)^n z^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n z^{2n}\,(2n+1)}{(2n+1)(2n)!}$$

Notes

$$= \sum_{n=0}^{\infty}\frac{(-1)^n z^{2n}}{2n!}$$

$$= \cos z$$

Hence, Proved

Hijri 1441

Notes

NOV/DEC 2019

Q: Show that

$$\sum_{n=1}^{\infty}\bar{z}_n = \bar{S}$$

29 FRI

A:

Notes

we know that

$$\bar{z}_n = x_n - iy_n$$

30 SAT

& $S = x+iy$, $\bar{S} = x-iy$

$$\sum_{n=1}^{\infty} x_n = X \quad , \quad \sum_{n=1}^{\infty} y_n = Y$$

Notes

$$\sum_{n=1}^{\infty}\bar{z}_n = \sum_{n=1}^{\infty}(x_n - iy_n)$$

1 SUN

$$= \sum_{n=1}^{\infty} x_n - i\sum_{n=1}^{\infty} y_n$$

$$= X - iY$$

Important

$$\sum_{n=1}^{\infty}\bar{z}_n = \bar{S}$$

Hence, Proved

November 2019

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M	T	W	T	F	S	S
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can turn it

Notes

Q: Evaluate

$$\int_C \frac{\cosh z}{z^4}\, dz$$

$x = \pm 2, \quad y = \pm 2$

Notes

A: Let

$$f(z) = \int_{-2}^{2} \frac{\cosh z}{z^4}\, dz$$

By Applying cauchy integral formula

Notes

$$= \int_{-2}^{2} \frac{\cosh z}{(z-0)^{3+1}}\, dz \qquad \because z_0 = 0$$

$$= \frac{2\pi i}{3!}\left[\frac{d^3}{dz^3}\cosh z\right]_{z=-2}^{2}$$

Notes

$$= \frac{\pi i}{3}\left(\cosh(-2) - \cosh(2)\right)$$

$$= \frac{\pi i}{3}(0)$$

$$= 0$$

Notes

Q: State and Prove Liouville's Theorem

A:

Statement:

Prove that every entire bounded function is constant.

Notes

Proof:

$\because f(z)$ is analytic everywhere in complex plane and is bounded.

So, by cauchy inequality theorem

Notes

$$\left|f^{(n)}(a)\right| \le \frac{n!}{\gamma^n} M \qquad \text{where } |z-a| = \gamma$$

for $n = 1$ $\quad \because n \neq 0, \infty$ in indeterminate for $\gamma^n \quad r \to \infty$

Important

$$|f'(a)| \le \frac{M}{\gamma} \qquad \text{if } a = z$$

$$\Rightarrow |f'(z)| \le \frac{M}{\gamma} \qquad -(1)$$

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9 MON 11 R. Sani

If $r \to \infty$ then from (1) we conclude that $f'(z) \to 0$

So, $f(z) =$ constant

Notes

10 TUE 12 R. Sani

Q: If $f = 8x^3yz$, $P = (2, 0, 3)$

$v = (1, 1, 1)$

then find $V_p[f]$

A:

$f = 8x^3yz$

Notes

11 WED 13 R. Sani

$\frac{\partial f}{\partial x} = 24x^2yz$, $\frac{\partial f}{\partial y} = 8x^3z$, $\frac{\partial f}{\partial z} = 8x^3y$

At Point $(2, 0, 3)$

$\left.\frac{\partial f}{\partial x}\right|_{(2,0,3)} = 0$, $\left.\frac{\partial f}{\partial y}\right|_{(2,0,3)} = 192$, $\left.\frac{\partial f}{\partial z}\right|_{(2,0,3)} = 0$

Notes

12 THU 14 R. Sani

So,

$V_p[f] = 1(0) + 1(192) + 1(0)$

$= 192$

Q: $\log(1-i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}$

A:

L.H.S

$= \log(1-i)$

Notes

13 FRI 15 R. Sani

$= \log|1-i| + i\,\mathrm{Arg}(1-i)$

$= \ln\sqrt{2} - \frac{\pi}{4}i$

$= \frac{1}{2}\ln 2 - \frac{\pi}{4}i$

$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

$|1-i| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

14 SAT 16 R. Sani

R.H.S

Notes

Q: $\log(-ei) = 1 - \frac{\pi}{2}i$

L.H.S

$= \log(-ei)$

Important

$= \ln|-ei| + i\,\mathrm{Arg}(-ei)$

$= \ln e - \frac{\pi}{2}i$

$= 1 - \frac{\pi}{2}i$

$\theta = \tan^{-1}\left(\frac{-1}{0}\right) = -\frac{\pi}{2}$

$|-ei| = \sqrt{(e)^2+(0)^2} = e$

15 SUN 17 R. Sani

R.H.S

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can turn it

16 MON 18 R. Sani

Notes

Q: Show that

$2Re(z) = z + \bar{z}$

A: R.H.S

$= z + \bar{z}$

$= x+iy + \overline{x+iy}$

17 TUE 19 R. Sani

Notes

$= x+iy + x-iy$

$= 2x$

$= 2\,Re(z)$

18 WED 20 R. Sani

Notes

Q: Find Harmonic conjugate

$u(x,y) = 5x(4-2y)$

A:

$u(x,y) = 5x(4-2y)$

19 THU 21 R. Sani

Notes

$\frac{\partial u}{\partial x} = 20 - 10y$, $\frac{\partial u}{\partial y} = -10x$

$\frac{\partial^2 u}{\partial x^2} = 0$, $\frac{\partial^2 u}{\partial y^2} = 0$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Notes

Fuction is harmonic

20 FRI 22 R. Sani

To find v

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 20 - 10y$

Notes

By integrating w.r.t y

$v(x,y) = 20y - 5y^2 + g(x)$

21 SAT 22 R. Sani

Taking derivative w.r.t x

$\frac{\partial v}{\partial x} = g'(x)$

Notes

Put $\frac{\partial v}{\partial x} = 10x$

22 SUN 24 R. Sani

$g'(x) = 10x$

$g(x) = 5x^2 + c$

Important $v(x,y) = 20y - 5y^2 + 5x^2 + c$

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Notes

23 MON 25 R. Sani

Q: Show that

$\log(1+i)^2 = 2\log(1+i)$

A:

$(1+i) = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

Notes

24 TUE 26 R. Sani

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= \sqrt{2}\, e^{i\frac{\pi}{4}}$

$(1+i)^2 = \left(\sqrt{2}\, e^{i\pi/4}\right)^2 = 2e^{i\pi/2}$

therefore $\arg(1+i) = \frac{\pi}{4}$ & $\arg(1+i)^2 = \frac{\pi}{2}$

Notes

25 WED 27 R. Sani

$\Rightarrow \log(1+i)^2 = \ln|(1+i)^2| + i\arg(1+i)^2$

$= \ln|2e^{i\pi/2}| + i\frac{\pi}{2}$

$= \ln\left|2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right| + i\frac{\pi}{2}$

Notes

26 THU 28 R. Sani

$= \ln|2| + i\frac{\pi}{2}$

$= 2\left(\frac{1}{2}\ln|2| + i\frac{\pi}{4}\right)$

$= 2\left(\ln\sqrt{2} + \frac{i\pi}{4}\right)$

Notes

$= 2\log(1+i)$

Q: show that

$\log(-1+i)^2 \neq 2\log(-1+i)$

Notes

27 FRI 29 R. Sani

A:

L.H.S

$-1+i = \sqrt{2}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

28 SAT 1 J. Awwal

$= \sqrt{2}\, e^{i3\pi/4}$

Notes $(-1+i)^2 = \left(\sqrt{2}e^{i3\pi/4}\right)^2 = 2e^{i3\pi/2}$

therefore $\arg(-1+i) = \frac{3\pi}{4}$, $\arg(-1+i)^2 = \frac{3\pi}{2}$

29 SUN 2 J. Awwal

$\Rightarrow \log(-1+i)^2 = \ln|(-1+i)^2| + i\arg(-1+i)^2$

$= \ln|2e^{i3\pi/2}| + i\frac{3\pi}{2}$

Important

$= \ln\left|2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right| + i\frac{3\pi}{2}$

$= \ln|2| + i\frac{3\pi}{2}$

$\neq 2\log(-1+i)$

December 2019

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30 MON 3 J. Awwal

Notes

Q: Prove that
$u(x,y) = e^{-y}\sin x$
is harmonic or not?

A:

$u(x,y) = e^{-y}\sin x$

31 TUE 4 J. Awwal

Notes

$\frac{\partial u}{\partial x} = e^{-y}\cos x \quad , \quad \frac{\partial u}{\partial y} = -e^{-y}\sin x$

$\frac{\partial^2 u}{\partial x^2} = -e^{-y}\sin x \quad , \quad \frac{\partial^2 u}{\partial x^2} = e^{-y}\sin x$

1 WED 5 J. Awwal

Notes

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -e^{-y}\sin x + e^{-y}\sin x$

$= 0$

Hence, Proved

---

2 THU 6 J. Awwal

Notes

Q: $f(z) = (x-y)^2 + 2i(x+y)$
Find analytic region

A:

$f(z) = (x-y)^2 + 2i(x+y)$

Notes

let

$u = (x-y)^2 \quad , \quad v = 2(x+y)$

$\frac{\partial u}{\partial x} = 2(x-y) \quad , \quad \frac{\partial v}{\partial x} = 2$

3 FRI 7 J. Awwal

Notes

$\frac{\partial u}{\partial y} = -2(x-y) \quad , \quad \frac{\partial v}{\partial y} = 2$

Now, by C.R equations

4 SAT 8 J. Awwal

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

Notes

$2(x-y) = 2 \quad \& \quad 2 = 2(x-y)$

$x-y = 1 \quad \& \quad x-y = 1$

5 SUN 9 J. Awwal

So, Analytic region is $x-y=1$

Important

January 2020

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		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
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6 MON 10 J. Awwal

Notes

Q: show that the function $f(z) = \bar{z}$ is nowwhere differentiable.

A:

As

$$z = x + iy$$
$$\bar{z} = x - iy$$

7 TUE 11 J. Awwal

Notes

let

$$u + iv = x - iy$$

$$u = x \quad , \quad v = -y$$

8 WED 12 J. Awwal

Notes

$$\frac{\partial u}{\partial x} = 1 \quad , \quad \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial u}{\partial y} = 0 \quad , \quad \frac{\partial v}{\partial y} = -1$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

9 THU 13 J. Awwal

Notes

CR equations are not satisfied.

