Mth641 Mega for Final term Fall (2022)

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Final objective (15files) in 1 File

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- Mth641 Final clear file solved by Moin pg42
- File 641 by raza pg42
- Confirmed by sunny mth641 pg42
- Mth641 with reference pg63
- MTH641 Final term MCQs pg85
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- ZE+BA official File(Mth641) solved by Best Vu Fellows pg86
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Mth641 Quiz#3 Fall (2022)

Module#57-90

Due Date 28Feb-2Mar

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- M@l!k MTH641 quizz 5 solution 27aug2021 pg47
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- Mth641 quiz 25.08.2021 correct pg51
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- Mth641 Quiz3(topic 57to90) (2mar2022)made by stylish pg22
- Mth641 Quiz3(topic 57to90)(2mar2022)Made by stylish pg44
- Mth641 Quiz3(topic 57to90)(3mar2022)Made by stylish pg56

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Question # 3 of 10 (Start time: 09:10:27 PM, 02 March 2022)

Null space is also a

Select the correct option

○	Linear functional.
○	Canonical mapping.
○	Vector space.  
○	Metric space.

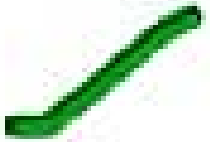
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Question # 3 of 10 (Start time: 10:06:35 PM, 07 March 2022)

If T_1 and T_2 are equal operators defined on a normed space X, then for any $x \in X, T_1x = T_2x \Longrightarrow$

Select the correct option

○	$x \neq 0$ necessarily
●	$x = 0$.



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MTH641:Quiz NO.03

Question # 7 of 10 (Start time: 10:48:39 PM, 02 March 2022)

If T is a continuous linear operator from a normed space X to normed space Y, then T is ___

Select the correct option

- bounded as well provided that X and Y are finite dimensional ✓
- bounded as well irrespective of dimension of X and Y
- unbounded as well irrespective of dimension of X and Y
- unbounded as well provided that X and Y are infinite dimensional

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MTH641:Quiz NO.03

Question # 10 of 10 (Start time: 09:07:58 PM, 02 March 2022)

In an Inner Product space say X, if the sequences $\{x_n\}$ and $\{y_n\}$ are Cauchy, then $\langle x_n, y_n \rangle$ is --------.

Select the correct option

○	*necessarily a Cauchy Sequence in F* ✓ Module 90
○	*necessarily a Cauchy Sequence in X*
○	*not necessarily a Cauchy Sequence in F*
○	*not necessarily a Cauchy Sequence in X*

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MTH641:Quiz NO.03

Question # 5 of 10 (Start time: 09:12:01 PM, 02 March 2022)

If $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (y, -5x + 4y)$, then $T^{-1}(-2, 7) =$_____

Select the correct option

○	(3,2)
○	(-3,2)
○	(3,-2)
○	(-3,-2) ✓

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Question # 4 of 10 (Start time: 09:36:25 PM, 02 March 2022)

If $\{x_n\}$ is a sequence in a null space $N(T)$ of a bounded linear operator T on a normed space X, then

Select the correct option

○	Tx_n is neither unit vector nor zero vector
○	Tx_n is either unit vector or zero vector
○	$Tx_n =$ zero vector
○	$Tx_n =$ unit vector



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MTH641:Quiz NO.03

Question # 5 of 10 (Start time: 10:03:10 PM, 02 March 2022)

Let $T : X \to Y$ be a linear operator, then restriction of T is expressed as

Select the correct option

O	$T_{\mid B} : Y \to B \ , \ B \subseteq Y$
O	$T_{\mid B} : B \to B \ , B \subseteq X$
O	$T_{\mid B} : X \to B \ , \ B \subseteq Y$
O	$T_{\mid B} : B \to Y \ , \ B \subseteq X$ ✓

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MTH641:Quiz NO.03

Question # 10 of 10 (Start time: 10:10:20 PM, 02 March 2022)

The pair

$$(V, <.\ .\ .>)$$

is called

Select the correct option

O	Banach space.
O	Inner product space.
O	Metric space.
O	Complete space.



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MTH641:Quiz NO.03

Question # 9 of 10 (Start time: 10:14:14 PM, 02 March 2022)

Let S and T are linear operators , we have

Select the correct option

○	$(ST)^{-1} = ST$
○	$(ST)^{-1} = I$
○	$(ST)^{-1} = T^{-1}S^{-1}$ ✓
○	$(ST)^{-1} = S^{-1}T^{-1}$

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Question # 6 of 10 (Start time: 10:53:22 PM, 02 March 2022)

If $T : \mathbb{R}^2 \to \mathbb{R}$ is defined by $T(x, y) = y - x$, then $T^{-1}(6)$_____.

Select the correct option

Relo

○	$\{(t, t-6) : t \in \mathbb{R}\}$
○	$\{(6+t, t) : t \in \mathbb{R}\}$
○	Both $\{(6+t, t) : t \in \mathbb{R}\}$ and $\{(t, t-6) : t \in \mathbb{R}\}$
○	None of these

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MTH641:Quiz NO.03

Question # 8 of 10 (Start time: 07:32:33 PM, 02 March 2022)

Dot product is a

Select the correct option

O	operator.
O	Mapping.
O	Functional. ✓
O	Function.

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MTH641:Quiz NO.03

Question # 5 of 10 (Start time: 08:22:02 PM, 02 March 2022)

Dual space of a normed space is

Select the correct option

- functional.
- Metric space.
- Incomplete.
- Banach space.

