MTH641 Functional Analysis

Question No : 2 of 41

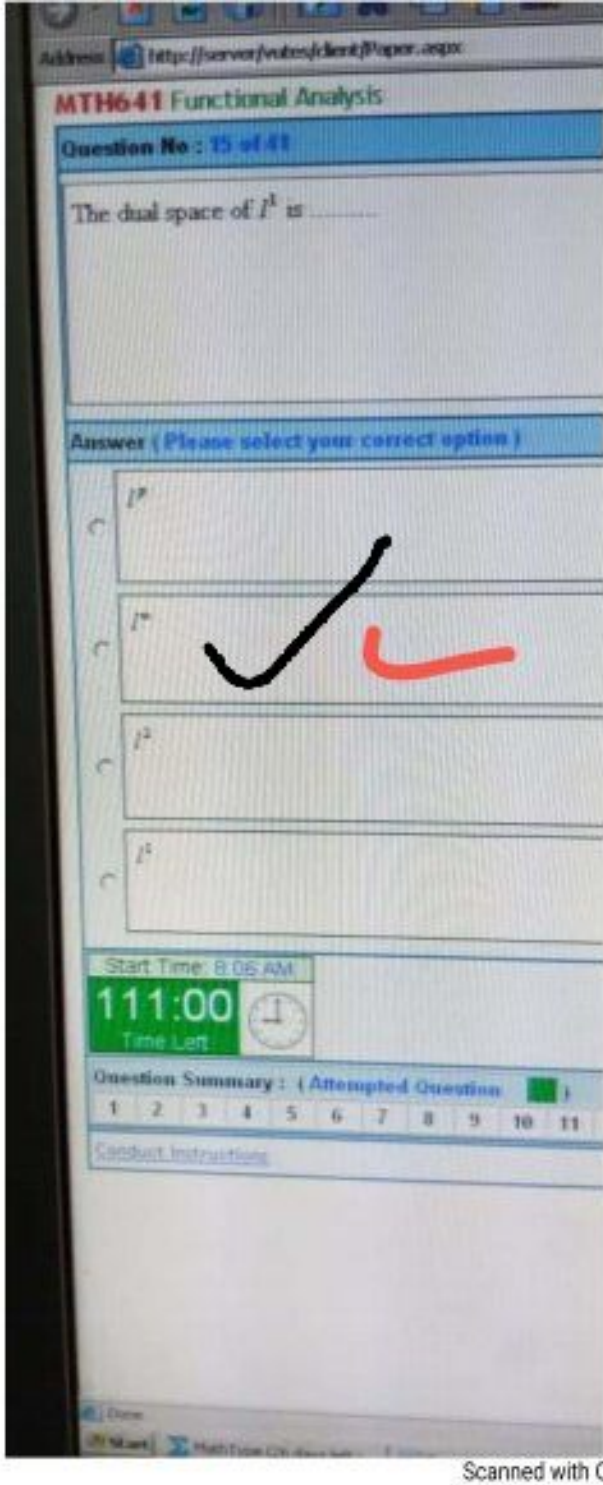
A mapping $T: X \to Y$ is said to be continuous if it is continuous at ..... point of X

Answer (Please select your correct option)

- Every
- atmost one
- exactly one
- atleast one

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MTH641 Functional Analysis

Question No : 13 of 41

A finite dimensional normed space is _____ to its second dual.

Answer (Please select your correct option)

- isometric
- isomorphic
- homomorphic
- none of these

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Question No : 5 of 41

Which one of the following axioms hold for any vectors u, v, w in vector space X and k, a, b in field K ?

Answer (Please select your correct option)

- $u + (-u) = (-u) + u$, for each $u \in X$
- $u + w = w - u$
- $k(u + v) = ku - kv$
- $(ab)u = a(bu)$

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Question No : 30 of 41

An orthonormal set M is an orthogonal set in an inner product space whose elements have norm ....

Answer (Please select your correct option)

- equal to 1
- >1
- <1
- None of these

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MTH641 Functional Analysis

Question No : 28 of 41

Inner product is the generalization of .....

Answer (Please select your correct option)

- Cross product
- Dot product
- Multiplication
- Distance function

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Question Summary

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MTH641 Functional Analysis

Question No : 24 of 41

Which of the following is not true for the space l^p with $p \neq 2$?

Answer (Please select your correct option)

- It is a complete metric space
- It is a complete norm space
- It is a complete inner product space
- None of these

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Question Summary : (Attempted Question)

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Question No : 29 of 41

Any pair of vectors in an orthogonal system is _____ to each other

Answer (Please select your correct option)

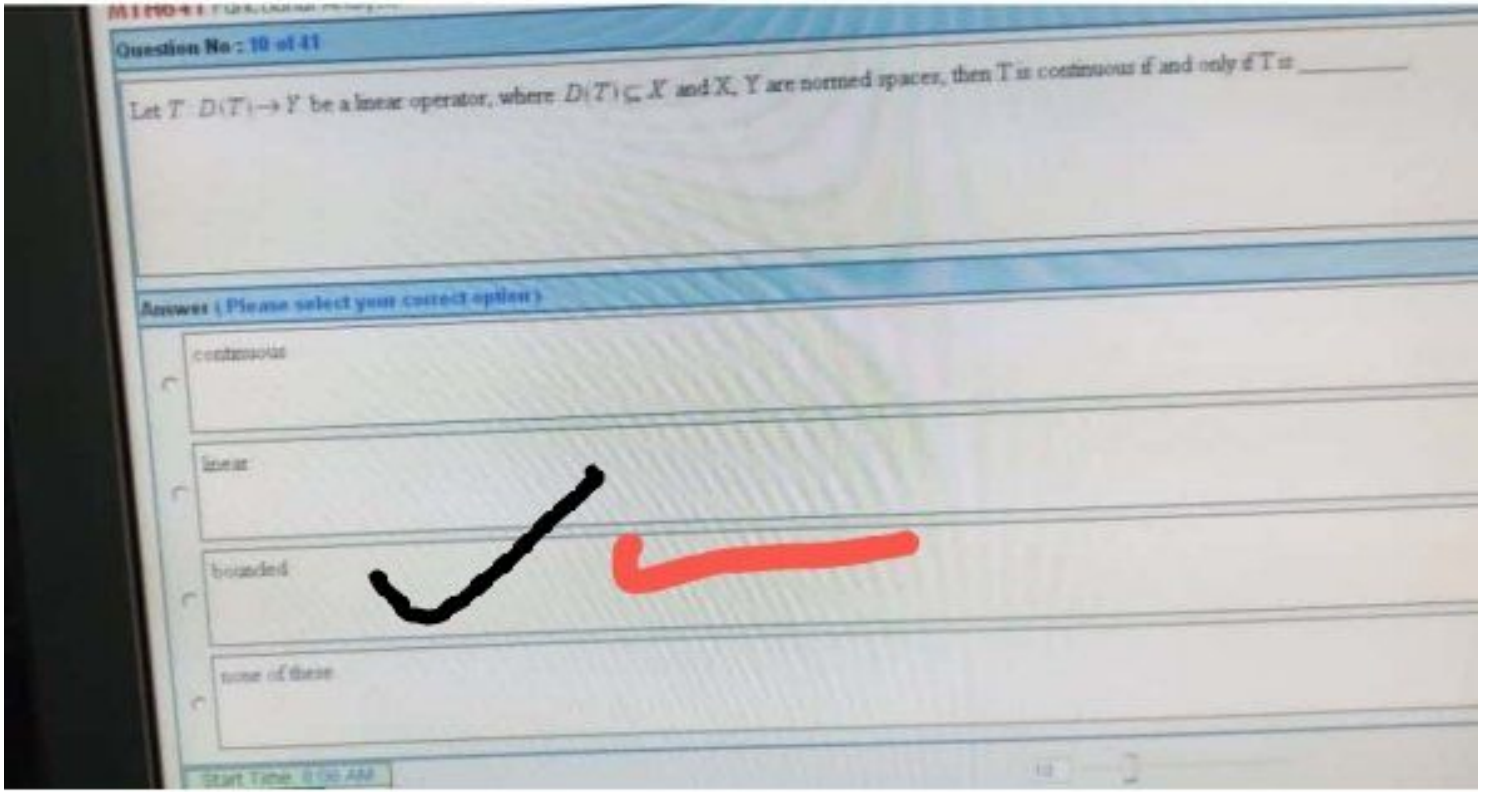
- Parallel
- Equal
- None of these
- Perpendicular





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Question No : 12 of 41 Marks: 1 (Budgeted Time Min)

Two operators T_1 and T_2 are defined to be equal, written $T_1 = T_2$, if _____ and if $T_1 x = T_2 x$ for all _____.

Answer (Please select your correct option)

- $D(T_1) = D(T_2)$, $x \in D(T_1) = D(T_2)$
- $D(T_1) \subset D(T_2)$, $x \in D(T_1) = D(T_2)$
- $D(T_1) \supseteq D(T_2)$, $x \in D(T_1) = D(T_2)$
- $D(T_1) \supset D(T_2)$, $x \in D(T_1) = D(T_2)$

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MTH641 Functional Analysis

Question No : 20 of 41

Every complete inner product space is _____ space.

Answer (Please select your correct option)

- Hilbert
- Banach
- Unitary
- Euclidean

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Conduct Instructions

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MTH641 Functional Analysis

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

- (a,b)
- $[a,b]$
- Space R^n
- None of these

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

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ATH641 Functional Analysis

Question No : 19 of 41

Which of the following is true about the inner product space?

Answer (Please select your correct option)

- Every normed space is an inner product space.
- Every inner product space is a normed space.
- Every inner product space is Hilbert space.
- None of these is true.

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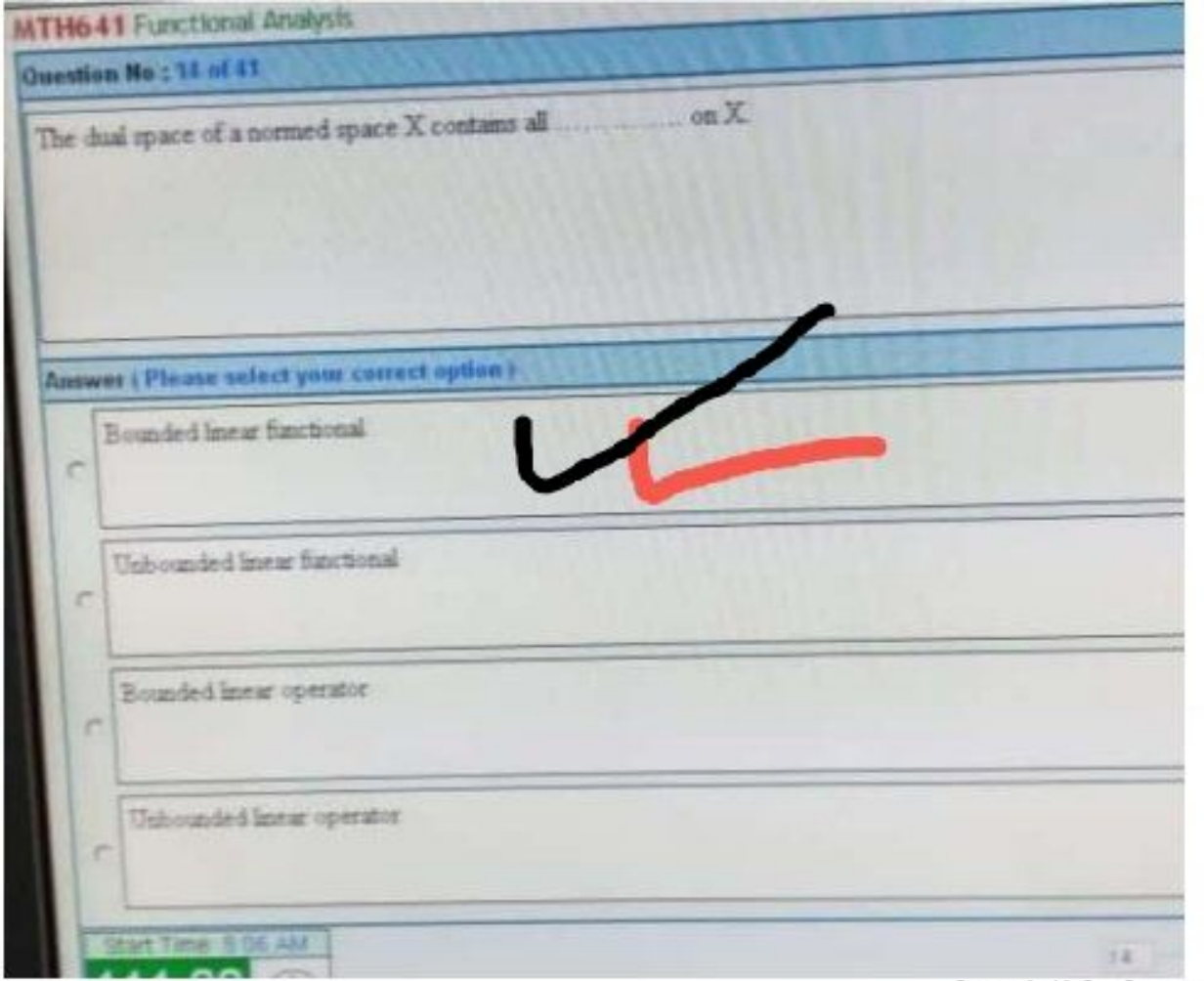
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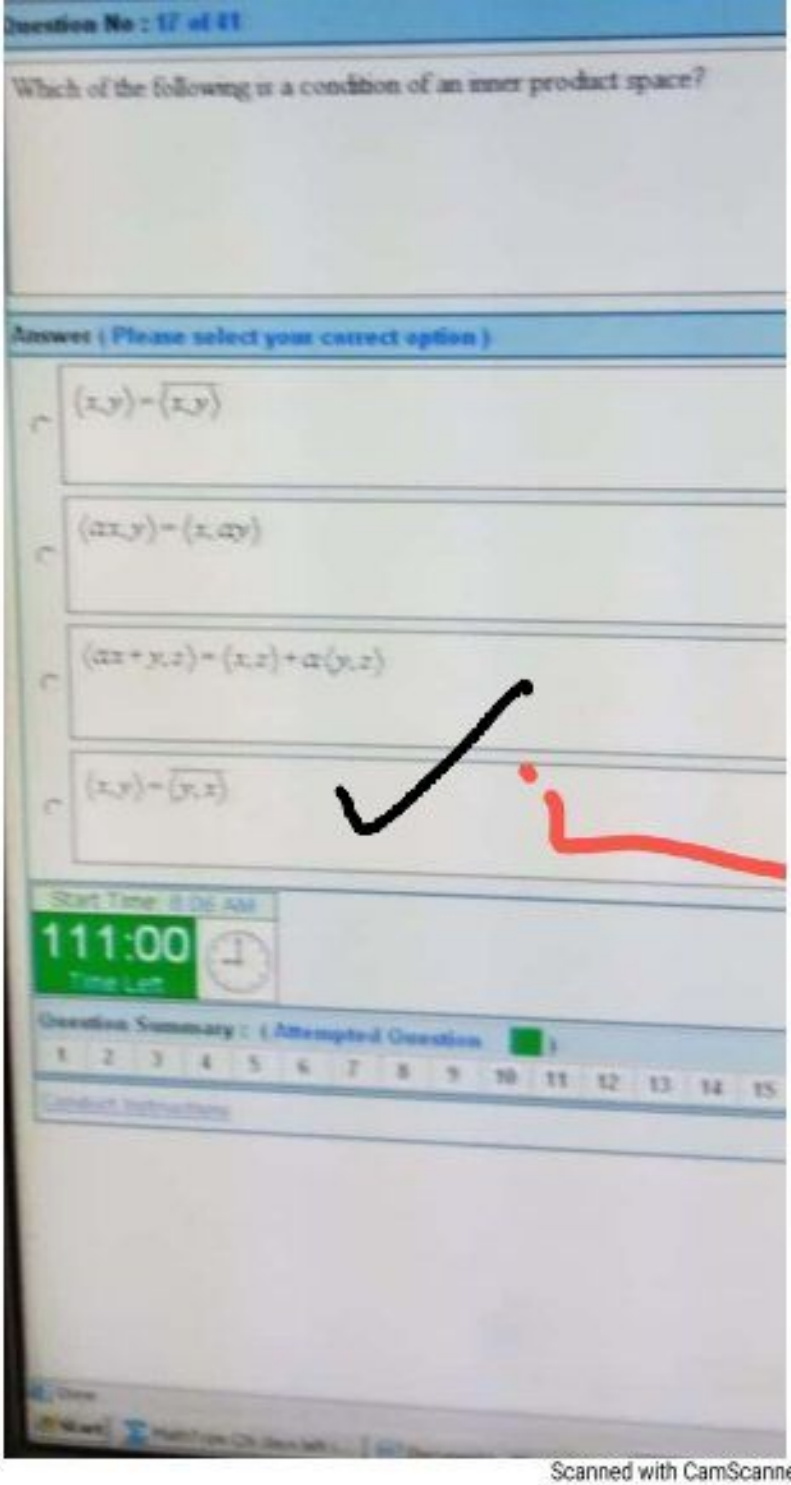
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In an inner product space V, the function $\| \ \| : V \to R^+$ given by _____ defines a _____ in V.

Answer (Please select your correct option)

- $|x| = \sqrt{\langle x, y \rangle}$, $x, y \in V$, norm
- $|x| = \sqrt{\langle x, x \rangle}$, $x \in V$, norm
- $|x| = \sqrt{\langle x, x \rangle}$, $x \in V$, metric
- None of these

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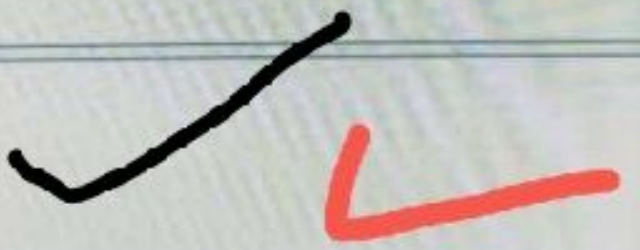
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Question No : 8 of 41

If a normed space X is _____ dimensional, then every linear operator on X is bounded.

Answer (Please select your correct option)

- infinite
- finite



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MTH641 Functional Analysis

Question No : 26 of 41

For a subset A of an inner product space X, x is orthogonal to A if .....

Answer (Please select your correct option)

- x is orthogonal to a for all $a \in A$
- x is orthogonal to any two elements a and b $\in A$
- x is orthogonal to a for a fixed $a \in A$
- None of these

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For any non empty subset A of (X,d), the diameter $\delta(A)$ of A is defined as

Answer (Please select your correct option)

- $\delta(A) = \inf_{x,y \in A} d(x,y)$
- $\delta(A) = d(x,y)$
- $\delta(A) = \sup_{x,y \in A} d(x,y)$
- $\delta(A) = d(x,y)$



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A sequence (x_n) in a metric space is said to be non-decreasing if for all n,

Answer (Please select your correct option)

- $x_n \geq x_{n+1}$
- $x_n \leq x_{n+1}$ ✓
- $x_n < x_{n+1}$
- $x_n > x_{n+1}$

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MTH641 Functional Analysis

Question No : 1 of 41

If d is a metric on X, Then for all $x, y \in X$, $d(x,y)$ is....

Answer (Please select your correct option)

- A non negative real number
- A negative real number
- Not a rational number
- A complex number

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MTH641 Functional Analysis

Question No : 16 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

- $\langle ax, y\rangle = a\langle x, y\rangle$
- $\langle ax+y, z\rangle = \langle x, z\rangle + a\langle y, z\rangle$
- $\langle ax, y\rangle = \langle x, ay\rangle$
- $\langle x, y\rangle = \overline{\langle x, y\rangle}$

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Marks: 1 (Budgeted Time

Question No : 7 of 45

If $S = \{v_1, v_2, v_3, \ldots, v_n\}$ is a set of vectors in a finite dimensional vector space X, then S is called a basis for $V \neq$ ---------- and S is linearly independent

Answer (Please select your correct option)

- S does not span V
- S is linearly dependent
- S is set of scalars
- S spans V

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Question No : 9 of 41

If a normed space X is finite dimensional, then every linear operator on X is_____

Answer (Please select your correct option)

- finite
- unbounded
- bounded
- none of these

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Every complete normed space is _____ space.

Answer (Please select your correct option)

- Hilbert
- Banach
- Unitary
- Euclidean

Start Time: 8:06 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

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MTH641 Functional Analysis

Question No : 27 of 41

For subsets A and B of an inner product space X, A is orthogonal to B if .....

Answer (Please select your correct option)

- a is orthogonal to b for all $a \in A and b \in B$ ✓
- a is orthogonal to b for a fixed $a \in A and$ a fixed $b \in B$
- a is orthogonal to b for a fixed $a \in A and b \in B$
- a is orthogonal to b for all $a \in A and$ a fixed $b \in B$

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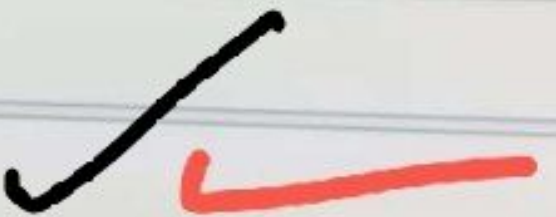
MTH641 Functional Analysis

Question No : 4 of 41

In a metric space (X,d) every convergent sequence is

Answer (Please select your correct option)

- Unbounded sequence
- Not a Constant sequence
- Oscillating sequence
- A Cauchy Sequence

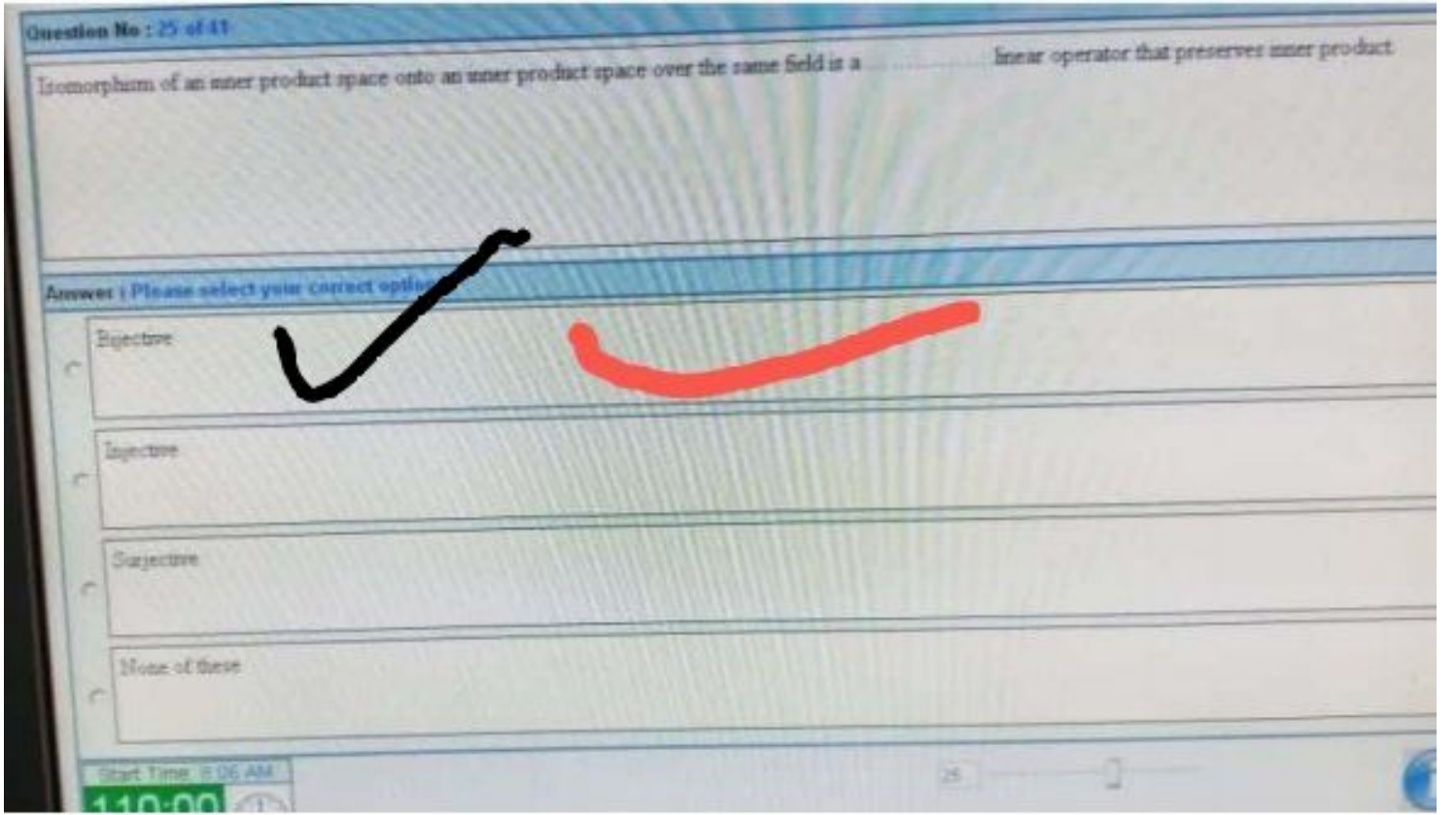


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MTH641 Functional Analysis

Question No : 11 of 41

Let T be a bounded linear operator. Then $x_n \to x$, where $x_n, x \in D(T)$ implies _____

Answer (Please select your correct option)

- $x_n \to x$
- $Tx_n \to Tx$
- $Tx \to Tx_n$
- None of these

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MTH641 Functional Analysis

Question No : 28 of 41

P_n is the collection of all .....

