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MTH633 Group Theory

Question No : 28 of 33 Marks: 2 (Budgete

State the Cauchy's Theorem. (Do not prove it)

Answer (Please click here to Add Answer)

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MTH633 Group Theory

Question No : 31 of 33 Marks: 3 (B

Show that the set of all inner automorphisms of G is a subgroup of $Aut(G)$.

Answer (Please click here to Add Answer)

100%

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MTH633 Group Theory

Question No : 27 of 33 Marks: 2 (Budgeted Ti

Show that every subgroup H containing just half the elements of a finite group G must be a normal subgroup

Answer (Please click here to Add Answer)

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MTH633 Group Theory

Question No : 29 of 33 Marks: 3 (Bu

Define a complete group.

Answer (Please click here to Add Answer)

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Answer (Please select your correct option)

- action
- identity
- commutativity
- associativity

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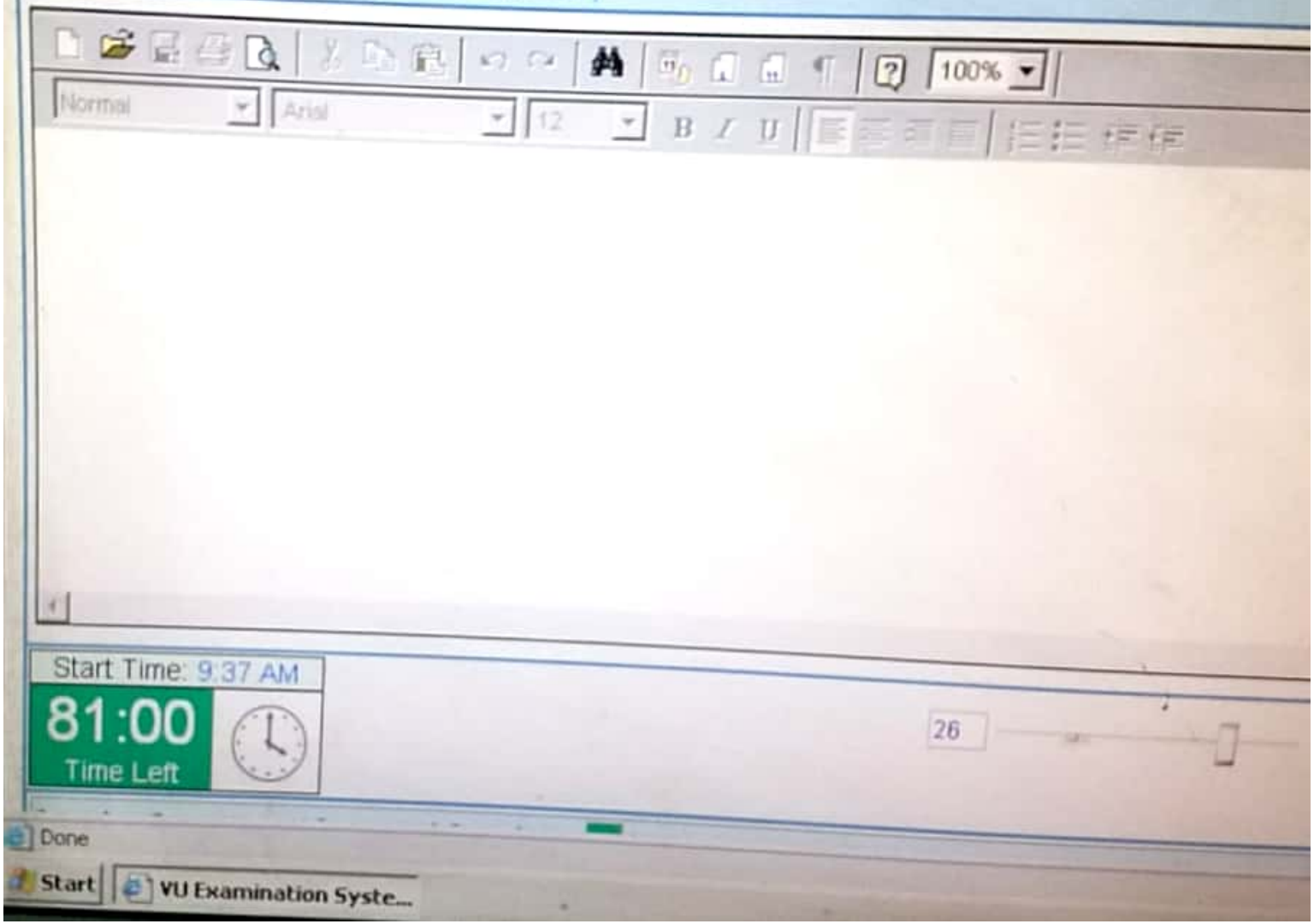
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We may sometimes write $\mathbf{Z}/n\mathbf{Z} = \mathbf{Z}_n$. How do you read $\mathbf{Z}_n$?

Answer (Please click here to Add Answer)



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Question No : 23 of 33

The center $Z(G)$ is defined by _____.

Answer (Please select your correct option)

- $Z(G) = \{z \in G \mid zg \neq gz \text{ for all } g \in G\}$
- $Z(G) = \{z \in G \mid zg = gz \text{ for all } g \in G\}$
- $Z(G) = \{z \in G \mid zg > gz \text{ for all } g \in G\}$
- $Z(G) = \{z \in G \mid zg < gz \text{ for all } g \in G\}$

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Answer (Please select your correct option)

- non-trivial proper
- non-trivial improper



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MTH633 Group Theory

Question No : 21 of 33

Marks: 1 (Budgeted T

Let $G = H \times K$ be the direct product of groups H and K. Then $\overline{H} = \{(h, e) | h \in H\}$ is _____ of G.

Answer (Please select your correct option)

- normal subgroup
- trivial subgroup
- isomorphic
- cyclic subgroup

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MTH633 Group Theory

Question No : 20 of 33

$G/G \cong$ _____ .

Answer (Please select your correct option)

- $\{e\}$
- G
- N
- $G \cap \{e\}$

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MTH633 Group Theory

Question No : 18 of 33

Ma

An isomorphism $\square: G \to G$ of a group G with itself is _____ of G.

Answer (Please select your correct option)

- isomorphism
- epimorphism
- indomorphism
- automorphism

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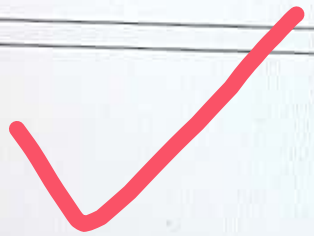
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Composition of group homomorphisms is again a group homomorphism.

Answer (Please select your correct option)

- False
- True



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MTH633 Group Theory

Question No : 16 of 33

Marks: 1 (Budgeted Time Min)

The direction $\rightarrow$ of the one-to-one correspondence $s \leftrightarrow g$ between $s \in S$ and $g \in G$ gives us a one-to-one function μ mapping S onto G.

Answer (Please select your correct option)

- False
- True

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MTH633 Group Theory

Question No : 14 of 33 Mark

The matrix A is invertible if and only if its determinant, $det(A)$ is _____.

Answer (Please select your correct option)

- 0
- 1
- 2
- non-zero ✓

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MTH633 Group Theory

Question No : 12 of 33 Marks: 1 (Budgeted Ti

The quotient group R / Z is isomorphic to the circle group _____.

Answer (Please select your correct option)

- $W = (0, 1, 2, \cdots)$
- $W = (a+ib \in \mathbb{Q} \mid a,b \in \mathbb{Q})$
- $W = (e^{i\theta} \in \mathbb{Q} \mid \theta \in \mathbb{Q})$
- $W = (1, -1, i, -i)$

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MTH633 Group Theory

Question No : 8 of 33

Marks:

If P is a Sylow p-subgroup, every conjugate gPg^{-1} of P is also _____.

Answer (Please select your correct option)

- sylow p - subgroup
- cyclic subgroups
- improper subgroups
- trivial subgroups

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The action of the group G on itself is called conjugation.

Answer (Please select your correct option)

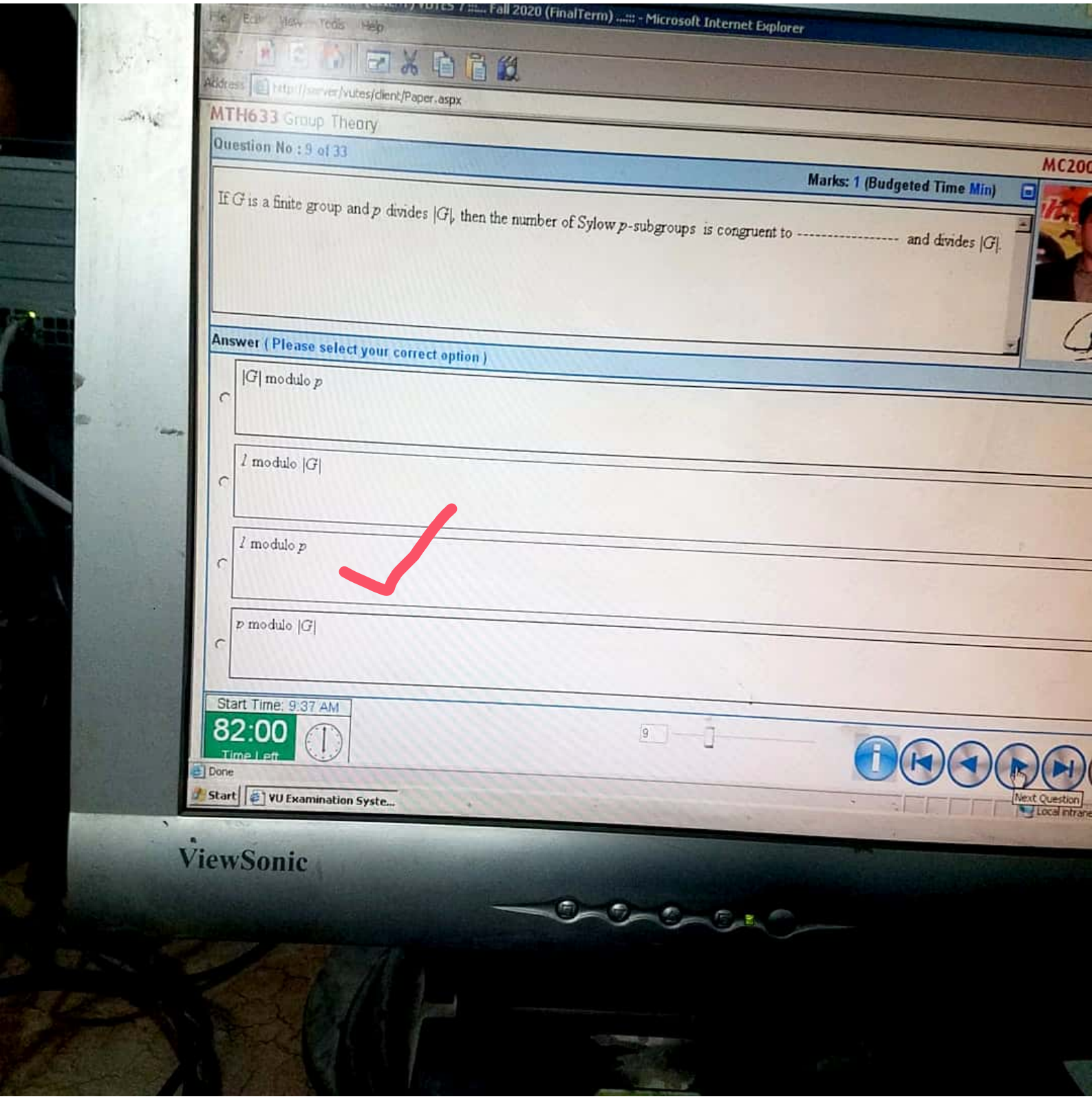
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- False

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MTH633 Group Theory

Question No : 7 of 33

Marks:

For a prime number p, every group G of order p^2 is _____.

Answer (Please select your correct option)

- abelian
- non-abelian
- infinite
- none of these

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MTH633 Group Theory

Question No : 2 of 33

Mar

There cannot be more than six automorphisms of S_3.

Answer (Please select your correct option)

- True
- False

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MTH633 Group Theory

Question No : 6 of 33 **Marks: 1**

The center of a finite non-trivial p-group G is _____.