So, $f(z) = \bar{z}$ nowhere differentiable.

Notes

Q: Evaluate $\int_C \frac{e^{-z}}{z - \frac{\pi}{2}i} dz$

by cauchy integral formula

where $x = \pm 2$, $y = \pm 2$

10 FRI 14 J. Awwal

Notes

A:

$$f(z) = \int_C \frac{e^{-z}}{z - (\frac{\pi}{2}i)} dz$$

11 SAT 15 J. Awwal

$$= 2\pi i \left[e^{-z} \right]_{z = \frac{\pi}{2}i}$$

Notes

$$= 2\pi i \, (-i)$$

12 SUN 16 J. Awwal

$$= 2\pi$$

Q: Show that $f(z) = |z|$ is differentiable at $z = 0$ but not analytic at $z = 0$.

Important

A: $f(z) = |z|$

January 2020

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13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

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Notes

13 MON 17 J. Awwal

~~$u+iv = |x+iy|$~~

$= \frac{f(z+h)-f(z)}{h}$

at $z=0$

$= \frac{f(h)-f(0)}{h} = \frac{|h|}{h} = 1$

Notes

14 TUE 18 J. Awwal

$\because$ $f(z)$ is differentiable.

$f(z) = |z| = \sqrt{x^2+y^2}$

$u = \sqrt{x^2+y^2}$, $v = 0$

Notes

15 WED 19 J. Awwal

$\frac{\partial u}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$, $\frac{\partial v}{\partial y} = 0$

$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

$\because$ $|z|$ is not analytic

Notes

16 THU 20 J. Awwal

Notes

Q: Find the arc length

$$S = \int_a^b \left|\frac{d\bar{x}}{dt}\right| dt$$

$\bar{x}(t) = (\cos t, \sin t, t)$

where $0 \le t \le 2\pi$

17 FRI 21 J. Awwal

Notes

A:

$\bar{x}(t) = (\cos t, \sin t, t)$

$|\bar{x}'(t)| = |-\sin t, \cos t, 1|$

$= \sqrt{2}$

18 SAT 22 J. Awwal

Notes

$$\int_0^{2\pi} |\bar{x}'(t)|\, dt = \int_0^{2\pi} \sqrt{2}\, dt$$

$$= \sqrt{2} \int_0^{2\pi} dt$$

19 SUN 23 J. Awwal

Important

$= \sqrt{2}(t)\Big|_0^{2\pi} = \sqrt{2}(2\pi - 0)$

$= \sqrt{2}(2\pi = 2\sqrt{2}\,\pi$

January 2020

M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29	30	31									

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20 MON 24 J. Awwal

Notes

Q: Prove that

$$v = \frac{1}{2}\log(x^2+y^2)$$

A:

$$v(x,y) = \frac{1}{2}\log(x^2+y^2)$$

21 TU E 25 J. Awwal

Notes

$$\frac{\partial v}{\partial x} = \frac{x}{x^2+y^2} \quad , \quad \frac{\partial v}{\partial y} = \frac{y}{x^2+y^2}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{-x^2+y^2}{(x^2+y^2)^2} \quad , \quad \frac{\partial^2 v}{\partial y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

22 WED 26 J. Awwal

Notes

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -\frac{x^2-y^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$= 0$$

Function is Harmonic

Now, to find v

23 THU 27 J. Awwal

Notes

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = \frac{x}{x^2+y^2}$$

By integrating w.r.t. y

$$v(x,y) = \tan^{-1}\left(\frac{y}{x}\right) + g(x)$$

Address & Telephone INDEX

Taking derivative w.r.t. x

$$\frac{\partial v}{\partial x} = -\frac{y}{x^2+y^2} + g'(x)$$

Put $\frac{\partial v}{\partial x} = -\frac{y}{x^2+y^2}$

$$-\frac{y}{x^2+y^2} = -\frac{y}{x^2+y^2} + g'(x)$$

$$g'(x) = 0$$

$$g(x) = c$$

$$v(x,y) = \tan^{-1}\left(\frac{y}{x}\right) + c$$

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Q: $\int_0^\infty e^{-zt}\,dt \qquad Re\,z > 0$

A:

$$I = \int_0^\infty e^{-zt}\,dt$$

$$= \lim_{b\to\infty} \int_0^b e^{-zt}\,dt$$

$$= \lim_{b\to\infty} \left[\frac{e^{-zt}}{-z}\right]_0^b$$

$$= -\frac{1}{z} \lim_{b\to\infty} \left(e^{-zb} - e^{-0b}\right)$$

$$= -\frac{1}{z} \lim_{b\to\infty} \left(e^{-zb} - e^{0}\right)$$

$$= -\frac{1}{z}(0-1)$$

$$= \frac{1}{z} \quad \text{when} \quad Re\,z > 0$$

Tel/Mob/Fax/E-mail | Name & Address

Q: $z_n = -2 + i\frac{(-1)^n}{n^2} \qquad (n = 1, 2, \ldots)$

$\lim_{n\to\infty} z_n$ converges or not?

A:

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty} \left(-2 + i\frac{(-1)^n}{n^2}\right)$$

$$= \lim_{n\to\infty} (-2) + \lim_{n\to\infty} \left[i\frac{(-1)^n}{n^2}\right]$$

$$= -2 + i \lim_{n\to\infty} \frac{(-1)^n}{n^2}$$

$$= -2 + i(0)$$

$$= -2 < 0$$

So, z_n converges.

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Q: $z_n = 2i - \frac{25}{n!}$ $(n=0,1,2,\dots)$

$\lim_{n\to\infty} z_n$ converges or not?

A:

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty}\left(2i - \frac{25}{n!}\right)$$

$$= \lim_{n\to\infty}(2i) - \lim_{n\to\infty}\left(\frac{25}{n!}\right)$$

$$= 2i - \frac{25}{\infty!}$$

$$= 2i - 0$$

$$= 2i > 0$$

(divergence)

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Q: $f(z) = \sin z$ is analytic or not?

A: $\sin z = \sin(x+iy)$

$= \sin x \cos iy + \cos x \sin iy$

$= \sin x \cosh y + i \cos x \sinh y$

$\because u = \sin x \cosh y$, $v = \cos x \sinh y$

$\frac{\partial u}{\partial x} = \cos x \cosh y$, $\frac{\partial v}{\partial x} = -\sin x \sinh y$

$\frac{\partial u}{\partial y} = \sin x \sinh y$, $\frac{\partial v}{\partial y} = +\cos x \cosh y$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

So, C.R equations are satisfied.

Hence, $\sin z$ is analytic

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Q: $u(x,y) = x^3 - 2x + 2$

Find Harmonic conjugate?

A:

$$u(x,y) = x^3 - 2x + 2$$

$$\frac{\partial u}{\partial x} = 3x^2 - 2 \quad , \quad \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial^2 u}{\partial x^2} = 6x \quad , \quad \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 0$$

$$= 6x \neq 0$$

Hence, this function is not Harmonic

Q: Test the analytic function

$$f(z) = e^z$$

A:

$$u + iv = e^{x+iy} \Rightarrow e^x \cdot e^{iy}$$

$$= e^x [\cos y + i \sin y]$$

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$u = e^x \cos y$, $v = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \cos y$, $\frac{\partial v}{\partial x} = e^x \sin y$

$\frac{\partial u}{\partial y} = -e^x \sin y$, $\frac{\partial v}{\partial y} = e^x \cos y$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

CR equations are satisfied

Hence, Function is analytic.

Q: $w = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$

Write 1-form as $\Sigma (df_i P) v_i$

A:

Let $f_1, f_2, f_3 : \mathbb{R}^3 \to \mathbb{R}$ be three functions.

Let

$$\phi = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$$

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MTH632 Complex Analysis and Differential Geometry

Question No : 35 of 41

Evaluate $\lim_{z \to i} \frac{(z^2 + 4z + 2)}{z + 1}$.

Answer (Please click here to Add Answer)

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Address http://server/vutes/client/Paper.aspx

MTH632 Complex Analysis and Differential Geometry

Question No : 32 of 41 **Marks: 2 (Budgeted**

Test whether the function $w = 2xy + i(x^2 - y^2)$ is analytic or not.

Answer (Please click here to Add Answer)

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$f(x+iy) = x^3 + ax^2y + bxy^2 + cy^3$

$a = 3i \quad b = -3 \quad c = -i$

$= x^3 + (3i)x^2y + (-3)xy^2 + (-i)y^3$

$= x^3 + 3ix^2y - 3xy^2 - iy^3$

$= (x^3 - 3xy^2) + i(3x^2y - y^3)$

$W = u + iv$

$u = x^3 - 3xy^2 \quad v = 3x^2y - y^3$

C.F is analytical if

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 \quad \frac{\partial v}{\partial y} = 3x^2 - 3y^2$

$\frac{\partial u}{\partial y} = -6y \quad \frac{\partial v}{\partial x} = 6y$

as clear.

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

function is analytical.

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MCQ

$U(x,y) = e^{ax}\cos y \quad , a=1$

put $a=1$

$U(x,y) = e^{x}\cos y$

$\frac{\partial u}{\partial x} = e^{x}\cos y$

Using C-R Equations

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$e^{x}\cos y = \frac{\partial v}{\partial y}$

Integrating

$e^{x}\sin y + c = v$

put $c=0$

$v(x,y) = e^{x}\sin y$

$\frac{\partial v}{\partial y} = e^{x}\cos y$

As

$\frac{\partial u}{\partial x} = e^{x}\cos y$

$\frac{\partial v}{\partial y} = e^{x}\cos y$

$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

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Mth632 f...jathol😍

$= 2(-1-1) + 2i\pi$

$= -4 + 2i\pi$

$\because e^{iz}$

$= \cos z + i\sin$

16

(17)

Ex: Find antiderivative of

$$\int_i^{i/2} e^{\pi z}\, dz$$

Solution:

$$= \int_i^{i/2} e^{\pi z}\, dz$$

$$= \left| \frac{e^{\pi z}}{\pi} \right|_i^{i/2}$$

$$= \frac{1}{\pi}\left(e^{\pi i/2} - e^{i\pi}\right)$$

$$= \frac{1}{\pi}\left(e^{i\pi/2} - e^{i\pi}\right)$$

$$= \frac{1}{\pi}\left(\cos(\pi/2) + i\sin(\pi/2) - (\cos\pi + i\sin\pi)\right)$$

$$= \frac{1}{\pi}\left((0+i) - (-1+0)\right)$$

$$= \frac{1}{\pi}(i+1)$$

$$= \frac{i+1}{\pi}$$

17

(18)

Ex: Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

satisfies Laplace's equation.