Answer (Please select your correct option)

- Polynomials of degree greater then equal to "n"
- Polynomials of degree less then equal to "n"
- Polynomials of degree equal to "n"
- Polynomials of degree less then "n"

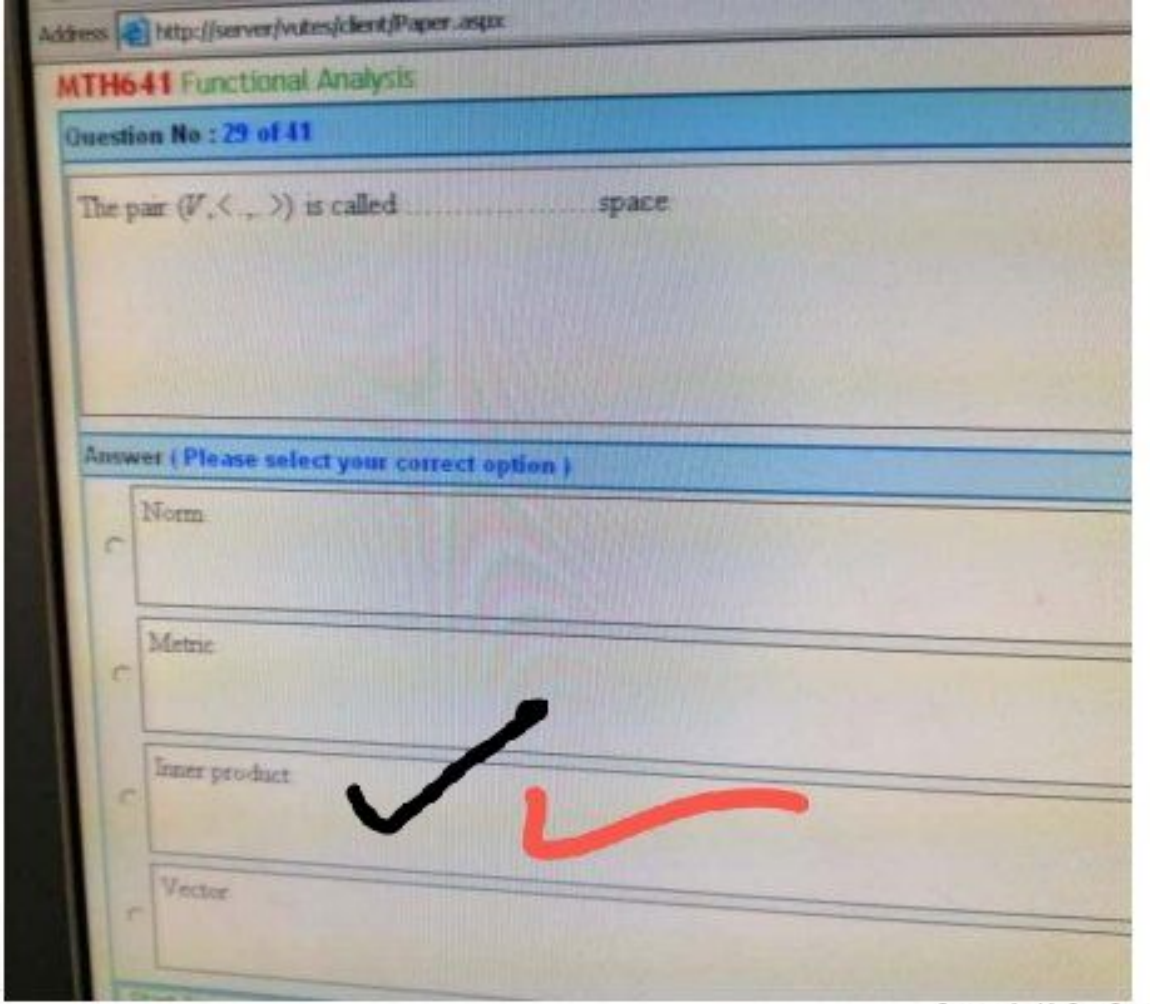
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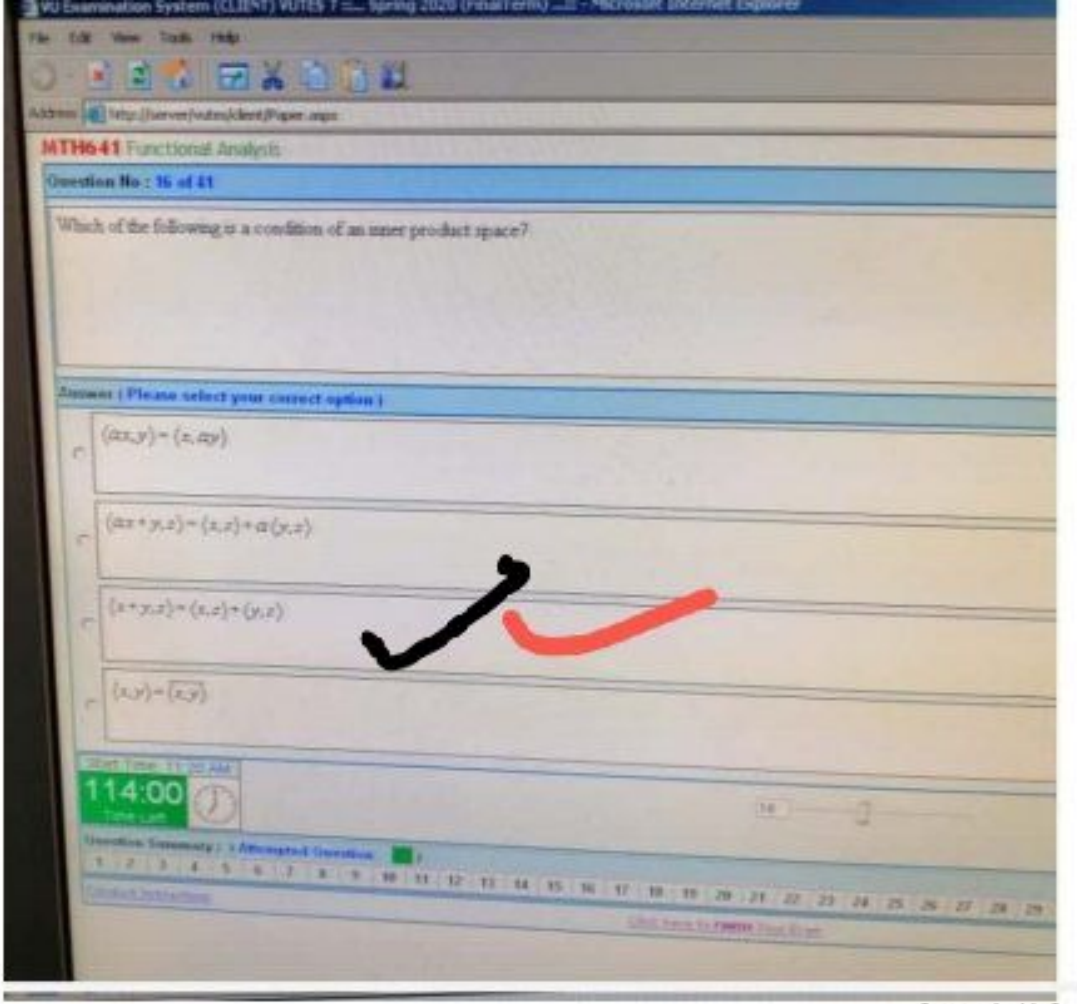
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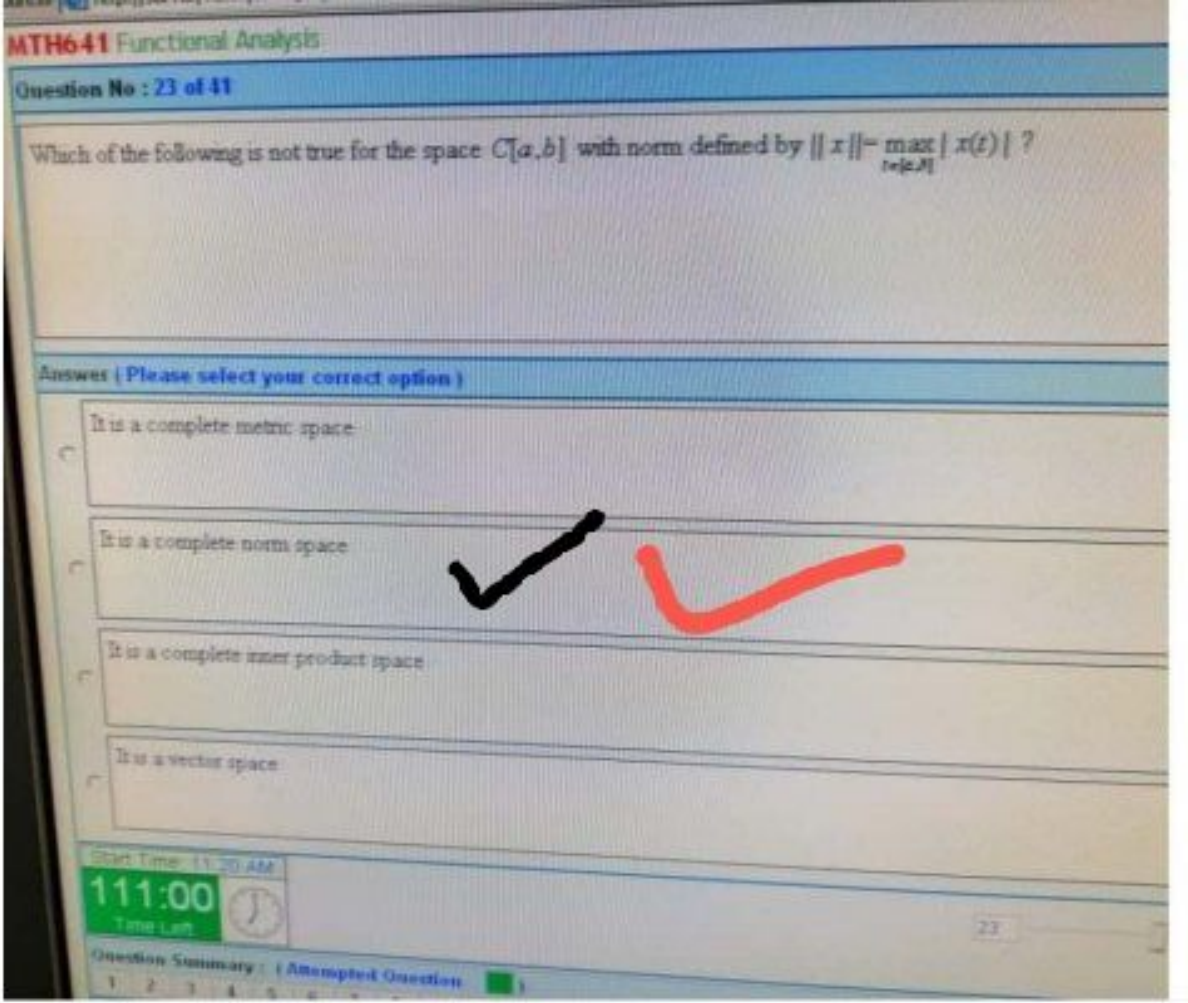
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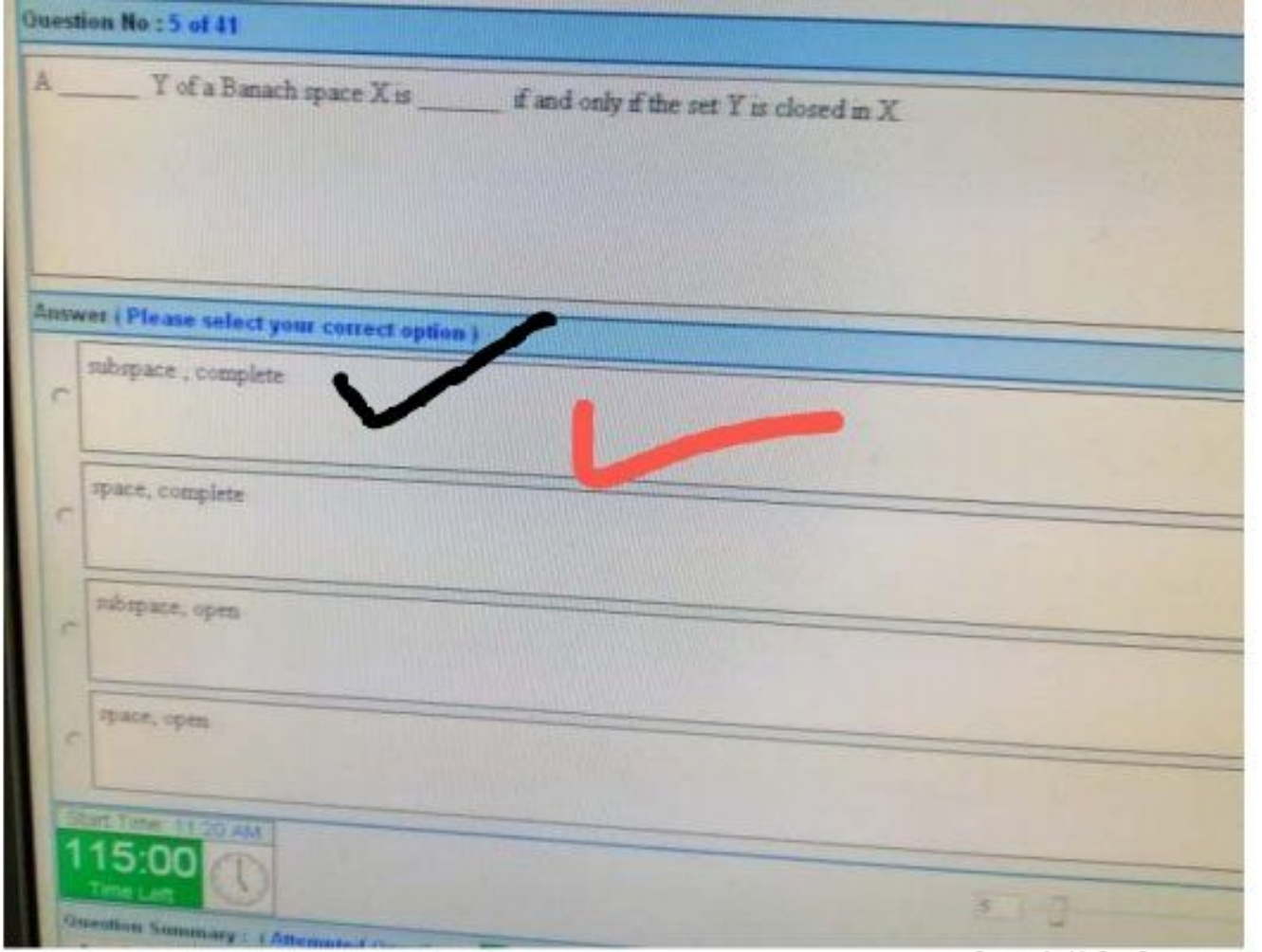
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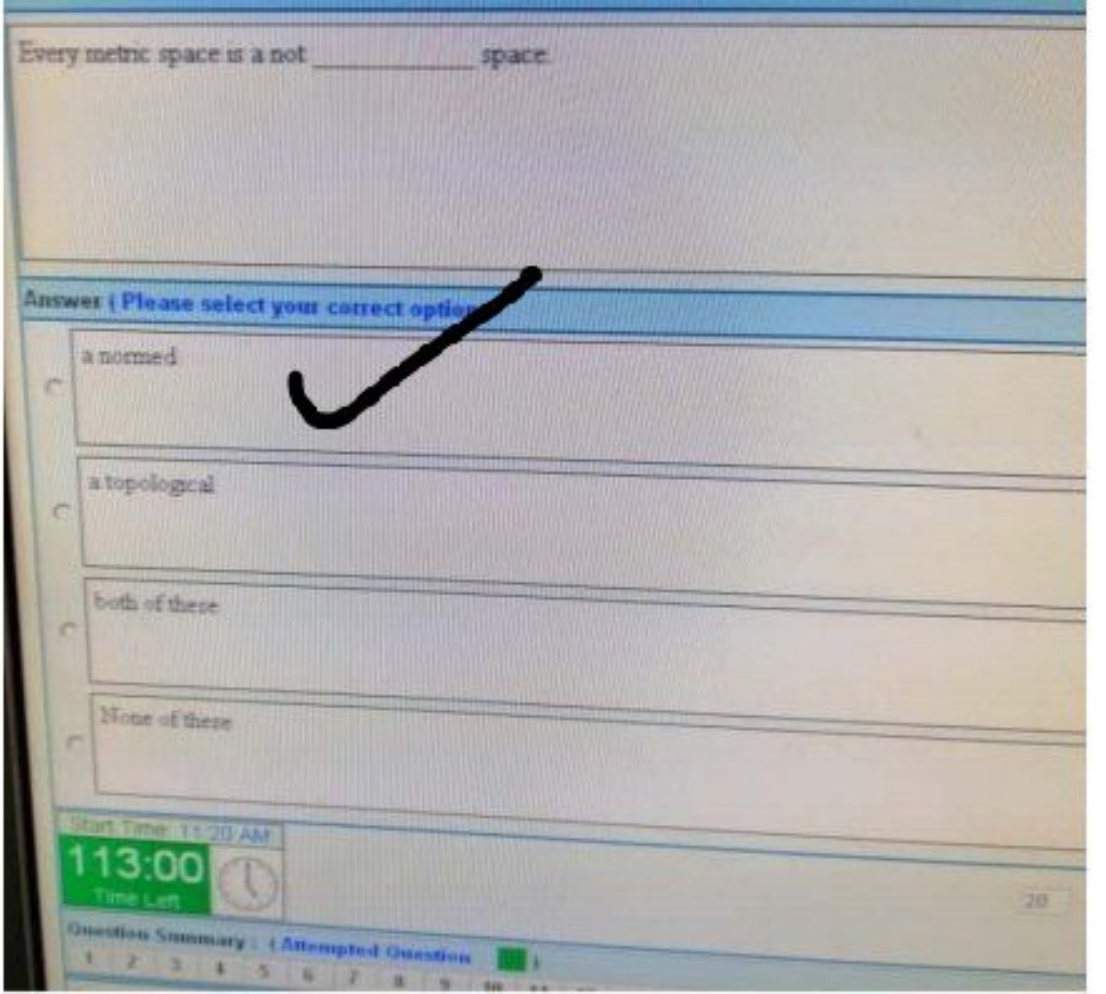
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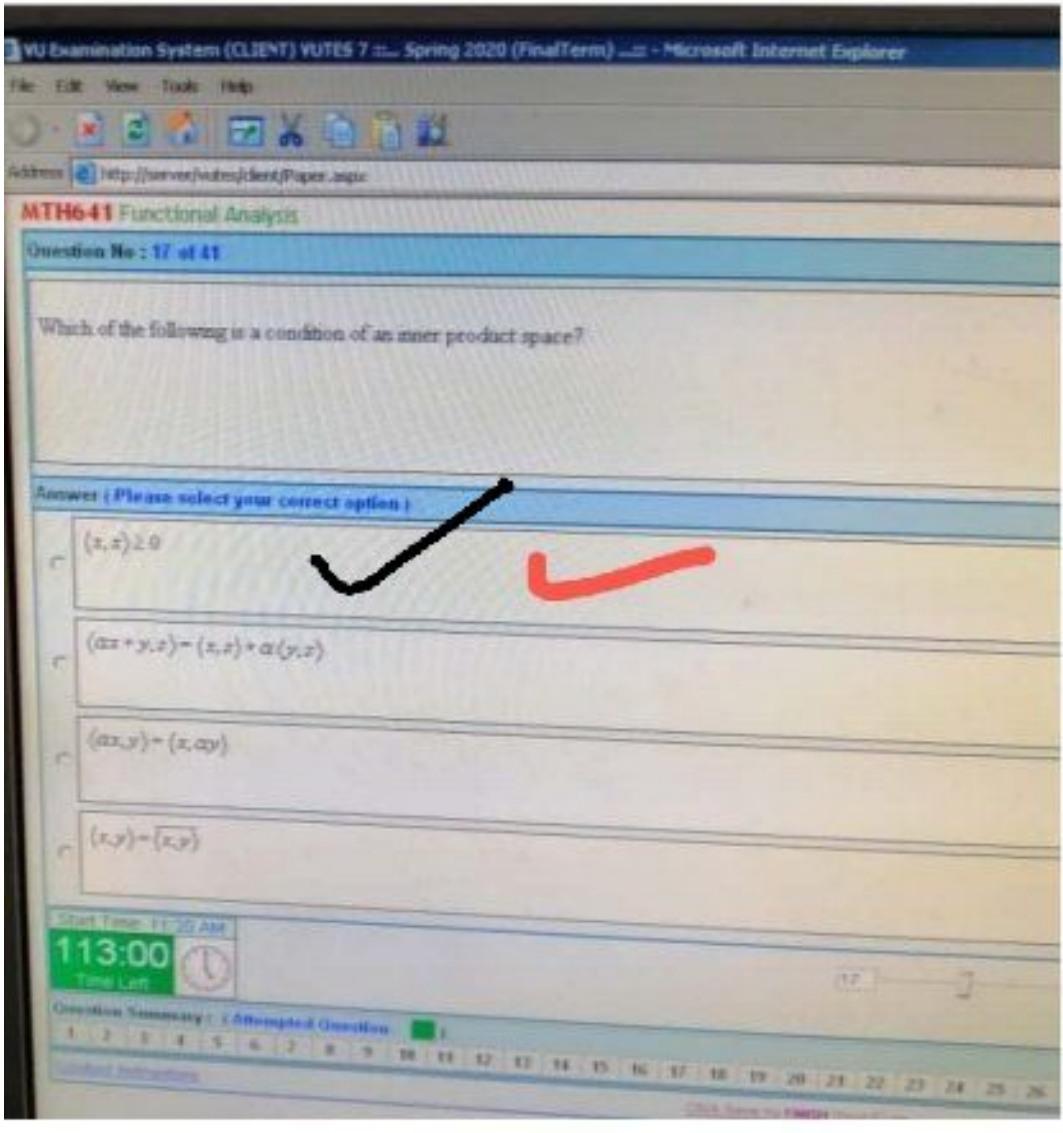
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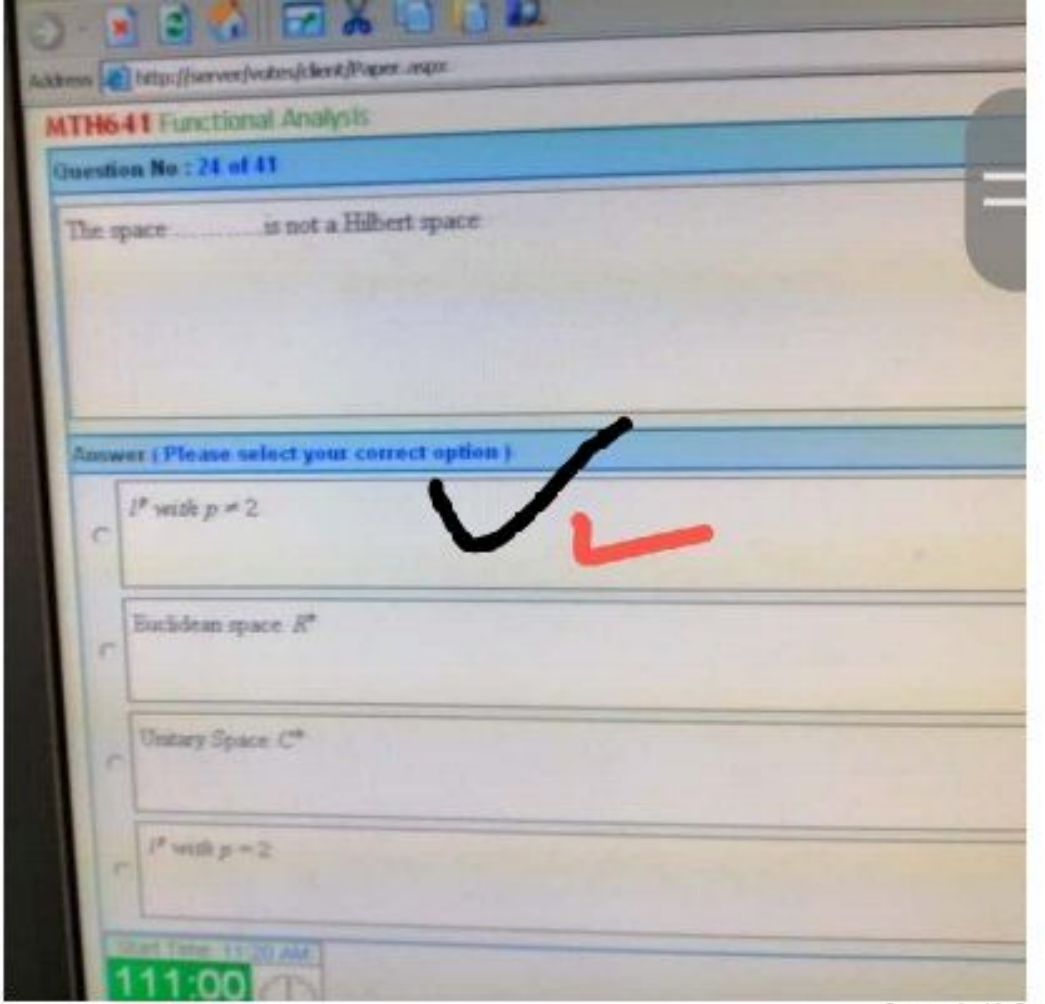
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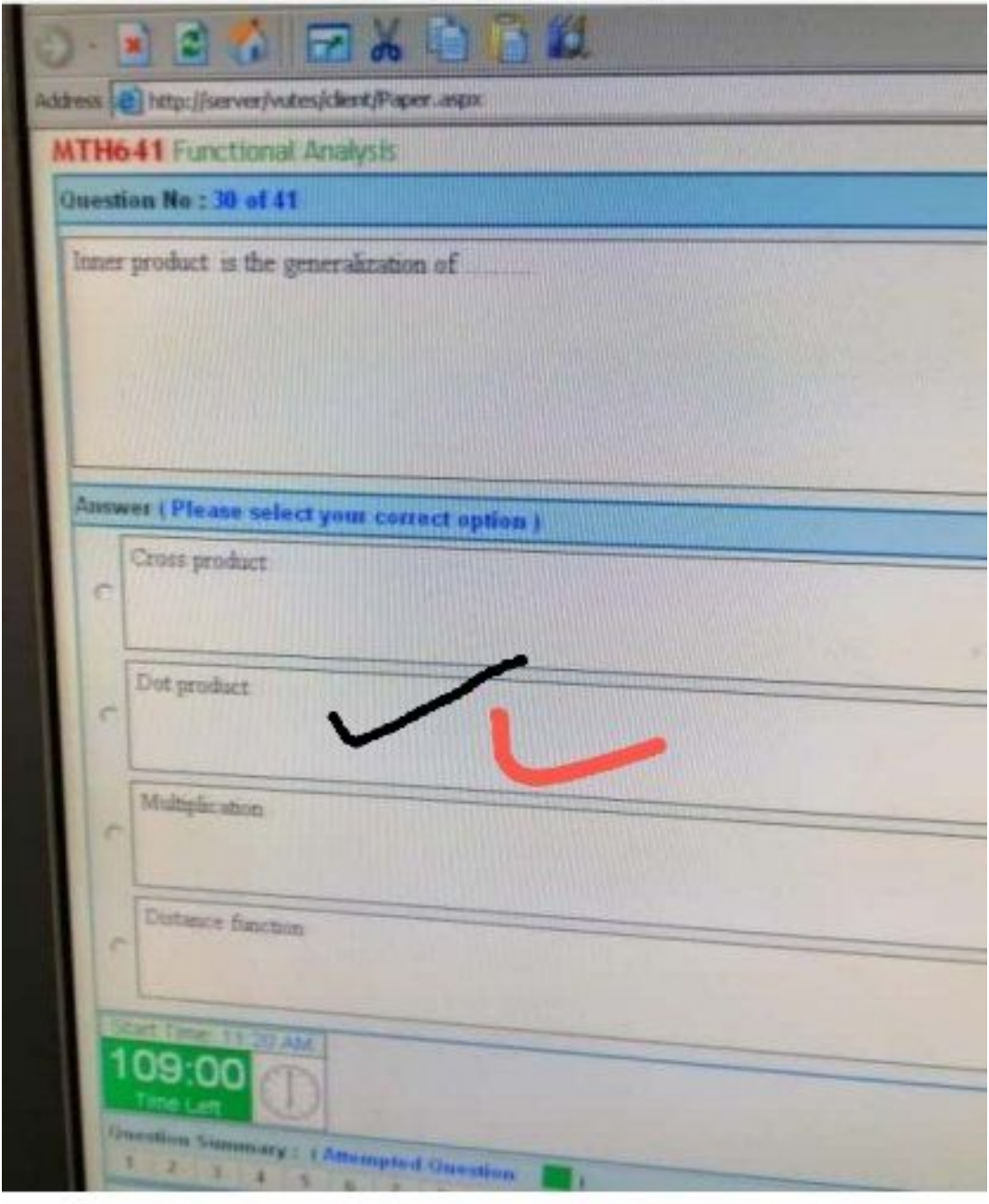
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MTH641 Functional Analysis

Question No : 18 of 41

Which of the following is true about the inner product space?