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MTH641:Quiz NO.03

Question # 7 of 10 (Start time: 08:37:22 PM, 02 March 2022)

Since a bounded linear operator T from the normed space X to normed space Y is defined and given as;

$\forall x \in D(T) \exists k > 0$, such that $\|Tx\| \leq k\|x\|$, then $\|T\| =$_____.

Select the correct option

O	$\sup_{\substack{x \in D(T) \\ \|x\|=1}} \|Tx\|$
O	both $\sup_{\substack{x \in D(T) \\ x \neq 0}} \frac{\|Tx\|}{\|x\|}$ and $\sup_{\substack{x \in D(T) \\ \|x\|=1}} \|Tx\|$ ✓
O	$\sup_{\substack{x \in D(T) \\ x \neq 0}} \frac{\|Tx\|}{\|x\|}$
O	None of these

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Question # 5 of 10 (Start time: 09:03:54 PM, 02 March 2022)

For n dimensional vector space and its dual space we have

Select the correct option

○	$R(X) = X^* = n$
○	$D(X) = X^* = n$
○	$\dim(X) \subset \dim(X^*)$
○	$\dim(X) = \dim(X^*) = n$

Module 75

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MTH641:Quiz NO.03

Quiz Start Time: 07:08 PM, 02 March 2022

Question # 10 of 10 (Start time: 07:17:02 PM, 02 March 2022)

Total Marks: 1

Which of the following an example of Linear Functional?

Select the correct option

Reload Math Equations

○	the integral operator $I : c[0,1] \to \mathbb{R}$, on the space of all contnuous functions on $[0,1]$ defined by $I(f) = \int_0^1 f(t)\,dt$
○	the derivative operator on space of all real polynomials

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sec(s)

MTH641:Quiz NO.03

Quiz Start Time: 11:55 PM, 02 March 2022

Question # 9 of 10 (Start time: 11:57:09 PM, 02 March 2022)

Total Marks: 1

If $T: c[0,1] \to c[0,1]$, which is defined and given as; $T(x)=\{\int_0^1 \kappa(t,\tau)x(\tau)d\tau : |\kappa| < k_o\}$, then $\forall\, x \in c[0,1]\ \ \exists k>0$ such that $\|Tx\| \leq k\|x\| \to$

Select the correct option

Reload Math Equations

○	T is non-linear bounded
○	T is linear unbounded
●	T is linear bounded ✓
○	T is non-linear unbounded

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Question # 3 of 10 (Start time: 11:58:31 PM, 02 March 2022)

Since a bounded linear operator T from the normed space X to normed space Y is defined and given as;

$\forall x \in D(T)\ \exists\ k > 0$, such that $\|Tx\| \leq k\|x\|$, then $\|T\| = 0 \Longrightarrow$

Select the correct option

○	$k = 0$
●	$x \in \{0\}$
○	$k < 0$
○	T is a zero operator



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Question # 10 of 10 (Start time: 11:57:19 PM, 02 March 2022)

Dual space of

$$l^1$$

is

Select the correct option

○	R
○	l^1
●	l^∞ ✓
○	R^n

Module 81, pg92

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MTH641:Quiz NO.03

Question # 5 of 10 (Start time: 11:50:06 PM, 02 March 2022)

If a sequence $x_n \to x$ in a normed space X, then for a bounded linear operator T on X, then

Select the correct option

○	$Tx_n \nrightarrow Tx$
●	$Tx_n \to Tx$ ✓

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MTH641:Quiz NO.03

Question # 3 of 10 (Start time: 11:49:36 PM, 02 March 2022)

The mapping; $T : V \to V$ defined by $T(v) = a + v$, where $v \in V$ is linear if

Select the correct option

○	$a = 0$
●	$a \neq 0$

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Question # 1 of 10 (Start time: 11:49:11 PM, 02 March 2022)

Norm in

$$V \text{ for } x \in V$$

is defined as

Select the correct option

●	$\|x\| = < x , x >$
○	$\|x\| = \sqrt{< x/2 , x/2 >}$
○	$\|x\| = \|< x , x >\|^2$
○ ✔	$\|x\| = \sqrt{< x , x >}$

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Question # 3 of 10 (Start time: 11:45:50 PM, 02 March 2022) Total Marks: 1

A linear functional f with domain D in a normed space is continuous if and only if f is

Select the correct option

O	Integrable.
O	Differentiable.
O	Real valued.
●	Bounded.



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MTH641:Quiz NO.03

Question # 4 of 10 (Start time: 11:56:13 PM, 02 March 2022)

Which of the following is not a condition of an inner product space?

Select the correct option

○	$\langle x, y\rangle = \overline{\langle y, x\rangle}$
●	$\langle \alpha x + y, z\rangle = \langle x, z\rangle + \alpha \langle y, z\rangle$ ✓
○	$\langle \alpha x, y\rangle = \alpha \langle x, y\rangle$
○	$\langle x, x\rangle \geqslant 0$

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Question # 4 of 10 (Start time: 11:43:06 PM, 02 March 2022)

Let $T : D(T) \to Y$ be a linear operator from normed space X to normed space Y, then

Select the correct option

●	$\forall x \in \overline{D(T)}$ $\exists$ a sequence $\{x_n\}$ in $D(T)$ such that $x_n \to x$ ✓
○	$\exists x \in \overline{D(T)}$ $\forall$ sequences $\{x_n\}$ in $D(T)$ such that $x_n \to x$
○	$\nexists x \in \overline{D(T)}$ $\forall$ sequences $\{x_n\}$ in $D(T)$ such that $x_n \to x$
○	$\forall x \in \overline{D(T)}$ $\nexists$ a sequence $\{x_n\}$ in $D(T)$ such that $x_n \to x$