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Evaluate the integral

$$\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$$

Solution:

$$\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$$

As

$$\left(\frac{1}{t} - i\right)^2 = \left(\frac{1}{t}\right)^2 + (i)^2 - 2\left(\frac{1}{t}\right)i$$

$$\left(\frac{1}{t} - i\right)^2 = \frac{1}{t^2} - 1 - \frac{2}{t}i$$

$$\int_1^2 \left(\frac{1}{t} - i\right)^2 dt = \int_1^2 \frac{1}{t^2} - 1 - \frac{2}{t}i \; dt$$

$$= \left| \frac{-1}{t} - t - 2i \ln|t| \right|_1^2$$

$$= \left[-\frac{1}{2} - 2 - 2i \ln|2| - \left(\frac{-1}{1} - 1 - 2i \ln|1| \right) \right]$$

$$= \left[-\frac{1}{2} - 2 - 2i \ln 2 + 1 + 1 \right] = -\frac{1}{2} - 2i \ln 2$$

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$$\frac{\partial v}{\partial y} = 2 - 2y$$

Integrating w.r.t. y

$$v = 2y - \frac{2y^2}{2} + c$$

$$v(x,y) = 2y - y^2 + c$$

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Test whether the function W

$W = 2xy + i(x^2 - y^2)$ is analytic

or not

Solution:

As $W = 2xy + i(x^2 - y^2)$

$u = 2xy$, $v = (x^2 - y^2)$

$\frac{\partial u}{\partial x} = 2y$ | $\frac{\partial v}{\partial x} = 2x$

$\frac{\partial u}{\partial y} = 2x$ | $\frac{\partial v}{\partial y} = -2y$

As $\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

Hence W is not analytic

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If $U = \log(x^2+y^2)$

find V and $f(z)$ s.t

$f(z) = U + iV$ is analytic

Solution.

As $U = \log(x^2+y^2)$

$$\frac{\partial U}{\partial x} = \frac{1}{x^2+y^2}(2x) = \frac{2x}{x^2+y^2}$$

Using C-R equations.

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}$$

$$\frac{2x}{x^2+y^2} = \frac{\partial V}{\partial y}$$

Integrating

$$\int \frac{2x}{x^2+y^2}\, dy = V$$

$$\log(x^2+y^2) + C = V$$

$C = 0$

$$\log(x^2+y^2) = V$$

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$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 6x + 6 - 6x - 6$$

(19)

$= 0$

$\therefore$ U satisfies the laplace equation.

Imp

Question: $z_n = \frac{1}{n^3} - i$ solve it for convergence also find value of ε

solution:

$$z_n = \frac{1}{n^3} - i$$

$$\lim_{n \to \infty} z_n = \lim_{n \to \infty} \left(\frac{1}{n^3} - i\right)$$

$$= \lim_{n \to \infty} \left(\frac{1}{n^3}\right) - \lim_{n \to \infty} (i)$$

$$= \frac{1}{\infty} - i$$

$= -i < 0$ (convergenes)

Imp (20)

Evaluate $\bar{z}_1 - \bar{z}_2$ when $z_1 = 5 + i$ and $z_2 = -8 + 3i$

Sol:

$z_1 = 5 + i$, $z_2 = -8 + 3i$

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Evaluate

$$\lim_{z \to i} \frac{(z^2+4z+2)}{(z+1)}$$

$$= \frac{(i)^2+4i+2}{i+1}$$

$$= \frac{-1+4i+2}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{(1+4i)(1-i)}{(1)^2-(i)^2} = \frac{1-i+4i-4i^2}{1-(-1)}$$

$$= \frac{1+3i-4(-1)}{2}$$

$$= \frac{5+3i}{2}$$

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$$\frac{d}{dz}\,\text{arc}\sin z = \frac{1}{\sqrt{1-z^2}}$$

Sol:- Let $w = \text{arc}\sin z$

$$w = \sin^{-1} z$$

$$\sin w = z$$

Diff w.r.t. z

$$\frac{d}{dz}(\sin w) = \frac{d}{dz}(z)$$

$$\cos w \cdot \frac{dw}{dz} = 1$$

$$\frac{dw}{dz} = \frac{1}{\cos w}$$

$$\frac{dw}{dz} = \frac{1}{\sqrt{1-\sin^2 w}}$$

$$\boxed{\frac{dw}{dz} = \frac{1}{\sqrt{1-z^2}}}$$

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$$v = \log(x^2+y^2)$$

$$\frac{\partial v}{\partial x} = \frac{1}{x^2+y^2}(2x) = \frac{2x}{x^2+y^2}$$

$$\frac{\partial v}{\partial y} = \frac{1}{x^2+y^2}\,2y = \frac{2y}{x^2+y^2}$$

$$\frac{\partial u}{\partial x} = \frac{2x}{x^2+y^2}$$

$$\frac{\partial u}{\partial y} = \frac{2y}{x^2+y^2}$$

As $$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

Hence $f(z)$ is not analytic.

Also

$$f(z) = u + iv$$

$$f(z) = \log(x^2+y^2) + i\log(x^2+y^2)$$

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$= \sin x \cosh y + i \cos x \sinh y$

Now

$u = \sin x \cosh y \; ; \; v = \cos x \sinh y$

$u_x = \cos x \cosh y \; ; \; v_x = -\sin x \sinh y$

$u_y = \sin x \sinh y \; ; \; v_y = \cos x \cosh y$

$u_x = v_y$ and $u_y = -v_x$

$\therefore$ function is analytic.

Show that $f(z) = e^z$ is analytic

Sol: $f(z) = e^z$

$u + iv = e^{x+iy}$

$= e^x \cdot e^{iy}$

$= e^x \cdot (\cos y + i \sin y)$

(28).

$u + iv = e^x \cos y + i e^x \sin y$

$u = e^x \cos y \; , \; v = e^x \sin y$

$u_x = e^x \cos y \; , \; v_x = e^x \sin y$

$u_y = -e^x \sin y \; , \; v_y = e^x \cos y$

here

$u_x = v_y$ and $u_y = -v_x$

So function is analytic.

Given an example such that u and v are harmonic but u+iv is not analytic.

Solution.

Consider the function

$f(z) = \bar{z}$

$u + iv = x - iy$

$u = x \; , \; v = -y$

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Q30

~~$d(fg) = (f dg) + (g df)$~~

$\boxed{d(fg) = g f d + f dg}$ Q

$d(fg) = \sum \frac{\partial}{\partial x_i}(fg)\, dx_i$

$= \sum \left(\frac{\partial f}{\partial x_i} g + \frac{\partial g}{\partial x_i} f\right) dx_i$

$= \sum \frac{\partial f}{\partial x_i} g\, dx_i + \sum \frac{\partial g}{\partial x_i} f\, dx_i$

$= g \sum \frac{\partial f}{\partial x_i} dx_i + f \sum \frac{\partial g}{\partial x_i} dx_i$

$= g\, df + f\, dg$ Ans

L·H·S = R·H·S

Q31

$\frac{d}{dx} \cos x = -\sin x$

Sol $\lim_{n \to 0} f(x) = \lim_{n \to 0} \cos x$

$\lim_{n \to 0} \left(\frac{\cos(x+h) - \cos x}{n}\right)$

$\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B$

so

$\lim_{n \to 0} \left(\frac{\cos x \cos h - \sin x \sin h - \cos x}{n}\right)$

$\lim_{n \to 0} \frac{\cos x(\cos h - 1)}{n} - \lim_{n \to 0} \frac{\sin x \sin h}{n}$

as $\lim_{a \to 0} \frac{\sin a}{a} = 1$

$= 0 - \sin x = -\sin x$ Ans.

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Q11 find antiderivative of $\int_i^{\pi/2} e^{\pi z}\, dz$

Sol $\int_i^{\pi/2} e^{\pi z}\, dz$

$= \left| \frac{e^{\pi z}}{\pi} \right|_i^{\pi/2} \Rightarrow \frac{1}{\pi}\left[e^{\pi \pi/2} - e^{\pi i} \right]$

$= \frac{1}{\pi}\left\{ \left[\cos(\pi/2) + i \sin\left(\frac{\pi}{2}\right) \right] - \left[\cos\pi + i \sin\pi \right] \right\}$

$= \frac{1}{\pi}\left[(0 + i) - (-1 + 0) \right]$

$= \frac{1}{\pi}(i + 1)$ Ans

Q 18

prove $U = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfy laplaces equation

Sol $\frac{\partial U}{\partial x} = 3x^2 - 3y^2 + 6x$

$\frac{\partial^2 U}{\partial x^2} = 6x + 6$ ——— (1)

$\frac{\partial U}{\partial y} = -6xy - 6y$

$\frac{\partial^2 U}{\partial y^2} = -6x - 6$ ——— (2)

(1) + (2)

$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 6x + 6 - 6x - 6 = 0$

(Satisfy)

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$+ \frac{\partial f}{\partial z}(P)$

$= 1(0) + 0(0) + (-3)(1)$

$= -3$

-3 is directional derivative.