Answer (Please select your correct option)

- non-trivial
- trivial
- non-abelian
- none of these

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True

False

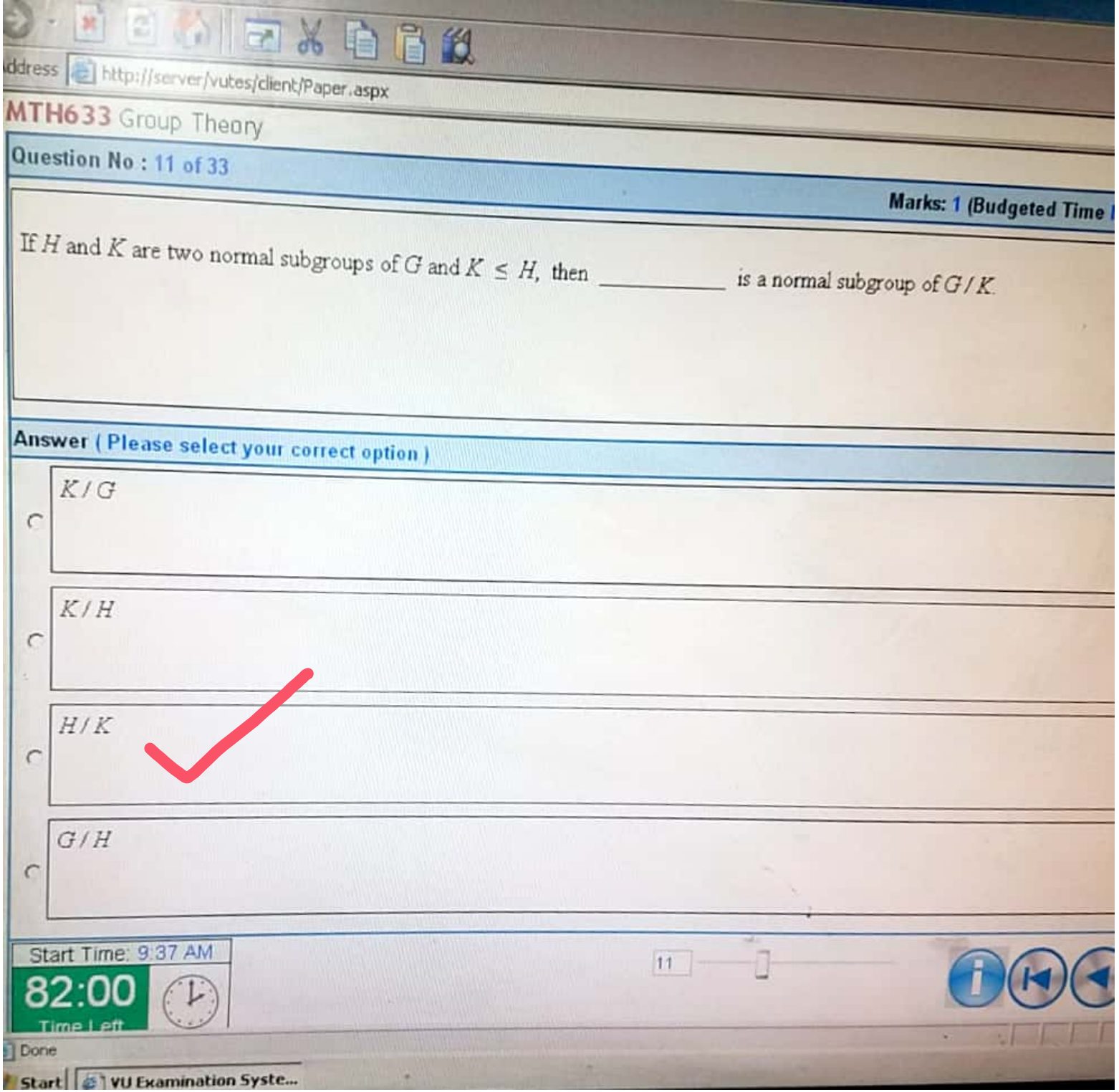
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MTH633 Group Theory

Question No : 1 of 33

If N is a normal subgroup of G, then G/N is abelian if and only if $C \leq N$.

Answer (Please select your correct option)

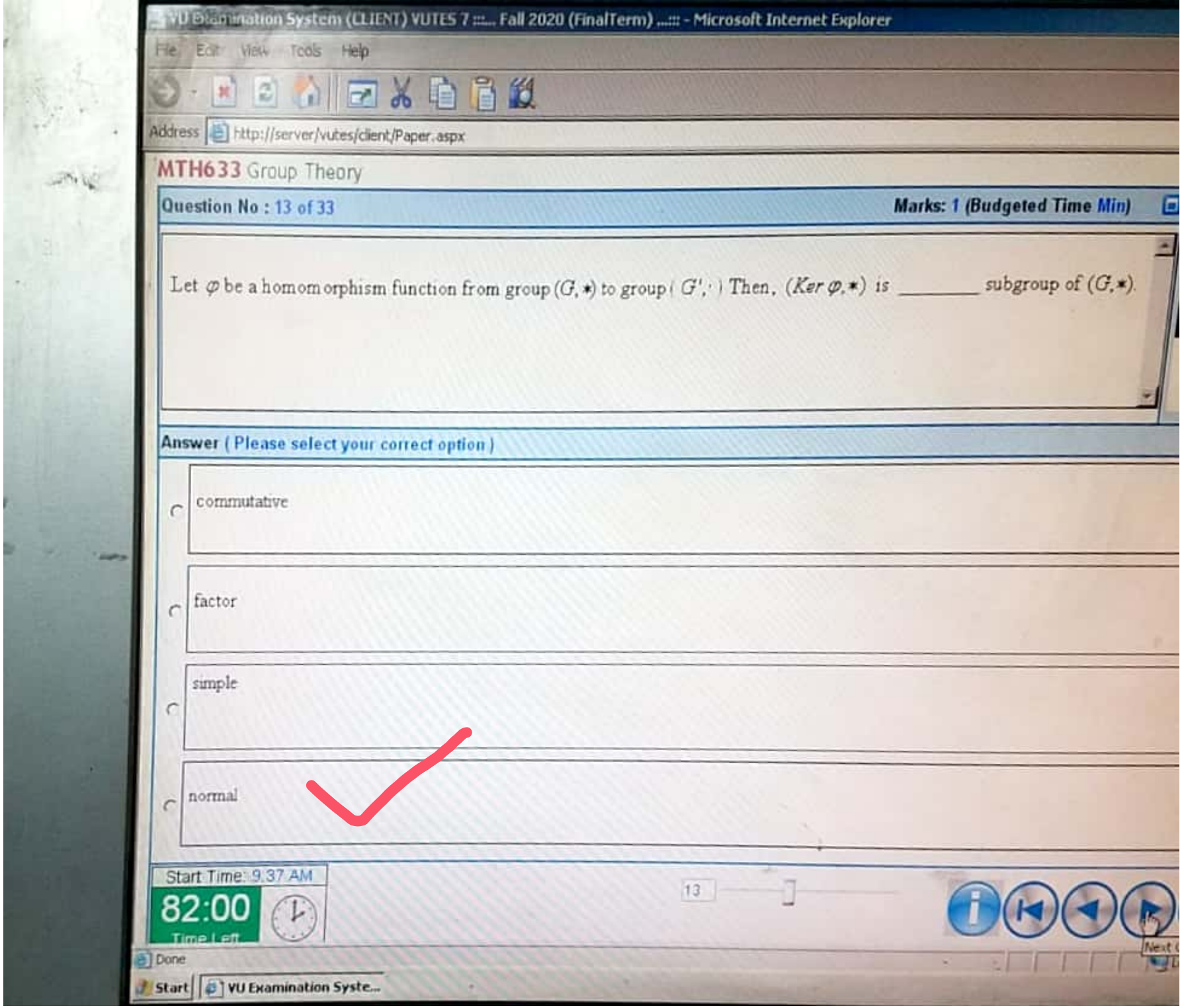
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- False

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MTH633 Group Theory

Question No : 10 of 33

Marks: 1 (Budgete

The Sylow theorems give us some important information about _____ groups.

Answer (Please select your correct option)

- finite abelian
- finite symmetric
- finite cyclic
- finite nonabelian

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MTH633 Group Theory

Question No : 5 of 33

Marks: 1 (Bud

If H and N are subgroups of a group G, then the their join _____ is the smallest subgroup of G containing b

Answer (Please select your correct option)

- $H \vee N$
- $H - N$
- H / N
- $H \cup N$

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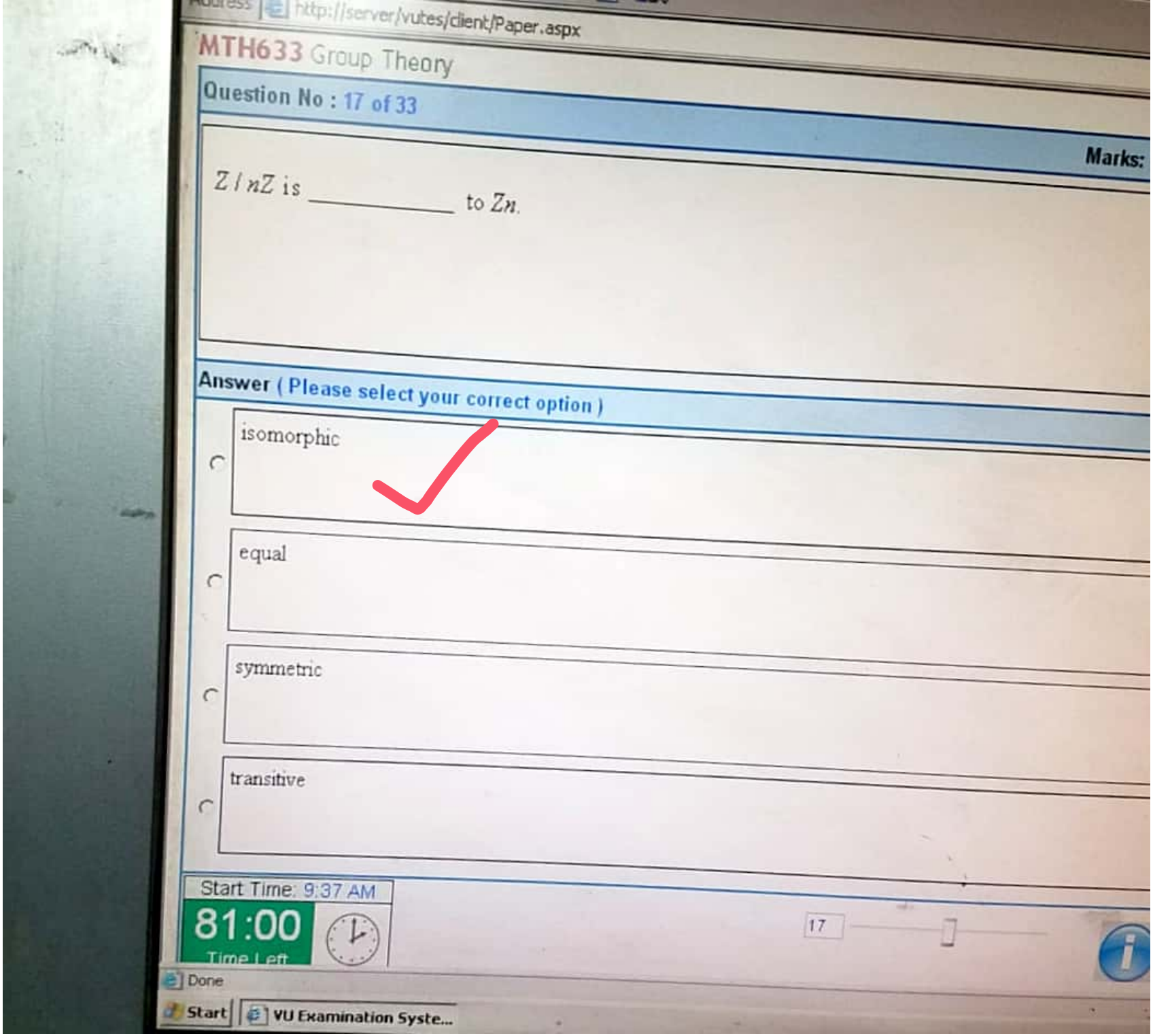
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The inverse of $a^2b^4a^{-3}b^2a^5$ is _____.

Answer (Please select your correct option)

- $a^{-5}b^{-2}b^{-4}a^3a^{-2}$
- $a^{-5}b^{-2}a^3a^{-2}b^{-4}$
- $a^{-5}b^{-2}a^3b^{-4}a^{-2}$ ✓
- $b^{-2}a^{-5}a^3b^{-4}a^{-2}$

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Question No : 19 of 33

Marks: 1 (Budgeted

Let N be a normal subgroup of G. In the factor group G/N, the subgroup N acts as _____ element.

Answer (Please select your correct option)

- identity
- non-identity
- inverse
- none of these

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MTH633 Group Theory

Marks: 5 (Bu

Question No : 32 of 33

Let φ be a homomorphism function from group $(G,*)$ to group $(G',\cdot)$. Then, $(Ker\ \varphi,*)$ is a normal subgroup of $(G,*)$.

Answer (Please click here to Add Answer)

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MTH633 Group Theory

Question No : 33 of 33

Marks: 5 (Bu

Let X be a G-set. Show that G_x is a subgroup of G for each $x \in X$.

Answer (Please click here to Add Answer)

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MTH633 Group Theory

Question No : 30 of 33

Marks: 3 (B

State second isomorphism theorem.

Answer (Please click here to Add Answer)

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MTH633 Group Theory

Question No : 24 of 33

$Z(G)=\{e\}$ is called the _____ of G is trivial

Answer (Please select your correct option)

- non-Identity elements
- inverse of each element
- center
- none of these

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 25 of 33

The Klein 4-group $V = \{e, a, b, c\}$ is generated by _____ since $a\,b = c$

Answer (Please select your correct option)

- $\{c\}$
- $\{b\}$
- $\{a\}$
- $\{a, b\}$

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

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MTH633 Group Theory

Question No : 21 of 33

If G is a finite group and G/N has just two elements, then we must have _____

Answer (Please select your correct option)

- $|G| = |N|$
- $|G| = 2\,|N|$
- $|G| = 3\,|N|$
- $|G| = 4\,|N|$

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 22 of 33

If H has 6 elements, then all cosets of H must have _____ elements.

Answer (Please select your correct option)

- 3
- 9
- 12
- 6

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 17 of 33

The direction $\leftarrow$ of $\leftrightarrow$ gives us the _____ function μ^{-1}.

Answer (Please select your correct option)

- Inverse
- Converse
- Bijection
- None of these

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23

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MTH633 Group Theory

Question No : 20 of 33

It is crucial to remember that multiplication of cosets in G/N can be computed by multiplying in G, using any representativ

Answer (Please select your correct option)

- False
- True

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 19 of 33

The set of all _____ of G is a subgroup of $Aut(G)$

Answer (Please select your correct option)

- automorphism
- endomorphism
- epimorphism
- inner automorphism

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

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MTH633 Group Theory

Question No : 23 of 33

The cyclic group $G = Z / pZ$ of congruence classes modulo p is _____, where p is a prime number.