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MTH641:Quiz NO.03

Question # 8 of 10 (Start time: 11:40:35 PM, 02 March 2022)

Every Inner product space is a metric space with norm given by;

Select the correct option

○	$\|x-y\| = \sqrt{\langle x-y, x-y\rangle}$
○	$d(x,y) = \|x-y\|$
●	All above are equivalent 
○	$d(x,y) = \sqrt{\langle x-y, x-y\rangle}$

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Question # 7 of 10 (Start time: 11:40:26 PM, 02 March 2022)

If $Tx_n = 0$ for a sequence $\{x_n\}$ in a null space of a bounded linear operator T on a normed space X, then $x_n \to x \Longrightarrow x =$

Select the correct option

Reload

○	is either unit vector or zero vector
○	unit vector
●	zero vector 
○	is neither unit vector nor zero vector

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MTH641:Quiz NO.03

Question # 4 of 10 (Start time: 11:39:54 PM, 02 March 2022)

If a normed space X is finite dimensional , then every linear operator on X is

Select the correct option

●	Bounded. ✓
○	Differentiable.
○	Integrable.
○	Continuous.

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MTH641:Quiz NO.03 Quiz Start Time: 11

Question # 3 of 10 (Start time: 11:39:46 PM, 02 March 2022)

Functionals defined on C[a,b] are

Select the correct option

- ● Linear and bounded.
- ○ Integrable.
- ○ Unbounded.
- ○ Canonical mappings.

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MTH641:Quiz NO.03

Question # 1 of 10 (Start time: 11:39:26 PM, 02 March 2022)

On a normed space X, for the identity operator $I : X \to X, \|I\| = \sup_{\substack{x \in D(I) \\ x \neq 0}} \frac{\|Ix\|}{\|x\|} =$_____.

Select the correct option

○	0
○	$\|kx\|, k > 0$
●	1 ✓ Module 62
○	$\|x\|$

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Question # 4 of 10 (Start time: 11:34:41 PM, 02 March 2022) T

For a fixed $t \in [0,1]$, defining the linear functional on the class of all continous functions on $[0,1]$, $f : c[0,1] \to \mathbb{R}$ as $f(x) = x(t)$, $\forall x \in c[0,1]$, then $\|f\| =$

Select the correct option

Reload Ma

○	1
●	0 ✓
○	$\max_{t\in[0,1]} x(t)$
○	$\min_{t\in[0,1]} x(t)$

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Question # 9 of 10 (Start time: 11:36:10 PM, 02 March 2022) Total Marks: 1

For an inner product space <x+y,z>=.....

Select the correct option

O	<x,z> - <y,z>
●	<x,z> + <y,z> ✓
O	None of these
O	<x,z> . <y,z>

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sec(s)

MTH641:Quiz NO.03

Quiz Start Time: 11:34 PM, 02 March 2022

Question # 1 of 10 (Start time: 11:34:05 PM, 02 March 2022)

Total Marks: 1

For a fixed $k = (k_1, k_2)$, defining the linear functional $f : \mathbb{R}^2 \to \mathbb{R}$ as $f(x) = x.k = x_1k_1 + x_2k_2$, $\forall (x_1, x_2) \in \mathbb{R}^2$, then $|f(x)| = |x.k| \leq$

Select the correct option

Reload Math Equations

●	$\|x\| \, \|k\|$
○	$\max_{x \in \mathbb{R}^1} \|f(x)\|$
○	$\max_{x \in \mathbb{R}} \|x\|$
○	$\|x\| \, \|k\|$

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MTH641:Quiz NO.03

Question # 10 of 10 (Start time: 11:31:38 PM, 02 March 2022)

Operator $T: D(T) \to Y$ is bounded if there is a real number c such that for all $x \in D(T)$,

Select the correct option

○	$\|Tx\| = c\|x\|$
○	$\|Tx\| \leqslant \|x\|$
●	$\|Tx\| \leqslant c\|x\|$ ✓ Module 60
○	$\|Tx\| \geqslant c\|x\|$

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MTH641:Quiz NO.03

Question # 3 of 10 (Start time: 11:29:54 PM, 02 March 2022)

On a normed space X, for the zero operator $O : X \to X, \|O\| = \sup_{\substack{x \in D(I) \\ x \neq 0}} \frac{\|Ox\|}{\|x\|} =$_____

Select the correct option

○	$\|x\|$
●	0
○	1
○	$\|kx\|, k > 0$

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MTH641:Quiz NO.03

Question # 2 of 10 (Start time: 11:29:39 PM, 02 March 2022)

Dual space of

R^n

is

Select the correct option

○	$\mathbb{C}^n$
● ✓	R^n
○	$\mathbb{Z}^n$
○	R

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Question # 8 of 10 (Start time: 11:24:00 PM, 02 March 2022)

If a sequence $x_n \to x$ in a normed space X, then for a bounded linear operator T on X_____.

Select the correct option

○	$\|Tx_n - Tx\| = 0$
●	$\|Tx_n - Tx\| \leq 0$
○	$\lim_{n \to \infty} \|Tx_n - Tx\| < 0$
○	$\lim_{n \to \infty} \|Tx_n - Tx\| > 0$

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MTH641:Quiz NO.03

Question # 7 of 10 (Start time: 11:23:49 PM, 02 March 2022)

For an element x belongs to an inner product space , $\langle x, x \rangle = \ldots\ldots\ldots$

Select the correct option

●	0 ✓
○	less than 0
○	infinity
○	greater than 0

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Question # 5 of 10 (Start time: 11:23:26 PM, 02 March 2022) Total Marks: 1

Zero vector lemma applies on

Select the correct option

●	Finite dimensional vector space. 
○	Canonical mapping.
○	Dual space of a vector space.
○	Infinite dimensional vector space.