(23)

Find Singularity if

$$f(z) = \frac{1}{z^2+2z+2}$$

Solution:

Here $z^2+2z+2=0$

by quadratic formula

$$z = \frac{-2 \pm \sqrt{4-4(1)(2)}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

$z = -1 \pm i$ is Singular point

Show that $\sum_{n=1}^{\infty} \bar{z}_n = \bar{S}$

Solution:

As we know That

(24)

$= (e + 1/e)$

$= R.H.S$

proved

Imp

find $\int_c f(z)\,dz$ if

$f(z) = \frac{z+2}{z}$, $z = e^{i\theta}$

Sol:

$$\int_c f(z)\,dz$$

$$= \int_0^{\pi} \left(\frac{z+2}{z}\right) dz$$

$$= \int_0^{\pi} \left(\frac{z+2}{z}\right)(i2e^{i\theta})\,d\theta$$

$$= \int_0^{\pi} \left(\frac{z}{z} + \frac{2}{z}\right)(i2e^{i\theta})\,d\theta$$

$$= \int_0^{\pi} \left(1 + \frac{2}{2e^{i\theta}}\right)(i2e^{i\theta})\,d\theta$$

$$= \int_0^{\pi} \left(1 + \frac{1}{e^{i\theta}}\right)(i2e^{i\theta})\,d\theta$$

$$= \int_0^{\pi} \left(\frac{e^{i\theta}}{e^{i\theta}} + 1\right)(i2e^{i\theta})\,d\theta$$

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Q19 $z_n = \frac{1}{n^3} - i$ Solve it for Convergence also find value of ϵ

Sol

$$z_n = \frac{1}{n^3} - i$$

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty}\left(\frac{1}{n^3} - i\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^3} - \lim_{n\to\infty} i$$

$$= \frac{1}{\infty} - i$$

$$= 0 - i \Rightarrow -i < 0 \text{ (Converg)}$$

Q20 Evaluate $\bar{z}_1 - \bar{z}_2$ when $z_1 = 5+2i$ and $z_2 = -8+3i$

Sol $\bar{z}_1 = 5-i$, $\bar{z}_2 = -8-3i$

$$z_1 - \bar{z}_2 = 5 - i + 8 + 3i$$

$$= 5 + 2i + 8$$

$$= 13 + 2i \quad \text{Ans}$$

Q21 If $x(t) = a\cos t\, e_1 + a\sin t\, e_2 + bt\, e_3$

Then solve $\frac{x'(t)}{|x'(t)|}$

$$x'(t) = -a\sin t\, e_1 + a\cos t\, e_2 + b e_3$$

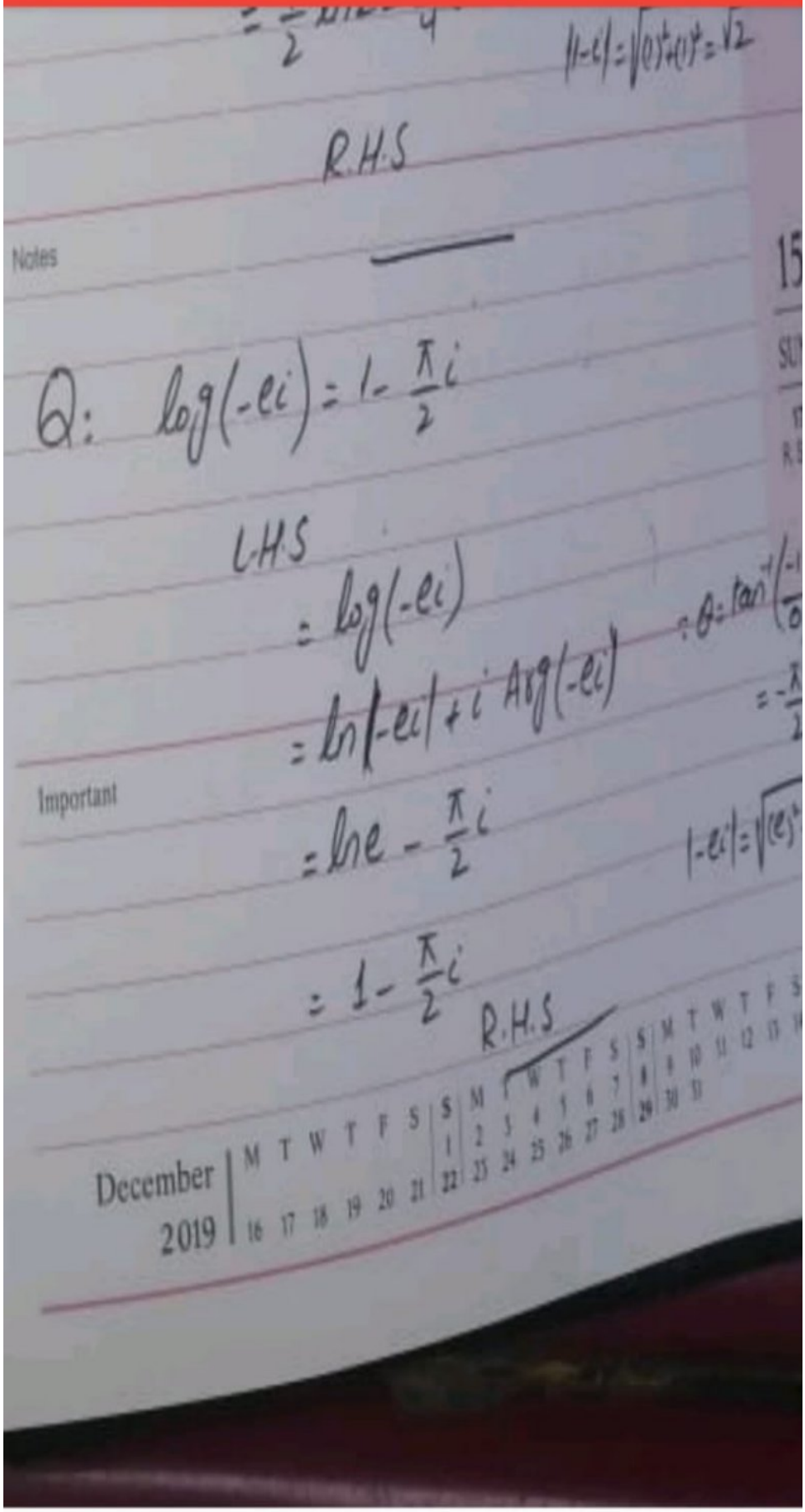
$$|x'(t)| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + b^2}$$

$$= \sqrt{a^2\sin^2 t + a^2\cos^2 t + b^2}$$

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$|1-i| = \sqrt{1^2+(-1)^2} = \sqrt{2}$

R.H.S

Notes

Q: $\log(-ei) = 1 - \frac{\pi}{2}i$

L.H.S

$= \log(-ei)$

$= \ln|-ei| + i\,\mathrm{Arg}(-ei)$ $\quad \theta = \tan^{-1}\left(\frac{-1}{0}\right)$

$= \ln e - \frac{\pi}{2}i$ $\quad = -\frac{\pi}{2}$

$= 1 - \frac{\pi}{2}i$ $\quad |-ei| = \sqrt{(e)^2}$

R.H.S

Important

December 2019

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differentiable.

(27)

Test the analyticity of function $w = \sin z$

Sol:

Let $w = f(z)$

$f(z) = \sin z$

$u + iv = \sin(x + iy)$

$= \sin x \cosh y + i \cos x \sinh y$

Now

$u = \sin x \cosh y \; ; \; v = \cos x \sinh y$

$u_x = \cos x \cosh y \; ; \; v_x = -\sin x \sinh y$

$u_y = \sin x \sinh y \; ; \; v_y = \cos x \cosh y$

$u_x = v_y$ and $u_y = -v_x$

$\therefore$ function is analytic.

Show that $f(z) = e^z$ is analytic

Sol: $f(z) = e^z$

$u + iv = e^{x+iy}$

$= e^x \cdot e^{iy}$

$= e^x \cdot (\cos y + i \sin y)$

(28).

$u + iv = e^x \cos y + i e^x \sin y$

$u = e^x \cos y \; , \; v = e^x \sin y$

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Edit 37

Q: Find Singularity for function

$$f(z) = \frac{1}{z^2+2z+2}$$

A: Here

$z^2+2z+2=0$

By applying quadratic formula

Q: Show that $\overline{\cos(iz)} = \cos(\bar{iz})$

A: L.H.S

$= \cos(\bar{iz})$

October 2019

can turn it into

(12)

OCTOBER 2019

$= \cos(-y+ix)$ ·

$= \cos y \cosh x - \sin y \sinh x$ — ①

$z = x+iy$

$iz = ix - y$

$\bar{iz} = -ix - y$

R.H.S

$= \cos(\bar{iz})$

$= \cos(-ix-y)$

$= \cos(-(y+ix))$

$= \cos(y+ix)$

$= \cos y \cosh x - \sin y \sinh x$ — ②

from eq ① & ②

Q: Prove that $\frac{d}{dz}(\cos z) = -\sin z$

A: L.H.S

$= \frac{d}{dz}(\cos z)$

(13)

NOVEMBER 2019

$= \frac{d}{dz} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$

$= \sum_{n=0}^{\infty} \frac{d}{dz} \frac{(-1)^n z^{2n}}{(2n)!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2n (z^{2n-1})}{(2n)(2n-1)!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n (z^{2n-1})}{(2n-1)!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^{n-1} \cdot (-1)^1 (z^{2n-1})}{(2n-1)!}$

$= -\sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^{(2n-1)}}{(2n-1)!}$

$= -\sin z$

Hence, Proved

November 2019

(14)

EMBER 2019

Q: Frenet - Seaset formula

Prove that

② $\vec{n} = t\vec{B} - k\vec{t}$

$n = B \times t$

differentiate w.r.t. s

$\vec{n} = B\vec{t} + t\vec{B}$

As $t = n \times b$

$n = b \times t$

(15)

NOVEMBER 2019

$k = \left|\frac{d\theta}{dt}\right| \cdot \left|\frac{dt}{ds}\right| = \left|\frac{d\theta}{dt}\right| = 1$

$k = \left|\frac{dt}{ds}\right|$

$= t = \frac{dt}{ds}$

$= \left|\frac{d\vec{s}'}{ds}\right| = |\vec{s}''| = k$

$\vec{s}''$ is colinear with $\vec{n}$

$|\vec{s}''| = \pm kn$

$\vec{s}'' = kn$

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$= -1 \pm i$

$z = -1 \pm i$ is Singular point

Show that $\sum_{n=1}^{\infty} \bar{z}_n = \bar{S}$

Solution:

As we know That

$z_n = x_n + iy_n$

$\bar{z}_n = x_n - iy_n$

and

$S = X + iY$

$\bar{S} = X - iY$

$\sum_{n=1}^{\infty} x_n = X \quad ; \quad \sum_{n=1}^{\infty} y_n = Y$

So

$$\sum_{n=1}^{\infty} \bar{z}_n = \sum_{n=1}^{\infty} (x_n - iy_n)$$

$$= \sum_{n=1}^{\infty} x_n - \sum_{n=1}^{\infty} y_n$$

$$= X - iY$$

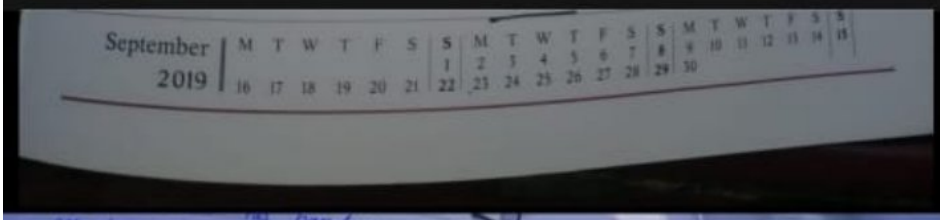
$$\sum_{n=1}^{\infty} \bar{z}_n = \bar{S}$$

Proved

Show that $(-1)^{\bar{y}_\pi} = \exp(1+2n)i$

Solution:

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MBER 2019

Q: $Z_n = \frac{1}{n^3} - i$ Solve it for convergence

A:

$$Z_n = \frac{1}{n^3} - i$$

$$\lim_{n\to\infty} Z_n = \lim_{n\to\infty}\left(\frac{1}{n^3} - i\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{n^3}\right) - \lim_{n\to\infty}(i)$$

$$= \frac{1}{\infty} - i = -i$$

(converges)

Q: Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies Laplace's equation.