Answer (Please select your correct option)

- Every normed space is an inner product space
- Every inner product space is a normed space
- Every inner product space is Hilbert space
- None of these is true

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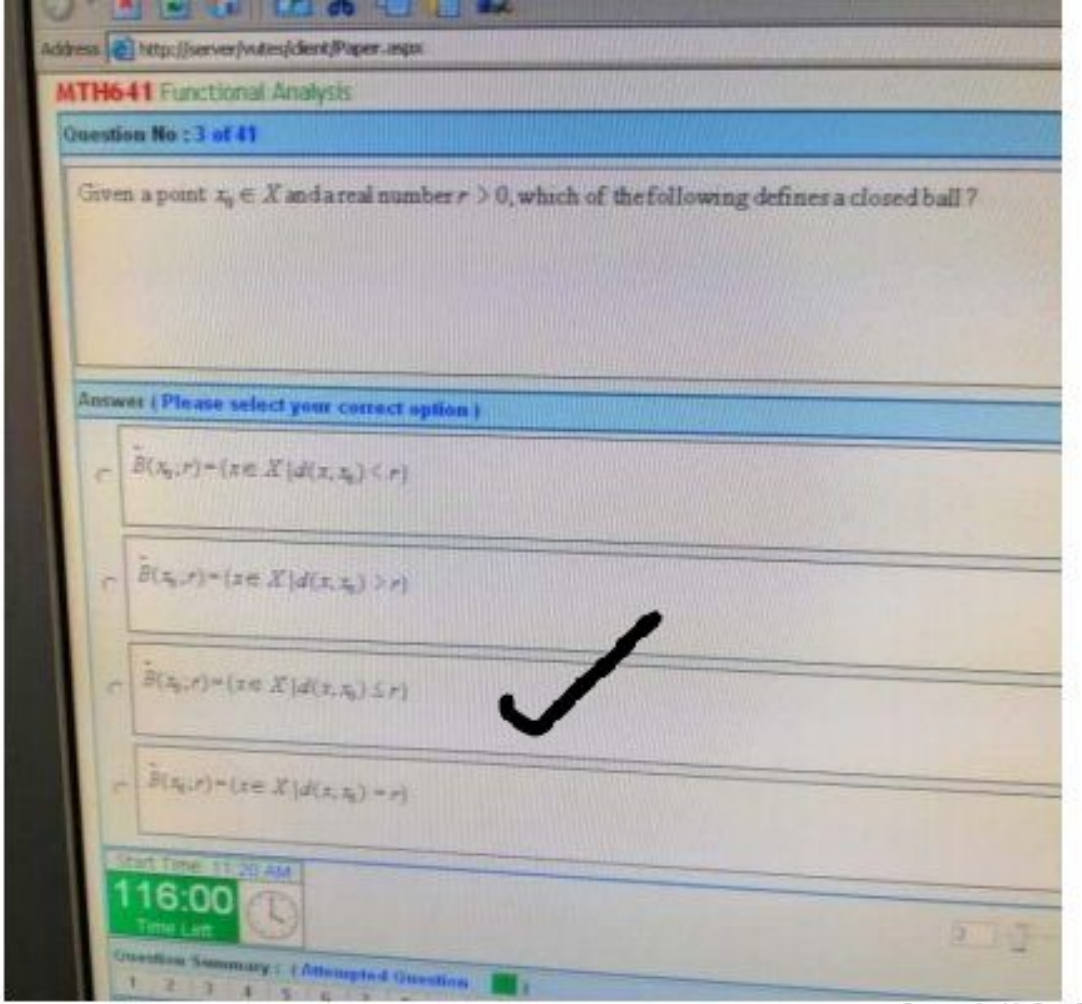
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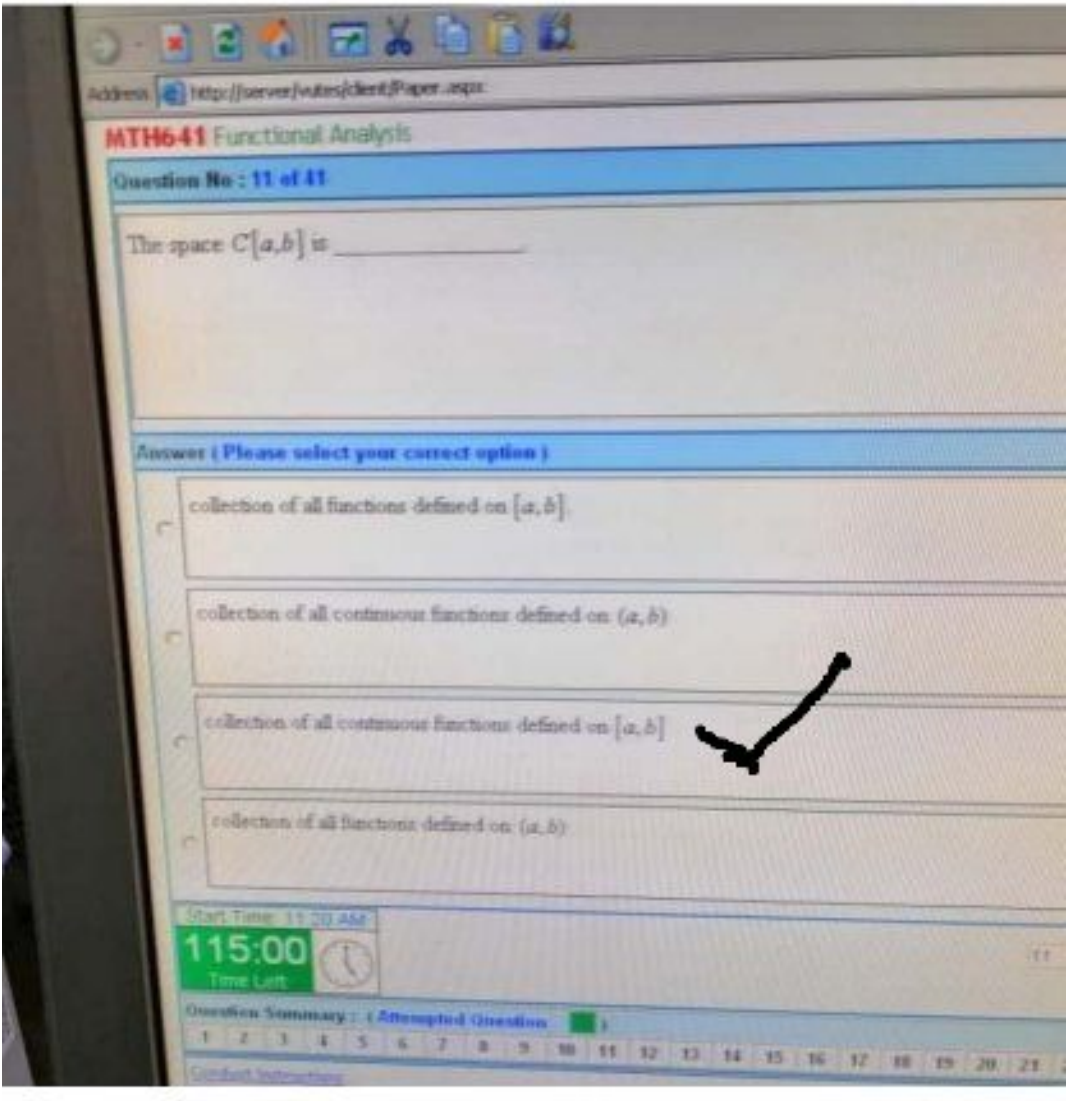
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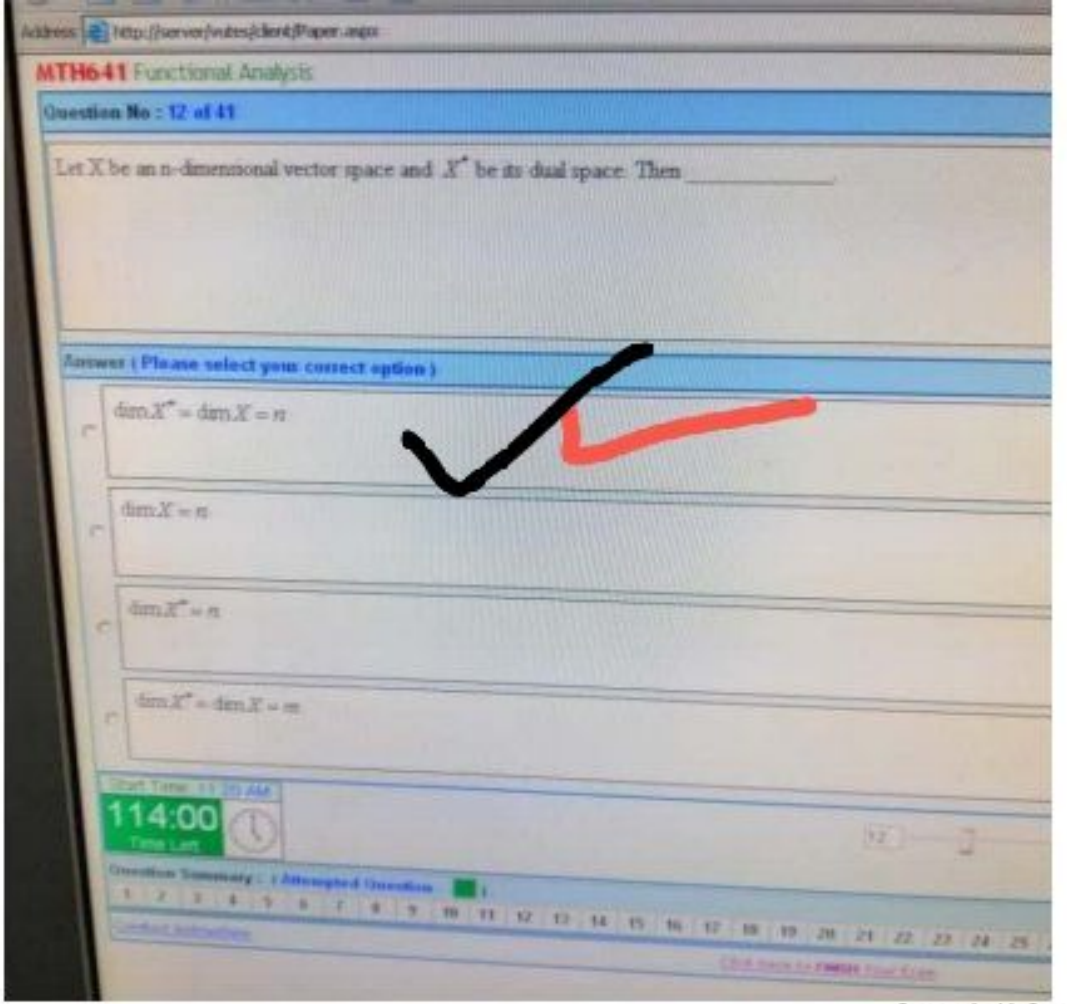
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MTH641 Functional Analysis

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

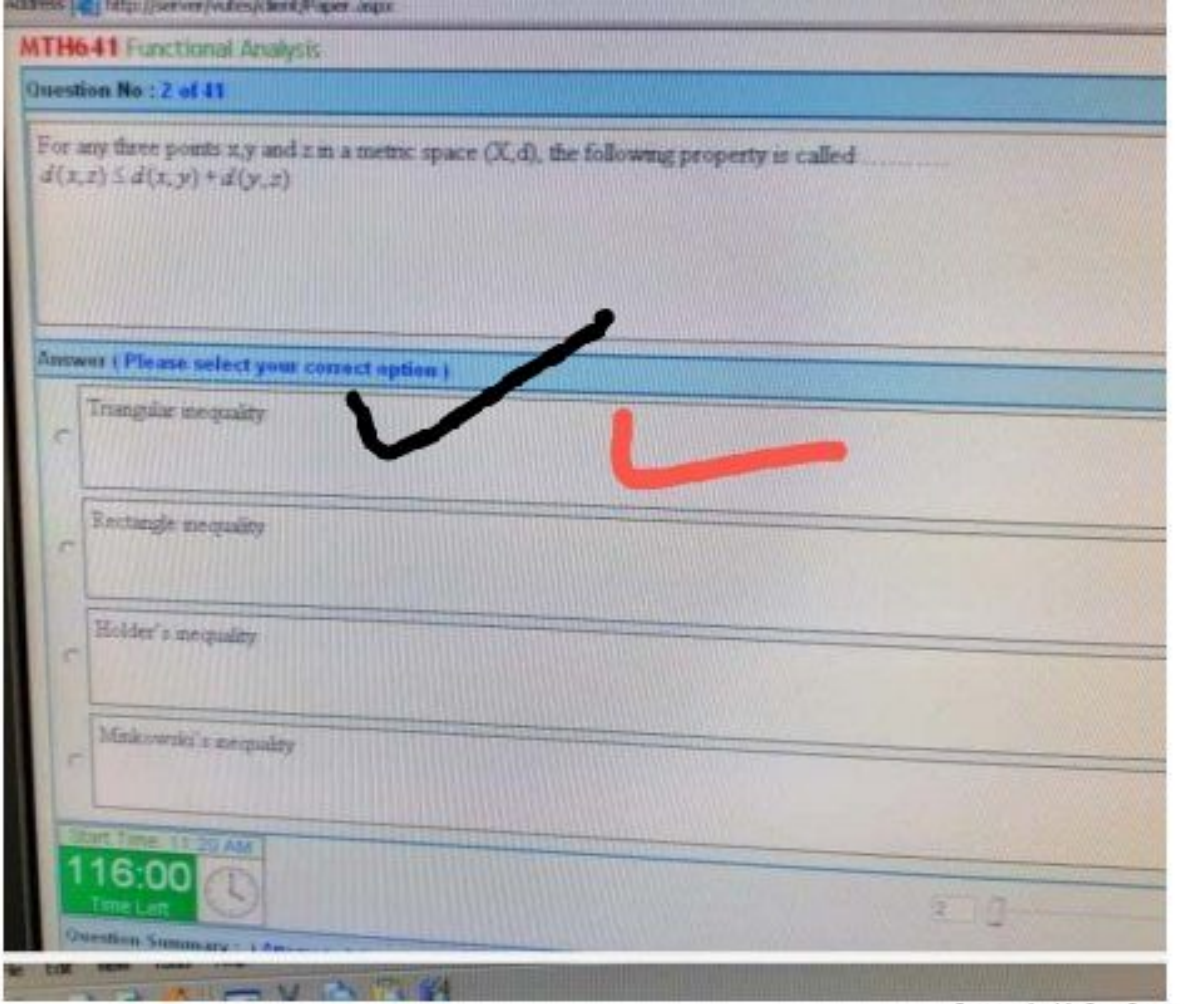
- (a,b)
- $[a,b]$
- Space R^n
- None of these

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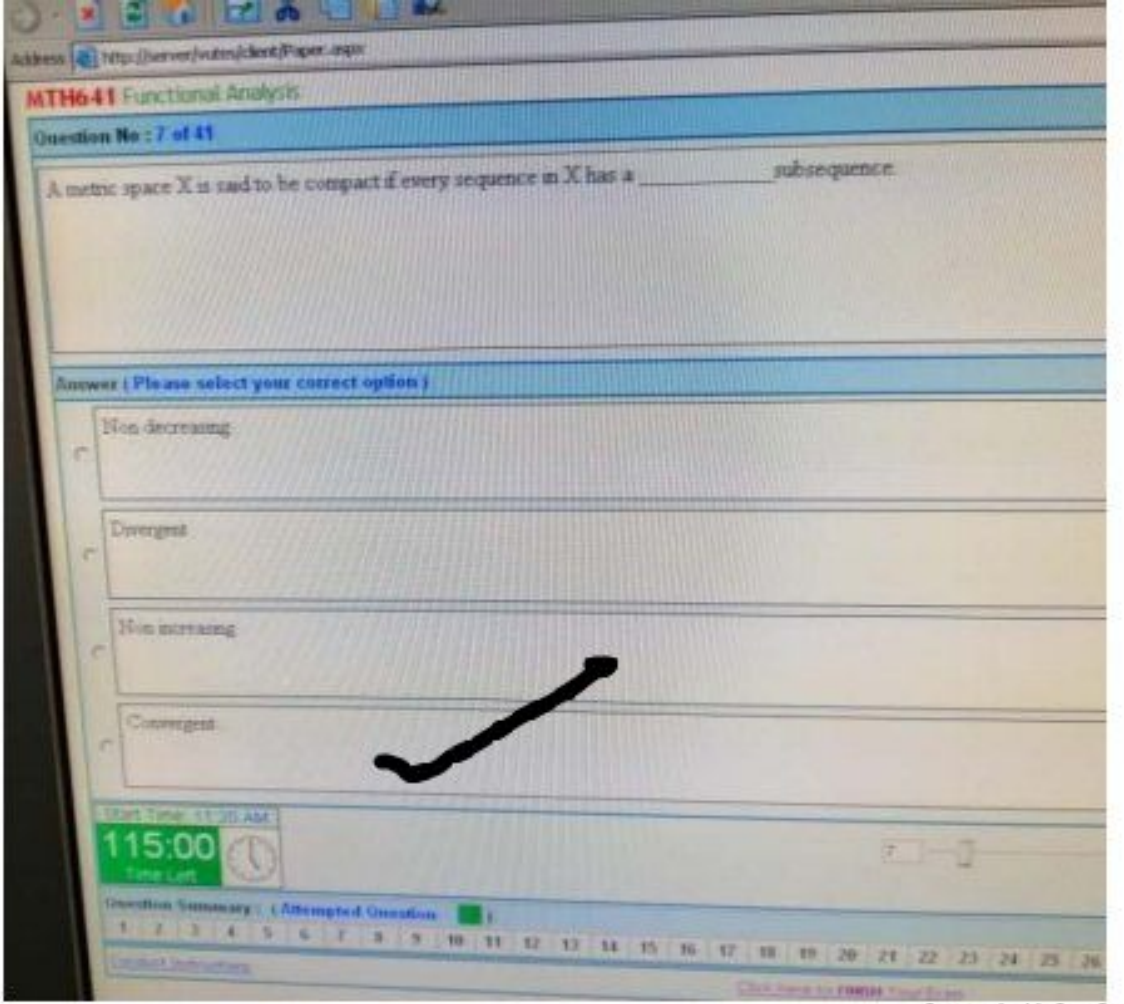
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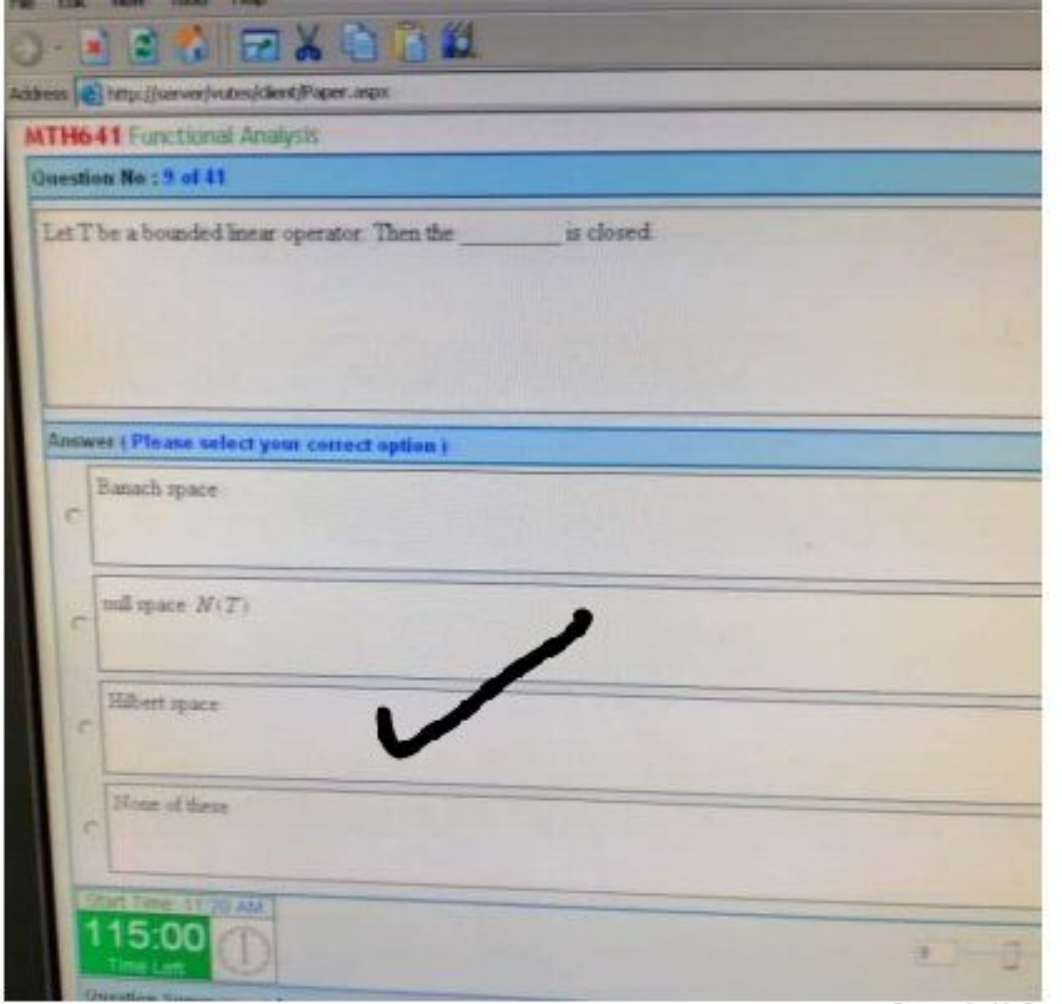
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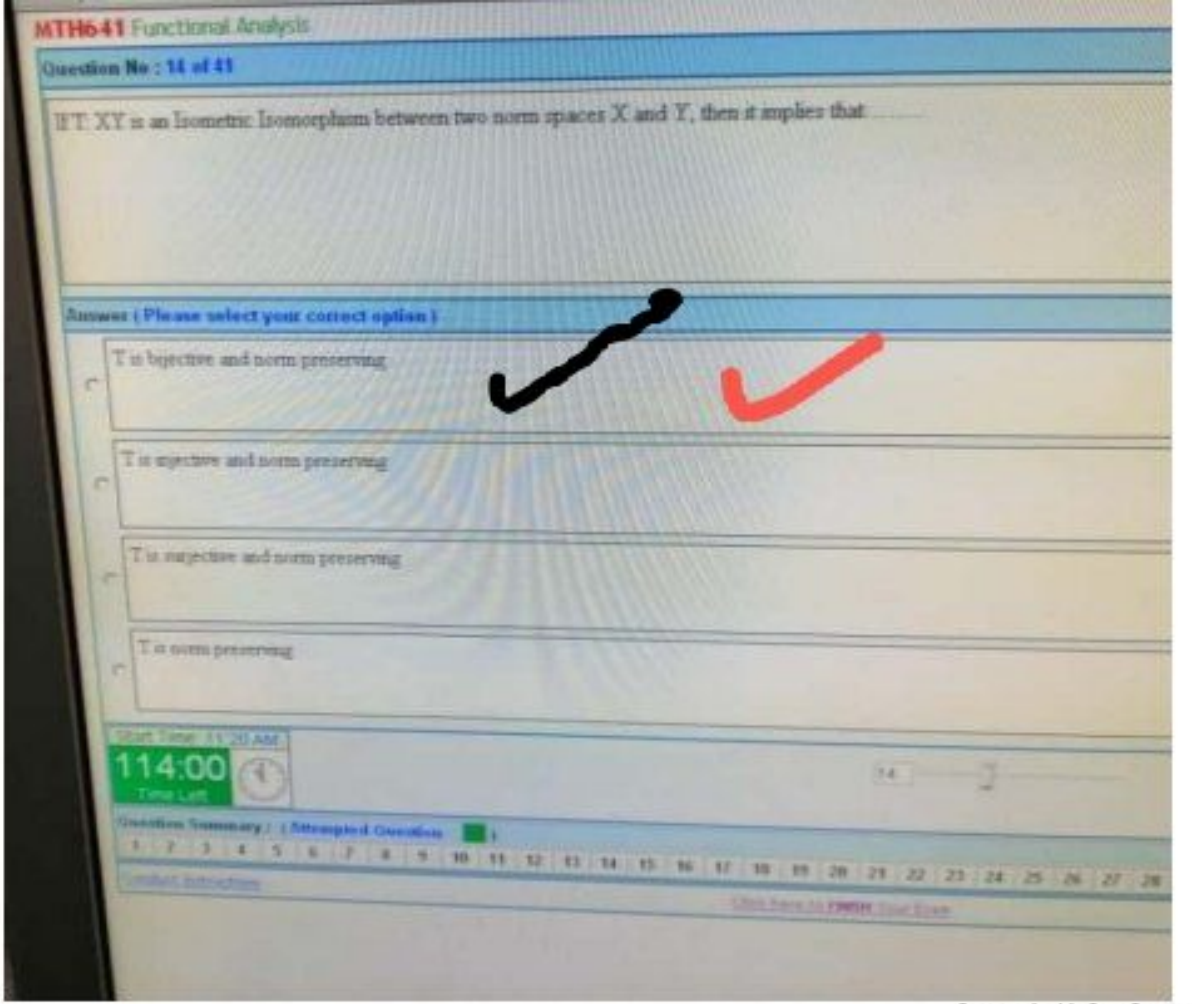
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MTH641 Functional Analysis

Question No : 6 of 41 Marks: 1 (Budg

Let $\{x_1, \ldots, x_n\}$ be _____ set of vectors in a normed space X . Then there is a number $c > 0$ such that for every choice of scalars $\alpha_1, \ldots, \alpha_n$, we have

$\|\alpha_1 x_1 + \ldots + \alpha_n x_n\| \geq c(|\alpha_1| + \ldots + |\alpha_n|)$

Answer (Please select your correct option)

- linearly independent
- linearly dependent
- orthogonal
- orthonormal

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 21 of 41

Every complete normed space is _____ space.