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Question # 10 of 10 (Start time: 11:15:02 PM, 02 March 2022)

Let X be a normed space, $f : X \to \mathbb{R}$ and $g : X \to \mathbb{R}$ be the linear functionals, then $f + g : X \to \mathbb{R}$ defined by $(f + g)(x) = f(x) + g(x) \forall x \in X$, is_____ a linear functional.

Select the correct option

- ○ not necessarily
- ○ never
- ● bounded
- ○ essentially

C or D

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Question # 4 of 10 (Start time: 11:23:07 PM, 02 March 2022) Total M

If T is a linear opertor on a finite dimensional normed space X having basis $\{e_1, e_2, \ldots, e_n\}$,then for any $x \in X \exists \{\alpha_i\}_{i=1}^{n} \subset F$, such that

Select the correct option

Reload Math Equ

○	$x = \sqrt{\sum_{i=1}^{n} \alpha_i e_i}$
○	$x = \alpha_i \sum_{i=1}^{n} e_{i_1}$ for any fixed α_i
○	$x = e_i \sum_{i=1}^{n} \alpha_{i_1}$ for any fixed e_i
●	$x = \sum_{i=1}^{n} \alpha_i e_i$ ✔

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MTH641:Quiz NO.03

Question # 7 of 10 (Start time: 11:14:20 PM, 02 March 2022)

For a bounded linear operator T , the null space $\mathrm{N}(T)$ is

Select the correct option

●	closed. 
○	open.
○	unbounded.
○	bounded.

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MTH641:Quiz NO.03

Question # 6 of 10 (Start time: 11:13:58 PM, 02 March 2022)

If $T : \mathbb{R}^2 \to \mathbb{R}$ is defined by $T(x, y) = y - x$, then $T^{-1}(6) = \{(t, t-6) : t \in \mathbb{R}\} \Longrightarrow$

Select the correct option

●	T is one-one
○	T is not one-one

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MTH641:Quiz NO.03

Question # 5 of 10 (Start time: 11:13:32 PM, 02 March 2022)

In an Inner Product space say X, for any sequences $\{x_n\}$ and $\{y_n\}$, if $x_n \longrightarrow x$ and $y_n \longrightarrow y$, then it ——.

Select the correct option

○	$\Longrightarrow \langle x_n, y_n \rangle = \langle x, y \rangle$
○	$\not\Rightarrow \langle x_n, y_n \rangle \longrightarrow \langle x, y \rangle$
●	$\Longrightarrow \langle x_n, y_n \rangle \longrightarrow \langle x, y \rangle$ ✓ Module 90
○	$\Longrightarrow \langle x_n, y_n \rangle \neq \langle x, y \rangle$

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MTH641:Quiz NO.03

Question # 4 of 10 (Start time: 11:13:07 PM, 02 March 2022)

Which of the following is a condition of an inner product space?

Select the correct option

○	$\langle \alpha x + y, z \rangle = \langle x, z \rangle + \alpha \langle y, z \rangle$
○	$\langle x, y \rangle = \overline{\langle x, y \rangle}$
○	$\langle \alpha x, y \rangle = \langle x, \alpha y \rangle$
●	$\langle x, x \rangle \geqslant 0$

Module83

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MTH641:Quiz NO.03

Question # 3 of 10 (Start time: 11:12:57 PM, 02 March 2022)

Let $T : \mathbb{R} \to \mathbb{R}$ be defined by $T(x) = x^2$, then the restriction $T|_A$ is one-one on $A =$

Select the correct option

○	$(-\infty, 0)$
●	$(-\infty, \infty)$ ✓
○	$(-\infty, 1)$
○	$(-1, \infty)$

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Question # 10 of 10 (Start time: 11:09:19 PM, 02 March 2022)

On a normed space X of all n degree polynomials defined on $[-1,1]$,if D is a differential linear operator defined and given as;

$D(x(t)) = \{\frac{d}{dt}x(t) : x(t) \in X, -1 \leq t \leq 1\}$, then $\forall x \in X \; \exists \; k > 0$ such that $\|Dx\| \leq k\|x\| \Longrightarrow$

Select the correct option

○	D is a non-linear bounded
○	D is non-linear unbounded
●	D is a linear bounded ✓
○	D is linear unbounded

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MTH641:Quiz NO.03

Question # 9 of 10 (Start time: 11:08:59 PM, 02 March 2022)

Since a bounded linear operator T from the normed space X to normed space Y is defined and given as;

$\forall x \in D(T)\ \exists\ k > 0$, such that $\|Tx\| \leq k\|x\|$, then for any $\alpha \in F$, $\|\alpha T\| =$

Select the correct option

○	$\sup_{\substack{x \in D(T) \\ \|X\|=1}} (-\|\alpha\|)\|Tx\|$
●	$\sup_{\substack{x \in D(T) \\ \|x\|=1}} \|\alpha\|\|Tx\|$ ✓
○	$\sup_{\substack{x \in D(T) \\ \|x\|=1}} \alpha\|Tx\|$
○	$\sup_{\substack{x \in D(T) \\ x \neq 0}} \frac{\alpha\|Tx\|}{\|x\|}$

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MTH641:Quiz NO.03

Question # 1 of 10 (Start time: 11:29:20 PM, 02 March 2022)

If T_1 and T_2 are bounded linear operators from the normed space X into Y, then $\|T_1 + T_2\|$_____.