$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x$, $\frac{\partial u}{\partial y} = -6xy - 6y$

$6x + 6$, $\frac{\partial u}{\partial}$ = 0

SEPTEMBER

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 6 - 6x - 6 = 0$$

Hence, u satisfies Laplace's equation.

Q: Evaluate $\bar{z}_1 - \bar{z}_2$ when $z_1 = 5+i$ & $z_2 = -8+3i$.

A:

$z_1 = 5+i$, $z_2 = -8+3i$

$\bar{z}_1 = 5-i$, $\bar{z}_2 = -8-3i$

$\bar{z}_1 - \bar{z}_2 = 5 - i + 8 + 3i$

$= 13 + 2i$

Q: If $\int_C f(z)dz$ & $f(z) = \frac{z+2}{z}$ & $z = 2e^{i\theta}$

A: $I = \int_C f(z)dz$



SEP/OCT 2019

30 MON

$$I = \int_0^{\pi} \left(\frac{z+2}{z}\right)dz \qquad z = 2e^{i\theta}$$

$$= \int_0^{\pi}\left(\frac{z}{z} + \frac{2}{z}\right)dz \qquad dz = 2ie^{i\theta}d\theta$$

1 TUE

$$= \int_0^{\pi}\left(1 + \frac{2}{z}\right)dz$$

$$= \int_0^{\pi}\left(1 + \frac{2}{2e^{i\theta}}\right)2ie^{i\theta}d\theta$$

$$= \int_0^{\pi}\left(1 + \frac{1}{e^{i\theta}}\right)2ie^{i\theta}d\theta$$

2 ED

$$= 2i\int_0^{\pi}\left(\frac{e^{i\theta}+1}{e^{i\theta}}\right)\cdot e^{i\theta}\,d\theta$$