Answer (Please select your correct option)

- Hilbert
- Banach
- Unitary
- Euclidean

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Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then _____

Answer : Please select your correct option

- $x_0 = 0$
- $x_0 \neq 0$
- $x_0 = 1$
- $x_0 \neq 1$

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Question Summary : (Attempted Question)

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Question No : 15 of 41

The dual space of a normed space X contains all ..... on X.

Answer (Please select your correct option)

- Bounded linear functional
- Unbounded linear functional
- Bounded linear operator
- Unbounded linear operator

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Question Summary : (Attempted Question)

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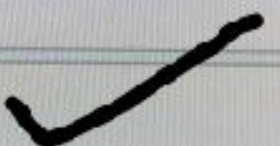
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Question No : 19 of 41

Every inner product space is a _____ space

Answer (Please select your correct option)

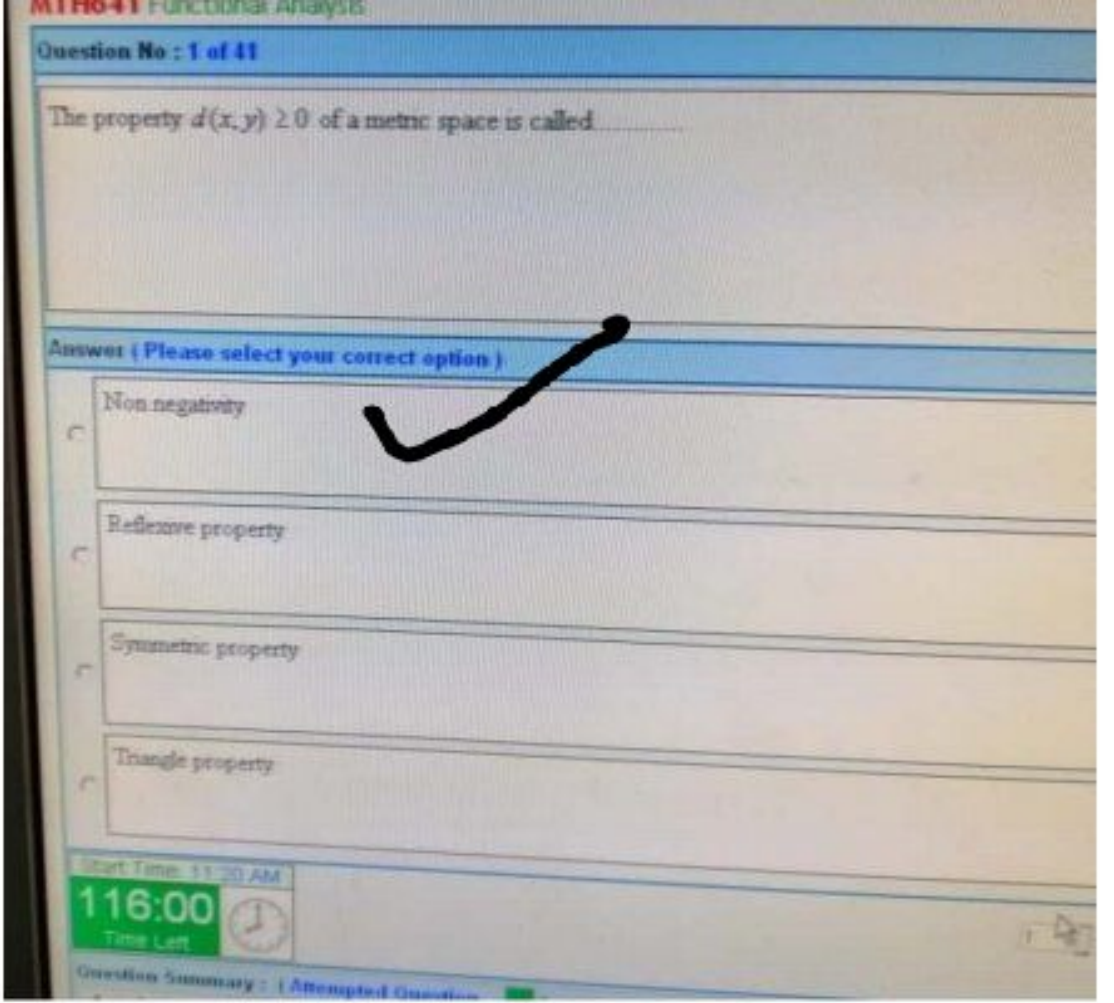
- Metric
- Normed
- Topological
- All of above



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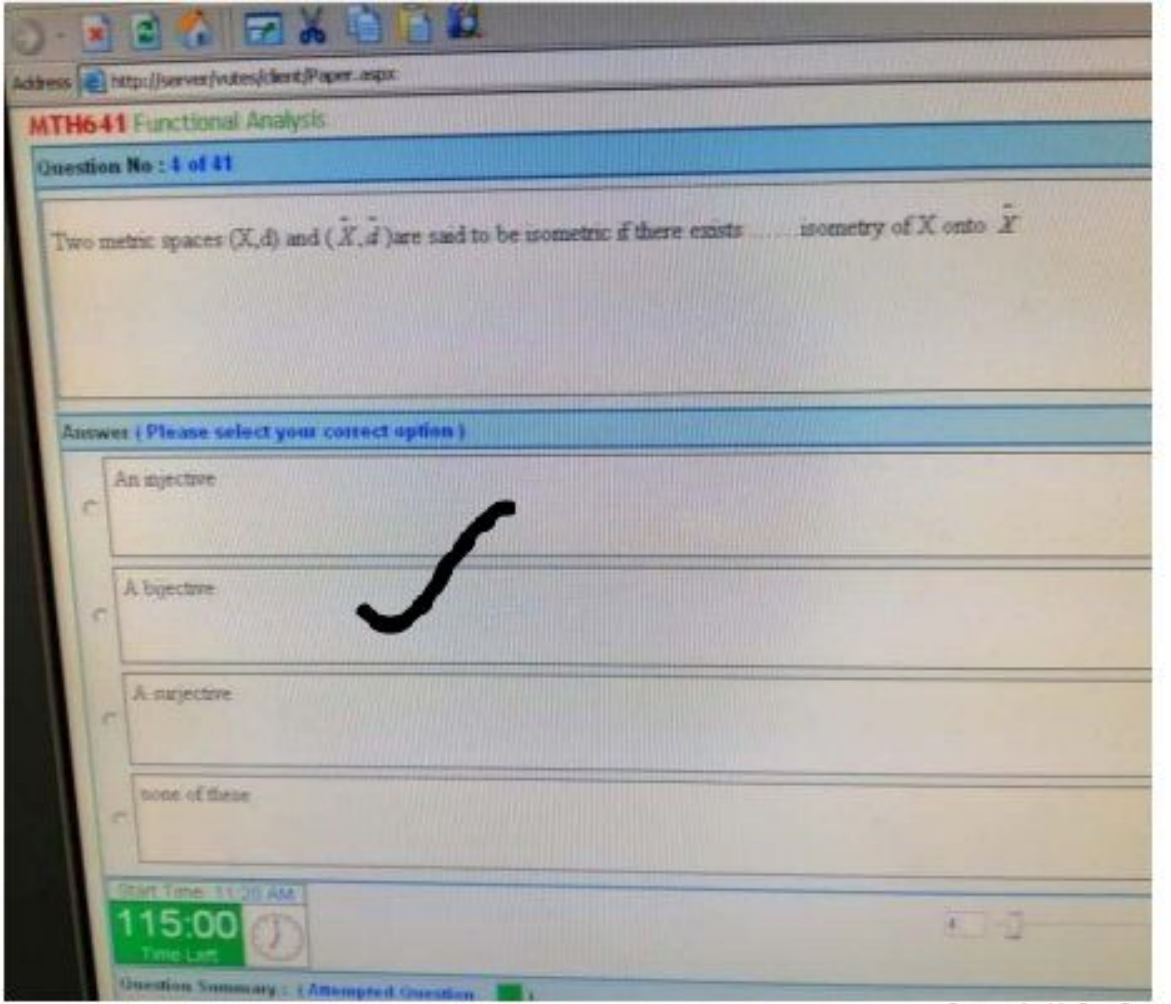
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MTH641 Functional Analysis

Question No : 26 of 41

$\|x+y\| \leq \|x\| + \|y\|$ is called.....

Answer (Please select your correct option)

- Triangle inequality
- Minkowski's inequality
- Holder's inequality
- Schwarz inequality

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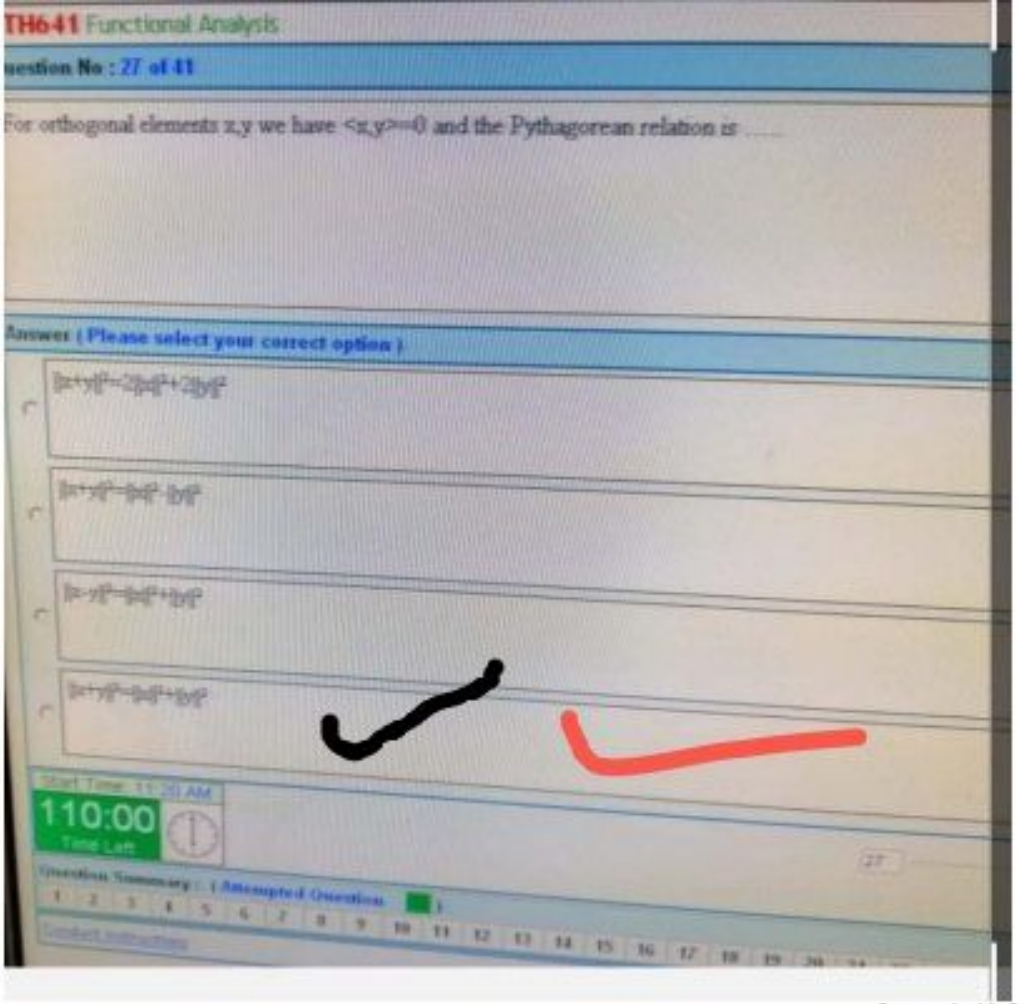
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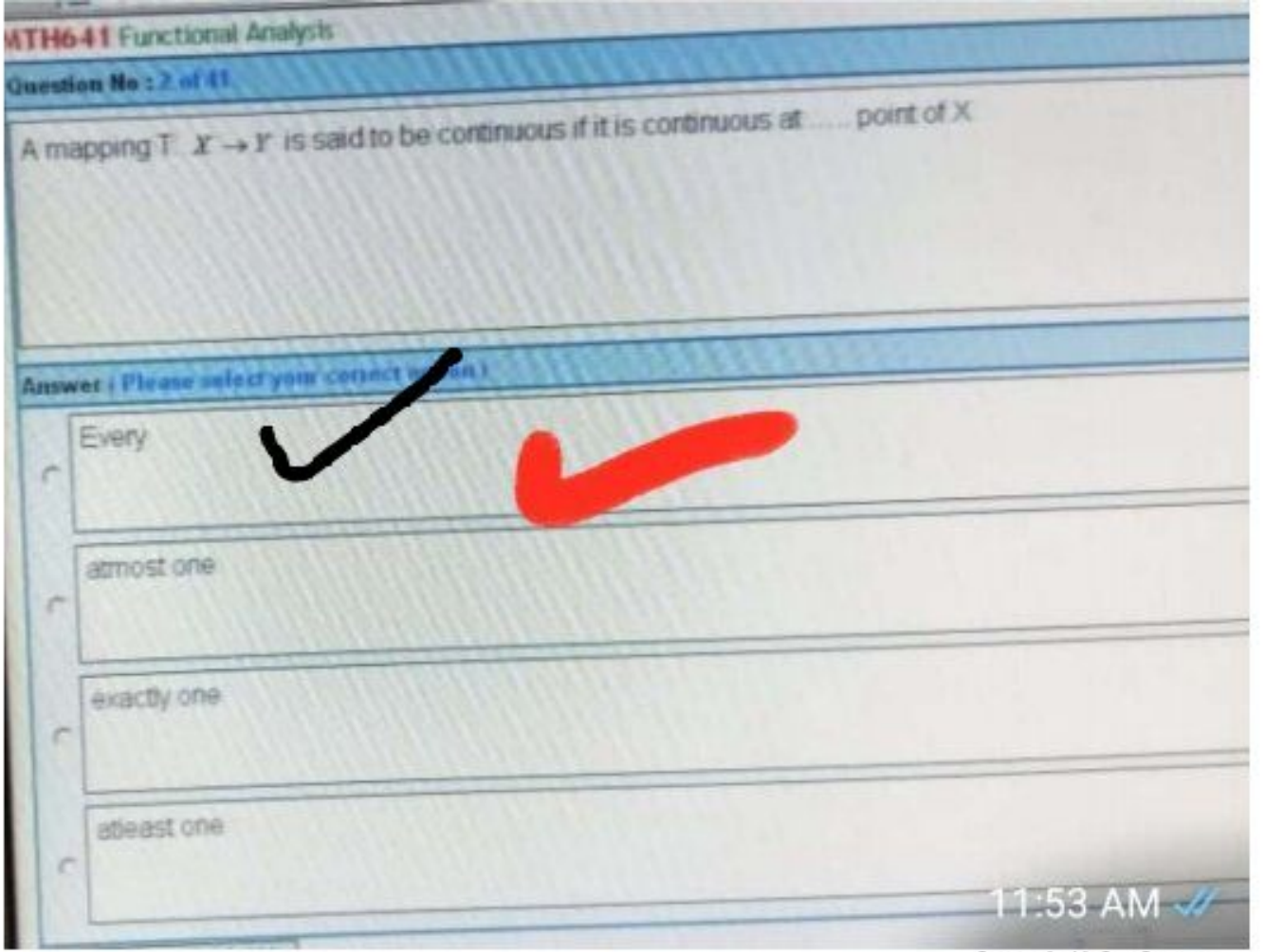
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MTH641 Functional Analysis

Question No : 13 of 41

A finite dimensional normed space is _____ to its second dual.

Answer (Please select your correct option)

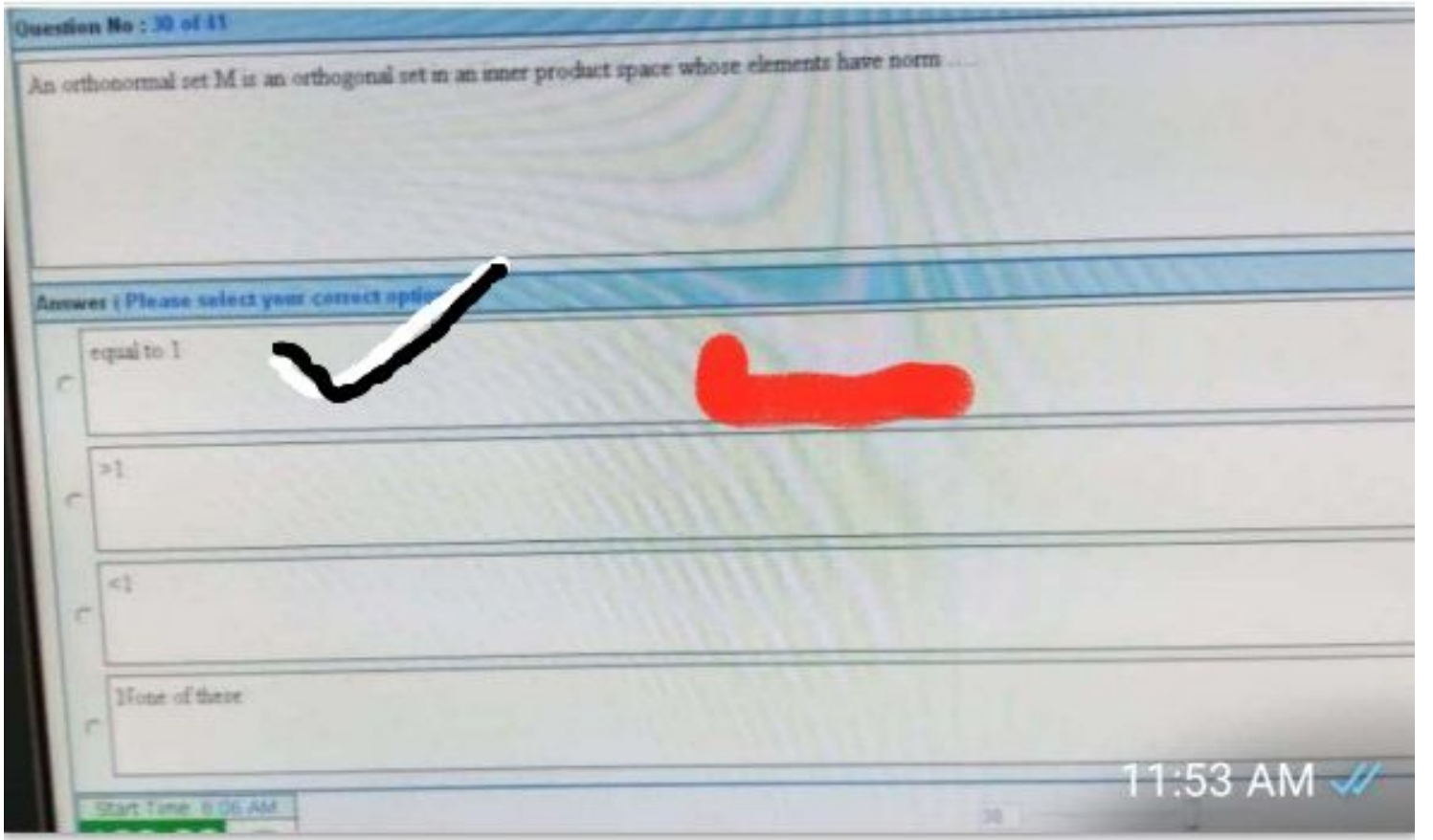
- isometric
- isomorphic
- homomorphic
- none of these



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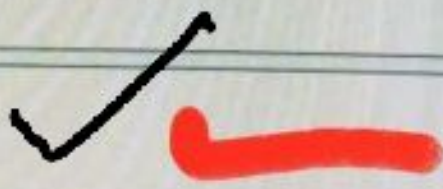
MTH641 Functional Analysis

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

- (a,b)
- $[a,b]$
- Space R^n
- None of these



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Question Summary : (Attempted Question)

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In an inner product space V, the function $\| \ \| : V \to R^+$ given by _____ defines a _____ on V

Answer (Please select your correct option):

- $|x| = \sqrt{(x,y)}$, $x, y \in V$, norm
- $|x| = \sqrt{(x,x)}$, $x \in V$, norm
- $|x| = \sqrt{(x,x)}$, $x \in V$, metric
- None of these

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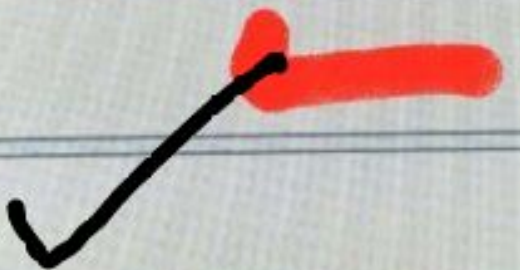
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A sequence (x_n) in a metric space is said to be non-decreasing if for all n,

Answer (Please select your correct option)

- $x_n \geq x_{n+1}$
- $x_n \leq x_{n+1}$
- $x_n < x_{n+1}$
- $x_n > x_{n+1}$



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MTH641 Functional Analysis

Question No : 1 of 41

If d is a metric on X, Then for all $x, y \in X$, $d(x,y)$ is....

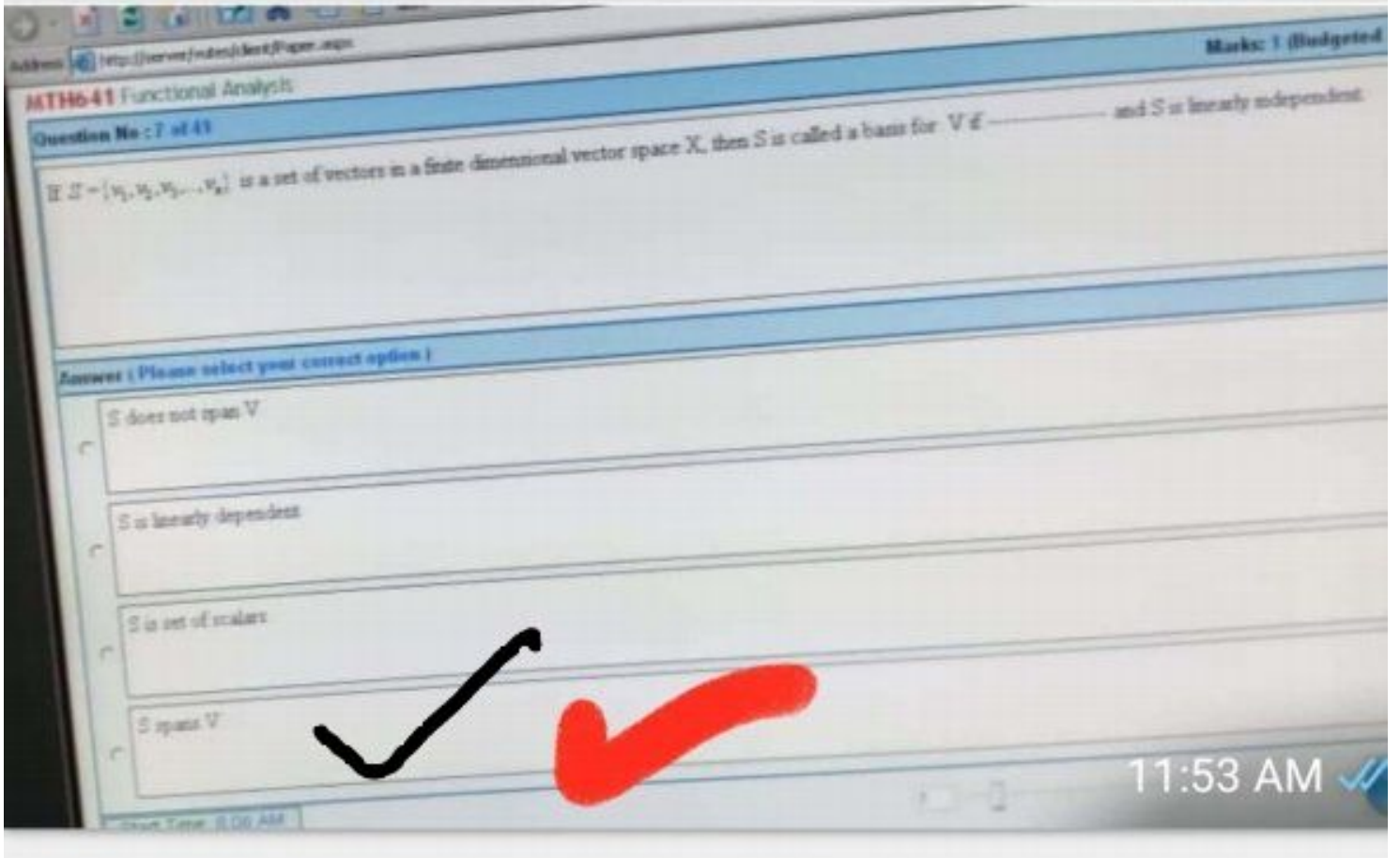
Answer (Please select your correct option)

- A non negative real number
- A negative real number
- Not a rational number
- A complex number

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MTH641 Functional Analysis

Question No : 13 of 41

A finite dimensional normed space is ..... to its second dual.