Select the correct option

○	$= \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|(T_1 + T_2)\, x\|$
●	All options are equivalent ✓
○	$\leq \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|T_1 x\| + \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|T_2 x\|$
○	$= \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|T_1 x + T_2 x\|$

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MTH641:Quiz NO.03

Question # 8 of 10 (Start time: 11:08:43 PM, 02 March 2022)

If T is a bounded linear operator from a normed space X to normed space Y, then T is _____.

Select the correct option

○	unbounded as well provided that X and Y are infinite dimensional
○	bounded as well irrespective of dimension of X and Y
○	unbounded as well irrespective of dimension of X and Y
●	continuous as well provided that X and Y are finite dimensional 

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Question # 7 of 10 (Start time: 11:08:32 PM, 02 March 2022)

If T_1 and T_2 are linear operators from $\mathbb{R}$ into $\mathbb{R}$, defined by $T_1(x) = -x$ and $T_2(x) = x, \forall x \in \mathbb{R}$, then $(T_1 T_2)^{-1}(x) =$____

Select the correct option

●	$-\frac{1}{x}$
○	$\frac{1}{x}$
○	x
○	$-x$

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MTH641:Quiz NO.03

Quiz Start Time: 11:07 PM, 02 March 2022

Question # 6 of 10 (Start time: 11:08:19 PM, 02 March 2022)

Total Marks: 1

Let X and Y be normed spaces. A linear operator $T:X \to Y$ is said to be bounded if ——— such that $\|Tx\| \leq k\|x\|$.

Select the correct option

Reload Math Equations

○	$\exists\ k>0, \forall\ x \in X$
●	$\forall\ k>0, \exists\ x \in X$
○	$\forall\ k>0, \exists\ x \in X$
○	$\exists\ k>0, \forall\ x \in X$

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MTH641:Quiz NO.03

Quiz Start Time: 11:07 PM, 02 March 2022

Question # 5 of 10 (Start time: 11:08:08 PM, 02 March 2022)

Total Marks: 1

A hilbert space is a /an.....

Select the correct option

○	Incomplete norm space
○	Incomplete Inner product space
●	complete Inner product space ✓
○	complete norm space

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MTH641:Quiz NO.03

Quiz Start Time: 11:07 PM, 02 March 2022

Question # 4 of 10 (Start time: 11:07:59 PM, 02 March 2022)

Total Marks: 1

Let $T : D(T) \to Y$ be a bounded linear operator from normed space X to Banach space Y, and $\forall x \in \overline{D(T)}$ $\exists$ a sequence $\{x_n\}$ in $D(T)$ such that $x_n \to x$,then $\|Tx_m - Tx_n\| \Longrightarrow$

Select the correct option

Reload Math Equations

○	$\{Tx_n\}$ may not converges in Y
○	$\{Tx_n\}$ converges in $\overline{Y}$
●	$\{Tx_n\}$ converges in Y ✓
○	$\{Tx_n\}$ converges in $\overline{R(T)}$

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MTH641:Quiz NO.03

Question # 3 of 10 (Start time: 11:07:46 PM, 02 March 2022)

The following expression represents ..... inequality $\frac{|x-z|}{1+|x-z|} \leqslant \frac{|x-y|}{1+|x-y|} + \frac{|y-z|}{1+|y-z|}$

Select the correct option

○	Minkowski's inequality
○	Holder's inequality
●	Triangular inequality ✓
○	Rectangle inequality

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MTH641:Quiz NO.03

Question # 2 of 10 (Start time: 11:07:35 PM, 02 March 2022)

Since a bounded linear operator T from the normed space X to normed space Y is defined and given as;

$\forall x \in D(T)\ \exists\ k > 0$, such that $\|Tx\| \leq k\|x\|$, then the minimum value of k is _____.

Select the correct option

●	any arbitrary non negative real number ✔
○	$\inf\limits_{\substack{x \in D(T) \\ X \neq 0}} \frac{\|Tx\|}{\|x\|}$
○	$\sup\limits_{\substack{x \in D(T) \\ X \neq 0}} \frac{\|Tx\|}{\|x\|}$
○	not defined

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Question # 1 of 10 (Start time: 11:07:26 PM, 02 March 2022)

Which of the following is an example of Linear Functional?

Select the correct option

- ● the projection function $\pi_i : \mathbb{R}^n \to \mathbb{R}$ defined by $\pi_i(x_1, x_2, \ldots, x_i, \ldots x_n) = x_i$ ✓
- ○ the determinant function $D : V \to \mathbb{R}$ from the space of n-square matrices to $\mathbb{R}$

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MTH641:Quiz NO.03

Question # 10 of 10 (Start time: 11:05:22 PM, 02 March 2022)

If T is a linear opertor on a finite dimensional normed space X having basis $\{e_1, e_2, \ldots, e_n\}$,then_____, such that $x = \sum_{i=1}^{n} \alpha_i e_i$.