OCTOBER

$$I = 2i\left[\frac{e^{i\theta}}{i}\Big|_0^{\pi} + \theta\Big|_0^{\pi}\right]$$

$$= 2i\left[\frac{e^{i\pi} - e^{0}}{i}\right] + 2i(\pi - 0)$$

$$= 2e^{i\pi} - 2 + 2i\pi \qquad e^{i\pi} = \cos\pi + i\sin\pi = -1 + 0 = -1$$

$$= 2(-1) - 2 + 2i\pi$$

$$= 2i\pi - 4$$

Q: Prove that $\int_0^{\pi+2i}\cos\left(\frac{z}{2}\right)dz = e + \frac{1}{e}$

A: L.H.S

$$I = \int_0^{\pi+2i}\cos\left(\frac{z}{2}\right)dz$$

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$$e^{\log 1^{2i}}$$

$$e^{2i \log(1)}$$

$$e$$

Ag

$$\text{Log}(z) = \ln|z| + i\,\text{Arg}(z)$$

$$= \ln|1| + i \tan^{-1}\left(\frac{0}{1}\right)$$

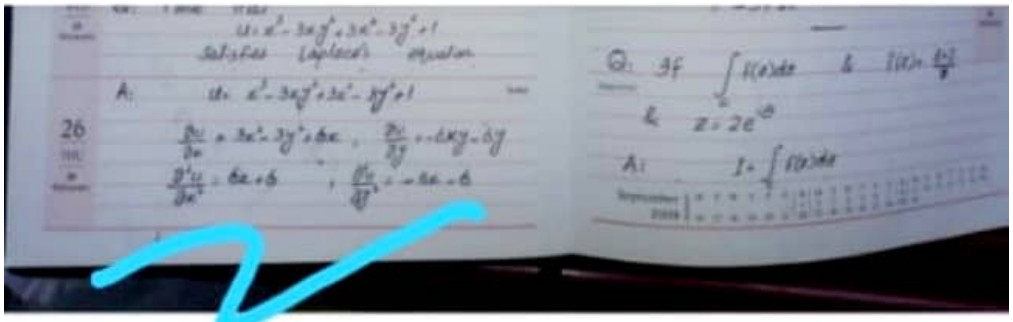
$$= 0 + i(0)$$

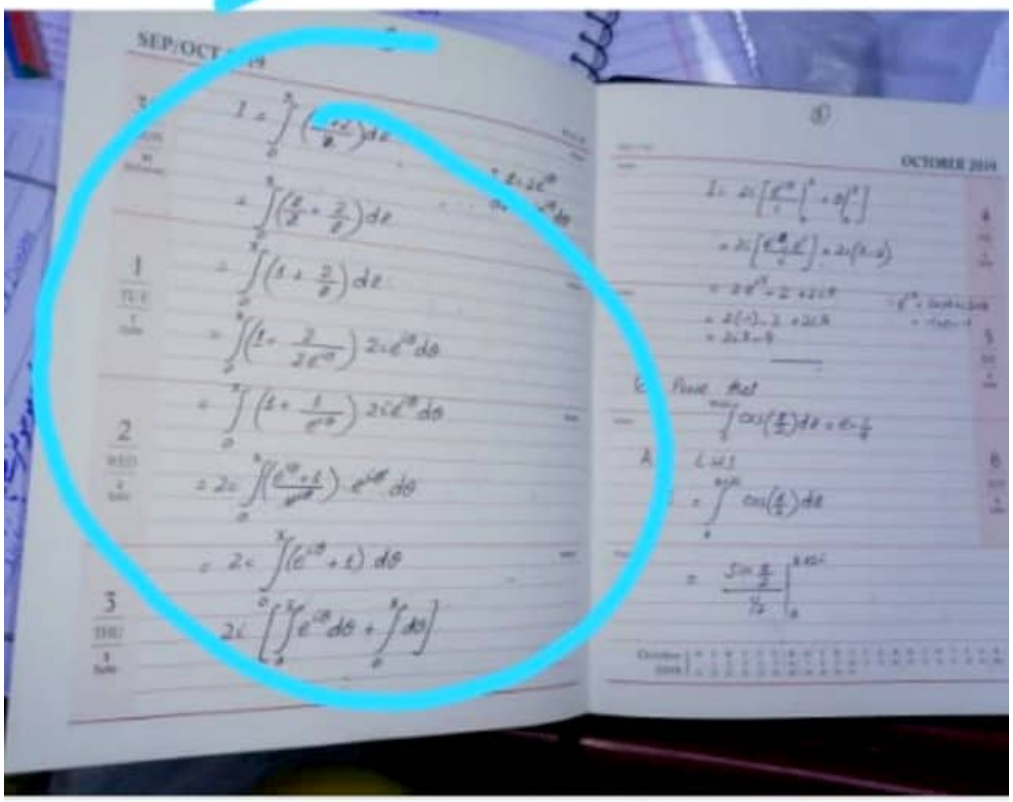
$$= 0$$

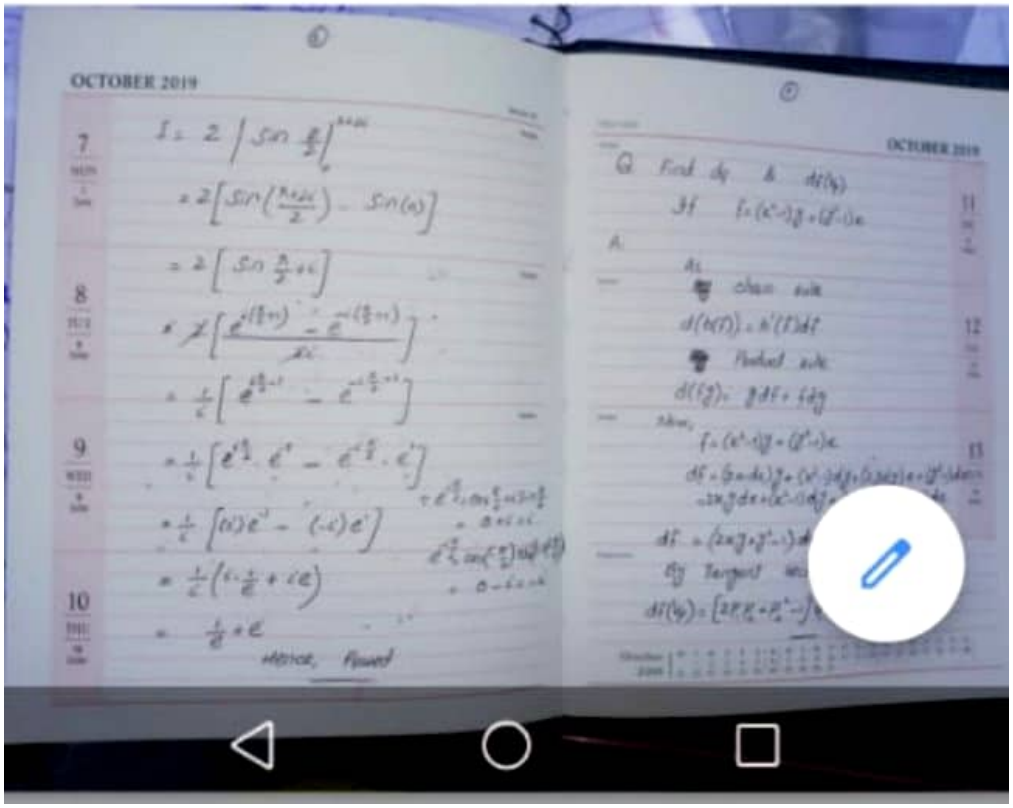
$$e^{2i(0)} = e^{0} = 1$$

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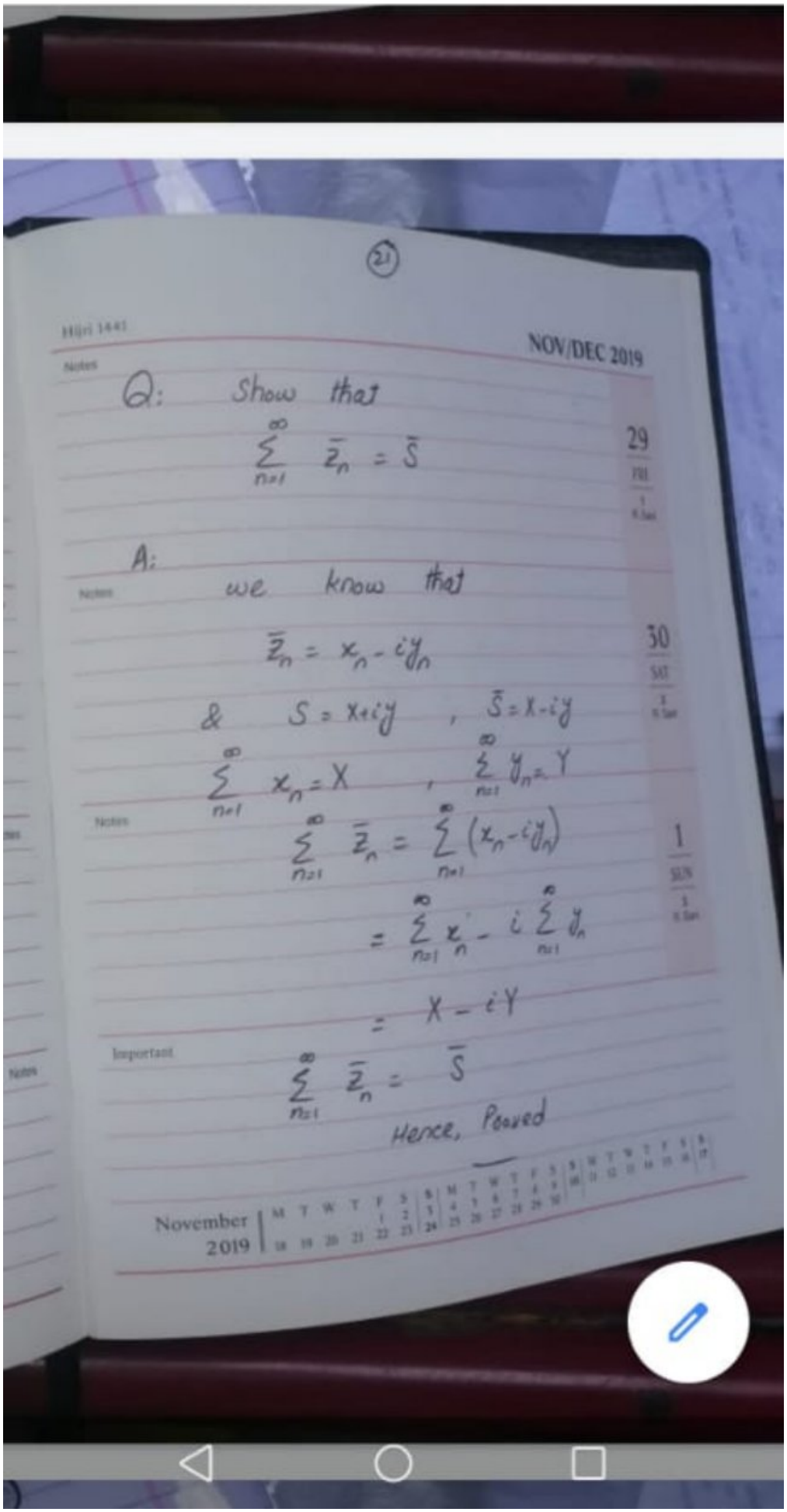






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Hence, function

Q: $w = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$

Write 1-form as $\Sigma (df_i P) v_i$

A:

Let $f_1, f_2, f_3 : \mathbb{R}^3 \to \mathbb{R}$ be three functions.

Let

$$\phi = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$$

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$= \cos(y+z)$

$= \cos y \cos z - \sin y \sin z \quad -(3)$

from eq (2) & (3)

LHS = RHS

Q: Prove that $\frac{d}{dz}(\cos z) = -\sin z$

A: LHS

$= \frac{d}{dz}(\cos z)$

$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (-1)^1 \cdot (z^{2n-1})}{(2n-1)!}$

$= -\sum_{n=1}^{\infty} \frac{(-1)^n z^{2n-1}}{(2n-1)!}$

$= -\sin z$

Hence, Proved

---

(10)

BER 2019

Q: Frenet Searet formula

Prove that

① $\bar{n} = t\bar{B} - k\bar{t}$

$n = B \times t$

differentiate wrt s

$\bar{n}' = B\bar{t}' + t\bar{B}'$

As $t = n \times b$

$n = b \times t$

$B = t \times n$

$\bar{n}' = k(b \times n) + (-\tau(n \times t)) \quad \because t' = k\bar{n}$

$= -k\bar{t} - \tau(-B) \quad b = -\tau\bar{n}$

$= -k\bar{t} + B\tau$

$\bar{n}' = B\tau - k\bar{t}$

② Prove that

$\bar{t}' = k\bar{n}$

$k = \frac{d\theta}{ds}$

$k = \frac{d\theta}{dt} \cdot \frac{dt}{ds}$

(11)

NOVEMBER 20

$k = \left|\frac{d\bar{A}}{dt}\right| \cdot \left|\frac{dt}{ds}\right| = \left|\frac{d\bar{t}}{ds}\right|$

$k = \left|\frac{d\bar{r}}{ds}\right| \quad \because t = \frac{d\bar{r}}{ds}$

$= \left|\frac{d\bar{t}'}{ds}\right| = |\bar{t}''| = k$

$\bar{n}'$ is ortho... to $\bar{t}$

$\cdot |\bar{t}''| = \pm kn$

$\bar{n}' = kn \quad \because \bar{t} \perp \bar{n}'$

Hence, $\bar{t}' = k\bar{n}$

③ $b = -\tau\bar{n}$

As $t \cdot b = 0$

differentiate wrt s $t \cdot b$

$t \cdot \bar{b}' + \bar{t} \cdot b = 0$

$t \cdot \bar{b}' + k\bar{n} \cdot b = 0$

$t \cdot \bar{b}' = 0$

---

(12)

EMBER 2019

$\because$ $\bar{b}$ is perpen to $\bar{t}$

also $\bar{b}$ is ortho... with n

$\bar{b} = \pm \bar{t} \times \bar{n}$

$\bar{b} = -\bar{t} \times \bar{n}$

b has opposite direction

so -ve sign

Q: If x, y & ϕ, ψ then

Prove that

$d(fg) = gdf + fdg$

A: $d(fg) = \sum \frac{\partial}{\partial x_i}(fg)\,dx_i$

$= \sum \left(\frac{\partial f}{\partial x_i} g + \frac{\partial g}{\partial x_i} f\right) dx_i$

$= \sum \frac{\partial f}{\partial x_i} g\,dx_i + \sum \frac{\partial g}{\partial x_i} f\,dx_i$

(13)

NOVEMBER 2

Q: If $P_1(1,1,0)$ & $V_1(1,2,-3)$

$f = x^2yz$, then find $\nabla_{V_1}(f)$

A: $f = x^2yz$

$\frac{\partial f}{\partial x} = 2xyz, \quad \frac{\partial f}{\partial y} = x^2z, \quad \frac{\partial f}{\partial z} = x^2y$

at Point $P_1(1,1,0)$

$\frac{\partial f}{\partial x}\Big|_{(1,1,0)} = 0, \quad \frac{\partial f}{\partial y}\Big|_{(1,1,0)} = 0, \quad \frac{\partial f}{\partial z}\Big|_{(1,1,0)} =$