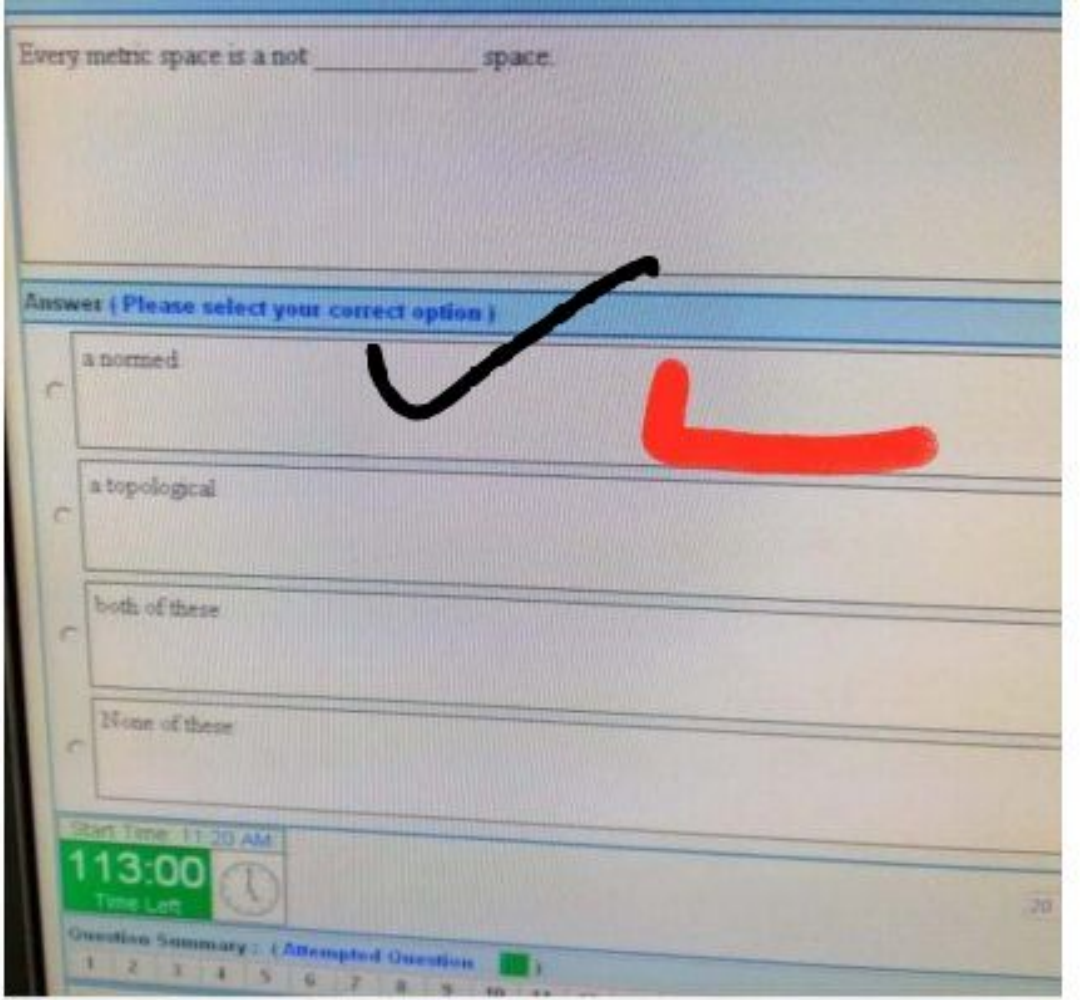
Answer (Please select your correct option)

- isometric
- isomorphic
- homomorphic
- none of these

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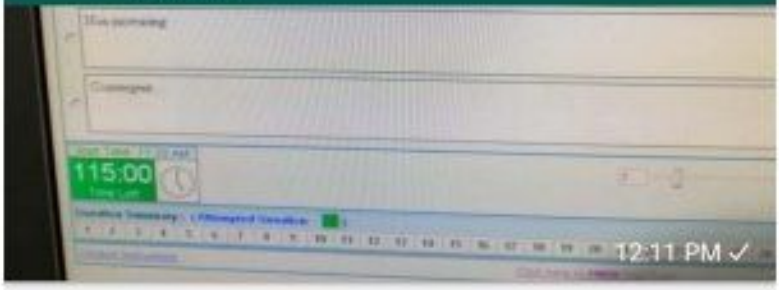
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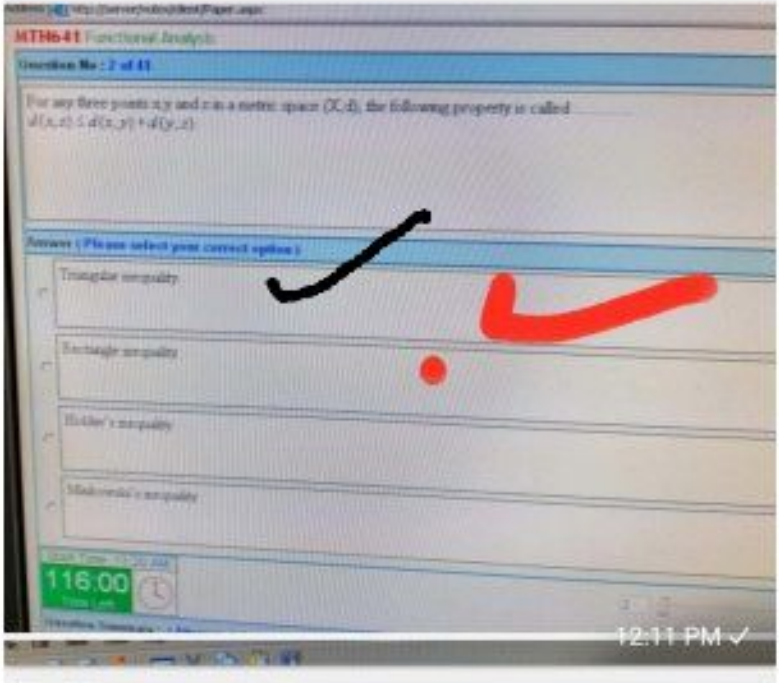
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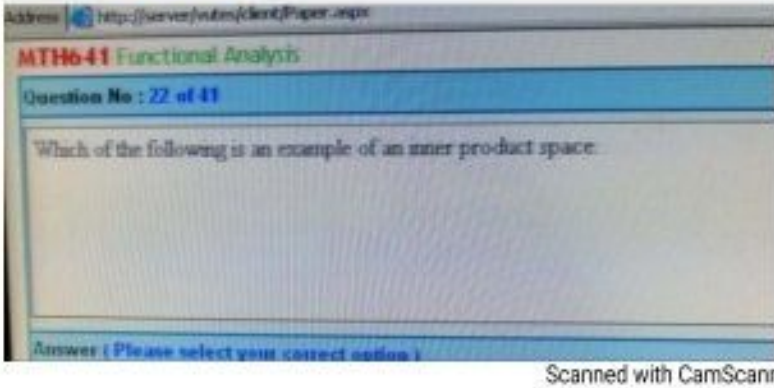


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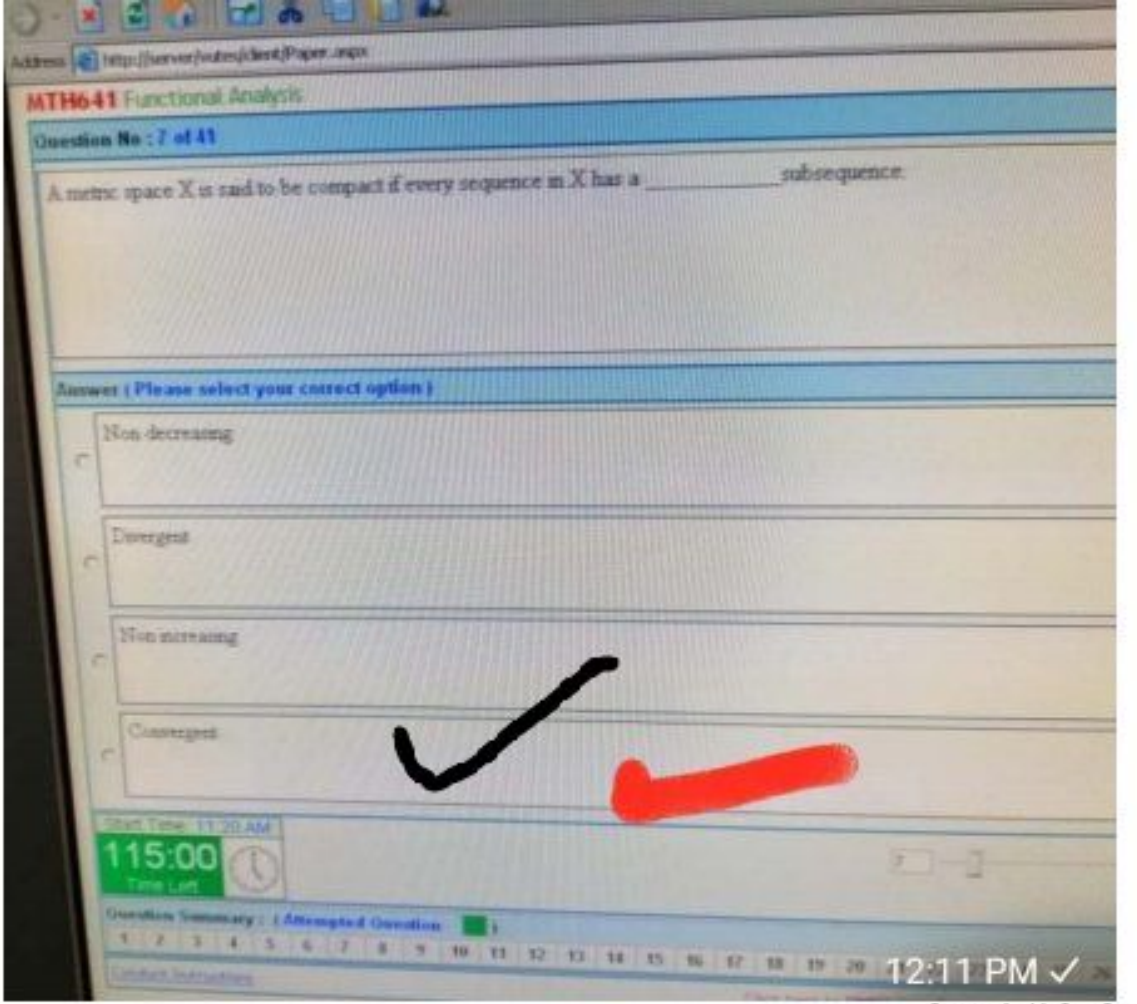






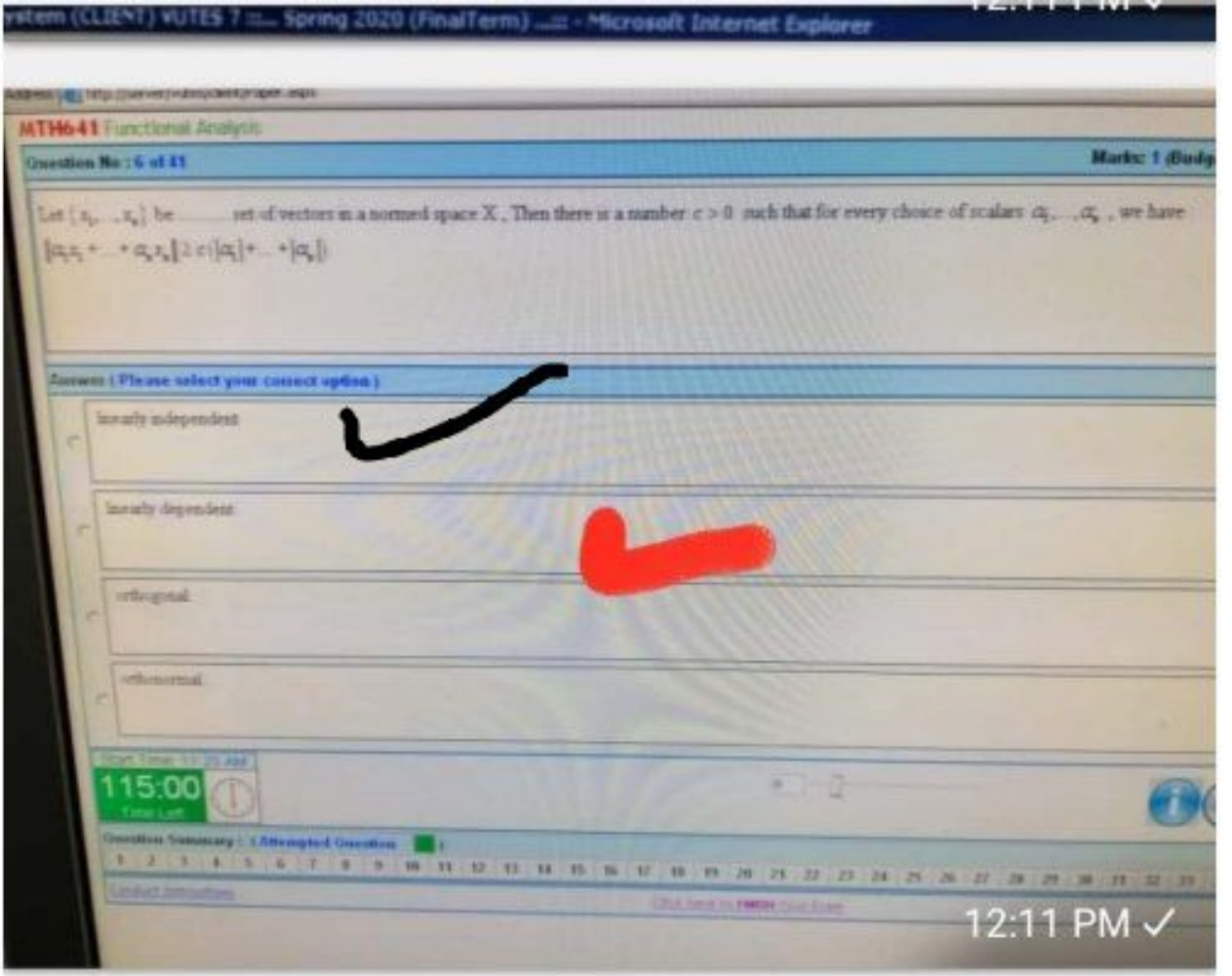
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Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then _____

Answer : Please select your correct option :

- $x_0 = 0$
- $x_0 \neq 0$
- $x_0 = 1$
- $x_0 \neq 1$

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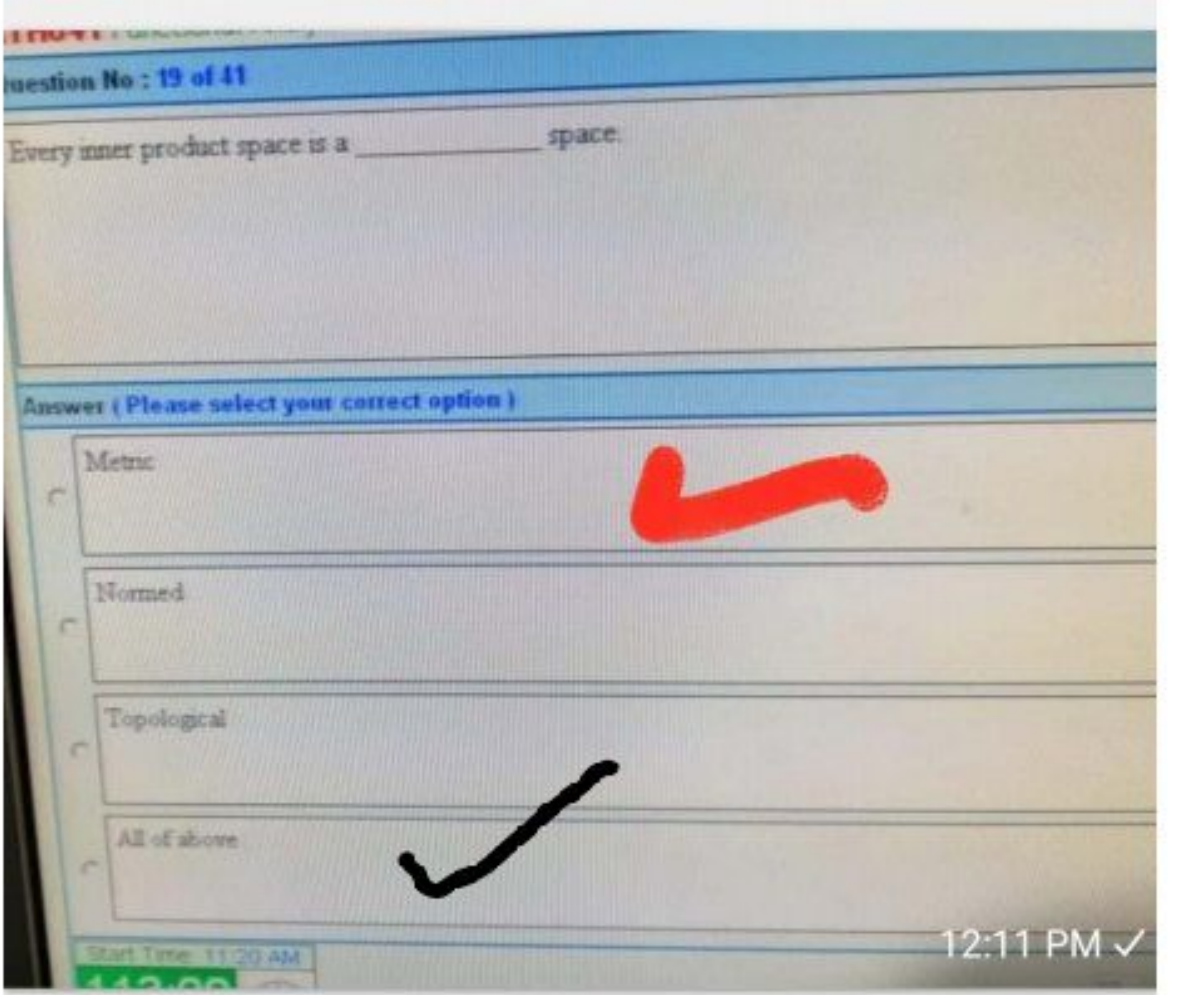
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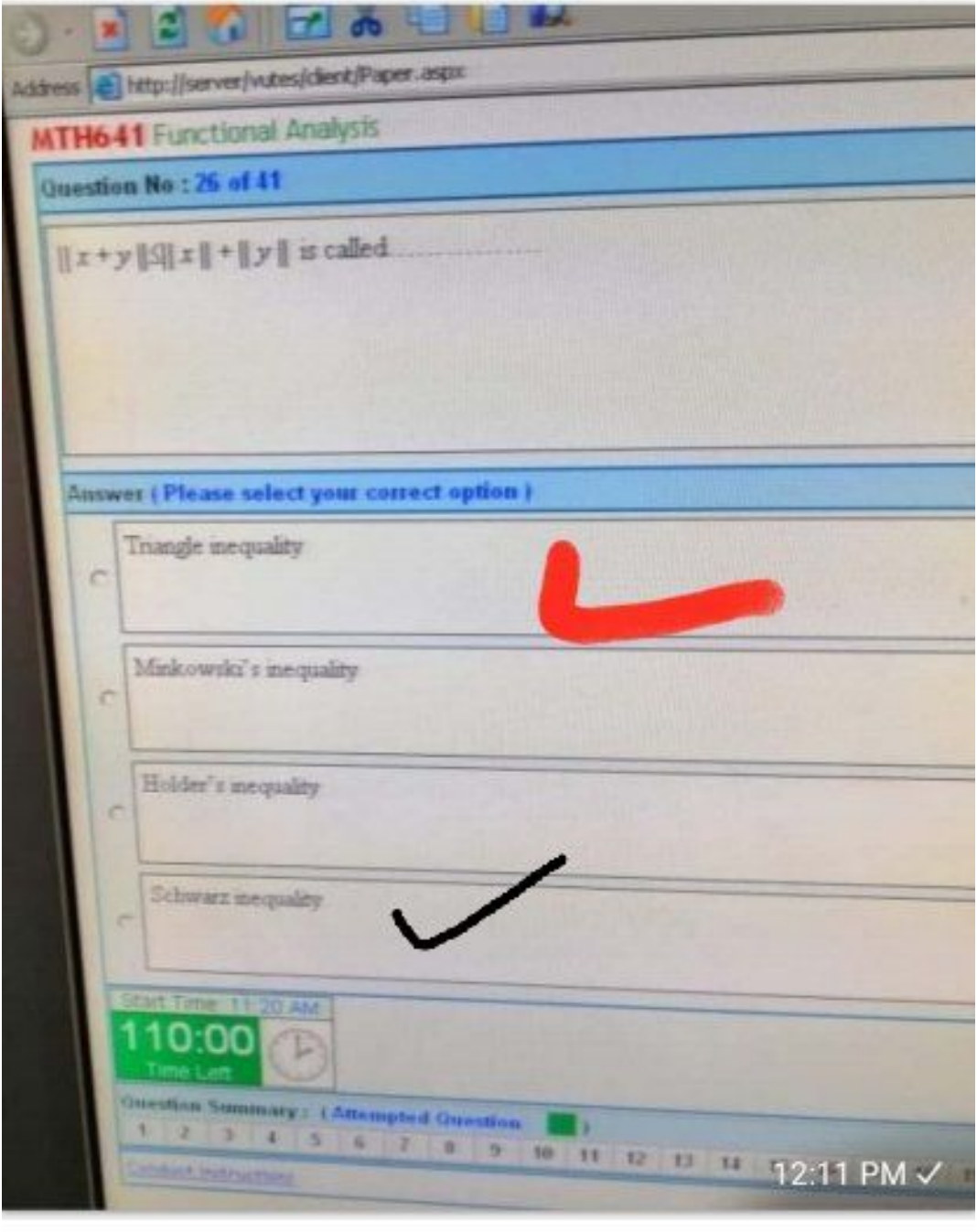
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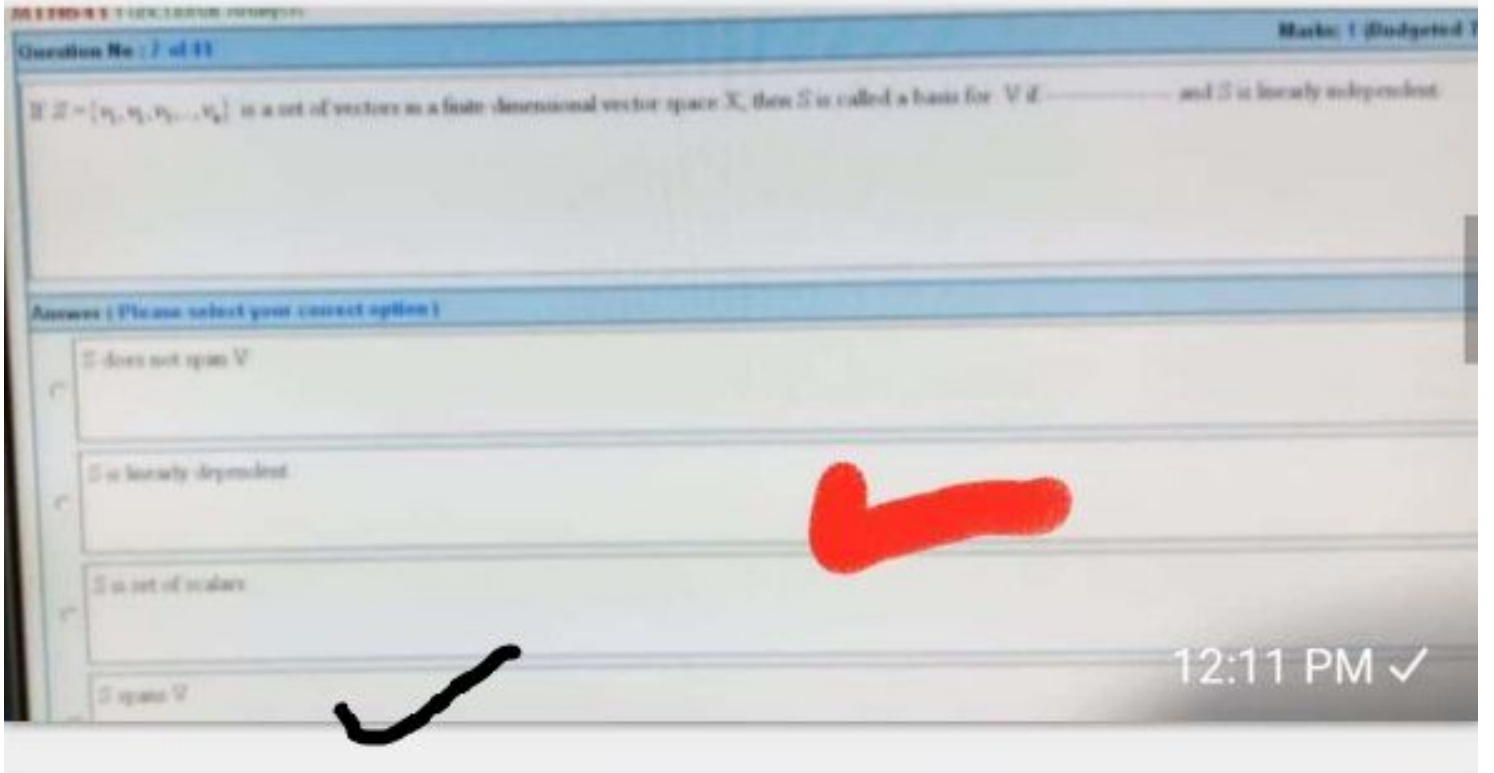
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MTH641 Functional Analysis

Question No : 11 of 41

Let T be a bounded linear operator. Then $x_n \to x$ (where $x_n, x \in D(T)$) implies _____

Answer (Please select your correct option)

- $x_n \to x$
- $Tx_n \to Tx$
- $Tx \to Tx_n$
- None of these

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MTH641 Functional Analysis

Question No : 27 of 41

For subsets A and B of an inner product space X, A is orthogonal to B if .....