Select the correct option

○	$\forall x \in X \nexists \{\alpha_i\}_{i=1}^{n} \subset F$
○	$\nexists x \in X \forall \{\alpha_i\}_{i=1}^{n} \subset F$
○	$\exists x \in X \forall \{\alpha_i\}_{i=1}^{n} \subset F$
●	$\forall x \in X \exists \{\alpha_i\}_{i=1}^{n} \subset F$ ✓

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MTH641:Quiz NO.03

Question # 9 of 10 (Start time: 11:05:05 PM, 02 March 2022)

If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T(x, y) = (y - x, y, x - 1)$, then the value of $T(0, 0) \Longrightarrow$

Select the correct option

○	T^{-1} does not exists
●	T is not linear 
○	T is linear
○	T^{-1} exists

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MTH641:Quiz NO.03

Question # 8 of 10 (Start time: 11:04:48 PM, 02 March 2022)

If T is a linear opertor on a finite dimensional normed space X having basis $\{e_1, e_2, \ldots, e_n\}$,then for any $x \in X \exists \{\alpha_i\}_{i=1}^n$, $x = \sum_{i=1}^n \alpha_i e_i$, then $\|T\left(\sum_{i=1}^n \alpha_i e_i\right)\|$

Select the correct option

○	$\leq \sum_{i=1}^n \|T\alpha_i\| \, \|Te_i\|$
○	$\leq T \, \|\sum_{i=1}^n \alpha_i e_i\|$
●	$\leq \sum_{i=1}^n \|\alpha_i\| \, \|Te_i\|$ ✓ Module 74, pg86
○	$\leq \sum_{i=1}^n \|T\alpha_i\| \, \|e_i\|$

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Question # 6 of 10 (Start time: 11:04:25 PM, 02 March 2022)

On a normed space X of all polynomials of form $x(t) = t^{n+1}, n \in \mathbb{N}$ defined on $[-1, 1]$,if D is a differential linear operator defined and given as;

$D(x(t)) = \left\{ \frac{d}{dt} x(t) : x(t) \in P[-1,1], -1 \leq t \leq 1 \right\}$, then $\|D\| =$_____

Select the correct option

Reload

- ○ 0
- ○ 1
- ● $n+1$
- ○ n

Module 62

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MTH641:Quiz NO.03

Question # 2 of 10 (Start time: 11:03:32 PM, 02 March 2022)

Every Inner Product space is a Metric Space as well.

Select the correct option

○	True
●	False

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MTH641:Quiz NO.03

Question # 1 of 10 (Start time: 11:03:21 PM, 02 March 2022)

Every complete Inner product space is -------.

Select the correct option

●	Hilbert space 
○	Euclidean space
○	Banach space
○	Complex space

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Question # 10 of 10 (Start time: 11:02:24 PM, 02 March 2022)

If T_1 and T_2 are equal operators defined on a normed space X, then for any $x \in X, T_1x = T_2x \Longrightarrow$

Select the correct option

●	$T_1 - T_2$ is a zero operator
○	$T_1 - T_2$ is not necessarily a zero operator

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Question # 8 of 10 (Start time: 11:01:50 PM, 02 March 2022)

Let $(V, \langle .,. \rangle)$ be an inner product space over a field F, then .....

Select the correct option

Reload

- ● $\langle x, \alpha . y \rangle = \bar{\alpha} \langle x, y \rangle , \quad \forall x, y \in V, \alpha \in F.$
- ○ $\langle x, \alpha . y \rangle = \bar{\alpha} \langle y, x \rangle , \quad \forall x, y \in V, \alpha \in F.$
- ○ $\langle x, \alpha . y \rangle = \alpha \langle x, y \rangle , \quad \forall x, y \in V, \alpha \in F.$
- ○ $\langle x, \alpha . y \rangle = \bar{\alpha} \langle x, x \rangle , \quad \forall x, y \in V, \alpha \in F.$

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MTH641:Quiz NO.03

Question # 7 of 10 (Start time: 11:01:39 PM, 02 March 2022)

Let $T : [0, \infty) \to \mathbb{R}$ be defined by $T(x) = x$, then its extension $\tilde{T}$ on $M = \mathbb{R}$ is

Select the correct option

●	All above are valid
○	$\tilde{T}(x) = \frac{x+\|x\|}{2}$
○	$\tilde{T}(x) = \|x\|$
○	$\tilde{T}(x) = x$

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MTH641:Quiz NO.03

Question # 5 of 10 (Start time: 11:01:18 PM, 02 March 2022)

If T is a bounded linear operator on a normed space X, then for the NuW space $N(T)$;

Select the correct option

●	$N(T) \subseteq \overline{N(T)}$ ✓
○	$N(T) \not\subseteq \overline{N(T)}$

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Question # 4 of 10 (Start time: 11:01:06 PM, 02 March 2022) To

A bounded linear operator T from the normed space X to normed space Y is defined and given as: $\forall x \in D(T)\ \exists\ k > 0$, such that $\|Tx\| \leq k\|x\|$ .If $D(T) = \{0\}$, then $\|T\| =$

Select the correct option

Reload Mat

○	k^2
●	0
○	∞
○	$\frac{1}{k}$

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MTH641:Quiz NO.03

Question # 2 of 10 (Start time: 11:00:41 PM, 02 March 2022)

Two operators T_1 and T_2 are equal if

Select the correct option

- (●) $D(T_1) = D(T_2)$ and $\forall\ x \in D(T_1) = D(T_2) \ \Rightarrow\ T_1(x) = T_2(x)$ ✓
- ( ) $\forall\ x \in D(T_1) = D(T_2) \ \Rightarrow\ T_1(x) = T_2(x)$
- ( ) $D(T_1) = D(T_2)$ and $\forall\ x \in R(T_1) = R(T_2) \ \Rightarrow\ T_1(x) = T_2(x)$
- ( ) $D(T_1) = D(T_2)$

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MTH641:Quiz NO.03

Question # 1 of 10 (Start time: 11:00:26 PM, 02 March 2022)

If T_1 and T_2 are bounded linear operators from the normed space X into Y, then $\|T_1 + T_2\|$_____.