Answer (Please select your correct option)

- simple
- non-abelian
- non-cyclic
- non-commutative

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 15 of 33

Let $h: G \to G'$ be a homomorphism of groups. The subgroup $h^{-1}[(e')] = (x \in G \mid h(x) = e')$ is the kernel of h, denoted by $Ker(h)$

Answer (Please select your correct option)

- False
- True

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 13 of 33

If N is a normal subgroup of $(G, \cdot)$, the set of cosets $G/N = \{Ng \mid g \in G\}$ forms a group $(G/N, \cdot)$, where by $(Ng_1) \cdot (Ng_2) = N(g_1 \cdot g_2)$. This group is called the _____ of G by N.

Answer (Please select your correct option)

- commutative group
- simple group
- factor group
- non-abelian group

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 18 of 33

A homomorphism h: G→G' is injective if and only if _____.

Answer (Please select your correct option)

- $Ker\ h = \{\}$
- $h(G) = \{e\}$
- $G' = \{e\}$
- $Ker\ h = \{e\}$

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 14 of 33

The relation $a \equiv b \bmod H$ is the _____ relation on G

Answer (Please select your correct option)

- equivalence
- only reflexive
- only symmetric
- only transitive

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 11 of 33

Let G be a group of order p^n and let X be a finite G-set. Then _____

Answer (Please select your correct option)

- $|X| \equiv |G_x| \pmod{p}$
- $|G| \equiv |X_0| \pmod{p}$
- $|X| \equiv |G| \pmod{p}$
- $|X| \equiv |X_0| \pmod{p}$

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 2 of 33

There cannot be more than six automorphisms of S_3

Answer (Please select your correct option)

- True
- False

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 12 of 33

Let H and K be normal subgroups of a group G with $K \leq H$. Then $G/H \simeq$ _____.

Answer (Please select your correct option)

- $(G/K)/(H)$
- $(G/K)/(H/K)$
- $(G/K)/(K)$
- $(G)/(H/K)$

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 16 of 33

Composition of group homomorphisms is again a group homomorphism

Answer (Please select your correct option)

- False
- True

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 3 of 33

A binary operation * on a set S is a function mapping SxS into S.

Answer (Please select your correct option)

- True
- False

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 8 of 33

No group of order 15 is _____

Answer (Please select your correct option)

- commutative
- simple
- finite
- cyclic

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

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MTH633 Group Theory

Question No : 9 of 33

If P is a Sylow p-subgroup, every conjugate gPg^{-1} of P is also _____.

Answer (Please select your correct option)

- sylow p - subgroup
- cyclic subgroups
- improper subgroups
- trivial subgroups

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 10 of 33

Let P_1 and P_2 be Sylow p-subgroups of a finite group G. Then P_1 and P_2 are ---------- of G.

Answer (Please select your correct option)

- conjugate subgroups
- cyclic subgroups
- improper subgroups
- trivial subgroups

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 5 of 33

Let G be a group acting on a set X, and let $x \in X$. Then the set $Gx = \{ax \mid a \in G\}$

Answer (Please select your correct option)

- True
- False

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23

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MTH633 Group Theory

Question No : 1 of 33

If N is a normal subgroup of G, then G/N is abelian if and only if $C \leq N$.

Answer (Please select your correct option)

- True
- False

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

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MTH633 Group Theory

Question No : 7 of 33

Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $HVK = G$. Then G is isomorphic to _____

Answer (Please select your correct option)

- $H \cap K$
- $H \cup K$
- $H \times K$
- H / K

Start Time 9:42 AM

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 4 of 33

The map $\square$: G → S_g defined by $\square(g) = \sigma_g$ is a homomorphism with the property that $\square(g)(x) = gx$

Answer (Please select your correct option)

- True
- False

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Question Summary : (Attempted Question)

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Question No : 17 of 33

The direction $\leftarrow$ of $\leftrightarrow$ gives us the _____ function μ^{-1}.

Answer (Please select your correct option)

- Inverse
- Converse
- Bijection
- None of these

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14

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MTH633 Group Theory

Question No : 33 of 33

Show that Z(G) is a normal and an abelian subgroup of G.

Answer (Please click here to Add Answer)

100%

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Question Summary : (Attempted Question)

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MTH633 Group Theory

Question No : 32 of 33 Marks: 5 (Budgeted Time Min)

Let $G=G_1 \times \cdots \times G_i \times \cdots \times G_n$ be a direct product of groups. Show that the **projection map** $\pi_i : G \to G_i$ where $\pi_i(g_1, \ldots, g_i, \ldots, g_n) = g_i$ is a homomorphism for each $i = 1, \ldots, n$.

Answer (Please click here to Add Answer)

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Question Summary : (Attempted Question)

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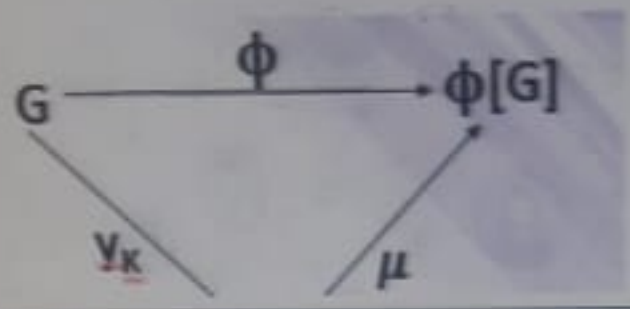
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MTH633 Group Theory

Question No : 6 of 33



Answer (Please select your correct option)

- First
- Second
- Third
- Fourth

Start Time: 9:42 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

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MTH633 Group Theory

Question No : 30 of 33

If N is a normal subgroup of G, then show that G / N is abelian if and only if $C \leq N$, where C is commutator subgroup.

Answer (Please click here to Add Answer)

100%

Start Time 9:42 AM

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Time Left

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

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Answer (Please select your correct option)

- non-Identity elements
- inverse of each element
- center
- none of these

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

Done

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MTH633 Group Theory

Question No : 26 of 33

Define Factor or quotient group.

Answer (Please click here to Add Answer)

Start Time: 9:42 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11

Conduct Instructions

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MTH633 Group Theory

Question No : 27 of 33

Write down all the generating sets of the Klein 4-group $V = \{e, a, b, c\}$.

Answer (Please click here to Add Answer)

Start Time: 9:42 AM

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Question Summary : (Attempted Question)

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ssweet (Please select your correct option)

- First
- Second
- Third
- Fourth

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Questions Summary : (Attempted Question)

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Conduct Instructions

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MTH633 Group Theory

Question No : 23 of 33

The cyclic group $G = Z / pZ$ of congruence classes modulo p is _____, where p is a prime number.

Answer (Please select your correct option)

- simple
- non-abelian
- non-cyclic
- non-commutative

Start Time: 9:42 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	2

Done

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Question No : 11 of 35

Let G be a group of order p^n and let X be a finite G-set. Then _____

Answer (Please select your correct option)

- $|X| \equiv |G_x| \pmod{p}$
- $|G| \equiv |X_G| \pmod{p}$
- $|X| \equiv |G| \pmod{p}$
- $|X| \equiv |X_G| \pmod{p}$

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Question Summary : (Attempted Question)

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TH633 Group Theory

Question No : 29 of 33

Let $r \in \mathbb{Z}$ and let $\phi_r : \mathbb{Z} \to \mathbb{Z}$ be defined by $\phi_r(n) = rn$ for all $n \in \mathbb{Z}$.

Show that ϕ_r is homomorphism.

Answer (Please click here to Add Answer)

Start Time: 9:42 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13

Conduct Instructions

MTH633 Group Theory

Question No : 11 of 41

$\langle Z, + \rangle$ is ---------- of $\langle R, \cdot \rangle$

page 305

Answer (Please select your correct option)

- subgroup
- not subgroup
- trivial subgroup
- non-trivial subgroup

MTH633 Group Theory

Question No : 12 of 41

Every subgroup of a cyclic group is ---------- group.

page 385

Answer (Please select your correct option)

- Cyclic
- Non- cyclic
- Non-abelian
- Finite

MTH633 Group Theory

Question No : 13 of 41

S_3 is NOT the group D_3 of symmetries of an equilateral triangle.

page 441

Answer (Please select your correct option)

- Yes
- No



Question No : 11 of 41

$\prod_{i=1}^{n} G_i$ is the ---------- notation for all the group operations for $G_1, \cdots, G_n$.

page 484

Answer (Please select your correct option)

- additive
- multiplicative
- subtractive
- division

Start Time 11:07 AM

14

Question No : 15 of 41

$Z_2 \times Z_3$ is cyclic group with generator ----------

page 496

Answer (Please select your correct option)

- (1, 1)
- (0, 0)
- (1, 0)
- (0, 1)

MTH633 Group Theory

Question No : [illegible]

Let H be a subgroup of G and a is in G, then _____

page 564

Answer (Please select your correct option)

- $|aH| = |bH|$ ⇦
- $|aH| \neq |bH|$
- $|aH| < |bH|$
- $|aH| > |bH|$

MTH633 Group Theory

Question No : 17 of 41

If N is _____ subgroup of a group G, the left cosets of N in G are the same as the right cosets of N in G.

page 652

Answer (Please select your correct option)

- normal
- proper
- improper
- trivial

Let φ be a homomorphism function from group $(G,\bullet)$ to group $(G',\cdot)$ Then, $(Ker\,\varphi,\bullet)$ is _____ subgroup of $(G,\bullet)$

page 675

Answer (Please select your correct option)

- commutative
- factor
- simple
- normal

MTH633 Group Theory

Question No : 19 of 41

$Ker(h)$ is called the null space of A if it consists of all $v \in \mathbb{R}^n$ such that _____

page 700

Answer (Please select your correct option)

- $Av = a$
- $Av = b$
- $Av = c$
- $Av = 0$ ⇦

MTH633 Group Theory

Question No : 20 of 41

Let the complex numbers of magnitude 1 form a subgroup U of C^*, then the cosets of U are _____

page 718

Answer (Please select your correct option)

- Parallel lines
- circles with center at the origin. ⇦
- Non-parallel lines
- None of these

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Amir Vu
August 26, 12:11 PM

MTH633 Group Theory

Question No : 1 of 41

A_3 is a normal subgroup of S_3

page 919

Answer (Please select your correct option)

Yes

No

MTH633 Group Theory

Question No : 2 of 41

Marks: 1 (Budgeted Time: 1 Min)

Generally, for any sets A, B, and C, we can view a map * A x B→ C as defining a "multiplication," where any element a of A times any element b of B has as value some element c of C

page 946

Answer (Please select your correct option)

True



MTH633 Group Theory

Question No : 3 of 41

Sometimes a set X is used to study G via a group action of G on X

page 953

Answer (Please select your correct option)

C Yes



No

MTH033 Group Theory

Question No : 4 of 41

Let G be a group acting on a set X, and let $x \in X$. Then the set $Gx = \{ax \mid a \in G\}$

is called the orbit of x in G

page 976

Answer (Please select your correct option)

- True
- False

Question No : 5 of 41

If H and N are subgroups of a group G, then the their join H V N is the smallest subgroup of G containing both H and N.

page 1004

Answer (Please select your correct option)

- True
- False

Question No : 6 of 41

For a prime number p, every group G of order p^2 is _____.

page 1091

Answer (Please select your correct option)

- abelian
- non-abelian
- infinite
- none of these

MTH633 Group Theory

Question No : 7 of 41

Each orbit in G under conjugation by G is a _____ in G.

page 1082

Answer (Please select your correct option)

- conjugate class
- subgroup
- trivial group
- none of these

MTH633 Group Theory

Question No : 8 of 11

A _____ contained in no larger p-subgroup.

page 1064

Answer (Please select your correct option)

- p - subgroup
- cyclic subgroups
- improper subgroups
- trivial subgroups

MTH633 Group Theory

Question No : 9 of 41

A non-abelian group G may have _____ of some order d dividing $|G|$.

page 1035

Answer (Please select your correct option)

- abelian subgroup
- subset
- no subgroup
- none of these

MTH633 Group Theory

Question No : 10 of 41

Let H be a subgroup of G and let N be a normal subgroup of G Then $(HN) / N =$ _____

page 1009

Answer (Please select your correct option)

- $H / (H \cup N)$
- $H / (H \cap N)$ ⇦
- $H \cup (H \cap N)$
- $H \cap (H \cap N)$

Start Time: 11:07 AM

Question # 1 of 10 (Start time: 06:58:05 PM, 18 August 2018)

The group G is a copy of the group H, if the function

$$f : G \to H$$

is ----------

Select the correct option

○	homomorphic
○	isomorphic
○	onto
○	bijective

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Time Left 58 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 2 of 10 (Start time: 06:58:47 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of (G, •), the set of cosets G/N = {Ng|g ∈ G} forms a
group (G/N, •), where the operation is defined by (Ng1) • (Ng2) = N(g1 • g2). This group is called the quotient group or factor group of G by N.

Select the correct option

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Time Left 28 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 3 of 10 (Start time: 06:59:36 PM, 18 August 2018) Total Marks: 1

The quotient group R/Z is isomorphic to the circle group

$$W = \{e^{i\theta} \in \mathbb{C} | \, \theta \in \mathbb{R}\}.$$

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Time Left 74 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:00:44 PM, 18 August 2018) **Total Marks: 1**

The converse of Lagrange's theorem holds for abelian groups.

Select the correct option

- (●) True
- ( ) False

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Quiz Start Time: 06:58 PM, 18 August 2018

Question # 5 of 10 (Start time: 07:01:04 PM, 18 August 2018) **Total Marks: 1**

Let ϕ be a homomorphism of a group G into a group G'.
If H is a subgroup of G, then $\phi[H]$ is a subgroup of G'.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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Time Left 75 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:02:16 PM, 18 August 2018) Total Marks: 1

Let $(G, \cdot)$ be a group with subgroup H. For $a, b \in G$,
we say $a \equiv b \, mod \, H$ if and only if $ab^{-1} \in H$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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Time Left 81 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:02:35 PM, 18 August 2018) **Total Marks: 1**

The relation

$$a \equiv b$$

mod H is an equivalence relation on G.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Quiz Start Time: 06:58 PM, 18 August 2018

Question # 8 of 10 (Start time: 07:02:47 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of a group G, the left cosets of N in G are the same as the right cosets of N in G.