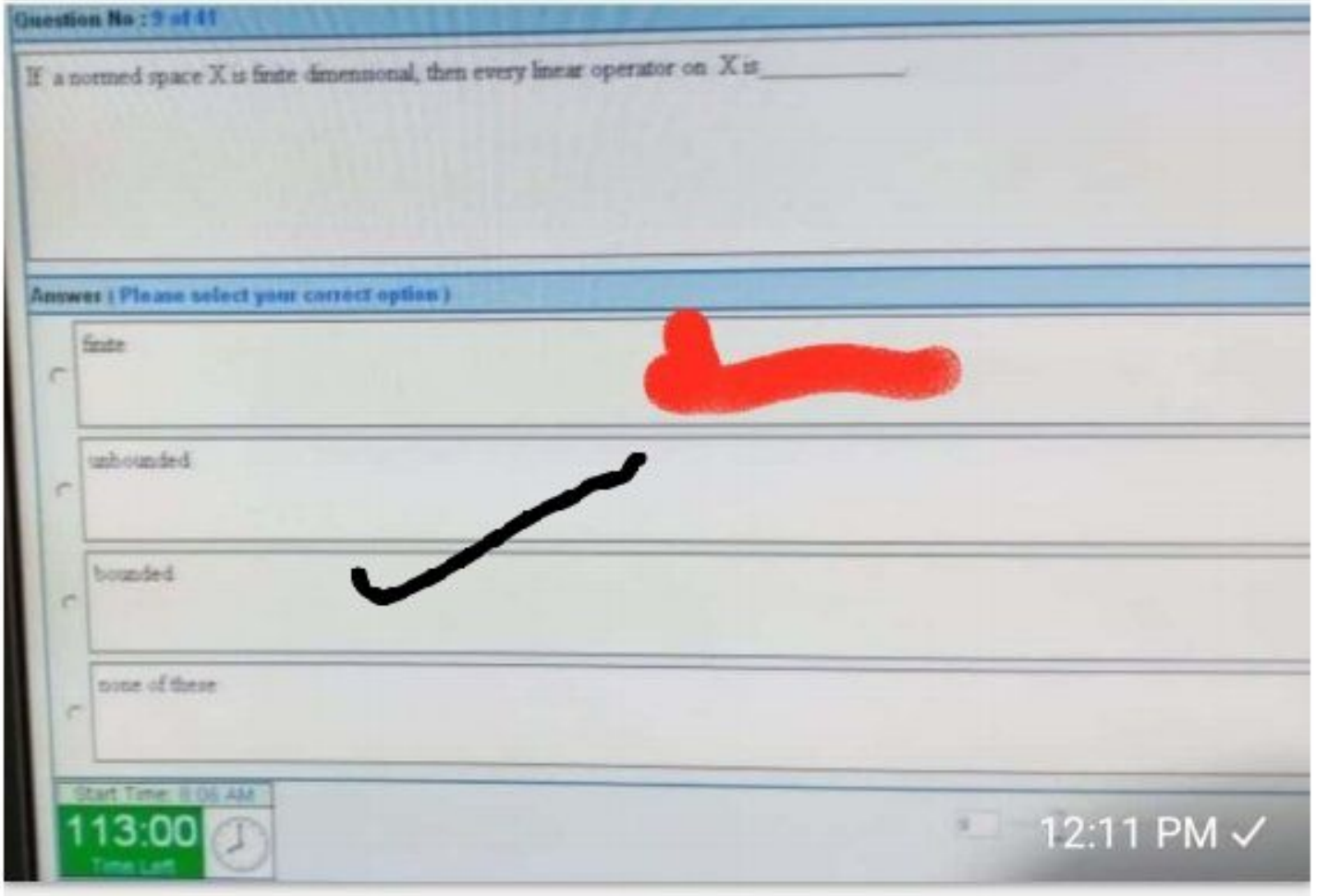
Answer (Please select your correct option)

- a is orthogonal to b for all $a \in A and b \in B$
- a is orthogonal to b for a fixed $a \in A and\ a\ fixed\ b \in B$
- a is orthogonal to b for a fixed $a \in A and\ b \in B$
- a is orthogonal to b for all $a \in A and\ a\ fixed\ b \in B$

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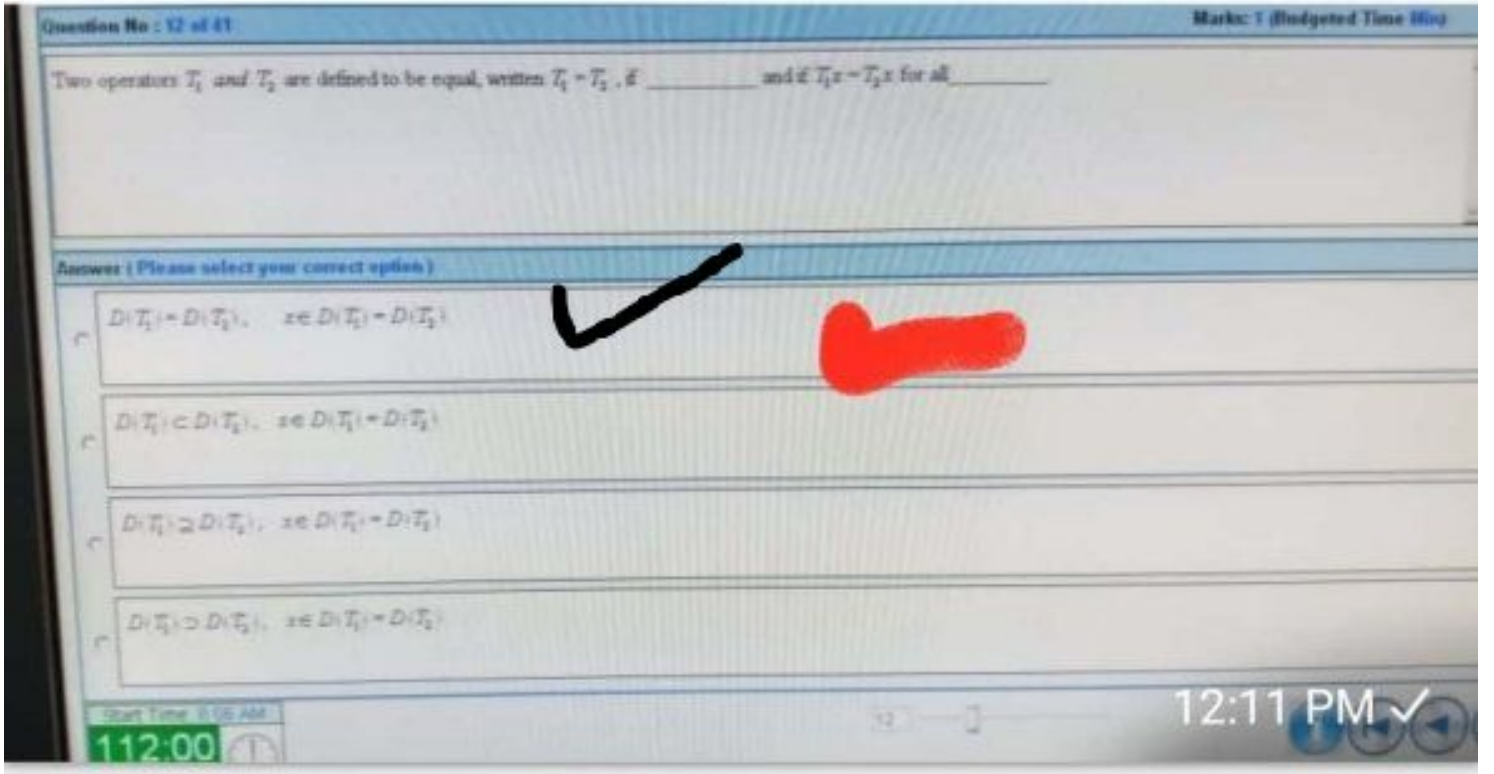
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Any pair of vectors in an orthogonal system is _____ to each other

Answer (Please select your correct option)

- Parallel
- Equal
- None of these
- Perpendicular

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MTH641 Functional Analysis

Question No : 18 of 41

Which of the following is true about the inner product space?

Answer (Please select your correct option)

- Every normed space is an inner product space.
- Every inner product space is a normed space.
- Every inner product space is Hilbert space.
- None of these is true.

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Question Summary : (Attempted Question)

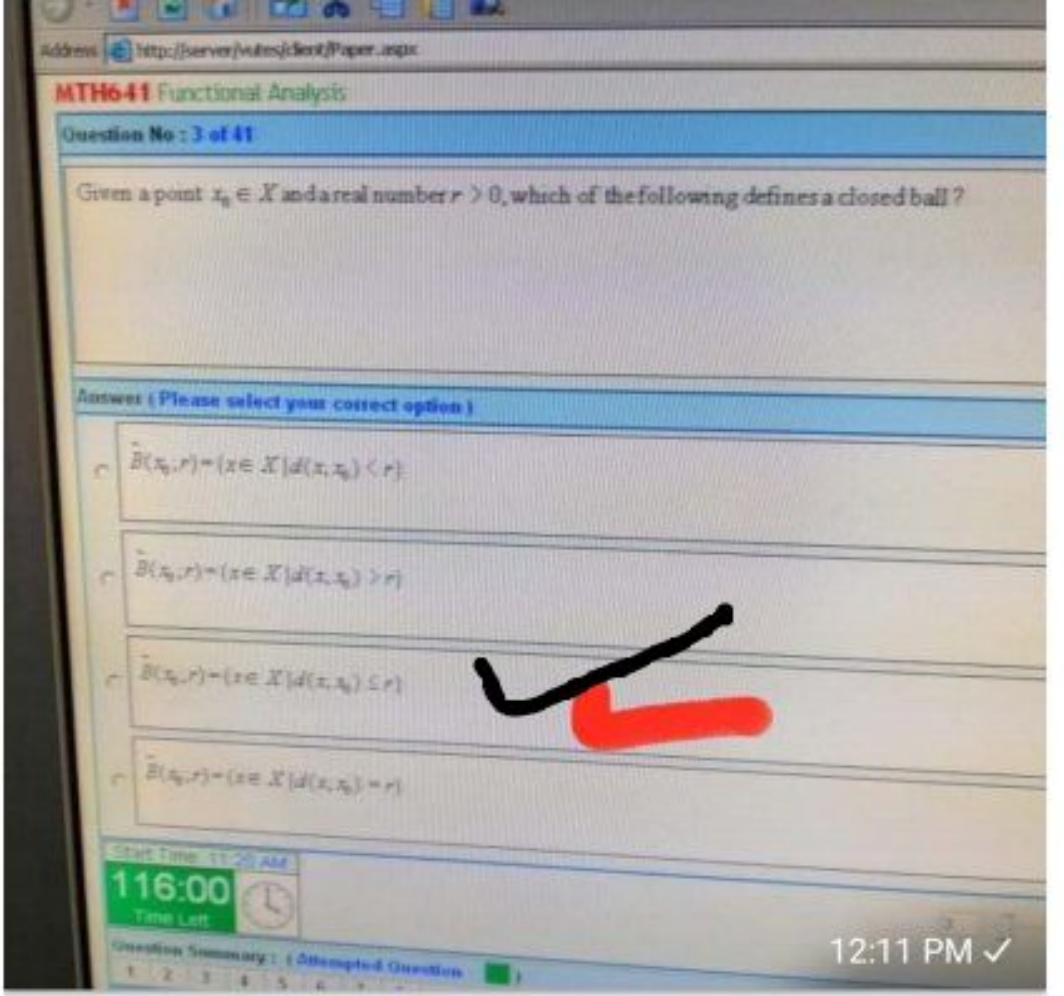
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MTH641 Functional Analysis

Question No : 35 of 41

Prove that if a set M is closed in X then it forms a complete subspace of X

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 28 of 41

P_n is the collection of all.....

Answer (Please select your correct option)

- Polynomials of degree greater then equal to 'n'
- Polynomials of degree less then equal to 'n'
- Polynomials of degree equal to 'n'
- Polynomials of degree less then 'n'

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Question Summary : (Attempted

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MTH641 Functional Analysis

Question No : 29 of 41

The pair $(V, \langle .,. \rangle)$ is called .....space

Answer (Please select your correct option)

- Norm
- Metric
- Inner product
- Vector

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MTH641 Functional Analysis

Question No : 16 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

- $\langle ax, y\rangle = \langle x, ay\rangle$
- $\langle ax + y, z\rangle = \langle x, z\rangle + a\langle y, z\rangle$
- $\langle x + y, z\rangle = \langle x, z\rangle + \langle y, z\rangle$
- $\langle x, y\rangle = \overline{\langle z, y\rangle}$

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 23 of 41

Which of the following is not true for the space $C[a,b]$ with norm defined by $\| x \| = \max_{t \in [a,b]} | x(t) |$?

Answer (Please select your correct option)

- It is a complete metric space
- It is a complete norm space
- It is a complete inner product space
- It is a vector space

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 5 of 41

A _____ Y of a Banach space X is _____ if and only if the set Y is closed in X

Answer (Please select your correct option)

- subspace . complete
- space. complete
- subspace. open
- space. open

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Question Summary : (Attempted Quest...

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MTH641 Functional Analysis

Question No : 20 of 41

Every metric space is a not _____ space.

Answer (Please select your correct option)

- a normed
- a topological
- both of these
- None of these

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 39 of 41

Show that canonical mapping is linear.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 17 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

- $\langle x,x\rangle \geq 0$
- $\langle \alpha x+y,z\rangle = \langle x,z\rangle + \alpha\langle y,z\rangle$
- $\langle \alpha x,y\rangle = \langle x,\alpha y\rangle$
- $\langle x,y\rangle = \overline{\langle x,y\rangle}$

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Question Summary : (Attempted Question)

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Question No : 37 of 41

Define norm and inner product for which the space R^2 is an inner product space.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 24 of 41

The space .....is not a Hilbert space

Answer (Please select your correct option)

- l^p with $p \neq 2$
- Euclidean space R^n
- Unitary Space C^n
- l^p with $p = 2$

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MTH641 Functional Analysis

Question No : 30 of 41

Inner product is the generalization of .....

Answer (Please select your correct option)

- Cross product
- Dot product
- Multiplication
- Distance function

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 32 of 41

Define linear functional

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 18 of 41

Which of the following is true about the inner product space?

Answer (Please select your correct option)

- Every normed space is an inner product space.
- Every inner product space is a normed space. ✓
- Every inner product space is Hilbert space.
- None of these is true.

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 10 of 41

A linear functional f with domain $D(f)$ in a normed space is continuous if and only if f is _____.

Answer (Please select your correct option)

- continuous
- bounded
- linear
- non linear

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 3 of 41

Given a point $x_0 \in X$ and a real number $r > 0$, which of the following defines a closed ball ?

Answer (Please select your correct option)

- $\bar{B}(x_0;r)=\{x\in X \mid d(x,x_0) < r\}$
- $\bar{B}(x_0;r)=\{x\in X \mid d(x,x_0) > r\}$
- $\bar{B}(x_0;r)=\{x\in X \mid d(x,x_0) \leq r\}$ ✓
- $\bar{B}(x_0;r)=\{x\in X \mid d(x,x_0) = r\}$

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 11 of 41

The space $C[a,b]$ is _____.

Answer (Please select your correct option)

- collection of all functions defined on $[a,b]$.
- collection of all continuous functions defined on (a,b)
- collection of all continuous functions defined on $[a,b]$ ✓
- collection of all functions defined on (a,b)

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

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Question No : 41 of 41

Marks: 5 (Budgeted Time Min)

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $(ST)^* = T^* S^*$ (assuming $H_2 = H_1$).

Answer (Please click here to Add Answer)

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 34 of 41

Define unitary operator.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 36 of 41 **Marks: 3 (Budgete**

Let $T : D(T) \to Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then show that if T is continuous at a single point, it is continuous.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 12 of 41

Let X be an n-dimensional vector space and X^* be its dual space. Then _____.

Answer (Please select your correct option)

- $\dim X^* = \dim X = n$
- $\dim X = n$
- $\dim X^* = n$
- $\dim X^* = \dim X = m$

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

- $[a,b]$
- $[a,b]$
- Space R^n
- None of these

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MTH641 Functional Analysis

Question No : 2 of 41

For any three points x,y and z in a metric space (X,d), the following property is called .....
$d(x,z) \leq d(x,y) + d(y,z)$

Answer (Please select your correct option)

- Triangular inequality
- Rectangle inequality
- Holder's inequality
- Minkowski's inequality

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Question Summary : (Attempted

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MTH641 Functional Analysis

Question No : 7 of 41

A metric space X is said to be compact if every sequence in X has a _____ subsequence.

Answer (Please select your correct option)

- Non decreasing
- Divergent
- Non increasing
- Convergent

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 9 of 41

Let T be a bounded linear operator. Then the _____ is closed.

Answer (Please select your correct option)

- Banach space
- null space $N(T)$
- Hilbert space
- None of these

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Question Summary : (Attempted Question

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MTH641 Functional Analysis

Question No : 14 of 41

If T: XY is an Isometric Isomorphism between two norm spaces X and Y, then it implies that.....

Answer (Please select your correct option)

- T is bijective and norm preserving
- T is injective and norm preserving
- T is surjective and norm preserving
- T is norm preserving

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 6 of 41 **Marks: 1 (Budgeted**

Let $\{x_1,....,x_n\}$ be .....set of vectors in a normed space X. Then there is a number $c > 0$ such that for every choice of scalars $\alpha_1,....,\alpha_n$, we have

$|\alpha_1 x_1 + ... + \alpha_n x_n| \geq c(|\alpha_1| + ... + |\alpha_n|)$

Answer (Please select your correct option)

- linearly independent
- linearly dependent
- orthogonal
- orthonormal

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 21 of 41

Every complete normed space is _____ space.

Answer (Please select your correct option)

- Hilbert
- Banach
- Unitary
- Euclidean

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MTH641 Functional Analysis

Question No : 13 of 41 **Marks:**

Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then _____.

Answer (Please select your correct option)

- $x_0 = 0$
- $x_0 = 0$
- $x_0 = 1$
- $x_0 = 1$

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13

Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 15 of 41

The dual space of a normed space X contains all ..... on X.

Answer (Please select your correct option)

- Bounded linear functional
- Unbounded linear functional
- Bounded linear operator
- Unbounded linear operator

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 19 of 41

Every inner product space is a _____ space.

Answer (Please select your correct option)

- Metric
- Normed
- Topological
- All of above ✓

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MTH641 Functional Analysis

Question No : 33 of 41

Write generalized form of orthogonal system.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 1 of 41

The property $d(x, y) \geq 0$ of a metric space is called.....

Answer (Please select your correct option)

- Non negativity
- Reflexive property
- Symmetric property
- Triangle property

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MTH641 Functional Analysis

Question No : 31 of 41

Show that the sequence $(x_n) = (1 + \frac{1}{n})$ is a Cauchy sequence with the usual metric defined on the set of real numbers.

Answer (Please click here to Add Answer)

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Ma

T and X, Y are normed spaces, then show that if T is continuous at a single point, t

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MTH641 Functional Analysis

Question No : 25 of 41 **Marks: 1 (Budge**

Isomorphism of an inner product space onto an inner product space over the same field is a .....linear operator that preserves inner product.

Answer (Please select your correct option)

- Bijective
- Injective
- Surjective
- None of these

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 4 of 41

Two metric spaces (X,d) and $(\bar{X},\bar{d})$ are said to be isometric if there exists ..... isometry of X onto $\bar{X}$

Answer (Please select your correct option)

- An injective
- A bijective
- A surjective
- none of these

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 38 of 41

Let H_1 and H_2 be Hilbert spaces. $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that

$\langle T^* y, x \rangle = \langle y, Tx \rangle \qquad (x \in H_1, y \in H_2)$.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 8 of 41 **Marks: 1 (Budgeted T**

Let $T : D(T) \to Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then T is _____ if and only if T is bounded

Answer (Please select your correct option)

- continuous
- linear
- bounded
- none of these

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 40 of 41

Show that if $\{x_n\}, \{y_n\}$ are Cauchy sequences in V, then $\{x_n, y_n\}$ is a Cauchy sequence in F.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 26 of 41

$\| x + y \| \leq \| x \| + \| y \|$ is called.....

Answer (Please select your correct option)

- Triangle inequality
- Minkowski's inequality
- Holder's inequality
- Schwarz inequality

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 27 of 41

For orthogonal elements x,y we have $\langle x,y\rangle=0$ and the Pythagorean relation is .....

Answer (Please select your correct option)

- $\|x+y\|^2=2\|x\|^2+2\|y\|^2$
- $\|x+y\|^2=\|x\|^2-\|y\|^2$
- $\|x-y\|^2=\|x\|^2+\|y\|^2$
- $\|x+y\|^2=\|x\|^2+\|y\|^2$ ✓

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Question Summary : (Attempted Question)

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Question 1

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Let the operators $U : H \to H$ and $V : H \to H$ be unitary; here, H is a Hilbert space. Then show that U is normal.

Prove:

$$UU^* = U^*U$$

Since we know that

$$U^* = U^{-1}$$

So

$$UU^{-1} = U^{-1}U$$

$(UV)^* = (UV)^{-1} = I$ Hence its proved that U is normal.

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Question 2

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that $\|U\| = 1$, provided $H \neq \{0\}$.

OR

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that $U^{-1}(= U^*)$ is unitary.

Prove:

Since U is bijective it shows that U^{-1} is bijective

$$(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}.$$

Hence it implies that U^{-1} is unitary.

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Question 3

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that U is isometric; thus $\|Ux\| = \|x\|$ for all $x \in H$.

Prove:

$$\| U_x \|^2 = < U_x, U_x >$$
$$\| U_x \|^2 = < x, U^* U_x >$$
$$\| U_x \|^2 = < x, Ix >$$
$$\| U_x \|^2 = < x, x >$$
$$\| U_x \|^2 = \| x \|^2$$

It proved.

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Question 4

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $(ST)^* = T^*S^*$ (assuming $H_2 = H_1$) .

Prove:

$$< x, (ST)^* y > = < (ST)x, y >$$

$$< x, (ST)^* y \geq < (SxTx), y >$$

$$< x, (ST)^* y > = < Sx, y >< Tx, y >$$

$$< x, (ST)^* y > = < x, S^* y >< x, T^* y >$$

$$< x, (ST)^* y > = < x, (S^* + T^* y) >$$

$$< x, (ST)^* y > = < x, (S^* T^*) y >$$

Hence we have proved that

$$(ST)^* = S^* T^*$$

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Question 5

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $(\alpha T)^* = \bar{\alpha} T^*$.

Prove:

$$< x, (\alpha T^*)^* y > = < (\alpha T)x, y >$$
$$< x, (\alpha T^*)^* y > = < \alpha(Tx), y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < Tx, y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < x, T^* y >$$
$$< x, (\alpha T^*)^* y > = < x, \bar{\alpha} T^* y >$$
$$(\alpha T^*)^* = \bar{\alpha} T^*$$

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Question 6

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $(T^*)^* = T$.

Prove:

$$< x, (T^*)^* y \geq < xT^*, y >$$

$$< x, (T^*)^* y \geq < x, Ty >$$

It implies that

$(T^*)^* = T$ so its proved.

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Question 7

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $\|T^*T\| = \|TT^*\| = \|T\|^2$.

Prove:

$$\|Tx\|^2 = \langle Tx, Tx\rangle = \langle T^*Tx, x\rangle \leq \|T^*Tx\|\,\|x\| \leq \|T^*T\|\,\|x\|^2.$$

Taking the supremum over all x of norm 1, we obtain $\|T\|^2 \leq \|T^*T\|$.

we have

$$\|T\|^2 \leq \|T^*T\| \leq \|T^*\|\,\|T\| = \|T\|^2.$$

Hence $\|T^*T\| = \|T\|^2$. Replacing T by T^* and we also have

$$\|T^{**}T^*\| = \|T^*\|^2 = \|T\|^2.$$

Here $T^{**} = T$

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Question 8

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $\langle T^* y, x \rangle = \langle y, Tx \rangle \quad (x \in H_1, y \in H_2)$.

Prove:

$$< T^* y, x > = < \overline{y, T^* x} >$$
$$< T^* y, x > = < \overline{Tx, y} >$$
$$< T^* y, x > = < y, Tx >$$

Hence its proved.

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Question 9

Let X and Y be inner product spaces and $Q: X \to Y$ a bounded linear operator. Then Show that $Q = 0$ if and only if $\langle Qx, y \rangle = 0$ for all $x \in X$ and $y \in Y$.

Prove:

$$Q = 0 \Longrightarrow Qx = 0 \; for \; all \; x \epsilon X$$
$$< Qx, y > = < 0, y >$$
$$< Qx, y > = < 0.w, y >$$
$$< Qx, y > = \; 0 < w, y >$$
$$< Qx, y > = \; 0. \quad for \; all \; x \epsilon X$$

It implies that Qx=0 and by definition Q=0. Its proved.

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Question 10

Let (T_n) be a sequence of bounded self-adjoint linear operators $T_n : H \to H$ on a Hilbert space H. Suppose that (T_n) converges. Say, $T_n \to T$, *that*, $\|T_n \to T\| \to 0$, where $\|\cdot\|$ is the norm on the space $B(H,H)$; then show that the limit operator T is a bounded self-adjoint linear operator on H.

Prove:

We show that $T^* = T$. This follows from $\|T - T^*\| = 0$.

$$\|T_n^* - T^*\| = \|(T_n - T)^*\| = \|T_n - T\|$$

and obtain by the triangle inequality in $B(H, H)$

$$\begin{aligned}\|T - T^*\| &\leq \|T - T_n\| + \|T_n - T_n^*\| + \|T_n^* - T^*\| \\ &= \|T - T_n\| + 0 + \|T_n - T\| \\ &= 2\|T_n - T\| \longrightarrow 0 \quad (n \longrightarrow \infty).\end{aligned}$$

Hence $\|T - T^*\| = 0$ and $T^* = T$.