Select the correct option

○	$\geq \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|(T_1 + T_2)\, x\|$
●	$= \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|T_1 x\| + \sup_{\substack{x \in D(T) \\ \|X\|=1}} \|T_2 x\|$  Module 61
○	None of these
○	$\leq \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|T_1 x + T_2 x\|$

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MTH641:Quiz NO.03

Question # 10 of 10 (Start time: 10:56:52 PM, 02 March 2022)

If $A : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as $Ax = y$ and given by;

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \Longrightarrow \eta_j = \sum_{i=1}^{2} \alpha_{ji}\xi_i$$, then A is

Select the correct option

●	linear operator 
○	non-linear operator

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Question # 9 of 10 (Start time: 10:56:42 PM, 02 March 2022)

Let $T : X \to Y$ be an invertible linear operator from X to Y, where $\dim X, \dim Y < \infty$, then

Select the correct option

●	$\dim X = \dim Y$
○	$\dim X < \dim Y$
○	$\dim X > \dim Y$
○	$\dim X = -\dim Y$

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Question # 7 of 10 (Start time: 10:56:17 PM, 02 March 2022) Total Marks: 1

Matrix operator is

Select the correct option

O	Linear.
O	Nonlinear and unbounded.
O	Non linear.
●	Linear and bounded. 

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MTH641:Quiz NO.03

Question # 6 of 10 (Start time: 10:56:04 PM, 02 March 2022)

For an inner product space defined on a real vector space $\langle x, y\rangle = \ldots\ldots\ldots$

Select the correct option

○	$\langle x, y\rangle$
○	$\langle y, -x\rangle$
○	$\langle -y, x\rangle$
●	$\langle y, x\rangle$ 

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MTH641:Quiz NO.03

Quiz Start Time: 10:55 PM, 02 March 2022

Question # 5 of 10 (Start time: 10:55:55 PM, 02 March 2022)

Total Marks: 1

For a fixed $k = (k_1, k_2)$, defining the linear functional $f : \mathbb{R}^2 \to \mathbb{R}$ as $f(x) = x.k = x_1k_1 + x_2k_2, \ \forall (x_1, x_2) \in \mathbb{R}^2$, then $\|f\| =$

Select the correct option

Reload Math Equations



A or B

●	$\max(x_1k_1, x_2k_2)$
○	$\|x\|$
○	$\|k\|$
○	$\min(x_1k_1, x_2k_2)$

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sec(B)

MTH641:Quiz NO.03

Quiz Start Time: 10:55 PM, 02 March 2022

Question # 4 of 10 (Start time: 10:55:48 PM, 02 March 2022)

Total Marks: 1

Let $B(X,Y)$ be the set of all _____operators from a normed space X to a normed space Y. If Y is a Banach space, then $B(X,Y)$ is a Banach space.

Select the correct option

Reload Math Equations

○	Non linear
○	Linear
●	Bounded linear
○	Unbounded linear

Module 82

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MTH641:Quiz NO.03

Quiz Start Time: 10:55 PM, 02 March 2022

Question # 2 of 10 (Start time: 10:55:27 PM, 02 March 2022)

Total Marks: 1

For a fixed $k = (k_i)_{i=1}^{\infty} \in l^2$, defining the linear functional $f : l^2 \to \mathbb{R}$ as $f(x) = \sum_{i=1}^{\infty} x_i k_i$, $\forall (x_i)_{i=1}^{\infty} \in l^2$, then $|f(x)| \leq$

Select the correct option

Reload Math Equations

○	$\|f\| \, \|x\|$
●	$\|\|x\|\| \, \|\|k\|\|$
○	$\|\|f\|\| \, \|\|k\|\|$
○	$\|x\| \, \|k\|$

Not confirm

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Question # 1 of 10 (Start time: 10:55:12 PM, 02 March 2022)

For algebraically reflexive mapping

Select the correct option

○	$X \subset X^{**}$
○	$D(C) = X^{**}$
○	$X = X^{**}$
●	$R(C) = X^{**}$ ✓

Module 73

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MTH641:Quiz NO.03

Question # 9 of 10 (Start time: 10:51:33 PM, 02 March 2022)

Canonical mapping is defined as

Select the correct option

○	$C : X \to R(Y)$
○	$C : X \to X$
●	$C : X \to X^{**}$ ✓
○	$C : X \to R$

Module 72, pg 85

Scanned with CamScanner

MTH641:Quiz NO.03

Quiz Start Time: 10:49 PM, 02 March 2022

Question # 10 of 10 (Start time: 10:51:50 PM, 02 March 2022)

Total Marks: 1

If the integral operator $I : c[0,1] \to \mathbb{R}$, on the space of all contnuous functions on $[0,1]$ defined by $f(x) = \int_0^1 x(t)\,dt$, is a linear functional, then $|f(x)| \leq$

Select the correct option

Reload Math Equations

○	$\|x\|$
○	1
●	$\max_{0 \leq t \leq 1} \|x(t)\|$ ✓
○	$\|x\|$



Module 62, pg81

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MTH641:Quiz NO.03

Question # 8 of 10 (Start time: 10:51:18 PM, 02 March 2022)

Since a bounded linear operator T from the normed space X to normed space Y is defined and given as;

$\forall x \in D(T)\ \exists\ k > 0$, such that $\|Tx\| \leq k\|x\|$, then the maximum value of k is_____.

Select the correct option

○	$\inf\limits_{\substack{x \in D(T) \\ X \neq 0}} \frac{\|Tx\|}{\|x\|}$
○	not defined
○	any arbitrary non negative real number
●	$\sup\limits_{\substack{x \in D(T) \\ X \neq 0}} \frac{\|Tx\|}{\|x\|}$  

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MTH641:Quiz NO.03

Question # 7 of 10 (Start time: 10:51:05 PM, 02 March 2022)

T is a linear operator . If T^{-1} exists, it is a

Select the correct option

●	linear operator. ✓ Module 58, pg75
○	discontinuous operator.
○	Zero operator.
○	non linear operator.