$\nabla_{V_1}(f) = \sum_i \frac{\partial f}{\partial x_i}(P)$

$= 1(0) + 2(0) - 3(1)$

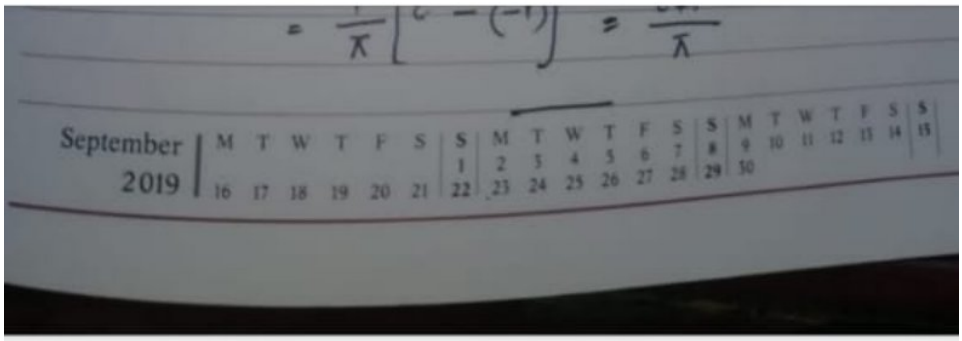
$= -3$

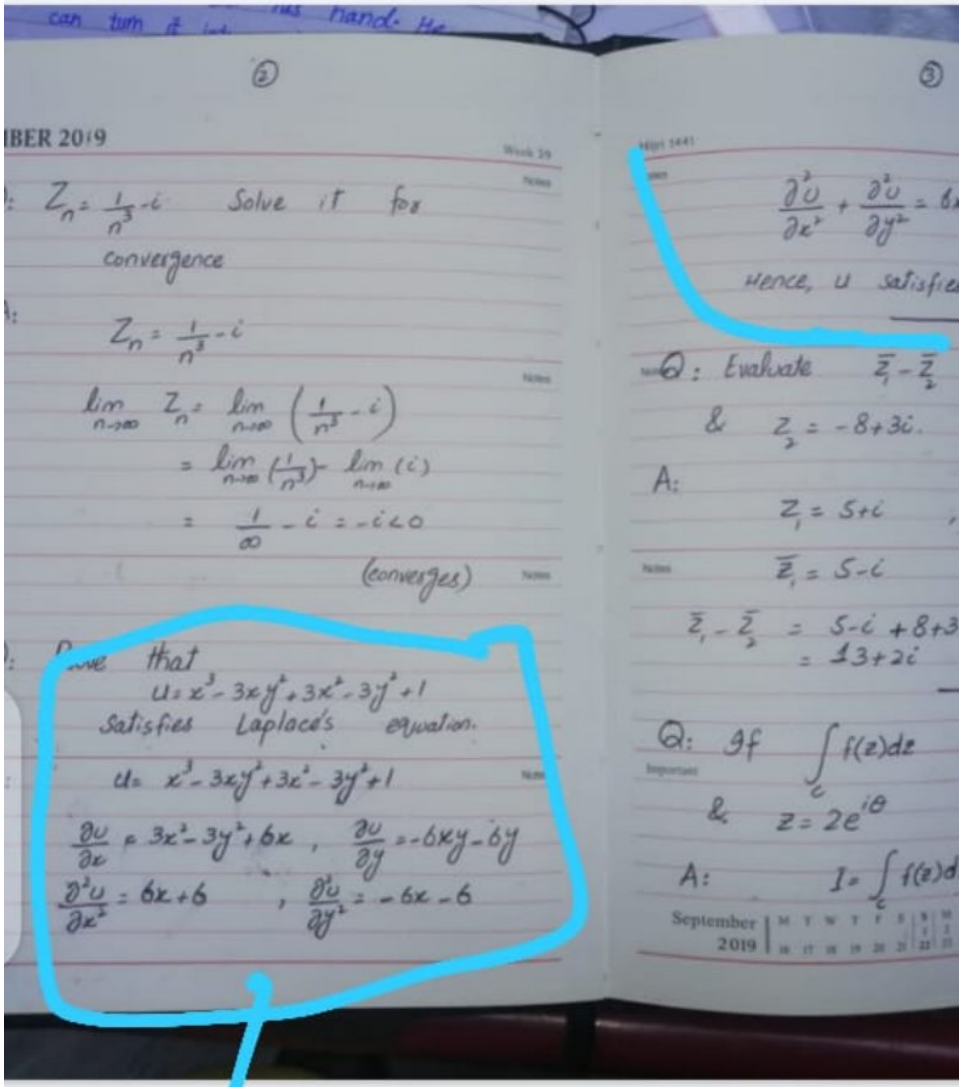
-3 is directional derivative

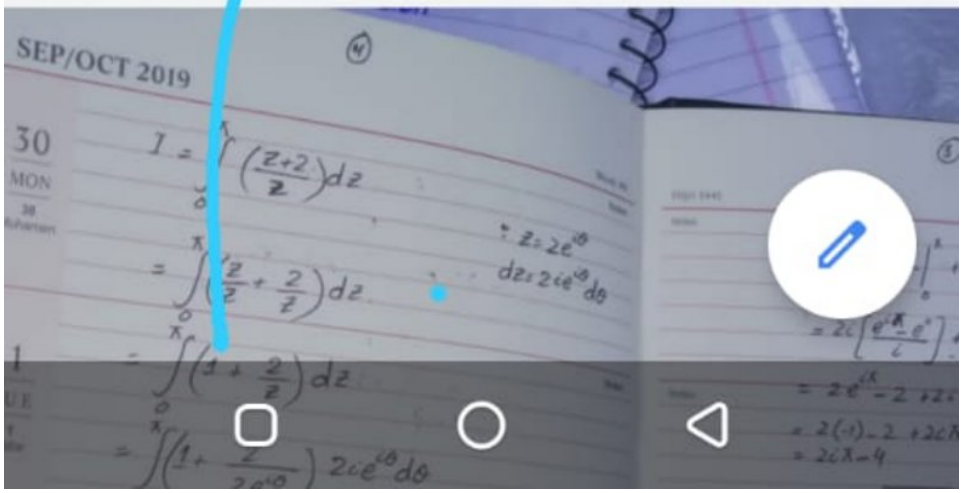
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we know that

$$\hat{t} = x = \frac{x'(t)}{|x'(t)|} = \frac{-a\sin(t)\hat{e}_1 + a\cos(t)\hat{e}_2 + b\hat{e}_3}{\sqrt{a^2+b^2}}$$

By integrating w.r.t y

$$v(x,y) = -e^y \sin x + g(x)$$

October 2019

(10)

TOBER 2019

By taking derivative w.r.t. x

$$\frac{\partial v}{\partial x} = -e^y \cos x + g'(x)$$

Put $\frac{\partial v}{\partial x} = -e^y \cos x$

$$-e^y \cos x = -e^y \cos x + g'(x)$$

$$g'(x) = 0$$

$$g(x) = c$$

Hence,

$$v(x,y) = -e^y \sin x + c$$

Q: Find Singularity for function

$$f(z) = \frac{1}{z^2+2z+2}$$

A: Here

$$z^2+2z+2 = 0$$

By applying quadratic formula

(11)

OCTOBE

$a=1$, $b=2$, $c=2$

$$z = \frac{(-2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{(-2) \pm \sqrt{4-8}}{2}$$

$$z = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

is singular Point

Q: Show that

$$\overline{\cos(iz)} = \cos(\bar{iz})$$

A: LHS

$$= \overline{\cos(iz)}$$

2019

(12)

TOBER 019

$$= \overline{\cos(-y+ix)} \quad \because z = x+iy$$

$$= \cos y \cosh x - \sin y \sinh x \quad \text{— ①} \qquad iz = ix - y$$

$$\bar{iz} = -ix - y$$

R.H.S

$$= \cos(\bar{iz})$$

$$= \cos(-ix - y)$$

$$= \cos(-(y+ix))$$

$$= \cos(y+ix)$$

$$= \cos y \cosh x - \sin y \sinh x \quad \text{— ②}$$

from EN ① & ②

LHS = R.H.S

Q: Prove that $\frac{d}{dz}(\cos z) = -\sin z$

A: L.H.S

$$= \frac{d}{dz}(\cos z)$$

(13)

NOVEMBE

$$= \frac{d}{dz} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{d}{dz} \frac{(-1)^n z^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2n (z^{2n-1})}{(2n-1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (z^{2n-1})}{(2n-1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \cdot (-1)^1 (z^{2n-1})}{(2n-1)!}$$

$$= -\sum_{n=1}^{\infty} \frac{(-1)^{n+1} z^{2n-1}}{(2n-1)!}$$

$$= -\sin z$$

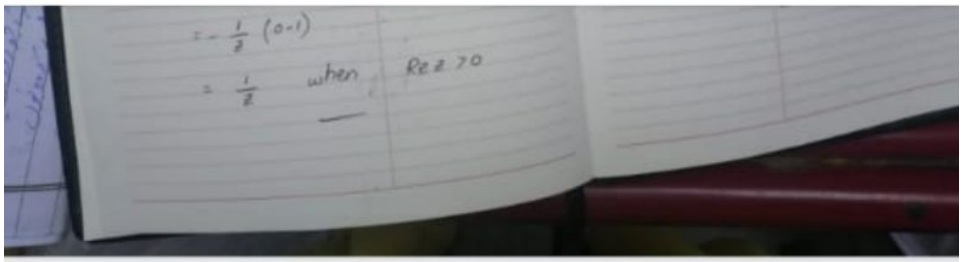
Hence, Proved

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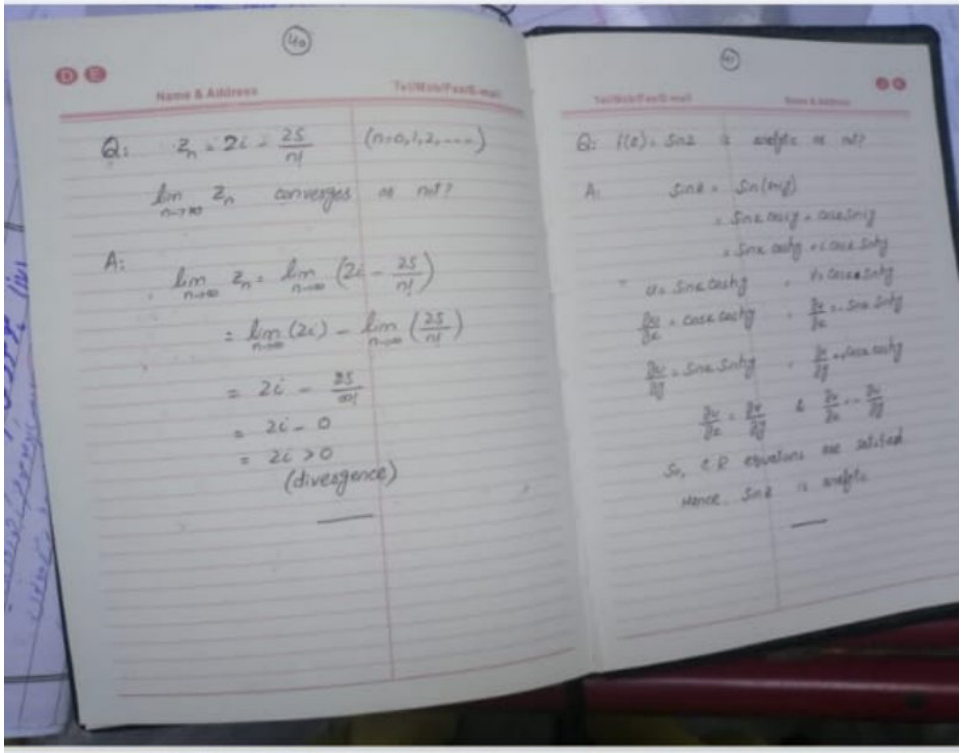
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$= -\frac{1}{z}(0-1)$

$= \frac{1}{z}$ when $Re\, z > 0$



Q: $z_n = 2i - \frac{25}{n!}$ $(n=0,1,2,\ldots)$

$\lim_{n\to\infty} z_n$ converges or not?

A:

$\lim_{n\to\infty} z_n = \lim_{n\to\infty}\left(2i - \frac{25}{n!}\right)$

$= \lim_{n\to\infty}(2i) - \lim_{n\to\infty}\left(\frac{25}{n!}\right)$

$= 2i - \frac{25}{\infty!}$

$= 2i - 0$

$= 2i > 0$

(divergence)

Q: $f(z) = \sin z$ is analytic or not?