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Question 11

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that a bounded linear operator T on a complex Hilbert space H is unitary if and only if T is isometric and surjective.

Prove:

Suppose that T is isometric and surjective. Isometry implies injectivity, so that T is bijective. We show that $T^* = T^{-1}$. By the isometry,

$$\langle T^*Tx, x\rangle = \langle Tx, Tx\rangle = \langle x, x\rangle = \langle Ix, x\rangle.$$

Hence

$$\langle (T^*T - I)x, x\rangle = 0$$

and $T^*T - I = 0$ so that $T^*T = I$. From this,

$$TT^* = TT^*(TT^{-1}) = T(T^*T)T^{-1} = TIT^{-1} = I.$$

Together, $T^*T = TT^* = I$. Hence $T^* = T^{-1}$, so that T is unitary. The converse is clear since T is isometric and surjective by

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Question 12

Let $T : H \to H$ be bounded linear operator on a Hilbert space H . Then show that if H is complex and $\langle Tx, x\rangle$ is real for all $x \in H$, the operator T is self-adjoint.

Prove:

If $\langle Tx, x\rangle$ is real for all x, then

$$\langle Tx, x\rangle = \overline{\langle Tx, x\rangle} = \overline{\langle x, T^*x\rangle} = \langle T^*x, x\rangle.$$

Hence

$$0 = \langle Tx, x\rangle - \langle T^*x, x\rangle = \langle (T - T^*)x, x\rangle$$

and $T - T^* = 0$

since H is complex.

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Question 13

Prove that the product of two bounded self-adjoint linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute, $ST = TS$.

Prove:

$$< x, (ST)^* y >=< (ST)x, y >$$
$$< x, (ST)^* y \geq< (SxTx), y >$$
$$< x, (ST)^* y >=< Sx, y >< Tx, y >$$
$$< x, (ST)^* y >=< x, S^* y >< x, T^* y >$$
$$< x, (ST)^* y >=< x, (S^*+T^* y) >$$
$$< x, (ST)^* y >=< x, (S^* T^*) y >$$

Hence we have proved that

$$(ST)^* = S^* T^*$$

$$(ST)^* = T^* S^* = TS.$$

Hence

$$ST = (ST)^* \iff ST = TS.$$

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Question 14

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $T^*T = 0 \Leftrightarrow T = 0$.

Prove:

$$T^*T = 0 \; iff \; T = 0$$

$$e = \|T^*T\| = \|T\|$$

(f) is true by e.

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Queestion 1

Show that $(\alpha T)^* = \alpha^{-1} T^*$



Solution:

$$< x, (\alpha T^*)^* y > = < (\alpha T)x, y >$$
$$< x, (\alpha T^*)^* y > = < \alpha(Tx), y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < Tx, y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < x, T^* y >$$
$$< x, (\alpha T^*)^* y > = < x, \bar{\alpha} T^* y >$$
$$(\alpha T^*)^* = \bar{\alpha} T^*$$

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Question 2

Prove that $(S+T)^* = S^* + T^*$

Prove:

$$< x, (S+T)^* y > = < (S+T)x, y >$$
$$< x, (S+T)^* y \geq < (Sx+Tx), y >$$
$$< x, (S+T)^* y > = < Sx, y > + < Tx, y >$$
$$< x, (S+T)^* y > = < x, S^* y > + < x, T^* y >$$
$$< x, (S+T)^* y > = < x, (S^* y + T^* y) >$$
$$< x, (S+T)^* y > = < x, (S^* + T^*) y >$$

Hence we have proved that

$$(S+T)^* = S^* + T^*$$

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Question 3

Find matrix multiplication if $(\xi_1, \xi_2, \xi_3) \leftrightarrow (\xi_1, \xi_2, -\xi_1 - \xi_2)$

Solution:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ -\xi_1, -\xi_2 \end{bmatrix}$$

Hence required matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

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Question 4

$$\|x_1 + x_2 + \ldots\ldots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \ldots\ldots + \|x_n\|^2$$

Prove:

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \left\langle \left\|\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i\right\| \right\rangle$$

$$= < x_1 + \ldots\ldots + x_n, x_1 + \ldots\ldots + x_n >$$

$$= < x_1, x_1 + \ldots\ldots + x_n > + \ldots\ldots + < x_n, x_1 + \ldots\ldots + x_n >$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} < x_i, x_j >$$

$< x_i, x_j >= \|x_i\|^2$, if $i \neq j$ $< x_i, x_j >= 0$ and for $i = j$ then $< x_i, x_j >= \|x_i\|^2$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

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Question 5

Prove that $(ST)^* = S^*T^*$

Prove:

$$< x, (ST)^*y >=< (ST)x, y >$$
$$< x, (ST)^*y \geq< (SxTx), y >$$
$$< x, (ST)^*y >=< Sx, y >< Tx, y >$$
$$< x, (ST)^*y >=< x, S^*y >< x, T^*y >$$
$$< x, (ST)^*y >=< x, (S^*+T^*y) >$$
$$< x, (ST)^*y >=< x, (S^*T^*)y >$$

Hence we have proved that

$$(ST)^* = S^*T^*$$

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Question 6

Show that $\overline{N(T)} < N(T)$

Prove:

$$x \in N(T)$$
$$\exists\ (x_n) such\ that\ x_n\ \in N(T) + x_n \to x$$

Therefore

$$Tx_n \to Tx$$
$$x_n\ \in N(T)$$
$$Tx_n = 0$$
$$T_n = 0$$
$$x\ \in N(T)$$
$$\overline{N(T)} < N(T)$$
$$\overline{N(T)} = N(T)$$
$$N(T) is\ closed.$$

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Question 7

Prove that $< x, ay + bz >= \bar{a} < x, y > + \bar{b} < x, z >$

Prove:

$$< x, ay + bz > = < \overline{ay + bz, x} >$$
$$< x, ay + bz > = < \overline{ay, x} > + < \overline{bz, x} >$$
$$< x, ay + bz > = \bar{a} < \overline{y, x} > + \bar{b} < \overline{z, x} >$$
$$< x, ay + bz > = \bar{a} < x, y > + \bar{b} < x, z >$$

Hence proved.

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Question 8

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $(T^*)^* = T$.

Prove:

$$< x, (T^*)^* y \geq < xT^*, y >$$

$$< x, (T^*)^* y \geq < x, Ty >$$

It implies that

$(T^*)^* = T$ so its proved.

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Question 9

Prove that the product of two bounded self-adjoint linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute, $ST = TS$.

Prove:

$$< x, (ST)^* y >=< (ST)x, y >$$
$$< x, (ST)^* y \geq< (SxTx), y >$$
$$< x, (ST)^* y >=< Sx, y >< Tx, y >$$
$$< x, (ST)^* y >=< x, S^* y >< x, T^* y >$$
$$< x, (ST)^* y >=< x, (S^*+T^* y) >$$
$$< x, (ST)^* y >=< x, (S^* T^*) y >$$

Hence we have proved that

$$(ST)^* = S^* T^*$$

$$(ST)^* = T^* S^* = TS.$$

Hence

$$ST = (ST)^* \iff ST = TS.$$

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Question 10

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $\|T^*T\| = \|TT^*\| = \|T\|^2$.

Prove:

$$\|Tx\|^2 = \langle Tx, Tx\rangle = \langle T^*Tx, x\rangle \leqq \|T^*Tx\|\,\|x\| \leqq \|T^*T\|\,\|x\|^2.$$

Taking the supremum over all x of norm 1, we obtain $\|T\|^2 \leqq \|T^*T\|$.

we have

$$\|T\|^2 \leq \|T^*T\| \leqq \|T^*\|\,\|T\| = \|T\|^2.$$

Hence $\|T^*T\| = \|T\|^2$. Replacing T by T^* and we also have

$$\|T^{**}T^*\| = \|T^*\|^2 = \|T\|^2.$$

Here $T^{**} = T$

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Question 11

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that U is isometric; thus $\|Ux\| = \|x\|$ for all $x \in H$.

Prove:

$$\| U_x \|^2 = < U_x, U_x >$$
$$\| U_x \|^2 = < x, U^* U_x >$$
$$\| U_x \|^2 = < x, Ix >$$
$$\| U_x \|^2 = < x, x >$$
$$\| U_x \|^2 = \| x \|^2$$

It proved.

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Question 12

Show that $< T^*y, x > = < y, Tx >$

Prove:

$$< T^*y, x > = < \overline{y, T^*x} >$$
$$< T^*y, x > = < \overline{Tx, y} >$$
$$< T^*y, x > = < y, Tx >$$

Hence its proved.

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Question 13

Let X and Y be inner product spaces and $Q: X \to Y$ a bounded linear operator. Then Show that $Q=0$ if and only if $\langle Qx, y\rangle = 0$ for all $x \in X$ and $y \in Y$.

Prove:

$$Q = 0 \Longrightarrow Qx = 0 \; for \; all \; x \epsilon X$$

$$< Qx, y > = < 0, y >$$

$$< Qx, y > = < 0.w, y >$$

$$< Qx, y > = \; 0 < w, y >$$

$$< Qx, y > = \; 0. \quad for \; all \; x \epsilon X$$

It implies that Qx=0 and by definition Q=0. Its proved.

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Question 14

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that $\|U\| = 1$, provided $H \neq \{0\}$.

OR

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that $U^{-1}(= U^*)$ is unitary.

Prove:

Since U is bijective it shows that U^{-1} is bijective

$$(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}.$$

Hence it implies that U^{-1} is unitary.

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Question 15

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that U is normal.

Prove:

$$UU^* = U^*U$$

Since we know that

$$U^* = U^{-1}$$

So

$$UU^{-1} = U^{-1}U$$

$(UV)^* = (UV)^{-1} = I$ Hence its proved that U is normal.

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Question 16

In an inner product space V and x, y in V if $x \perp y$ then

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2$$

Prove:

$$\|x+y\|^2 = < x+y, x+y >$$

$$= < x,x > + < x,y > + < y,x > + < y,y >$$

As x and y are perpendicular so $< x,y >= 0, < y,x >= 0$

$$\|x+y\|^2 = < x,x > + < y,y > = \|x\|^2 + \|y\|^2$$

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Question 17

$$\|x+y\|^2+\|x-y\|^2=2\left(\|x\|^2+\|y\|^2\right) \quad \text{for all x,y } \in V$$

Prove:

$$\|x+y\|^2 = < x+y, x+y >$$
$$= < x,x > + < x,y > + \overline{< x,y >} + < y,y >$$
$$= \|x\|^2 + 2\,\mathrm{Re} < x,y > + \|y\|^2 \qquad \text{....(i}$$

Replace y=-y

$$\|x-y\|^2 = < x+y, x+y >$$
$$= < x,x > - < x,y > - \overline{< x,y >} + < y,y >$$
$$= \|x\|^2 - 2\,\mathrm{Re} < x,y > + \|y\|^2 \qquad \text{.....(ii}$$

Adding (i and (ii

$$\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$$

That we have to prove.

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Question 17

$$\|x+y\|^2+\|x-y\|^2=2\left(\|x\|^2+\|y\|^2\right) \quad \text{for all x,y} \in V$$

Prove:

$$\|x+y\|^2 = < x+y, x+y >$$
$$= < x,x > + < x,y > + \overline{< x,y >} + < y,y >$$
$$= \|x\|^2 + 2\,\mathrm{Re} < x,y > + \|y\|^2 \quad \text{....(i}$$

Replace y=-y

$$\|x-y\|^2 = < x+y, x+y >$$
$$= < x,x > - < x,y > - \overline{< x,y >} + < y,y >$$
$$= \|x\|^2 - 2\,\mathrm{Re} < x,y > + \|y\|^2 \quad \text{.....(ii}$$

Adding (i and (ii

$$\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$$

That we have to prove.

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Question18

For any two elements x,y is an inner product space V,

$$|< x, y >| \leq \|x\| . \|y\|$$

Prove:

If x=y=0 then 0=0

Let at least one of x and y is not equal to zero

Let $|< x+\lambda y, x+\lambda y >| \geq 0$ by definition

$$< x, x+\lambda y > + < \lambda y, x+\lambda y >$$

$$< x, x+\lambda y > + y < y, x+\lambda y >$$

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Question 19

SHOW THAT T is uniquely determinal if the image $y_k = Te_k$ of n basis vectors $e_1, \ldots\ldots, e_n$ are prescribed.

Prove:

$$y=Tx \;;\; y \in Y \quad \{b_1, \ldots\ldots, b_n\}$$

$$y=\eta_1 b_1 + \eta_2 b_2 + \ldots\ldots\ldots + \eta_r b_r$$

$$Te_k \in Y, \qquad Te_1 = \tau_{11} b_1 + \tau_{12} b_2 + \ldots\ldots\ldots + \tau_{1r} b_r$$

$$Te_k = \sum_{j=1}^{r} \tau_{kj} b_j$$

$$y=\sum_{j=1}^{r} \eta_j b_j = \sum_{k=1}^{n} \xi_k Te_k = \sum_{k=1}^{n} \xi_k \sum_{j=1}^{r} \tau_{kj} b_j$$

Comibinig these two summation

$$y = \sum_{j=1}^{r} \left(\sum_{k=1}^{n} \tau_{kj} \xi_k \right) b_j$$

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_k$$

The image $y=Tx=\sum \eta_j b_j$ of $x = \sum \xi_k Te_k$ can be obtained from

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_k$$

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Question 20

Sum and Scalar multipication of Dual.

Prove:

as follows. The SUM $f_1 + f_2$ of two **functionals** f_1 **and** f_2 is the functional s whose value **at every** $x \in X$ **is**

$$s(x) = (f_1 + f_2)(x) = f_1(x) + f_2(x);$$

the Product αf of a scalar α and **a functional** f is the functional p whose value **at** $x \in X$ **is**

$$p(x) = (\alpha f)(x) = \alpha f(x).$$

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Question 21

Define Inner Product Space:

The pair $(V, <\bullet,\bullet>)$ is called an inner product space.

Define Appolonius Identity:

$$\|z-x\|^2+\|z-y\|^2=\frac{1}{2}\|x-y\|^2+2\left\|z-\frac{1}{2}(x+y)\right\|^2 \quad , \quad x,y,z\in V$$

Define Equal Operators:

Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have the same domain i.e. $D(T_1) = D(T_2)$



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Question 22

Define Orthonormal System.

Orthonormal systems are those functions which are real or complex whose inner product with itself results in 1 and with other functions result in 0.

Define Polarization Identity.

For any x, y in complex inner product space

$$< x, y >= \frac{1}{4}\left\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\right\}$$

We have to prove this complex inner product space.

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Question 23

Define Bounded Linear Function.

Let X and Y be normed spaces and $T : D(T) \to Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$\|Tx\| \leq c\|x\|$

Define Canonical Mapping.

$C : X \to X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

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Question 24

Define Unitary Operator.

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $T^* = T^{-1}$.

Define Normal Operator.

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $TT^* = T^*T$

Define Extension of T.

Extension of T on a set $M \supset D(T)$ is defined as $\bar{T}: M \to Y$ or

$$\bar{T}x = Tx \; for \; all \; x \in D(T)$$

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Question 25

Write examples of Bounded Linear Operators

Identity Operator, Zero Operator, Differentiation Operator, Integral Operator

Define Self Adjoint Operator or Hermitian Operator.

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Self Adjoint Operator or Hermitian Operator if $T^* = T$.

Give an example of function which is not linear.

$\| . \|: x \to R$ is functional but not linear.

Find basis of R(T).

$(\xi_1, \xi_2, -\xi_1, -\xi_2) = \xi_1(1,0,-1) + \xi_2(0,1,-1)$

So the basis of R(T)= $\{(1,0,-1),(0,1,-1)\}$

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Question 26

Give any two examples of Linear Functional.

Space C[a, b] and Space l^2

Define Linear Functional:

A linear functional f is a linear operator with domain in a vector space X and range in the scalar field K of X; thus $f: D(f) \to K$. where K=R if X is real and K=C if X is complex.

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Define Bounded Linear Extension:

$T: D(T) \to Y$ be a bounded linear operator, where D(T) lies in a normal space X and Y is a Banach Space then T has an extension $\bar{T}: \overline{D(T)} \to Y$. Where $\bar{T}$ is a bounded linear operator of norm.

$$\|\bar{T}\| = \|T\|$$

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MTH-641

Final Term Solution

Q#01:-

Define Extension of T.

Extension of T on a Set

$$M \supset D(T)$$

$$\overline{T} : M \to T$$

$$\overline{T}x = Tx \quad \forall x \in D(T)$$

Q#02:-

Prove That

$$\langle x, ay + bz \rangle = \bar{a}\langle x, y \rangle + \bar{b}\langle x, z \rangle$$

Solution:-

$$\langle x, ay + bz \rangle = \overline{\langle ay + bz, x \rangle}$$

$$= \overline{\langle ay, x \rangle} + \overline{\langle bz, x \rangle}$$

$$= \bar{a}\,\overline{\langle y, x \rangle} + \bar{b}\,\overline{\langle z, x \rangle}$$

$$= \bar{a}\langle x, y \rangle + \bar{b}\langle x, z \rangle$$

Proved

"Remember me in your Prayers"
"Jazak Allah"

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Q#03.-

Appollonius Identity:-

$x, y, z \in V$

$$\|z-x\|+\|z-y\| = \frac{1}{2}\|x-y\|^2 + 2\left\|z-\frac{1}{2}(x+y)\right\|^2$$

Q#04:-

Define bounded linear function:

A bounded linear function F is a bounded linear operator with range in a scalar field of the normed space X.

$$|F(x)| \leq c\|x\|$$

Q#05

Find basis of $N(T)$.

$x = (e_1, e_2, e_3) \Rightarrow x \in N(T)$

$\Rightarrow (0, 0, e_3)$

$\Rightarrow e_3(0, 0, 1)$

Q#6:-

Define self adjoint operator:

A bounded linear operator $T: H \to H$ on a Hilbert space

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His said to be, self adjoint or Hesmition if

$$T^* = T$$

Dmf

Q#07

Prove that U is normal.

To Show U is normal.

$$UU^* = U^*U$$

$$U^* = U^{-1}$$

$$UU^{-1} = U^{-1}U = I$$

Proved

Q#08

Give Two examples of orthogonal System.

Two examples of orthogonal System is

* Space R^n
* Space R^3

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V.V. Imp / M.T. Imp

Note
9 marks
3 marks
5 marks
ma a re ha)

Q#9:- If $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3) \to (\mathcal{E}_1, \mathcal{E}_2 - \mathcal{E}_1, -\mathcal{E}_1 - \mathcal{E}_2)$ then find matrix representation.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \\ \mathcal{E}_3 \end{bmatrix} = \begin{bmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \\ -\mathcal{E}_1 - \mathcal{E}_2 \end{bmatrix}$$

Hence required matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$

Q#10

Prove that

$$(\alpha T^*)^* = \bar{\alpha} T^*$$

$$\langle x, (\alpha T^*)^* y \rangle = \langle (\alpha T) x, y \rangle$$

$$= \langle \alpha (Tx), y \rangle$$

$$= \bar{\alpha} \langle Tx, y \rangle$$

$$= \bar{\alpha} \langle x, T^* y \rangle$$

$$= \langle x, \bar{\alpha} T^* y \rangle$$

$$\langle x, (\alpha T^*)^* y \rangle = \langle x, \bar{\alpha} T^* y \rangle$$

$$\Rightarrow (\alpha T)^* = \bar{\alpha} T^*$$

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Q NO 11:-

Write to examples of linear functional

1) space $C[a\ b]$

2) space l^2

Q NO 12:-

Orthonormal System Definition:-

orthonormal system are those function which are real and complex whose inner product with itself result in 1 and with othes functions result in 0.

Imp Q NO 13

Let $T: R^3 \to R^3$ define by

$(\xi_1, \xi_2, \xi_3) \longmapsto (\xi_1, \xi_2, -\xi_1 - \xi_2)$

Then find basis of $R(T)$.

$(\xi_1, \xi_2, -\xi_1 - \xi_2) = \xi_1(1, 0, -1) + \xi_2(0, 1, -1)$

$D \Rightarrow \{(1, 0, -1), (0, 1, -1)\}$

Imp Q NO: 14

Show that $U^{-1} = U^*$ is unitary. OR $\|U\| = 1$

Since U is bijective $\Rightarrow U^{-1}$ is bijective

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$(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}$

$(U^{-1})^* = (U^{-1})^{-1}$

$\Rightarrow U^{-1}$ is Unitary.

Q NO:15.

Difference b/w Algebraic dual Space and Conjugate space.

"Algebraic dual space of X":

Set of all linear functionals defined on X.

"Dual or Conjugat Space of X":

X^* Set of all continuous or bounded linear functionals defined on X.

We take algebraic dual when there is no condition of Continuous or bounded linear functions.

Q NO:16

Show that $\langle x, \alpha y \rangle = \bar{\alpha} \langle x, y \rangle$

For all $x, y \in V \quad \alpha \in F$

$\langle x, \alpha y \rangle = \overline{\langle \alpha y, x \rangle} = \overline{\alpha \langle y, x \rangle}$

$\bar{\alpha} \overline{\langle y, x \rangle} = \bar{\alpha} \langle x, y \rangle$

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QNO17:- Topic 64

poove that if a normed space X is finite dimentional then every linear operator on X is bounded.

Proof:-

X is finite dimension.

⇒ every linear operator on X is bounded.

Let $\dim X = n \Rightarrow \{e_1, e_2, \ldots e_n\}$ is a basis for X

Let $x \in X$

$\Rightarrow x = \sum_{j=1}^{n} \xi_j e_j$

Let T be any linear operator on X.

$$\|Tx\| = \|T(\sum_{j=1}^{n} \xi_j e_j)\| = \|\sum_{j=1}^{n} \xi_j Te_j\|$$

↓ linear

$$\leq \sum_{j=1}^{n} |\xi_j| \|Te_j\| \leq N_3\{\|Te_1\|, \ldots \|Te_n\|\}$$

$$\leq \max \|Te_k\| \sum_{j=1}^{n} |\xi_j| \quad \text{---} (1)$$

$$\|\sum_{j=1}^{n} \xi_j e_j\| \geq c \left(\sum_{j=1}^{n} |\xi_j|\right)^k$$

$$\sum_{j=1}^{n} |\xi_j| \leq \frac{1}{c} \|\sum_{j=1}^{n} \xi_j e_j\| = \frac{1}{c}\|x\|$$

$$(1) \Rightarrow \|Tx\| \leq \max \|Te_k\| \frac{1}{c}\|x\|$$

$$\Rightarrow \|Tx\| \leq c'\|x\|$$

$\Rightarrow$ T is bounded.

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Q NO 18:-

Show that $(S+T)^* = S^* + T^*$

Proof

$$\langle x, (S+T)^* y \rangle = \langle (S+T)x, y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle (Sx + Tx), y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle Sx, y \rangle + \langle Tx, y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, S^* y \rangle + \langle x, T^* y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, (S^* y + T^* y) \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, (S^* + T^*) y \rangle$$

Hence we have prove that

$$(S+T)^* = S^* + T^*$$

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imp

QNO:19

Show that

$$\|x_1+x_2+x_3+---+x_n\|^2=\|x_1\|^2+\|x_2\|^2+\cdots+\|x_n\|^2$$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2=\left\langle \sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i\right\rangle$$

$$=\langle x_1, x_2+---x_n, x_1+x_2+---x_n\rangle$$

$$=\langle x_1, x_2+---x_n\rangle+---\langle x_1+x_2+-x_n\rangle$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{n}\langle x_i, x_j\rangle$$

$$\because \langle x_i, x_j\rangle=\|x_i\|^2$$

$$\because \langle x_i, x_j\rangle=0 \quad i\neq j \text{ so,}$$

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(7)

$$\left\| \sum_{i=1}^{n} x_i \right\|^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \langle x_i, x_j \rangle$$

$$\langle x_i, x_j \rangle = 0$$

QNO 20:-

Give an example of function which is not linear?

$\|\cdot\| : X \longrightarrow R$ Functional but not linear.

QNO 21:-

Generalization of orthogonal System:-

$\{x_1, x_2, \ldots x_n\}$ be nonzero vectors in V innes product space such that

$\langle x_i, x_j \rangle = 0 \quad i \neq j$

This system $\{x_1, x_2, \ldots x_n\}$ is called as all vectors inside it are perpendicular to each other.

QNO:22

Isomorphsim of metric Space:-

A function $f : (X, d) \longrightarrow (Y, \rho)$

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is an isomorphism if f is an isometry and f is surjective. Two metric space are called isomorphic if there is an isomorphism between them.

QNO 23:- Define Canonical mapping

$C: X \to X^{**}$ this mapping is called Canonical mapping of X into X^{**} defined as $x \mapsto \hat{x}$.

∴ made by baloch (R+H)

Def

QNO 24:- Define Normal Operator:-

A bounded linear operator $T: H \to H$ on a Hilbert space H is said to be Unitary operator if T is bijective and $TT^* = T^*T$

Def

Define equal operator:-

Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have Same domain i.e $D(T_1) = D(T_2)$

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Q NO 28:-

Examples of Inner product Spaces:-

- ★ Space R^n
- ★ Space C^n
- ★ Space $C[a,b]$
- ★ Space l^2
- ★ Space P_n

Imp N.T

Q NO 29:-

Prove that null space is Closed.

OR

$$\overline{N(T)} = N(T)$$

Solution:-

$$x \in N(T)$$

$\exists (x_n)$ such that $x_n \in N(T) \to x_n \to x$

Therefore

$$Tx_n \to Tx$$

$$x_n \in N(T)$$

$$Tx_n = 0$$

$$Tn = 0$$

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$x \in N(T)$

$\overline{N(T)} \subset N(T)$

$N(T) = N(T)$

$N(T)$ is closed.

Q No 30:- Define Polarization Identity:-

For any x, y in complex inner product space

$$\langle x, y \rangle = \frac{1}{4}\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\}$$

we have prove this complex inner product space.