Select the correct option

- (●) True
- ( ) False

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Time Left 71 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 9 of 10 (Start time: 07:03:07 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of a group G, the left cosets of N in G are not the same as the right cosets of N in G

Select the correct option

- (●) False
- ( ) True

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Quiz Start Time: 06:58 PM, 18 August 2018

Question # 10 of 10 (Start time: 07:03:31 PM, 18 August 2018) **Total Marks: 1**

Let

$$\phi : G \to H$$

be a homomorphism, then $\phi(a^{-1}) = \phi(a)^{-1}$ for $a \in G$.

Select the correct option

Reload Math Equations

- ● True
- ○ False

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Quiz Start Time: 07:19 PM, 18 August 2018

Question # 3 of 10 (Start time: 07:19:31 PM, 18 August 2018) **Total Marks: 1**

$(Z_n, +)$ is a cyclic group with 1 as a generator.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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Time Left 83 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:19:59 PM, 18 August 2018) **Total Marks: 1**

The group G is a copy of the group H, if the function

$$f : G \to H$$

is ----------

Select the correct option

Reload Math Equations

- homomorphic
- isomorphic
- onto
- bijective

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Quiz Start Time: 07:19 PM, 18 August 2018

Question # 5 of 10 (Start time: 07:20:10 PM, 18 August 2018) **Total Marks: 1**

Let H be a subgroup of a group G. Then (a H) (b H) = a b H is not always true when G is non-abelian.

Select the correct option

- ○ False
- ● True

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Quiz Start Time: 07:19 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:20:27 PM, 18 August 2018) **Total Marks: 1**

Any subgroups of Abelian group G are not normal subgroups of G.

Select the correct option

- (●) False
- ( ) True

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Quiz Start Time: 07:19 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:20:46 PM, 18 August 2018) **Total Marks: 1**

$Hg = gH$, *for all* $g \in G$, *if and only if* H *is a normal subgroup of* G.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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Quiz Start Time: 07:19 PM, 18 August 2018

Question # 9 of 10 (Start time: 07:21:08 PM, 18 August 2018) **Total Marks: 1**

Let

$$\phi : G \to H$$

be an isomorphism, then if G is abelian, then H is not abelian.

Select the correct option

Reload Math Equations

- ○ True
- ● False

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Quiz Start Time: 07:19 PM, 18 August 2018

Question # 10 of 10 (Start time: 07:21:23 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of a group G, the left cosets of N in G are the same as the right cosets of N in G.

Select the correct option

- (●) True
- ( ) False

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Quiz Start Time: 07:31 PM, 18 August 2018

Question # 3 of 10 (Start time: 07:32:09 PM, 18 August 2018) **Total Marks: 1**

Any subgroups of Abelian group G are not normal subgroups of G.

Select the correct option

- (●) False
- ( ) True

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Quiz Start Time: 07:31 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:32:26 PM, 18 August 2018) **Total Marks: 1**

Let ϕ be a homomorphism of a group G into a group G'.
If K' is a subgroup of G', then $\phi^{-1}[K']$ is a subgroup of G.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Time Left 77 sec(s)

Quiz Start Time: 07:31 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:32:51 PM, 18 August 2018) **Total Marks: 1**

Let

$$\phi : G \to H$$

be an isomorphism, then if G is abelian, then H is not abelian.

Select the correct option

Reload Math Equations

- ○ True
- ● False

Click to Save Answer & Move to Next Question

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quiz.vu.edu.pk/QuizQuestion.aspx?ver=6f609dda-2255-441f-b6dc-011ed2cc333a
Google

AHMAD MSC(MATH)
Areeb2113@gmail.com

Time Left 55 sec(s)

Quiz Start Time: 07:48 PM, 18 August 2018

Question # 1 of 10 (Start time: 07:48:23 PM, 18 August 2018) **Total Marks: 1**

Let

$$f : G \to H$$

be a homomorphism, then $f(e_G) = ----$ for e_G and e_H are identities of G and H respectively.

Select the correct option

Reload Math Equations

- ○ e_G
- ● e_H

Click to Save Answer & Move to Next Question

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quiz.vu.edu.pk/QuizQuestion.aspx?ver=67b751ff-918f-4e67-93b0-54350be7ed51
Google

AHMAD MSC(MATH)
Areeb2113@gmail.com

Time Left 77 sec(s)

Quiz Start Time: 07:48 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:50:22 PM, 18 August 2018) **Total Marks: 1**

Let φ be a homomorphism function from group $(G,*)$ to group $(G',\cdot)$. Then, $(Ker\,\varphi,*)$ is a normal subgroup of $(G,*)$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Jazz ZONG 2G 3G 1:36 pm

Quiz

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MC170200830: Kainat Ashfaq

Time Left 68 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 01:30 PM, 24 August 2018

Question # 10 of 10 (Start time: 01:35:47 PM, 24 August 2018)

Total Marks: 1

If N is a normal subgroup of (G, •), the set of cosets G/N = {Ng|g ∈ G} forms a group (G/N, •), where the operation is defined by (Ng1) • (Ng2) = N(g1 • g2). This group is called the quotient group or factor group of G by N.

Select the correct option

- True
- False

Click to Save Answer & Move to Next Question

LMS-Virtual University of Paki
Quiz
quiz.vu.edu.pk/QuizQuestion.aspx?ver=049dbdb9-4658-425a-9144-eba88f5c4f16
Google

MC170202886: Aqsa Zunaira

Time Left 75 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:02:16 PM, 18 August 2018)

Total Marks: 1

Let $(G, \cdot)$ be a group with subgroup H. For $a, b \in G$,
we say $a \equiv b \, mod \, H$ if and only if $ab^{-1} \in H$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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quiz.vu.edu.pk/QuizQuestion.aspx?ver=1813f4c6-ea7f-425b-8cc1-b29d88e33c5c
Google

MC170202886: Aqsa Zunaira

Time Left 26 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 5 of 10 (Start time: 07:01:04 PM, 18 August 2018)

Total Marks: 1

Let ϕ be a homomorphism of a group G into a group G'.
If H is a subgroup of G, then $\phi[H]$ is a subgroup of G'.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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quiz.vu.edu.pk/QuizQuestion.aspx?ver=126ee9ab-bd64-4724-82cc-7bddad52f906
Google

MC170202886: Aqsa Zunaira

Time Left 81 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:02:35 PM, 18 August 2018) **Total Marks: 1**

The relation

$$a \equiv b$$

mod H is an equivalence relation on G.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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8/18/2018

MC170202458: Aneeta Shaheen

Time Left 83 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:19:59 PM, 18 August 2018) **Total Marks: 1**

The group G is a copy of the group H, if the function

$$f : G \to H$$

is ----------

Select the correct option

Reload Math Equations

- homomorphic
- isomorphic
- onto
- bijective

Click to Save Answer & Move to Next Question

MTH633:Quiz No. 2

Question # 4 of 10 (Start time: 12:03:39 PM, 23 August 2018)

Let ϕ be a homomorphism of a group G into a group G'.
If H is a subgroup of G, then $\phi[H]$ is a subgroup of G'.

Select the correct option

- True
- False

MTH633:Quiz No. 2

Question # 7 of 10 (Start time: 12:05:32 PM, 23 August 2018)

Let ϕ be a homomorphism of a group G into a group G'.
If K' is a subgroup of G', then $\phi^{-1}[K']$ is a subgroup of G.

Select the correct option

- True
- False

MTH633:Quiz No. 2

Question # 6 of 10 (Start time: 12:04:50 PM, 23 August 2018)

Let $(G, \cdot)$ be a group with subgroup H. For $a, b \in G$, we say $a \equiv b \bmod H$ if and only if $ab^{-1} \in H$.

Select the correct option

- True
- False

MTH633:Quiz No. 2

Question # 5 of 10 (Start time: 12:04:15 PM, 23 August 2018)

If N is a normal subgroup of a group G, the left cosets of N in G are the same as the right cosets of N in G.

Select the correct option

○	True
○	False

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Google

MC170202458: Aneeta Shaheen

Time Left 65 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 3 of 10 (Start time: 07:19:31 PM, 18 August 2018)

Total Marks: 1

$(Z_n, +)$ is a cyclic group with 1 as a generator.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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MTH633:Quiz No. 2

Question # 1 of 10 (Start time: 12:02:01 PM, 23 August 2018)

Let φ be a homomorphism function from group $(G, *)$ to group $(G', \cdot)$. Then, $(Ker\,\varphi, *)$ is a normal subgroup of $(G, *)$.

Select the correct option

- True
- False

MTH633 Quiz No. 2

Question # 10 of 10 (Start time: 12:06:47 PM, 23 August 2018)

$$Hg = gH, \text{ for all } g \in G, \text{ if and only if } H \text{ is a normal subgroup of } G.$$

Select the correct option

- True
- False

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Quiz
quiz.vu.edu.pk/QuizQuestion.aspx?ver=11a70d80-f729-45f2-9ac0-2ddd98159084
Google

MC170202458: Aneeta Shaheen

Time Left 77 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 5 of 10 (Start time: 07:20:10 PM, 18 August 2018) **Total Marks: 1**

Let H be a subgroup of a group G. Then (a H) (b H) = a b H is not always true when G is non-abelian.

Select the correct option

- ○ False
- ● True

Click to Save Answer & Move to Next Question

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Google

MC170200563: Alia Jamil

Time Left 77 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:31 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:32:26 PM, 18 August 2018)

Total Marks: 1

Let ϕ be a homomorphism of a group G into a group G'.
If K' is a subgroup of G', then $\phi^{-1}[K']$ is a subgroup of G.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Question # 3 of 10 (Start time: 12:03:06 PM, 23 August 2018)

Let

$$f : G \to H$$

be a homomorphism, then $f(e_G) = ----$ for e_G and e_H are identities of G and H respectively.

Select the correct option

○	e_G
○	e_H

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Google

MC170201745: Shakeel Ahmad

Time Left 77 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:48 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:50:22 PM, 18 August 2018)

Total Marks: 1

Let φ be a homomorphism function from group $(G, *)$ to group $(G', \cdot)$. Then, $(Ker\,\varphi, *)$ is a normal subgroup of $(G, *)$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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8/18/2018

MTH603 Quiz No. 2

Question # 9 of 10 (Start time: 12:06:27 PM, 23 August 2018)

The converse of Lagrange's theorem holds for abelian groups.

Select the correct option

- True
- False

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Quiz
quiz.vu.edu.pk/QuizQuestion.aspx?ver=a68f4346-d2de-405b-9032-82bd374740fa
Google

MC170202458: Aneeta Shaheen

Time Left 81 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:20:46 PM, 18 August 2018)

Total Marks: 1

$Hg = gH$, *for all* $g \in G$, *if and only if* H *is a normal subgroup of* G.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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MTH633 Quiz No. 2

Question # 8 of 10 (Start time: 12:06:06 PM, 23 August 2018)

$(Z_n, +)$ is a cyclic group with 1 as a generator.

Select the correct option

- True
- False

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Quiz
quiz.vu.edu.pk/QuizQuestion.aspx?ver=9ab712bb-d3a8-4901-b78c-d32ca4f17347
Google

MC170202886: Aqsa Zunaira

Time Left 73 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 8 of 10 (Start time: 07:02:47 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of a group G, the left cosets of N in G are the same as the right cosets of N in G.

Select the correct option

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Google

MC170202458: Aneeta Shaheen

Time Left 79 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 9 of 10 (Start time: 07:21:08 PM, 18 August 2018) **Total Marks: 1**

Let

$$\phi : G \to H$$

be an isomorphism, then if G is abelian, then H is not abelian.

Select the correct option

Reload Math Equations

- ○ True
- ● False

Click to Save Answer & Move to Next Question

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Google

MC170201745: Shakeel Ahmad

Time Left 55 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:48 PM, 18 August 2018

Question # 1 of 10 (Start time: 07:48:23 PM, 18 August 2018) Total Marks: 1

Let

$$f : G \to H$$

be a homomorphism, then $f(e_G) = ----$ for e_G and e_H are identities of G and H respectively.

Select the correct option

Reload Math Equations

- e_G
- e_H

Click to Save Answer & Move to Next Question

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Google

MC170202886: Aqsa Zunaira

Time Left 74 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:00:44 PM, 18 August 2018) **Total Marks: 1**

The converse of Lagrange's theorem holds for abelian groups.

Select the correct option

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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8/18/2018

MC170200563: Alia Jamil

Time Left 80 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:31 PM, 18 August 2018

Question # 3 of 10 (Start time: 07:32:09 PM, 18 August 2018) Total Marks: 1

Any subgroups of Abelian group G are not normal subgroups of G.

Select the correct option

- (●) False
- ( ) True

Click to Save Answer & Move to Next Question

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quiz.vu.edu.pk/QuizQuestion.aspx?ver=344b3cc8-83ba-4a36-bb88-01f36a312d9e
Google

MC170202458: Aneeta Shaheen

Time Left 79 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 10 of 10 (Start time: 07:21:23 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of a group G, the left cosets of N in G are the same as the right cosets of N in G.

Select the correct option

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Google

MC170202886: Aqsa Zunaira

Time Left 58 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 2 of 10 (Start time: 06:58:47 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of (G, •), the set of cosets G/N = {Ng|g ∈ G} forms a
group (G/N, •), where the operation is defined by (Ng1) • (Ng2) = N(g1 • g2). This group is called the quotient group or factor group of G by N.