A: $\sin z = \sin(x+iy)$

$= \sin x \cos iy + \cos x \sin iy$

$= \sin x \cosh y + i \cos x \sinh y$

$u = \sin x \cosh y$ $\quad v = i \cos x \sinh y$

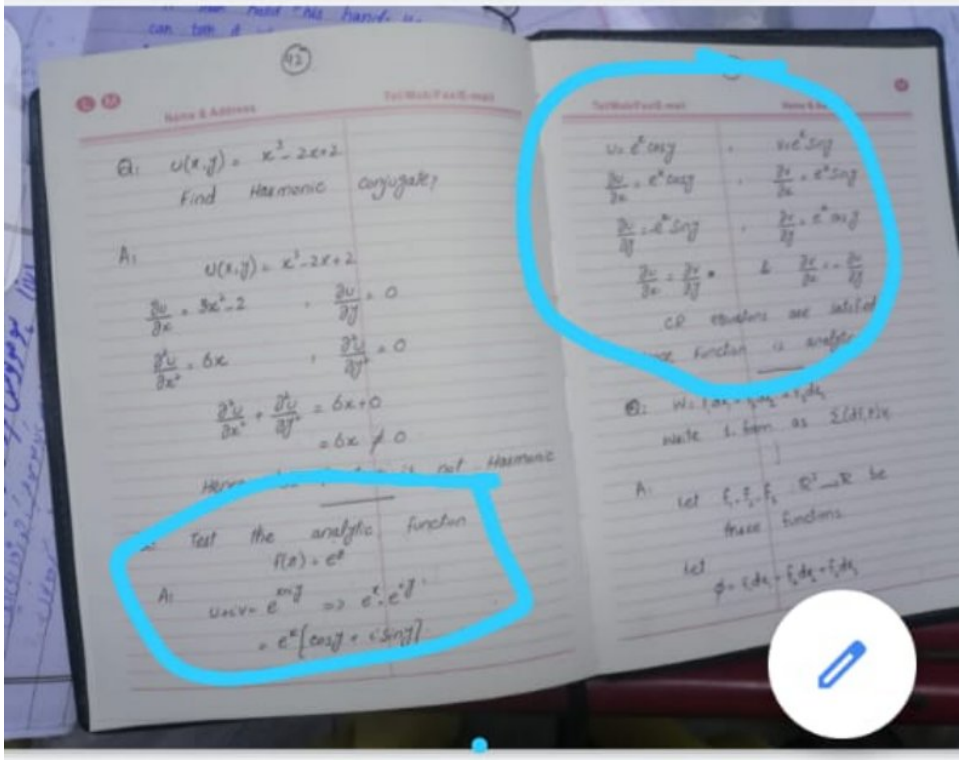
$\frac{\partial u}{\partial x} = \cos x \cosh y$ $\quad \frac{\partial v}{\partial x} = -\sin x \sinh y$

$\frac{\partial u}{\partial y} = \sin x \sinh y$ $\quad \frac{\partial v}{\partial y} = \cos x \cosh y$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

So, C-R equations are satisfied

Hence $\sin z$ is analytic



Q: $u(x,y) = x^3 - 2x + 2$

Find Harmonic conjugate

A:

$u(x,y) = x^3 - 2x + 2$

$\frac{\partial u}{\partial x} = 3x^2 - 2$ $\quad \frac{\partial u}{\partial y} = 0$

$\frac{\partial^2 u}{\partial x^2} = 6x$ $\quad \frac{\partial^2 u}{\partial y^2} = 0$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + 0$

$= 6x \neq 0$

Harmonic

Test the analytic function

$f(z) = e^z$

A: $u + iv = e^{x+iy} \Rightarrow e^x \cdot e^{iy}$

$= e^x(\cos y + i \sin y)$

$u = e^x \cos y$ $\quad v = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \cos y$ $\quad \frac{\partial v}{\partial x} = e^x \sin y$

$\frac{\partial u}{\partial y} = -e^x \sin y$ $\quad \frac{\partial v}{\partial y} = e^x \cos y$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

C-R equations are satisfied

Function is analytic

Q: $w = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$

write it form as $\sum f_i dx_i$

A: Let $f_1, f_2, f_3 : \mathbb{R}^3 \to \mathbb{R}$ be three functions

Let

$\phi = f_1 dx_1 + f_2 dx_2 + f_3 dx_3$

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MTH632 Complex Analysis and Differential Geometry

Question No : 39 of 41

Show that

(a) $Log(1+i)^2 = 2Log(1+i)$ and (b) $Log(-1+i)^2 \neq 2Log(-1+i)$

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry **MC180**

Question No : 34 of 41 **Marks: 2 (Budgeted Time Min)**

If tangent vector along the helix $\vec{x}(t) = a(\cos t)\hat{e}_1 + a(\sin t)\hat{e}_2 + b(t)\hat{e}_3$, $a, b \neq 0$ is $\vec{x}'(t) = -a(\sin t)\hat{e}_1 + a(\cos t)\hat{e}_2 + b\hat{e}_3$.

Then evaluate the unit tangent vector $\vec{t}(s) = \dot{x} = \dfrac{\vec{x}'(t)}{|\vec{x}'(t)|}$.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 32 of 41

Test whether the function $w = 2xy + i(x^2 - y^2)$ is analytic or not.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 31 of 41

Find $f'(z)$, when $f(z)=(1-4z^2)^3$.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 36 of 41 Ma

If $\lim_{n\to\infty} z_n = z$ then by using the inequality, $\big||z_n|-|z|\big| \le |z_n - z|$, show that $\lim_{n\to\infty} |z_n| = |z|$.

Answer (Please click here to Add Answer)

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MTH632 Complex Analy[illegible] Differential Geometry

Question No : 32 of 41 **Mar[illegible]**

Test whether the function $w = 2xy + i(x^2 - y^2)$ is analytic or not.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 33 of 41

Evaluate the integral: $\int_{1}^{2}\left(\frac{1}{t}-i\right)^{2} dt$.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 38 of 41 **Ma**

Let f and g be functions, ϕ and ψ be 1-forms. Then show that $d(fg) = df\,g + f\,dg$.

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MTH632 Complex Analysis and Differential Geometry

Question No : 37 of 41

Show that $\frac{d}{dx}(\operatorname{arcsinh}(x)) = \frac{1}{\sqrt{1+x^2}}$

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 40 of 41

If $u = \log(x^2 + y^2)$ find v and $f(z)$ such that $f(z) = u + iv$ is analytic.

Answer (Please click here to Add Answer)



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MTH632 Complex Analysis and Differential Geometry **MC1**

Question No : 41 of 41 **Marks: 5 (Budgeted Time Min)**

Let C_1 denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 1, y = \pm 1$ and let C_2 be the positively oriented circle $|z| = 4$ (given in Fig. below). Explain why, $\int_C f(z)dz = \int_{C_1} f(z)dz$ where $f(z) = \dfrac{z+2}{\sin(z/2)}$?

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry **MC180**

Question No : 41 of 41 **Marks: 5 (Budgeted Time Min)**

Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate the integral $\int_C \frac{\tan(z/2)}{(z - x_0)^2} dz$, $(-2 < x_0 < 2)$ by using the extended Cauchy integral formula.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 40 of 41 **Marks: 5 (Budgeted Time**

Verify that $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$ whenever $|z| < 1$.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 39 of 41 **Marks: 5 (**

Find a function w such that $w = u + iv$ is analytic, if $u = e^x \sin y$.

Answer (Please click here to Add Answer)



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MTH632 Complex Analysis and Differential Geometry **MC180**

Question No : 38 of 41 **Marks: 3 (Budgeted Time Min)**

Verify the Frenet formula for the relation, $\vec{n}' = \tau\vec{b} - k\vec{t}$, where $k > 0$ and τ are curvature and torsion of a curve respectively.

Answer (Please click here to Add Answer)

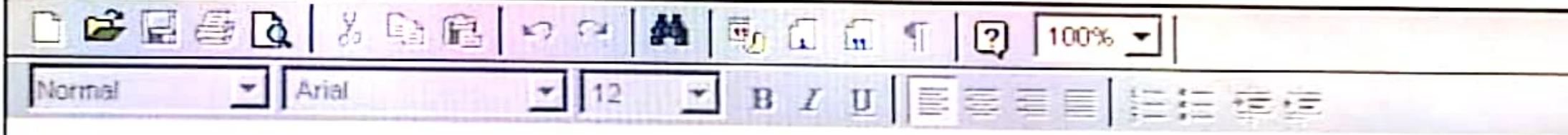


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Question No : 37 of 41 **Marks: 3 (B**

Evaluate the integral $\int_0^{\pi/6} e^{i2t}\,dt$.

Answer (Please click here to Add Answer)



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MTH632 Complex Analysis and Differential Geometry

Question No : 36 of 41 **Marks: 3 (Budgeted Tim**

Prove that $u = e^{-y} \sin x$ is harmonic.

Answer (Please click here to Add Answer)

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MTH632 Complex Analysis and Differential Geometry

Question No : 35 of 41 **Marks: 3 (Budgeted Time**

Evaluate, $\lim_{z \to i} \frac{z^4 - 1}{z^2 + 1}$.

Answer (Please click here to Add Answer)

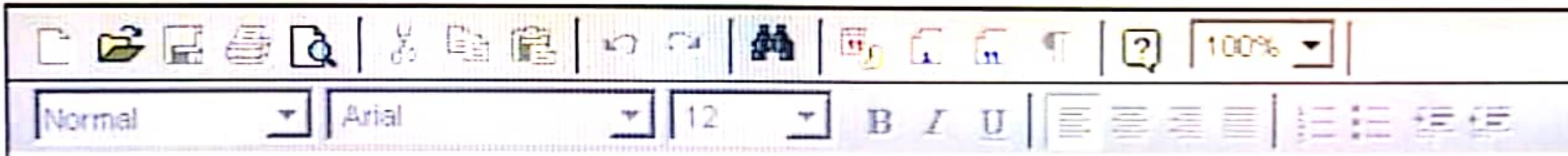
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MTH632 Complex Analysis and Differential Geometry

Question No : 33 of 41

Show that $\frac{d}{dz}(\sin(z)) = \frac{1}{\sqrt{1-z^2}}$.

Answer (Please click here to Add Answer)



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MTH632 Complex Analysis and Differential Geometry **MC1**

Question No : 31 of 41 **Marks: 2 (Budgeted Time Min)**

Evaluate $\lim_{z \to i} f(z)$, where $f(z) = 3z^2 - iz + 4 - 5i$ is a polynomial, it is continuous at all points in the complex plane.

Answer (Please click here to Add Answer)

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