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Note:=

Find the Null space of $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $(\xi_1, \xi_2, \xi_3) \to (\xi_1, \xi_2, -\xi_1 - \xi_2)$

i) Basis of $R(T)$ PgNo: 5

2) Basis of $N(T)$ PgNo: 2

3) Matrix of representation PgNo: 4

یہ Questions باہر نکل میں آرہے ہیں.

Module No # 78

made by Baloch (R+H)

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Bounded Linear Operator:

Let X and Y be normed spaces and $T : D(T) \to Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$$\|Tx\| \leq c\|x\|$$

Let T be a bounded linear operator as defined before then an alternate formula for the norm of T is

$$\|T\| = \sup_{x \in D(T),\ x \neq 0} \|Tx\|$$

The norm defined on T satisfies (N1) to (N4).

Examples of Bounded Linear Operators

Identity Operator
Zero Operator
Differentiation Operator
Integral Operator
Matrix

Linear Function:

Algebraic Dual Space
Second Algebraic Dual Space
Canonical Mapping

Algebraic Dual Space

Set of all linear function defined on a vector space X is itself a vector space and called Algebraic Dual Space and denoted by X^*

Conical Mapping:

$C : X \to X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

Linear Function (Examples):

Space $C[a\ b]$
Space l^2

Linear Operators and functional on finite Dimensional Spaces:

Finite dimensions mean basis which have finite many elements.

Let X and Y bef.gfinite dimension vector spaces over the same field.

Let $T : X \to Y$ be a linear operator. let $E = \{e_1, \ldots\ldots, e_n\}$ be the basis for X and $B = \{b_1, \ldots\ldots, b_m\}$ be the basis for Y.

$x \in X$, $\quad x = \xi_1 e_1 + \xi_2 e_2 + \ldots\ldots\ldots + \xi_n e_n$

$$y = Tx = T\left(\sum_{k=1}^{n} \xi_k e_k\right) = \sum_{k=1}^{n} T(\xi_k e_k) = \sum_{k=1}^{n} \xi_k T(e_k)$$

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T is uniquely determinal if the image $y_k = Te_k$ of n basis vectors $e_1, \ldots\ldots, e_n$ are prescribed.

$y=Tx \; ; \; y \in Y \quad (b_1, \ldots\ldots, b_m)$

$y= \eta_1 b_1 + \eta_2 b_2 + \ldots\ldots\ldots + \eta_r b_r$

$Te_k \in Y, \qquad Te_i = \tau_{i1} b_1 + \tau_{i2} b_2 + \ldots\ldots\ldots + \tau_{ir} b_r$

$$Te_k = \sum_{j=1}^{r} \tau_{kj} b_j$$

$$y=\sum_{j=1}^{r} \eta_j b_j = \sum_{k=1}^{n} \xi_{ik} Te_k = \sum_{k=1}^{n} \xi_{ik} \sum_{j=1}^{r} \tau_{kj} b_j$$

Combinig these two summation

$$y = \sum_{j=1}^{r} \left(\sum_{k=1}^{n} \tau_{kj} \xi_{ik} \right) b_j$$

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_{ik}$$

The image $y=Tx=\sum \eta_j b_j$ of $x = \sum \xi_{ik} Te_k$ can be obtained from

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_{ik}$$

Let X be an n-dimensional vector space and X^* be its dual space. Then

$$\dim X^* = \dim X = n.$$

X^* is collection of linear functions or linear operater while X may be any space.

Proof:

Let $\dim X = n$.

Let basis of X be $B = \{e_1, \ldots\ldots\ldots, e_n\}$

We define a function.

$$f_i(e_i) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j : i, j = 1, \ldots\ldots, n \end{cases}$$

e.g. j=1, $\quad f(e_1) = 1, f(e_2) = 0, f(e_3) = 0, \ldots\ldots\ldots\ldots, f(e_n) = 0$

j=2, $f(e_1) = 0, f(e_2) = 1, f(e_3) = 0, \ldots\ldots\ldots\ldots, f(e_n) = 0$

but each n-tuples f_i in this case can be extended as linear functions on X.

Let X be a finite deimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$ then $x_0 = 0$.

B^* is the basis of X^*

$\{f_1, f_2, \ldots\ldots, f_n\}$

$$\Rightarrow f_i(e_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$= \delta_{ij}$$

Proof:

For all $x_0 = 0$,

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$$x_0 = \sum_{i=1}^{n} x_i e_i \quad ; \quad f \in X^* \quad ,$$

$$f(x_0) = 0 \quad \Rightarrow \quad \sum_{i=1}^{n} f\left(\sum_{i=1}^{n} x_i e_i\right) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{n} x_i f(e_i) = 0 \quad , \quad j=1,\ldots\ldots,n$$

$$\Rightarrow \quad x_j = 0 \quad , \quad \forall\, j=1,\ldots\ldots,n$$

$$x_0 = \sum_{i=1}^{n} x_i e_i = 0 \quad \Rightarrow \quad x_0 = \delta$$

Theroem

Let $B(X, Y)$ be the set of all bounded linear operators form a normed space X to a normed space Y. If Y is a Banach space, then $B(X, Y)$ is also a Banach.

Proof:

Let $\{T_n\}$ be an arbitrary Cauchy seq. in $B(X,Y)$.

We will show that $\{T_n\}$ converges to an operator T in $B(X, Y)$.Since $\{T_n\}$ is Cauchy for every

$\varepsilon > 0 \quad \exists \;\; N$ such that $\|T_n - T_m\| < \varepsilon \qquad (m,n>N)$

For all $x \in X$ and (m,n>N) we have

$$\|T_n(x) - T_m(x)\| = \|(T_n - T_m)(x)\|$$
$$\leq \|T_n - T_m\|\|x\| < \varepsilon\|x\|$$

Thus for a fixed x and given ε

Hence B(X, Y) is complete and Banach space.

Inner product Space

Let V be a vector space over a field F where $F$is $\mathbb{R}$ or $\mathbb{C}$.

An inner product in V is a function $\langle\cdot,\cdot\rangle : V \times V \to F$ satisfying the following conditions:

Quiz:

Let $x, y, z \in V \;\; ; \;\; \alpha \in$ F where α may be real or complex.

i. $\langle x, x \rangle \geq 0; \quad \langle x, x \rangle = 0 \quad \Leftrightarrow x = 0$

ii. $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$; but not true for second value as $\langle x, \alpha y \rangle \neq \alpha \langle x, y \rangle$

iii. $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$

iv. $\langle x, y \rangle = \overline{\langle y, x \rangle}$

$\langle\cdot,\cdot\rangle : V \times V \to F$ inner product.

Inner Product Space

The pair $(V, \langle\cdot,\cdot\rangle)$ is called an inner product space.

a): $\langle ax + by, z \rangle \qquad where \quad x, y, z \in V \;, \;\; a, b \in F$

Using (iii) property $\qquad \langle ax + by, z \rangle = \langle ax, z \rangle + \langle by, z \rangle$

Using (ii) property $\qquad a\langle x, z \rangle + b\langle y, z \rangle$

$\langle 0, z \rangle = \langle 0.x, z \rangle = 0 \langle x, z \rangle = 0$

b): *Quiz:*

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for all $x, y \in V$, $a \in F$

$$< x, ay > = \overline{< ay, x >} = \overline{a < y, x >}$$

$$= \bar{a}\overline{< y, x >} = \bar{a} < x, y >$$

Theorem:

For any two elements x,y is an inner product space V,

$|< x, y >| \le \|x\|.\|y\|$, the define norm is $\|x\| = \sqrt{< x, x >}$, $x, y \in V$

Proof:

If x=y=0 then 0=0

Let at least one of x and y is not equal to zero

Let $|< x + \lambda y, x + \lambda y >| \ge 0$ by definition

$$< x, x + \lambda y > + < \lambda y, x + \lambda y >$$

$$< x, x + \lambda y > + y < y, x + \lambda y >$$

Theorem:

In an inner product space V, the function $\|.\| : V \to \mathbb{R}^+$ given by

$\|x\| = \sqrt{< x, y >}$, $x \in V$ defines a norm in V.

Proof:

N1: $\|x\| \ge 0$

For a $x \in V$, $\|x\| = \sqrt{< x, x >} \ge 0$ as $< x, x > \ge 0$

N2:

$\|x\| = 0 \Leftrightarrow \sqrt{< x, x >} = 0 \Leftrightarrow < x, x > = 0 \Leftrightarrow x = 0$

N3: $\|\alpha x\| = |\alpha| \|x\|$

now $\|\alpha x\| = \sqrt{< \alpha x, \alpha x >} \Rightarrow \|\alpha x\|^2 = < \alpha x, \alpha x >$

$\Rightarrow \|\alpha x\|^2 = \alpha\bar{\alpha} < x, x > = |\alpha|^2 \|x\|^2$

N4: $\|x + y\| \le \|x\| + \|y\| \quad \forall \; x, y \in V$

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$$\|x+y\|^2 = \langle x+y, x+y\rangle$$
$$= \langle x, x+y\rangle + \langle y, x+y\rangle$$
$$= \overline{\langle x+y, x\rangle} + \overline{\langle x+y, y\rangle}$$
$$= \overline{\langle x, x\rangle + \langle y, x\rangle} + \overline{\langle x, y\rangle + \langle y, y\rangle}$$
$$= \overline{\langle x, x\rangle} + \overline{\langle y, x\rangle} + \overline{\langle x, y\rangle} + \overline{\langle y, y\rangle}$$

Now $= \langle x, x\rangle + \langle x, y\rangle + \overline{\langle x, y\rangle} + \langle y, y\rangle$

$$= \|x\|^2 + 2\,\mathrm{Re}\langle x, y\rangle + \|y\|^2 \qquad \because \mathrm{Re}(z) \le |z|$$
$$\le \|x\|^2 + 2|\langle x, y\rangle| + \|y\|^2$$
$$\le \|x\|^2 + 2\|x\|\|y\| + \|y\|^2 \qquad \because |\langle x, y\rangle| \le \|x\|\|y\|$$
$$= (\|x\| + \|y\|)^2$$
$$\|x+y\|^2 \le \|x\| + \|y\|$$

Theorem:

$$\|x+y\|^2 + \|x-y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right) \quad \text{for all } x,y \in V$$

Proof:

$$\|x+y\|^2 = \langle x+y, x+y\rangle$$
$$= \langle x, x\rangle + \langle x, y\rangle + \overline{\langle x, y\rangle} + \langle y, y\rangle$$
$$= \|x\|^2 + 2\,\mathrm{Re}\langle x, y\rangle + \|y\|^2 \qquad \ldots(i$$

Replace y=-y

$$\|x-y\|^2 = \langle x+y, x+y\rangle$$
$$= \langle x, x\rangle - \langle x, y\rangle - \overline{\langle x, y\rangle} + \langle y, y\rangle$$
$$= \|x\|^2 - 2\,\mathrm{Re}\langle x, y\rangle + \|y\|^2 \qquad \ldots(ii$$

Adding (i and (ii

$$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

That we have to prove.

Polarization Identity

For any x, y in complex inner product space

$$\langle x, y\rangle = \frac{1}{4}\left(\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\right)$$

Appolonius Identity

$$\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z - \frac{1}{2}(x+y)\right\|^2, \quad x, y, z \in V$$

Using parallelogram law

$$\|x'+y'\|^2 + \|x'-y'\|^2 = 2\|x'\|^2 + 2\|y'\|^2 \quad \text{put} \quad x' = z-x,\ y' = z-y$$

Self-assignment

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Theorem:

Let V be any inner product space. For any sequences $\{x_n\}$ and $\{y_n\}$ in V

$x_n \to x$, $y_n \to y$ implies $< x_n, y_n > \to < x, y >$

Proof:

$$|< x_n, y_n > - < x, y >|$$
$$= |< x_n, y_n > - < x_n, y > + < x_n, y > - < x, y >|$$
$$= |< x_n, y_n - y > + < x_n - x, y >|$$
$$\leq |< x_n, y_n - y >| + |< x_n - x, y >|$$

Now from Cauchy Swarzinequality

$$|x, y| \leq \|x\| \|y\|$$
$$\leq \|x_n\| \|y_n - y\| + \|x_n - x\| \|y\|$$

Given that $x_n \to x$, $y_n \to y$ so,

$$\|y_n - y\| = \|y - y\| = 0 \quad , \quad \|x_n - x\| = \|x - x\| = 0 \quad \text{as } n \to \infty$$

As $n \to \infty$

$$|< x_n, y_n > - < x, y >| \leq 0$$
$$< x_n, y_n > \to < x, y > \quad \text{as } n \to \infty$$

Theorem:

If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in V, then the inner product $< x_n, y_n >$ is a Cauchy sequence in F.

Proof:

$\{x_n\}, \{y_n\}$ are Cauchy sequence

To show $< x_n, y_n >$ is also Cauchy Sequence.

$\Rightarrow$ $\|x_n - x_m\| \to 0$; $\|y_n - y_m\| \to 0$, $m,n \to \infty$

$$|< x_n, y_n > - < x_m, y_m >| = |< x_n, y_n > - < x_n, y_m > + < x_n, y_m > - < x_m, y_m >|$$
$$= |< x_n, y_n - y_m > + < x_n - x_m, y_m >|$$
$$\leq |< x_n, y_n - y_m >| + |< x_n - x_m, y_m >|$$
$$\leq \|x_n\| \|y_n - y_m\| + \|x_n - x_m\| \|y_m\|$$

$\Rightarrow$ $|< x_n, y_n > - < x_m, y_m >| \to 0$, as $n,m \to \infty$

$\Rightarrow$ $< x_n, y_n >$ is a Cauchy Sequence

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Theorem:

In an inner product space V and x, y in V if $x \perp y$ then

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2$$

Proof:

$$\|x+y\|^2 = \langle x+y, x+y \rangle$$

$$= \langle x, x \rangle + \langle x, y \rangle + \langle y, x \rangle + \langle y, y \rangle$$

As x and y are perpendicular so $\langle x, y \rangle = 0, \langle y, x \rangle = 0$

$$\|x+y\|^2 = \langle x, x \rangle + \langle y, y \rangle = \|x\|^2 + \|y\|^2$$

Generalized form:

$\{x_1, x_2, \ldots\ldots, x_n\}$ be nonzero vectors in V inner product space such that

$\langle x_i, x_j \rangle = 0 \quad , \quad i \neq j$

This system $\{x_1, x_2, \ldots\ldots, x_n\}$ is called orthogonal system as all vectors inside it are perpendicular to each other.

The generalized statement is $\|x_1 + x_2 + \ldots\ldots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \ldots\ldots + \|x_n\|^2$

The idea of proof is

$$\left\| \sum_{i=1}^{n} x_i \right\|^2 = \left\langle \sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i \right\rangle$$

$$= \langle x_1 + \ldots\ldots + x_n, x_1 + \ldots\ldots + x_n \rangle$$

$$= \langle x_1, x_1 + \ldots\ldots + x_n \rangle + \ldots\ldots + \langle x_n, x_1 + \ldots\ldots + x_n \rangle$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \langle x_i, x_j \rangle$$

$\langle x_i, x_j \rangle = \|x_i\|^2$, if $i \neq j$ $\langle x_i, x_j \rangle = 0$ and for $i = j$ then $\langle x_i, x_j \rangle = \|x_i\|^2$

$$\left\| \sum_{i=1}^{n} x_i \right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

Any sequence $\{x_n\}$ of non-zero mutually orthogonal vectors in an inner product space V is linearly independent.

Proof: do it yourself

Let $x = (x_1, x_2, \ldots\ldots, x_n)$ be the orthogonal sequence.

Remark:

If $\langle x, x_i \rangle = 0$, $\forall$ i=1,2,.....,n $\quad \Rightarrow \quad \left\langle \sum_{i=0}^{n} a_i x_i, x \right\rangle = 0$

$$\left\langle \sum_{i=0}^{n} a_i x_i, x \right\rangle = \langle a_1 x_1 + a_2 x_2 + \ldots\ldots + a_n x_n, x \rangle = a_1 \langle x_1, x \rangle + \ldots\ldots + a_n \langle x_n, x \rangle = 0$$

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IMPORTANT SUBJECTIVE FOR FINAL TERM

Q.No.01: Prove that $< x, ay + bz >= \bar{a} < x, y > + \bar{b} < x, z >$

Proof:

$$< x, ay + bz > = < \overline{ay + bz}, x >$$
$$< x, ay + bz > = < \bar{a}\bar{y}, x > + < \overline{bz}, x >$$
$$< x, ay + bz > = \bar{a} < \bar{y}, x > + \bar{b} < \bar{z}, x >$$
$$< x, ay + bz > = \bar{a} < x, y > + \bar{b} < x, z >$$

Hence proved.

Q.No.02: Prove that UV **is unitary OR** $(UV)^* = (UV)^{-1}$

Proof:

$$(UV)^* = U^*V^*$$

Since we know that

$$U^* = U^{-1} \text{ and } V^* = V^{-1}$$

So

$$(UV)^* = U^{-1}V^{-1}$$
$$(UV)^* = (UV)^{-1}$$

It shows that UV is unitary.

Q.No.03: Give any two examples of Linear Functional.

Solution:

Space C[a, b] and Space l^2

Q.No.04: Prove that $(S + T)^* = S^* + T^*$

Proof:

$$< x, (S + T)^* y >=< (S + T)x, y >$$
$$< x, (S + T)^* y >=< Sx + Tx, y >$$
$$< x, (S + T)^* y >=< Sx, y > + < Tx, y >$$
$$< x, (S + T)^* y >=< x, S^* y > + < x, T^* y >$$
$$< x, (S + T)^* y >=< x, (S^* y + T^* y) >$$
$$< x, (S + T)^* y >=< x, (S^* + T^*) y >$$

Hence we have proved that

$$(S + T)^* = S^* + T^*$$

Q.No.05: Find matrix multiplication if $(\xi_1, \xi_2, \xi_3) \leftrightarrow (\xi_1, \xi_2, -\xi_1 - \xi_2)$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ -\xi_1 - \xi_2 \end{bmatrix}$$

Hence required matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$

Q.No.06: Show that

$$< x_i, x_j >= \|x_i\|^2 \text{ if } i \neq j$$

$< x_i, x_j >= 0$ if $i = j$ then $< x_i, x_j >= \|x_i\|^2$

Proof:

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The generalized statement is $\|x_1 + x_2 + \ldots\ldots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \ldots\ldots + \|x_n\|^2$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \left\langle \sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i \right\rangle$$

$$= \langle x_1 + \ldots\ldots + x_n, x_1 + \ldots\ldots + x_n \rangle$$

$$= \langle x_1, x_1 + \ldots\ldots + x_n \rangle + \ldots + \langle x_n, x_1 + \ldots\ldots + x_n \rangle$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \langle x_i, x_j \rangle$$

$\langle x_i, x_j \rangle = \|x_i\|^2$, if $i \neq j$ $\langle x_i, x_j \rangle = 0$ and for $i = j$ then $\langle x_i, x_j \rangle = \|x_i\|^2$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

Q.No.07: Show that $\overline{N(T)} < N(T)$

Solution:

$$x \in N(T)$$
$$\exists\ (x_n) such\ that\ x_n \in N(T) + x_n \to x$$

Therefore

$$Tx_n \to Tx$$
$$x_n \in N(T)$$
$$Tx_n = 0$$
$$T_n = 0$$
$$x \in N(T)$$
$$\overline{N(T)} < N(T)$$
$$\overline{N(T)} = N(T)$$
$$N(T) is\ closed.$$

Q.No.08: Give an example of function which is not linear.

Solution: $\|\ .\ \|: X \to R$ is functional but not linear.

Q.No.09: Find basis of R(T).

Solution:

$$(\xi_1, \xi_2, -\xi_1, -\xi_2) = \xi_1(1,0,-1) + \xi_2(0,1,-1]$$

So the basis of R(T)= $\{(1,0,-1), (0,1,-1)\}$

Q.No.10: Generalization of Orthogonal

Solution: $\{x_1, x_2, \ldots\ldots x_n\}$ be nonzero vectors in V inner product space such that $\langle x_i, x_j \rangle = 0$, $i \neq j$ This system $\{x_1, x_2, \ldots\ldots x_n\}$ is called orthogonal system as all vectors inside it are perpendicular to each other. The generalized statement is $\|x_1 + x_2 + \ldots\ldots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \ldots\ldots + \|x_n\|^2$

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Q.No.11: Show that U is normal.

Solution:

$$UU^* = U^*U$$

Since we know that

$$U^* = U^{-1}$$

So

$$UU^{-1} = U^{-1}U$$

$(UV)^* = (UV)^{-1} = I$ Hence its proved that U is normal.

Q.No.12: Show that U^{-1} is Unitary or Prove that $U^* = U^{-1}$ or $\| U\| = 1$

Solution: Since U is bijective it shows that U^{-1} is bijective $(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}$. Hence it implies that U^{-1} is unitary.

Q.No.14: Show that U is Isometric.

Solution:

$$\| U_x\|^2 =< U_x, U_x >$$
$$\| U_x\|^2 =< x, U^*U_x >$$
$$\| U_x\|^2 =< x, Ix >$$
$$\| U_x\|^2 =< x, x >$$
$$\| U_x\|^2 = \| x\|^2 \text{ It proved.}$$

Q.No.15: Prove that $(ST)^* = S^*T^*$

Proof:

$$< x, (ST)^*y >=< (ST)x, y >$$
$$< x, (ST)^*y >=< (Sx)Tx], y >$$
$$< x, (ST]^*y >=< Sx, y >< Tx, y >$$
$$< x, (ST)^*y >=< x, S^*y >< x, T^*y >$$
$$< x, (ST]^*y >=< x, (S^*+T^*y) >$$
$$< x, (ST)^*y >=< x, (S^*T^*]y >$$

Hence we have proved that

$$(ST)^* = S^*T^*$$

Q.No.16: Show that $T^*T=0$ or $T=0$ is normal.

Solution:

$$T^*T = 0 \; iff \; T = 0$$
$$e = \|T^*T\| = \| T\|$$

(f) is true by e.

Q.No.17: Show that $(\alpha T)^* = \alpha^{-1}T^*$.

Solution:

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$$< x, (\alpha T^*)^* y > = < (\alpha T)x, y >$$
$$< x, (\alpha T^*)^* y > = < \alpha (Tx), y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < Tx, y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < x, T^* y >$$
$$< x, (\alpha T^*)^* y > = < x, \bar{\alpha} T^* y >$$
$$(\alpha T^*)^* = \bar{\alpha} T^*$$

Q.No.18: Show that $(T^*)^* = T$

Solution:

$$< x, (T^*)^* y \geq < xT^*, y >$$
$$< x, (T^*)^* y \geq < x, Ty >$$

It implies that

$(T^*)^* = T$ so its proved.

Q.No.19: Show that $\|T^*T\| = \|TT^*\| = \|T\|^2$

Solution:

$$\| U_x \|^2 = < U_x, U_x >$$

Q.No.20: Show that $< T^* y, x > = < y, Tx >$

Solution:

$$< T^* y, x > = < \overline{y, T^* x} >$$
$$< T^* y, x > = < \overline{Tx, y} >$$
$$< T^* y, x > = < y, Tx >$$

Hence its proved.

Q.No.21: Show that $Q = 0$ if and only if $< Qx, y >= 0$

Solution:

$$Q = 0 \Longrightarrow Qx = 0 \ for \ all \ x \epsilon X$$
$$< Qx, y > = < 0, y >$$
$$< Qx, y > = < 0.w, y >$$
$$< Qx, y > = 0 < w, y >$$
$$< Qx, y > = 0. \ for \ all \ x \epsilon X$$

It implies that Qx=0 and by definition Q=0. Its proved.

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Q.No.22: Write examples of Bounded Linear Operators

Solution: Identity Operator, Zero Operator, Differentiation Operator, Integral Operator

Q.No.23: Define Self Adjoint Operator or Hermitian Operator.

Solution:

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Self Adjoint Operator or Hermitian Operator if $T^* = T$.

Q.No.24: Define Self Adjoint Operator or Hermitian Operator.

Solution:

The Hilbert Adjoint Operator T^* of T is defined by $< Tx, y > = < x, T^* y >$ If T is self adjoint then it becomes $< Tx, y > = < x, Ty >$. If T is self adjoint or unitary then T is normal.

Q.No.25: Define Unitary Operator.

Solution:

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $T^* = T^{-1}$.

Q.No.26: Define Normal Operator.

Solution:

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $TT^* = T^*T$

Q.No.27: Define Extension of T.

Solution: Extension of T on a set $M \supset D(T)$ is defined as $\hat{T}: M \to Y$ or

$$\hat{T}x = Tx \; for \; all \; x \in D(T)$$

Q.No.28: Define Polarization Identity.

Solution:

For any x,y in complex inner product space we have

$$< x, y > = \frac{1}{4}\left(\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 \right)$$

Q.No.29: Define Bounded Linear Function.

Solution:

Let X and Y be normed spaces and $T: D(T) \to Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$\|Tx\| \leq c\|x\|$

Q.No.30: Define Canonical Mapping.

Solution:

$C: X \to X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

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Q.No.31: Define Orthonormal System.

Solution:

Orthonormal systems are those functions which are real or complex whose inner product with itself results in 1 and with other functions result in 0.

Q.No.32: Define Polarization Identity.

Solution:

For any x, y in complex inner product space

$$< x, y >= \frac{1}{4}\left\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\right\}$$

We have to prove this complex inner product space.

Q.No.33: Define Appolonius Identity:

Solution: $\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z-\frac{1}{2}(x+y)\right\|^2, \quad x, y, z \in V$

Q.No.34: Define Equal Operators:

Solution: Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have the same domain i.e. $D(T_1) = D(T_2)$

Q.No.35: Write examples of Inner Product Space:

Solution: Identity

Q.No.36: Define Inner Product Space:

Solution: The pair $(V, <\cdot,\cdot>)$ is called an inner product space.

Q.No.37: Define Linear Functional:

Solution: A linear functional f is a linear operator with domain in a vector space X and range in the scalar field K of X; thus $f: D(f) \to K$. where K=R if X is real and K=C if X is complex.

Q.No.38: Define Bounded Linear Extension:

Solution: $T: D(T) \to Y$ be a bounded linear operator, where D(T) lies in a normal space X and Y is a Banach Space then T has an extension $\hat{T}: \overline{D(T)} \to Y$. Where $\hat{T}$ is a bounded linear operator of norm.

$$\|\hat{T}\| = \|T\|$$

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