Select the correct option

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Google

MC170202458: Aneeta Shaheen

Time Left 79 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:20:27 PM, 18 August 2018) **Total Marks: 1**

Any subgroups of Abelian group G are not normal subgroups of G.

Select the correct option

- (●) False
- ( ) True

Click to Save Answer & Move to Next Question

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Google

MC170202886: Aqsa Zunaira

Time Left 82 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 10 of 10 (Start time: 07:03:31 PM, 18 August 2018) **Total Marks: 1**

Let

$$\phi : G \to H$$

be a homomorphism, then $\phi(a^{-1}) = \phi(a)^{-1}$ for $a \in G$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Google

MC170202886: Aqsa Zunaira

Time Left 71 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 9 of 10 (Start time: 07:03:07 PM, 18 August 2018) **Total Marks: 1**

If N is a normal subgroup of a group G, the left cosets of N in G are not the same as the right cosets of N in G

Select the correct option

- (●) False
- ( ) True

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MC170202886: Aqsa Zunaira

Time Left 28 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 3 of 10 (Start time: 06:59:36 PM, 18 August 2018)

Total Marks: 1

The quotient group R/Z is isomorphic to the circle group

$$W = \{e^{i\theta} \in \mathbb{C} | \, \theta \in \mathbb{R}\}.$$

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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MC170202886: Aqsa Zunaira

Time Left 77 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 1 of 10 (Start time: 06:58:05 PM, 18 August 2018) **Total Marks: 1**

The group G is a copy of the group H, if the function

$$f : G \to H$$

is ----------

Select the correct option

Reload Math Equations

- homomorphic
- isomorphic
- onto
- bijective

Click to Save Answer & Move to Next Question

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MC170202886: Aqsa Zunaira

Time Left 77 sec(s)

MTH633:Quiz No. 2

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 1 of 10 (Start time: 06:58:05 PM, 18 August 2018) **Total Marks: 1**

The group G is a copy of the group H, if the function

$$f : G \to H$$

is ----------

Select the correct option

Reload Math Equations

- homomorphic
- isomorphic
- onto
- bijective

Click to Save Answer & Move to Next Question

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Question # 4 of 10 (Start time: 06:47:50 AM, 24 August 2018)

The converse of Lagrange's theorem holds for abelian groups.

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Select the correct option

- True
- False

Question # 2 of 10 (Start time: 06:47:12 AM, 24 August 2018)

The group G is a copy of the group H, if the function

$$f : G \to H$$

is ---------

Select the correct option

○	homomorphic
●	isomorphic
○	onto
○	bijective

MTH633:Quiz No. 2

Question # 1 of 10 (Start time: 06:46:44 AM, 24 August 2018)

Let φ be a homomorphism function from group $(G, *)$ to group $(G', \cdot)$. Then, $(Ker\,\varphi, *)$ is a normal subgroup of $(G, *)$.

Select the correct option

●	True
○	False

Click to Sav

Question # 9 of 10 (Start time: 06:49:03 AM, 24 August 2018)

Let

$$\phi : G \to H$$

be a homomorphism, then $\phi(a^{-1}) = \phi(a)^{-1}$ for $a \in G$.

Select the correct option

●	True
○	False

Question # 7 of 10 (Start time: 06:48:33 AM, 24 August 2018)

The relation

$a \equiv b$ mod H is an equivalence relation on G.

Select the correct option

●	True
○	False

Question # 3 of 10 (Start time: 06:47:30 AM, 24 August 2018)

If N is a normal subgroup of (G, •). the set of cosets G/N = {Ng|g ∈ G} forms a group (G/N, •). where the operation is defined by (Ng1) • (Ng2) = N(g1 • g2). This group is called the quotient group or factor group of G by N.

Select the correct option

●	True
○	False

Question # 6 of 10 (Start time: 06:48:20 AM, 24 August 2018)

Let $(G, \cdot)$ be a group with subgroup H. For $a, b \in G$, we say $a \equiv b \ mod \ H$ if and only if $ab^{-1} \in H$.

Select the correct option

●	True
○	False

Question # 10 of 10 (Start time: 06:49:20 AM, 24 August 2018)

Let H be a subgroup of a group G. Then (a H) (b H) = a b H is not always true when G is non-abelian.

Select the correct option

○	False
●	True

Question # 8 of 10 (Start time: 06:48:48 AM, 24 August 2018)

The quotient group R/Z is isomorphic to the circle group

$$W = \{e^{i\theta} \in \mathbb{C} | \, \theta \in \mathbb{R}\}.$$

Select the correct option

●	True
○	False

Click to Save

Question # 5 of 10 (Start time: 06:48:03 AM, 24 August 2018)

Let ϕ be a homomorphism of a group G into a group G'. If H is a subgroup of G, then $\phi[H]$ is a subgroup of G'.

Select the correct option

- (●) True
- ( ) False

MC170203385: Maffia Bibi

Time Left 71 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 7 of 10 (Start time: 12:27:13 PM, 07 June 2018) Total Marks: 1

Which of the following is the subgroup of {1, -1, i, -i } under multiplication?

Select the correct option

- {1, i}
- {1, -i}
- {1, -1}
- {i, -i}

Click to Save Answer & Move to Next Question

MC170203385: Maffia Bibi

Time Left 48 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 5 of 10 (Start time: 12:26:05 PM, 07 June 2018) Total Marks: 1

How many tests are there to show a subset of a group to be a subgroup?

Select the correct option

- [ ] 1
- [ ] 2
- [x] 3
- [ ] 4

Click to Save Answer & Move to Next Question

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Google

MC170203385: Maffia Bibi

Time Left 83 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 10 of 10 (Start time: 12:29:50 PM, 07 June 2018) Total Marks: 1

The order of the group $\{1, \omega, \omega^2\}$ is _ _ _ _ _ _ _ _ _

Select the correct option

Reload Math Equations

- 2
- 3
- 4
- 5

Click to Save Answer & Move to Next Question

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6/7/2018

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MC170203385: Maffia Bibi

Time Left 74 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 2 of 10 (Start time: 12:23:51 PM, 07 June 2018) Total Marks: 1

The group (Z, +) is a finite group, where Z is the set of integers.

Select the correct option

- False
- True

Click to Save Answer & Move to Next Question

80%

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12:24 PM
6/7/2018

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quiz.vu.edu.pk/QuizQuestion.aspx?ver=af106355-4fca-42b6-a736-425f1a2d93b7
Google

MC170203385: Maffia Bibi

Time Left 44 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 3 of 10 (Start time: 12:24:24 PM, 07 June 2018) **Total Marks: 1**

A subset H of a group G under addition is a subgroup of G if and only if **whenever** $a, b \in H,$ **then** $a - b \in H$

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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47% 3:30 pm

Question # 1 of 10 (Start time: 03:29:56 PM, 07 June 2018) Total Marks: 1

A subset H of a group G under addition is a subgroup of G if and only if whenever $a, b \in H$, then $a - b \in H$

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Select the correct option

Reload Math Equations

- True
- False

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Click to Save Answer & Move to Next Question

MC170203385: Maffia Bibi

Time Left 71 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 6 of 10 (Start time: 12:26:51 PM, 07 June 2018) Total Marks: 1

A subgroup H of a group (G, *) is a subset of G, which is itself a group under * .

Select the correct option

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

Question # 6 of 10 (Start time: 03:33:30 PM, 07 June 2018) Total Marks: 1

Show that $H = \left\{ \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \mid n \in Z \right\}$ is the subgroup of G = GL (2, R) under multiplication.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

Question # 6 of 10 (Start time: 03:33:30 PM, 07 June 2018) Total Marks: 1

Show that $H = \left\{ \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \mid n \in Z \right\}$ is the subgroup of G = GL (2, R) under multiplication.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

47% 3:34 pm

Question # 9 of 10 (Start time: 03:34:33 PM, 07 June 2018) Total Marks: 1

The group (Z, +) is a finite group, where Z is the set of integers.

Select the correct option

- (●) False
- ( ) True

Click to Save Answer & Move to Next Question

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Google

MC170201745: Shakeel Ahmad

Time Left 46 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:39 PM, 07 June 2018

Question # 2 of 10 (Start time: 12:39:13 PM, 07 June 2018) Total Marks: 1

$\langle Z, + \rangle$ is the subgroup of $\langle R, + \rangle$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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MC170203385: Maffia Bibi

Time Left 44 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 4 of 10 (Start time: 12:25:14 PM, 07 June 2018) Total Marks: 1

$\langle \{1, -1\}, \cdot \rangle$ is the subgroup of $\langle \{1, -1, i, -i\}, \cdot \rangle$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

Click to Save Answer & Move to Next Question

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Question # 7 of 10 (Start time: 08:44:47 PM, 08 June 2018)

$\langle \{1, -1\}, \cdot \rangle$ is the subgroup of $\langle \{1, -1, i, -i\}, \cdot \rangle$.

Select the correct option

- (●) True
- ( ) False

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MC170202458: Aneeta Shaheen

Time Left 62 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:52 PM, 07 June 2018

Question # 4 of 10 (Start time: 12:52:47 PM, 07 June 2018) Total Marks: 1

$\langle Q, + \rangle$ is the subgroup of $\langle R, + \rangle$.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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MTH633:Quiz No. 1

Quiz Start Time: 03:29 PM, 07 June 2018

Question # 10 of 10 (Start time: 03:34:48 PM, 07 June 2018)

Total Marks: 1

A subset H of a group G is a subgroup of G if and only if H is non−empty and ----------

Select the correct option

Reload Math Equations

- whenever $a, b \in H$, then $ab \in H$
- whenever $a, b \in H$, then $b^{-1} \in H$
- whenever $a, b \in H$, then $ab^{-1} \in H$
- whenever $a, b \in H$, then $(ab)^{-1} \in H$

MC170202458: Aneeta Shaheen

Time Left 65 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:52 PM, 07 June 2018

Question # 9 of 10 (Start time: 12:54:33 PM, 07 June 2018) Total Marks: 1

The set of even integers is not a subgroup of the set of Integers under the same binary operation of addition.

Select the correct option

- ○ True
- ● False

Click to Save Answer & Move to Next Question

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MC170201745: Shakeel Ahmad

Time Left 76 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:39 PM, 07 June 2018

Question # 7 of 10 (Start time: 12:40:53 PM, 07 June 2018) Total Marks: 1

A non empty subset S of a finite group G is called subgroup of G if and only if ab is in S for all a, b in S.

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Select the correct option

- (●) True
- ( ) False

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MC170201745: Shakeel Ahmad

Time Left 78 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:39 PM, 07 June 2018

Question # 8 of 10 (Start time: 12:41:33 PM, 07 June 2018) Total Marks: 1

The order of the group {1, -1, i, -i, j, -j, k, -k} is ----------

Select the correct option

- 6
- 7
- 8
- 9

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Question # 4 of 10 (Start time: 03:32:40 PM, 07 June 2018) Total Marks: 1

The group ({1, −1}, .) is subgroup of the group (Z, +).

Select the correct option

- (●) True
- ( ) False

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Question # 8 of 10 (Start time: 03:34:23 PM, 07 June 2018) Total Marks: 1

The order of the group $\{1, \omega, \omega^2\}$ is - - - - - - - - - -

Select the correct option

Reload Math Equations

○	2
○	3
○	4
○	5

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MC170201745: Shakeel Ahmad

Time Left 38 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:39 PM, 07 June 2018

Question # 9 of 10 (Start time: 12:41:48 PM, 07 June 2018) Total Marks: 1

The group {1, -1, i, -i } is not a subgroup of itself.

Select the correct option

- (●) False
- ( ) True

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MC170201745: Shakeel Ahmad

Time Left 82 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:39 PM, 07 June 2018

Question # 4 of 10 (Start time: 12:40:10 PM, 07 June 2018) Total Marks: 1

The order of the group {e} is ----------

Select the correct option

- 0
- 1
- 2
- 3

Click to Save Answer & Move to Next Question

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47% 3:33 pm

Question # 5 of 10 (Start time: 03:33:11 PM, 07 June 2018) Total Marks: 1

The group (Q, +) is an infinite group, where Q is the set of rational numbers.

Select the correct option

- False
- True

Click to Save Answer & Move to Next Question

47% 3:33 pm

Question # 5 of 10 (Start time: 03:33:11 PM, 07 June 2018) Total Marks: 1

The group (Q, +) is an infinite group, where Q is the set of rational numbers.

Select the correct option

- False
- True

Click to Save Answer & Move to Next Question

MC170203385: Maffia Bibi

Time Left 81 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 1 of 10 (Start time: 12:23:34 PM, 07 June 2018) Total Marks: 1

In the group {1, -1, i, -i}, the order of -1 is -------

Select the correct option

- 1
- 2
- 3
- 4

Click to Save Answer & Move to Next Question

MTH633:Quiz No. 1

Quiz Start Time: 03:29 PM, 07 June 2018

Question # 2 of 10 (Start time: 03:32:01 PM, 07 June 2018)

Total Marks: 1

The group {1, −1, i, −i} is a finite group.

Select the correct option

- (●) True
- ( ) False

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MC170202458: Aneeta Shaheen

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Time Left 75 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:52 PM, 07 June 2018

Question # 6 of 10 (Start time: 12:53:26 PM, 07 June 2018) Total Marks: 1

In the group {1, -1, i, -i}, the order of i is -------

Select the correct option

- 1
- 2
- 3
- 4 (selected)

Click to Save Answer & Move to Next Question

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MC170201745: Shakeel Ahmad

Time Left 84 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:39 PM, 07 June 2018

Question # 5 of 10 (Start time: 12:40:25 PM, 07 June 2018) Total Marks: 1

Klein Four group is not cyclic group.

Select the correct option

- (●) True
- ( ) False

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MC170203385: Maffia Bibi

Time Left 54 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 9 of 10 (Start time: 12:29:09 PM, 07 June 2018)

Total Marks: 1

Show that $H = \left\{ \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \mid n \in Z \right\}$ is the subgroup of G = GL (2, R) under multiplication.

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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MC170202458: Aneeta Shaheen

Time Left 52 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:52 PM, 07 June 2018

Question # 7 of 10 (Start time: 12:53:45 PM, 07 June 2018) **Total Marks: 1**

A subset H of a group G is a subgroup of G if and only if (i) whenever $a, b \in H$, then $ab \in H$ (ii) whenever $b \in H$, then $b^{-1} \in H$

Select the correct option

Reload Math Equations

- (●) True
- ( ) False

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Question # 6 of 10 (Start time: 08:44:21 PM, 08 June 2018)

A non empty subset S of a finite group G is called subgroup of G if and only if ab is in S for all a, b in S.

Select the correct option

- True
- False

Question # 3 of 10 (Start time: 08:43:18 PM, 08 June 2018)

$\langle Z, + \rangle$ is the subgroup of $\langle R, \cdot \rangle$.

Select the correct option

- True
- False

Question # 9 of 10 (Start time: 08:45:11 PM, 08 June 2018)

The group ({1, -1}, .) is subgroup of the group (Z, +).

Select the correct option

- ○ True
- ○ False

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MC170203385: Maffia Bibi

Time Left 24 sec(s)

MTH633:Quiz No. 1

Quiz Start Time: 12:23 PM, 07 June 2018

Question # 8 of 10 (Start time: 12:27:53 PM, 07 June 2018) **Total Marks: 1**

The group (Q, +) is an infinite group, where Q is the set of rational numbers.

Select the correct option

- ○ False
- ● True

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Question # 10 of 10 (Start time: 08:46:04 PM. 08 June 2018)

The order of the group {e} is ------

Select the correct option

- 0
- 1
- 2
- 3

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Question # 3 of 10 (Start time: 03:32:25 PM, 07 June 2018) Total Marks: 1

The order of the group {1, −1} is ----------

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Select the correct option

- 1
- 2
- 3
- 4

MTH633:Quiz No. 1

Question # 5 of 10 (Start time: 08:43:47 PM, 08 June 2018)

$\langle Q, + \rangle$ is the subgroup of $\langle R, + \rangle$.

Select the correct option

- True
- False

Question # 8 of 10 (Start time: 08:44:59 PM, 08 June 2018)

The order of the group {1, -1, i, -i} is ------

Select the correct option

○	2
○	3
●	4
○	5

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Important questions Group theory (MTH633)

1) Give example of Quotient group?
$(Z_n, +)$ is the quotient group of $(Z, +)$ by subgroup.
$nz = \{nz | z \in Z\}$.

2) Define Simple group?
A group is simple if it is nontrivial and has no proper nontrivial normal subgroup.

Example:
The cyclic group $G = Z/pZ$ of congruence classes modulo p is simple where p is a prime number.

3) State Cauchy's theorem?
Let p be a prime. Let G be a finite group and let p divide $|G|$. Then G has an element of order p and consequently, a subgroup of order p.

4) Can we solve equations of the form $a * x = b$ in a group $\langle G, * \rangle$

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(2

Solution:

$a' * (a * x) = a' * b$

$(a' * a) * x = a' * b$

$e * x = a' * b$

$x = a' * b$

5) Define 1st Sylow theorem

Let G be a finite group and let $|G| = p^n m$ where $n \geq 1$ and where p does not divide m. Then

i) G contains a subgroup of order p^i for each i where $1 \leq i \leq n$,

ii) Every subgroup H of G of order p^i is a normal subgroup of a subgroup of order p^{i+1} for $1 \leq i \leq n$

6) Show that the set of all inner automorphisms of G is a subgroup of $Aut(G)$

Proof:

1) Let $i_a, i_b \in Inn(G)$

Then $i_a(i_b(x)) = a(i_b(x))a^{-1} = abxb^{-1}a^{-1}$

$= abx(ab)^{-1} = i_{ab} \in Inn(G)$

Hence The conjugation by b composed by conjugation by a is conjugation by ab.

(3

7)

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2) The inverse of i_a is conjugation by $a' = a^{-1}$. $i_a(i_{a'}(x)) = i_a(a'xa'^{-1}) = aa'xa'^{-1}a^{-1} = aa'x(aa')^{-1} = x$

Thus $Inn(G)$ is a subgroup.

7) Show That homomorphism $h: G \to G'$ is injective if and only if $\ker h = \{e\}$.

Proof:

Suppose h is injective, and let $x \in \ker h$.

Then $h(x) = e' = h(e)$

Hence $x = e$

Conversely, Suppose $\ker h = \{e\}$.

Then $h(x) = h(y)$

$\Rightarrow h(xy^{-1}) = h(x)h(y^{-1})$

$= h(x)h(y)^{-1} = e'$

$\Rightarrow xy^{-1} \in \ker h$

$\Rightarrow xy^{-1} = e$

$\Rightarrow x = y$

Hence, h is injective

8) **Theorem:** Show That set $Aut(G)$ of all automorphisms of a group G is a group under composition of mappings, and $Inn(G) \trianglelefteq Aut(G)$ Morever $G/Z(G) \simeq Inn(G)$.

4)

Proof:

Clearly, Aut(G) is nonempty.

Let $\sigma, \tau \in Aut(G)$.

Then for all $x, y \in G$, $\sigma\tau(xy) = \sigma(\tau(x)\tau(y))$

$= (\sigma\tau(x))(\sigma\tau(y))$.

Hence $\sigma\tau \in Aut(G)$. Again,

$\sigma(\sigma^{-1}(x)\sigma^{-1}(y)) = \sigma\sigma^{-1}(x)\sigma\sigma^{-1}(y) = xy$

Hence $\sigma^{-1}(x)\sigma^{-1}(y) = \sigma^{-1}(xy)$. Therefore, $\sigma^{-1} \in Aut(G)$. This proves that $Aut(G)$ is a subgroup of the symmetric group S_G and, hence, is itself a group

9) Let $G = \langle a \mid a^n = e \rangle$ be a finite cyclic group of order n. Then the mapping $\sigma : a \to a^m$ is an automorphism of G iff $(m, n) = 1$

Solution:

If $(m, n) = 1$, then ~~it has been shown in Exa~~

assume that σ is an automorphism. Then the order of $\sigma(a) = a^m$ is the same as that of a, which is n. Further, if $(m, n) = d$, then $(a^m)^{n/d} = (a^n)^{m/d} = e$. Thus the order of a^m divides n/d; that is, $n \mid n/d$.

Hence $d = 1$ and the solution is complete.

10) Define Commutator Subgroup.

The set of all commutators $aba^{-1}b^{-1}$ for $a, b \in G$ generates a subgroup C of G is called the commutator subgroup.

11) Theorem: Let G be a group. The set of all commutator $aba^{-1}b^{-1}$ for $a, b \in G$ generates a subgroup C of G.

Proof:

Let $a, b \in G$. Then

$(aba^{-1}b^{-1})(aba^{-1}b^{-1})^{-1}$

$= aba^{-1}b^{-1}bab^{-1}a^{-1}$

$= e \in C$

Since $e = eee^{-1}e^{-1}$ is a commutator.

12) Write down all the generating sets of group Z_6?

Solution:

The group Z_6 is generated by $\{1\}$ and $\{5\}$.

It is also generated by $\{2, 3\}$ Since $2+3=5$, So That any subgroup containing 2 and 3 must contain 5 and must therefore be Z_6.

{It is also generated by $\{3,4\}, \{2,3,4\}$ $\{1,3\}$ and $\{3,5\}$}

13) Theorem: The intersection of some subgroup H_i of a group G for $i \in I$ is again a subgroup of G.

Proof:

Let us show closure

Let $a \in \bigcap_{i \in I} H_i$ and $b \in \bigcap_{i \in I}$

So that $a \in H_i$ for all $i \in I$ and $b \in H_i$ for all $i \in I$.

Then $ab \in H_i \ \forall i \in I$

Since H_i is a group. Thus

$ab \in \bigcap_{i \in I} H_i$.

Since H_i is a subgroup $\forall i \in I$,

~~Wh~~ we have $e \in H_i \ \forall i \in I$, and

hence $e \in \bigcap_{i \in I} H_i$.

Finally, for $a \in \bigcap_{i \in I} H_i$, we have

$a \in H_i$, for all $i \in I$.

So $a^{-1} \in H_i \ \forall i \in I$, which implies

That

$a^{-1} \in \bigcap_{i \in I} H_i$.

14) Define Group action.

Let X be a set and G a group.

An action of G on X is a map

$*: G \times X \to X$ such that

$ex = x \ \forall x \in X$

$(g_1, g_2)(x) = g_1(g_2 x) \ \forall x \in X$, & $g_1, g_2 \in G$

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Under these conditions X is a G-set.

15) Define Stabilizer?

Let G be a group acting on a set X. and let $x \in X$. Then the set

$$G_x = \{ g \in G \mid gx = x \}.$$

which can be shown to be a subgroup is called the Stabilizer (or isotropy) group of x in G.

16) Theorem: Let X be a G-set. Then G_x is a subgroup of G $\forall x \in X$.

Proof:

Let $x \in X$ and $g_1, g_2 \in G_x$. Then $g_1 x = x$ and $g_2 x = x$.

Consequently, $(g_1 g_2)x = g_1(g_2 x)$

$= g_1 x = x$, so $g_1 g_2 \in G_x$, and G_x is closed under the induced operation of G.

of course $ex = x$, so $e \in G_x$.

If $g \in G_x$, Then $gx = x$ so $x = e x (g^{-1} g)x$

$= g^{-1}(gx) = g^{-1}x$.

and consequently $g^{-1} \in G_x$.

Thus G_x is a subgroup of G.

8)

17) Define Orbits ?

Let G be a group acting on a set X, and let $x \in X$.
Then the set
$G_x = \{ax \mid a \in G\}$
is called the orbit of x in G.

18) Theorem: Let G be a group of order p^n and let x be a finite G-set. Then $|X| \equiv |X_G| \pmod{p}$.

Recall $|X| = |X_G| + \sum_{i=s+1}^{r} |Gx_i|$.
In the notation of above Equation, we know that $|Gx_i|$ divides $|G|$.
Consequently p divides $|Gx_i|$ for $s+1 \le i \le r$.
Above equation then shows that
$|X| - |X_G|$ is divisible by p.
So $|X| \equiv |X_G| \pmod{p}$.

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Jathod

9)

19) Define kernel of a Homomorphism.

Let $h: G \to G'$ be a homomorphism of groups. The subgroup

$h^{-1}[\{e'\}] = \{x \in G \mid h(x) = e'\}$

is the kernel of h. denoted by ker (h).

20 Classify the group $(Z_4 \times Z_2)/(\{0\}, Z_2)$ according to the fundamental theorem of finitely generated abelian groups.

Solution:

The projection map $\pi_1: Z_4 \times Z_2 \to Z_4$ given by $\pi_1(x,y) = x$ is a homomorphism of $Z_4 \times Z_2$ onto Z_4 with kernel $\{0\} \times Z_2$.

By ~~for~~ fundamental theorem of homomorphism, we know that the given factor is isomorphic to Z_4.

The projection map $\pi_1: Z_4 \times Z_2 \to Z_4$ given by $\pi_1(x,y) = x$

$K = \ker \pi_1 = \{0\} \times Z_2$

$= \{(0,0), (0,1)\}$.

$(1,0) + K = \{(1,0)(1,1)\}$

$(2,0) + K = \{(2,0), (2,1)\}$

$(3,0) + K = \{(3,0), (3,1)\}$

21) Define inner Automorphisms?

An isomorphism $\phi: G \to G$ of a group G with itself is an automorphism of G. The automorphism of G. The automorphism

$i_g : G \to G$, where $i_g(x) = gxg^{-1}$ $\forall x \in G$ is inner automorphism of G by g, denoted by $Inn(G)$.

Performing i_g on x is called conjugation of x by g.

22) Theorem: A factor group of a cyclic group is cyclic.

Proof:

Let G be a cyclic ~~group~~ with generator a, and let N be a normal subgroup of G.

we claim the coset aN generator G/N. we must compute all powers of aN. But this amounts to computing, in G all powers of the representative a and all these powers give all elements in G.

Hence the powers of aN certainly give all cosets of N and G/N is cyclic.

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22)

23) Theorem: Show that $Z(G)$ is a normal and an abelian subgroup of G.

Proof:

For each $g \in G$ and $z \in Z(G)$ we have $gzg^{-1} = zgg^{-1} = ze = z$

we see at once that $Z(G)$ is a normal subgroup of G. It implies that $gz = zg$ for $g \in Z$ and $z \in Z(G)$

If G is abelian, then $Z(G) = G$;

In this case, the centre is not useful.

24) State 1st isomorphism theorem.

Let $\phi: G \to G'$ be a homomorphism with Kernel K, and let

$\gamma_K : G \to G/K$ be the canonical homomorphism.

There is a unique isomorphism

$\mu : G/K \to \phi[G]$ such that

$\phi(x) = \mu(\gamma_K(x))$ for each $x \in G$.

25) State 2nd isomorphism theorem.

Let H be a subgroup of G and let N be a normal subgroup of G. Then

$(HN)/N \cong H/(H \cap N)$.

22)

26) State 3rd isomorphism theorem.
Let H and K be normal subgroups of a group G with $K \leq H$.
Then $G/H \cong (G/K)(H/K)$.

27) Every subgroup of a cyclic group is cyclic.

Proof:

Let $G = \langle a \rangle$ and suppose $H \leq G$.

If H is trivial, then H is cyclic.

Suppose H is not trivial.

Let m be the smallest positive integer with a^m in H.

By ~~then~~ closure $\langle a^m \rangle$ is contained in H.

we claim that $H = \langle a^m \rangle$. To see this choose any $b = a^k$ in H. There exist integer q, r with $0 \leq r < m$ such that $a^k = a^{qm+r}$

Since $b = a^k = a^{qm} a^r$, we have $a^r = (a^m)^{-q} b$.

Since b and a^m are in H, so is a^r.

But $r < m$ (the smallest power of a in H)

So $r = 0$

Hence $b = (a^m)^q$ and b is in H.

13)

It follows That $H = \langle a^m \rangle$ as required.

28) Let $(G, *)$ and $(G', \cdot)$ be two groups and ϕ is homomorphism Then show that $\ker \phi$ is normal in $(G, *)$

Proof:

i) $\ker \phi$ is a subgroup of G $\forall a, b \in \ker \phi$.

$\phi(a) = e_{G'}$, $\phi(b) = e_{G'}$.

Then, $\phi(a * b) = \phi(a)\,\phi(b) = e_{G'}$.

Therefore $a * b \in \ker \phi$.

29) Inverse element:

$\forall a \in \ker \phi$, $\phi(a) = e_{G'}$.

Then

$\phi(a^{-1}) = \phi(a)^{-1}$

$= e_{G'}$. Therefore $a^{-1} \in \ker \phi$

ii) $\forall g \in G$, $a \in \ker \phi$, $\phi(a) = e_{G'}$.

Then

$\phi(g^{-1} * a * g)$

$= \phi(g^{-1})\,\phi(a)\,\phi(g)$

$= \phi(g)^{-1}\, e_{G'}\, \phi(g)$

$= e_{G'}$

Therefore

$g^{-1} * a * g \in \ker \phi$.

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14)

30) Find the Subgroup of $Z_4 \times Z_6$ which is generated by $(0,2)$

$(0,2)$ generates the subgroup

$H = \{(0,0), (0,2), (0,4)\}$

of $Z_4 \times Z_6$ of order 3.

Here the 1st factor Z_4 of $\mathbb{Z}_4 \times \mathbb{Z}_6$ is left alone.

The $\mathbb{Z}_6$ factor on the other hand is essentially collapsed by a subgroup of order 3, givin a factor group in the second factor of order 2 That must be isomorphic to Z_2.

Thus $(Z_4 \times Z_6)/\{(0,2)\}$ is isomorphic to $Z_4 \times Z_2$.

31) Equivalence relation $x_1 \sim x_2 \in X$.

For each $x \in X$, we have $ex = x$,

So $x \sim x$ and $\sim$ is reflexive.

Suppose $x_1 \sim x_2$ so $gx_1 = x_2$ for some $g \in G$.

Then $g^{-1}x_2 = g^{-1}(gx_1) = (g^{-1}g)x_1 = ex_1 = x_1$,

So $x_2 \sim x_1$ and $\sim$ is symmetric.

Finally, if $x_1 \sim x_2$ and $x_2 \sim x_3$, then

$g_1 x_1 = x_2$ and $g_2 x_2 = x_3$ for some $g_1, g_2 \in G$.

Then

15)

$(g_1 g_2)x_1 = h(g_1, x_1) = g_2 x_2 = x_3$.
So $x_1 \sim x_3$ and $\sim$ is transitive.

32) Define normal subgroup.?

A subgroup H of a group G is called a normal subgroup of G if $g^{-1}hg \in H \; \forall \; g \in G$ and $h \in H$.

33) Write two examples of normal subgroup.

i) H_4 is a normal subgroup of S_3

ii) The centre of a group is a normal.

iii) The commutator subgroup is a normal subgroup.

34) Is $(Z_n, +)$ is a cyclic group? If yes then tell its generator.

$(Z_n, +)$ is a cyclic group with 1 as a generator.
When there is no confusion we write the elements of Z_n as $0, 1, 2, 3, \ldots n-1$ instead of $[0], [1], [2], \ldots [n-1]$.

16)

35) Define subgroup with examples.
Let $(G, *)$ be a group. A subset H of G is called a subgroup of G if H also forms a group under the operation $*$

Examples

i) $(\mathbb{Z}, +)$ is a subgroup of $(\mathbb{R}, +)$

ii) $(\cdot, \{1, -1\})$ is a subgroup of $(\{1, -1, i, -i\}, \cdot)$

36) Give examples of group under multiplication.

i) $\langle \mathbb{R} - \{0\}, \cdot \rangle$

ii) $\langle Q - \{0\}, \cdot \rangle$

iii) $\langle C - \{0\}, \cdot \rangle$

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Question # 1 of 10 (Start time: 12:23:34 PM, 07 June 2018)

In the group {1, -1, i, -i}, the order of -1 is -------

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Select the correct option

○	1
●	2
○	3
○	4

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Question # 2 of 10 (Start time: 12:23:51 PM, 07 June 2018)

The group (Z, +) is a finite group, where Z is the set of integers.

Select the correct option

○	False
●	True

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A subset H of a group G under addition is a subgroup of G if and only if **whenever** $a, b \in H,$ **then** $a - b \in H$

Select the correct option

- (●) True
- ( ) False

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$\langle \{1, -1\}, \cdot \rangle$ is the subgroup of $\langle \{1, -1, i, -i\}, \cdot \rangle$.

Select the correct option

●	True
○	False

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How many tests are there to show a subset of a group to be a subgroup?

Select the correct option

○	1 AHMAD MSC(MATH) Areeb2113@gmail.com
○	2
●	3
○	4

Question # 6 of 10 (Start time: 12:26:51 PM, 07 June 2018)

A subgroup H of a group (G, *) is a subset of G, which is itself a group under * .

Select the correct option

- (●) True
- ( ) False

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Question # 7 of 10 (Start time: 12:27:13 PM, 07 June 2018)

Which of the following is the subgroup of {1, -1, i, -i } under multiplication?

Select the correct option

- ○ {1, i}
- ○ {1, -i}
- ● {1, -1}
- ○ {i, -i}

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Question # 8 of 10 (Start time: 12:27:53 PM, 07 June 2018)

The group (Q, +) is an infinite group, where Q is the set of rational numbers.

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Select the correct option

- ○ False
- ● True

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Question # 9 of 10 (Start time: 12:29:09 PM, 07 June 2018)

Show that $H = \left\{ \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \mid n \in Z \right\}$ is the subgroup of G = GL (2, R) under multiplication.

Select the correct option

- (●) True
- ( ) False

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Question # 10 of 10 (Start time: 12:29:50 PM, 07 June 2018)

The order of the group $\{1, \omega, \omega^2\}$ is _ _ _ _ _ _ _ _ _

Select the correct option

○	2
●	3
○	4
○	5

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$\langle Z, + \rangle$ is the subgroup of $\langle R, + \rangle$.

Select the correct option

●	True
○	False

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The order of the group {e} is ----------

Select the correct option

○	0
●	1
○	2
○	3

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Klein Four group is not cyclic group.

Select the correct option

- (●) True
- ( ) False

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A non empty subset S of a finite group G is called subgroup of G if and only if ab is in S for all a, b in S.

Select the correct option

- (●) True
- ( ) False

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Question # 8 of 10 (Start time: 12:41:33 PM, 07 June 2018)

The order of the group {1, -1, i, -i, j, -j, k, -k} is ----------

Select the correct option

○	6
○	7
●	8
○	9

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Question # 5 of 10 (Start time: 12:41:48 PM, 07 June 2018)

The group {1, -1, i, -i} is not a subgroup of itself.

Select the correct option

●	False
○	True

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$\langle Q, + \rangle$ is the subgroup of $\langle R, + \rangle$.

Select the correct option

- (●) True
- ( ) False

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Question # 6 of 10 (Start time: 12:53:26 PM, 07 June 2018)

In the group {1, -1, i, -i}, the order of i is -------

Select the correct option

○	1 AHMAD MSC(MATH) Areeb2113@gmail.com
○	2
○	3
●	4

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Question # 7 of 10 (Start time: 12:53:45 PM, 07 June 2018)

A subset H of a group G is a subgroup of G if and only if (i) whenever $a, b \in H$, then $ab \in H$ (ii) whenever $b \in H$, then $b^{-1} \in H$

Select the correct option

- (●) True
- ( ) False

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Question # 5 of 10 (Start time: 12:34:33 PM, 07 June 2018)

The set of even integers is not a subgroup of the set of Integers under the same binary operation of addition.

Select the correct option

○	True
●	False

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A subset H of a group G is a subgroup of G if and only if H is non-empty and ----------

Select the correct option

○	whenever $a, b \in H$, then $ab \in H$
○	whenever $a, b \in H$, then $b^{-1} \in H$
○	whenever $a, b \in H$, then $ab^{-1} \in H$
	whenever $a, b \in H$, then $(ab)^{-1} \in H$

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47% 3:33 pm

Question # 5 of 10 (Start time: 03:33:11 PM, 07 June 2018) Total Marks: 1

The group (Q, +) is an infinite group, where Q is the set of rational numbers.

Select the correct option

- False
- True

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Click to Save Answer & Move to Next Question

Question # 4 of 10 (Start time: 03:32:40 PM, 07 June 2018)

The group ({1, −1}, .) is subgroup of the group (Z, +).

Select the correct option

●	True
○	False

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MTH633:Quiz No. 1

Question # 5 of 10 (Start time: 08:43:47 PM, 08 June 2018)

$\langle Q, + \rangle$ is the subgroup of $\langle R, + \rangle$.

Select the correct option

- True
- False

47% 3:30 pm

Question # 1 of 10 (Start time: 03:29:56 PM, 07 June 2018) Total Marks: 1

A subset H of a group G under addition is a subgroup of G if and only if whenever $a, b \in H$, then $a - b \in H$

Select the correct option

Reload Math Equations

- True
- False

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Click to Save Answer & Move to Next Question

... 47% 3:33 pm

Question # 5 of 10 (Start time: 03:33:11 PM, 07 June 2018) Total Marks: 1

The group (Q, +) is an infinite group, where Q is the set of rational numbers.

Select the correct option

- False
- True

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Click to Save Answer & Move to Next Question

Question # 3 of 10 (Start time: 08:43:18 PM, 08 June 2018)

$\langle Z, + \rangle$ is the subgroup of $\langle R, \cdot \rangle$.

Select the correct option

- True
- False

Question # 8 of 10 (Start time: 03:34:23 PM, 07 June 2018)

The order of the group $\{1, \omega, \omega^2\}$ is - - - - - - - - - -

Select the correct option

○	2
○	3
○	4
○	5

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Question # 9 of 10 (Start time: 08:45:11 PM, 08 June 2018)

The group ({1, -1}, .) is subgroup of the group (Z, +).

Select the correct option

- True
- False

Question # 2 of 10 (Start time: 03:32:04 PM, 07 June 2018)

The group {1, −1, i, −i} is a finite group.

Select the correct option

●	True
○	False

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Question # 9 of 10 (Start time: 03:34:33 PM, 07 June 2018)

The group (Z, +) is a finite group, where Z is the set of integers.

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Select the correct option

- (●) False
- ( ) True

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Question # 8 of 10 (Start time: 08:44:59 PM, 08 June 2018)

The order of the group $\{1, -1, i, -i\}$ is _____

Select the correct option

- ○ 2
- ○ 3
- ● 4
- ○ 5

Question # 7 of 10 (Start time: 08:44:47 PM, 08 June 2018)

$\langle \{1, -1\}, \cdot \rangle$ is the subgroup of $\langle \{1, -1, i, -i\}, \cdot \rangle$.

Select the correct option

- (●) True
- ( ) False

The order of the group {1, −1} is ----------

Select the correct option

○	1
●	2
○	3
○	4

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Show that $H = \left\{ \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \mid n \in Z \right\}$ is the subgroup of G = GL (2, R) under multiplication.

Select the correct option

●	True
○	False

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Question # 6 of 10 (Start time: 08:44:21 PM, 08 June 2018)

A non empty subset S of a finite group G is called subgroup of G if and only if ab is in S for all a, b in S.

Select the correct option

- True
- False

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Example

Consider $\mathbb{Z}_3 \times \mathbb{Z}_3$. This is a group of nine elements. We claim that $\mathbb{Z}_3 \times \mathbb{Z}_3$ is *not* cyclic.

mcq

Since the addition is by components, and since in $\mathbb{Z}_3$ every element added to itself three times gives the identity, the same is true in $\mathbb{Z}_3 \times \mathbb{Z}_3$. Thus no element can generate the group, for a generator added to itself successively could only give the identity after nine summands. We have found another group structure of order 9. A similar argument shows that $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not cyclic. Thus $\mathbb{Z}_2 \times \mathbb{Z}_2$ must be isomorphic to the Klein 4-group.

Direct Products

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Example

The group $\mathbb{Z} \times \mathbb{Z}_2$ is generated by the elements (1, 0) and (0, 1). More generally, the direct product of n cyclic groups, each of which is either $\mathbb{Z}$ or $\mathbb{Z}_m$ for some positive integer m, is generated by then n-tuples

(1, 0,⋯, 0), (0, 1,⋯, 0),....,(0, 0,⋯, 1). Such a direct produ[illegible] also be generated by few[er] ele[illegible]ents. For example, $\mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_{35}$ is gener[illegible]ed by the single element (1, 1, 1).

MCQ

Orbits

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Definition

An orbit of a permutation p is an equivalence class under the relation:

$a \sim b \iff b = p^n(a)$,

for some n in $\mathbb{Z}$.

MCQ

Parity of a Permutation

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MCQ

Theorem

The parity of a permutation is even or odd, but not both.

MTH #633
Group Theory

MC170201745
26/06/2018

(1) Define Bijective Mapping 2 Marks

(2) Write identity permutation of D_4 and D_5 2 Marks

(3) Write Elements of Dihedral Group D_4 3 Marks

(4) Suppose $G = \langle a \rangle$ with $\langle a \rangle = 30$
Find $|a^{21}|$ and $\langle a^{21} \rangle$ 3 Marks

(5) Show that $(Z, +)$ is group under addition 5 Marks

(6) Let 'P' be permutation of set 'S'
Then prove following relation is an equivalence relation

$$a \sim b \iff b = P^n(a) \text{ for some } n \in Z$$

Anila



Mth633 mai cosets topicai se thy

Anila

Aik long qsn tha...
Every subgrp of a cyclic grp is also cyclic prove krnaa tha isyyyy



R mcqs mai ziada tr true false thy r easy he thy

Anila



And one step subgrp and two step subgrp tasks ko just state krna thaa

Us MN whole set ko prove krna tha k ye group nhe h

Infinite set k right coset describe krny thy

Aik prove tha subset say related pta nhe kahan say diya tha

Homomorphism ki example Deni the aik

Aik group btana tha k ye abilian h k nhe

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MTH633 Group theory.

Q1 Write three element of group.

Q2:- Write identity permutation of $(S_4, \circ)$.

Q3:- $\{1, -1, i, -i\}$ is abelian.

Q4:- if H is a subgroup and a, b is in G then prove $|aH| = |bH|$.

Q5:- Write two function of Homomorphism Give Reason.

Q6:- Suppose $G\langle a\rangle$ with $\langle a\rangle = 30$ find $|a^{21}|$ and $\langle a^{21}\rangle$

MTH 633

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① Write 2 groups that are abelian. 2 Marks.

② Write 2 properties of Langrange's Theorem. 2 Marks.

③ If $|a| = 30$ then find $|a^{21}|$ & $\langle a^{21} \rangle$ 3 Marks.

④ If $f : \langle R, + \rangle \to \langle R^+, \cdot \rangle$ then prove isomerism. 5-Marks.

⑤ Prove that $\langle R, + \rangle$ is a group. 5-Marks.

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MTH 633

1) Define Bijective mapping? (2) marks
2) Write identity of $(S_6, \cdot)$. (2) marks
3) Write right cosets of $3\mathbb{Z}$ of $\mathbb{Z}$. (3) marks
4) Draw Cayley's table of $\{1, -1, i, -i\}$. (3) marks
(5) Prove that $\{1, -1, i, -i\}$ is an abelian group? 5 marks
(6) Prove abelian group upto isomorphism of order 360? 5 marks.

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MTH633

(5) $(S_3, 0)$ is non abelian

---

(5) $\langle Z, + \rangle$ is group under addition

---

(3) Find all orbits of

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 2 & 1 & 3 \end{bmatrix}$$

---

(2) Define Sub group with example

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Practice Questions - Group Theory MTH633

-- These questions are for your practice only. We will not upload their solutions.

-- If you want to have your solution checked, please email your solution to mth633@vu.edu.pk

Modules (90 – 181) Lectures (23 – 45)

Q1: Define normal subgroup and give example.

Q2: Let φ be a homomorphism function from group $(G,*)$ to group $(G',\cdot)$. Then, $(Ker\,\varphi,*)$ is a normal subgroup of $(G,*)$.

Q3: Let H be a subgroup of a group G. Then H is normal if and only if

(aH)(bH) = (ab)H, for all a, b in G

Q4: A homomorphism h: G→G' is injective if and only if Ker h={e}.

Q5: Show that a factor group of a cyclic group is cyclic.

Q6: Define commutator subgroup

Q7: Let G be a group. The set of all commutators $aba^{-1}b^{-1}$ for $a, b \in G$ generates a subgroup C of G.

Q8: Let G be a group. Then, the commutator subgroup C of G is a normal subgroup of G.

Q9: What does mean by X is a G-set?

Q10: Give an example of group action on a set.

Q11: What is a p – group?

Q12: Define all Sylow Theorems.

QUESTION 16

Let G be a finite abelian group of order n, and let m be a positive integer relative prime to n. Then the mapping $\sigma: x \to x^m$ is an automorphism of G.

Solution:

(m,n) = 1 ⇒ there exist integers u and v such that mu + nv = 1 ⇒ for all $x \in G$, $x^{mu+nv} = x^{mu}\, x^{nv} = x^{um}$ since o(G)=n.

Now for all $x \in G$, $x=(x^u)^m$ implies that σ is onto. Further,

$x^m = e \Rightarrow x^{mu} = e \Rightarrow x = e$, showing that σ is 1-1.

That σ is a homomorphism follows from the fact that G is abelian. Hence, σ is an automorphism of G.

QUESTION 17

Let $G = \langle a \mid a^n = e \rangle$ be a finite cyclic group of order n. Then the mapping $\sigma : a \to a^m$ is an automorphism of G iff (m,n) = 1.Further, if (m,n)=d, then $(a^m)^{n/d} = (a^n)^{m/d} = e$. Thus, the order of a^m divides n/d; that is, n|n/d. Hence, d = 1, and the solution is complete.Let X be a G-set. Then G_x is a subgroup of G for each $x \in X$.

Proof:

Let $x \in X$ and let $g_1, g_2 \in G_x$. Then $g_1x=x$ and $g_2x=x$. Consequently, $(g_1g_2)x=g_1(g_2x)=g_1x=x$, so $g_1g_2 \in G_x$, and G_x is closed under the induced operation of G. Of course $ex=x$, so $e \in G_x$. If $g \in G_x$, then $gx = x$, so $x=ex=(g^{-1}g)x= g^{-1}(gx)=g^{-1}x$, and consequently $g^{-1} \in G_x$. Thus G_x is a subgroup of G.

QUESTION 18

Let X be a G-set. For x_1, $x_2 \in X$, let $x_1 \sim x_2$ if and only if there exists $g \in G$ such that $gx_1 = x_2$. Then $\sim$ is an equivalence relation on X.

Proof:

For each $x \in X$, we have $ex = x$, so $x \sim x$ and $\sim$ is reflexive.

Suppose $x_1 \sim x_2$, so $gx_1 = x_2$ for some $g \in G$. Then $g^{-1}x_2 = g^{-1}(gx_1) = (g^{-1}g)x_1 = ex_1 = x_1$, so $x_2 \sim x_1$, and $\sim$ is symmetric. Finally, if $x_1 \sim x_2$ and $x_2 \sim x_3$, then $g_1x_1 = x_2$ and $g_2x_2 = x_3$ for some $g_1, g_2 \in G$. Then $(g_2g_1)x_1 = g_2(g_1x_1) = g_2x_2 = x_3$, so $x_1 \sim x_3$ and $\sim$ is transitive.

QUESTION 19

If N is a normal subgroup of G, and if H is any subgroup of G, then

H V N=HN=NH. Furthermore, if H is also normal in G, then HN is normal in G.

Proof:

We show that HN is a subgroup of G, from which

H V N=HN follows at once. Let $h_1, h_2 \in H$ and $n_1, n_2 \in N$. Since N is a normal subgroup, we have $n_1 h_2 = h_2 n_3$ for some $n_3 \in N$. Then $(h_1 n_1)(h_2 n_2) = h_1(n_1 h_2)n_2 = h_1(h_2 n_3)n_2 = (h_1 h_2)(n_3 n_2) \in HN$, so HN is closed under the induced operation in G. Clearly $e=ee$ is in HN. For $h \in H$ and $n \in N$, we have $(hn)^{-1} = n^{-1} h^{-1} = h^{-1} n_4$ for some $n_4 \in N$, since N is a normal subgroup. Thus $(hn)^{-1} \in HN$, so $HN \leq G$.

A similar argument shows that NH is a subgroup, so NH=H V N=HN.

Now suppose that H is also normal in G, and let $h \in H$, $n \in N$, and $g \in G$. Then

$ghng^{-1} = (ghg^{-1})(gng^{1}) \in HN$, so HN is indeed normal in G.

QUESTION 20

Let G be a group such that for some fixed integer n >1, $(ab)^n = a^n b^n$ for all a, b∈G. Let $G_n = \{a \in G \mid a^n = e\}$ and $G^n = \{a^n \mid a \in G\}$. Then $G_n \lhd G$, $G^n \lhd G$, and $G/G_n \simeq G^n$.

Solution:

Let a, b∈G_n and x∈G. Then $(aba^{-1})^n = a^n (b^n)^{-1} = e$, so $aba^{-1} \in G_n$. Also, $(xax^{-1})^n = (xax^{-1})\ldots(xax^{-1}) = xa^n x^{-1} = e$ implies $xax^{-1} \in G_n$. Hence, $G_n \lhd G$.

Let a, b, x∈G. Then $a^n (b^n)^{-1} = (aba^{-1})^n \in G^n$.

Also, $xa^n x^{-1} = (xax^{-1})\ldots(xax^{-1}) = (xax^{-1})^n \in G^n$. Therefore, $G^n \lhd G$.

QUESTION 21

Third Isomorphism Theorem

Let H and K be normal subgroups of a group G with $K \leq H$. Then $G/H \simeq (G/K)/(H/K)$.

Proof:

Let $\phi: G \to (G/K)/(H/K)$ be given by $\phi(a) = (aK)(H/K)$ for $a \in G$. Clearly ϕ is onto $(G/K)/(H/K)$, and for $a, b \in G$,

$\phi(ab) = [(ab)K](H/K)$

$= [(aK)(bK)](H/K)$

$= [(aK)(H/K)][(bK)(H/K)] = \phi(a)\,\phi(b)$, so ϕ is a homomorphism.

The kernel consists of those $x \in G$ such that $\phi(x) = H/K$.

These x are just the elements of H.

Then first isomorphism theorem shows that $G/H \simeq (G/K)/(H/K)$.

QUESTION 22

Let G be a group of order p^n and let X be a finite G-set. Then

$|X| \equiv |X_G| \pmod{p}$.

Proof:

Recall $|X| = |X_G| + \sum_{i=s+1}^{r} |Gx_i|$. In the notation of above Equation,

we know that

$|Gx_i|$ divides $|G|$.

Consequently p divides $|Gx_i|$ for $s + 1 \leq i \leq r$. Above equation then shows that $|X| - |X_G|$ is divisible by p, so $|X| \equiv |X_G| \pmod{p}$.

QUESTION 23

Let H be a p-subgroup of a finite group G. Then

$$(N[H]:H) \equiv (G:H) \pmod{p}.$$

Proof:

Let $\mathcal{L}$ be the set of left cosets of H in G, and let H act on $\mathcal{L}$ by left translastion, so that h(xH) = (hx)H. Then $\mathcal{L}$ becomes an H-set. Note that $|\mathcal{L}|=(G:H)$. Let us determine $\mathcal{L}_H$, that is, those left cosets that are fixed under action by all elements of H. Now xH= h(xH) if and only if $H=x^{-1}hxH$, or if and only if $x^{-1}hx \in H$.

Thus xH=h(xH) for all h∈H if and only if $x^{-1}hx = x^{-1}h(x^{-1})^{-1} \in H$ for all h∈H, or if and only if $x^{-1} \in N[H]$, or if and only if x∈N[H]. Thus the left cosets in $\mathcal{L}_H$ are those contained in N[H]. The number of such cosets is (N[H]:H), so $|\mathcal{L}_H| = (N[H]:H)$.

Since H is a p-group, it has order a power of p. Then $|\mathcal{L}| \equiv |\mathcal{L}_H| \pmod{p}$, that is, $(G:H) \equiv (N[H]:H) \pmod{p}$.

QUESTION 24

Second Sylow Theorem

Let P_1 and P_2 be Sylow p-subgroups of a finite group G. Then P_1 and P_2 are conjugate subgroups of G.

Proof:

Here we will let one of the subgroups act on left cosets of the other. Let $\mathcal{L}$ be the collection of left cosets of P_1, and let P_2 act on $\mathcal{L}$ by $z(xP_1)=(zx)P_1$ for $z\in P_2$. Then $\mathcal{L}$ is a P_2-set. We have $|\mathcal{L}_{P_2}| \equiv |\mathcal{L}| \pmod p$, and $|\mathcal{L}|= (G: P_1)$ is not divisible by p, so $|\mathcal{L}_{P_2}|\neq 0$. Let $xP_1 \in \mathcal{L}_{P_2}$.

Then $zxP_1=xP_1$ for all $z\in P_2$, so $x^{-1}zxP_1=P_1$ for all $z\in P_2$. Thus $x^{-1}zx\in P_1$ for all $z\in P_2$, so $x^{-1}P_2x\leq P_1$. Since $|P_1|=|P_2|$, we must have $P_1=x^{-1}P_2x$, so P_1 and P_2 are indeed conjugate subgroups.

QUESTION 25

The center of a finite nontrivial p-group G is nontrivial.

Proof:

We have $|G|=c+n_{c+1}+\ldots+n_r$, where n_i is the number of elements in the ith orbit of G under conjugation by itself.

For G, each n_i divides $|G|$ for $c+1 \leq i \leq r$, so p divides each n_i, and p divides $|G|$. Therefore p divides c. Now $e \in Z(G)$, so $c \geq 1$. Therefore $c \geq p$, and there exists some $a \in Z(G)$ where $a \neq e$.

QUESTION 26

For a prime number p, every group G of order p^2 is abelian.

Proof:

If G is not cyclic, then every element except e must be of order p.

Let a be such an element. Then the cyclic subgroup <a> of order p does not exhaust G. Also let $b \in G$ with $b \notin \langle a \rangle$. Then $\langle a \rangle \cap \langle b \rangle = \{e\}$, since an element c in $\langle a \rangle \cap \langle b \rangle$ with $c \neq e$ would generate both <a> and <b>, giving $\langle a \rangle = \langle b \rangle$, contrary to construction.

From first Sylow theorem, <a> is normal in some subgroup of order p^2 of G, that is, normal in all of G. Likewise <b> is normal in G.

Now <a> V <b> is a subgroup of G properly containing <a> and of order dividing p^2. Hence <a> V <b> must be all of G.

Thus the hypotheses of last lemma are satisfied, and G is isomorphic to <a> x <b> and therefore abelian.