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MTH641:Quiz NO.03

Question # 6 of 10 (Start time: 10:50:52 PM, 02 March 2022)

Linear Functional f is a linear operator from a normed space X to _____.

Select the correct option

○	range of f i.e $f:X \to R(f) \subseteq Y$
●	scalar field F of given vector space $X(F)$ i.e. $f:X \to F$ ✓
○	any normed space Y i.e. $f:X \to Y$
○	a Banach space Y i.e. $f:X \to Y$

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Question # 5 of 10 (Start time: 10:50:42 PM, 02 March 2022) Total Marks: 1

..... is algebraically reflexive.

Select the correct option

- ● A finite dimensional vector space
- ○ A dual space of a vector space
- ○ An infinite dimensional vector space
- ○ Canonical mapping



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MTH641:Quiz NO.03

Question # 4 of 10 (Start time: 10:50:26 PM, 02 March 2022)

For all x, y belongs to an an inner product space

$$\langle \alpha x, y \rangle = \ldots\ldots\ldots\ldots$$

Select the correct option

○	$\alpha \langle x, -y \rangle$
● ✓	$\alpha \langle x, y \rangle$
○	$\alpha \langle -x, y \rangle$
○	$\alpha \langle y, x \rangle$

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Question # 3 of 10 (Start time: 10:50:20 PM, 02 March 2022) Total Marks: 1

Which of the following is/are bounded linear operator

Select the correct option

○	Identity operator .
●	All given options. ✔ Module 62
○	Zero operator.
○	Integral operator.

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MTH641:Quiz NO.03

Question # 2 of 10 (Start time: 10:50:01 PM, 02 March 2022)

If $A:R(R) \to R(R)$ is defined as $Ax=y$ and given by;$a\zeta=\eta$, and if $\|x\|=|\zeta|,\|y\|=|\eta|$,then A is

Select the correct option

●	bounded linear operator ✔
○	bounded non-linear operator
○	unbounded non-linear operator
○	unbounded linear operator

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MTH641:Quiz NO.03

Quiz Start Time: 10:49 PM, 02 March 2022

Question # 1 of 10 (Start time: 10:49:50 PM, 02 March 2022)

Total Marks: 1

Let X be a normed space, $f : X \to \mathbb{R}$ be the linear functional and for any $\alpha \in \mathbb{R}$, then $\alpha f : X \to \mathbb{R}$ defined by $(\alpha f)(x) = \alpha f(x) \forall x \in X$, is _____ a linear functional.

Select the correct option

Reload Math Equations

○	never
○	essentially
○	not necessarily
●	bounded

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MTH641 Functional Analysis

Question No : 28 of 41

P_n is the collection of all.....

Answer (Please select your correct option)

- Polynomials of degree greater then equal to 'n'
- Polynomials of degree less then equal to 'n'
- Polynomials of degree equal to 'n'
- Polynomials of degree less then 'n'

Start Time: 11:20 AM

110:00 Time Left

Question Summary

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MTH641 Functional Analysis

Question No : 29 of 41

The pair $(V, \langle .,. \rangle)$ is called ..... space

Answer (Please select your correct option)

- Norm
- Metric
- Inner product
- Vector

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MTH641 Functional Analysis

Question No : 16 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

- $\langle \alpha x, y \rangle = \langle x, \alpha y \rangle$
- $\langle \alpha x + y, z \rangle = \langle x, z \rangle + \alpha \langle y, z \rangle$
- $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$
- $\langle x, y \rangle = \overline{\langle x, y \rangle}$

Start Time: 11:20 AM

114:00 Time Left

16

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

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MTH641 Functional Analysis

Question No : 17 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

- $\langle x,x\rangle \geq 0$
- $\langle \alpha x+y,z\rangle = \langle x,z\rangle + \alpha\langle y,z\rangle$
- $\langle \alpha x,y\rangle = \langle x,\alpha y\rangle$
- $\langle x,y\rangle = \overline{\langle x,y\rangle}$

Start Time 11:20 AM

113:00 Time Left

17

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Product Instructions

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Question No : 9 of 41

Let T be a bounded linear operator. Then the _____ is closed.

Answer (Please select your correct option)

- Banach space
- null space $N(T)$
- Hilbert space
- None of these

Start Time: 11:20 AM

115:00 Time Left

9

Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 1 of 41

The property $d(x, y) \geq 0$ of a metric space is called.....

Answer (Please select your correct option)

- Non negativity
- Reflexive property
- Symmetric property
- Triangle property

Start Time: 11:20 AM

116:00
Time Left

Question Summary : (Attempted

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Question No : 8 of 41 **Marks: 1 (Budgeted**

Let $T : D(T) \to Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then T is _____ if and only if T is bounded.

Answer (Please select your correct option)

- continuous
- linear ✔
- bounded
- none of these

Start Time: 11:20 AM

115:00 Time Left

8

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

Conduct Instructions

Click here to FINISH Your Exam

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MTH641 Functional Analysis

Question No : 4 of 41

Two metric spaces (X,d) and $(\tilde{X},\tilde{d})$ are said to be isometric if there exists ..... isometry of X onto $\tilde{X}$

p=44 mid

Answer (Please select your correct option)

- An injective
- A bijective ✓
- A surjective
- none of these

Start Time: 11:20 AM

115:00 Time Left

4

Question Summary : (Attempted Question)

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MTH641 Functional Analysis **solved and rechecked by Khalil Sahir (referenced)**

Question No : 2 of 41

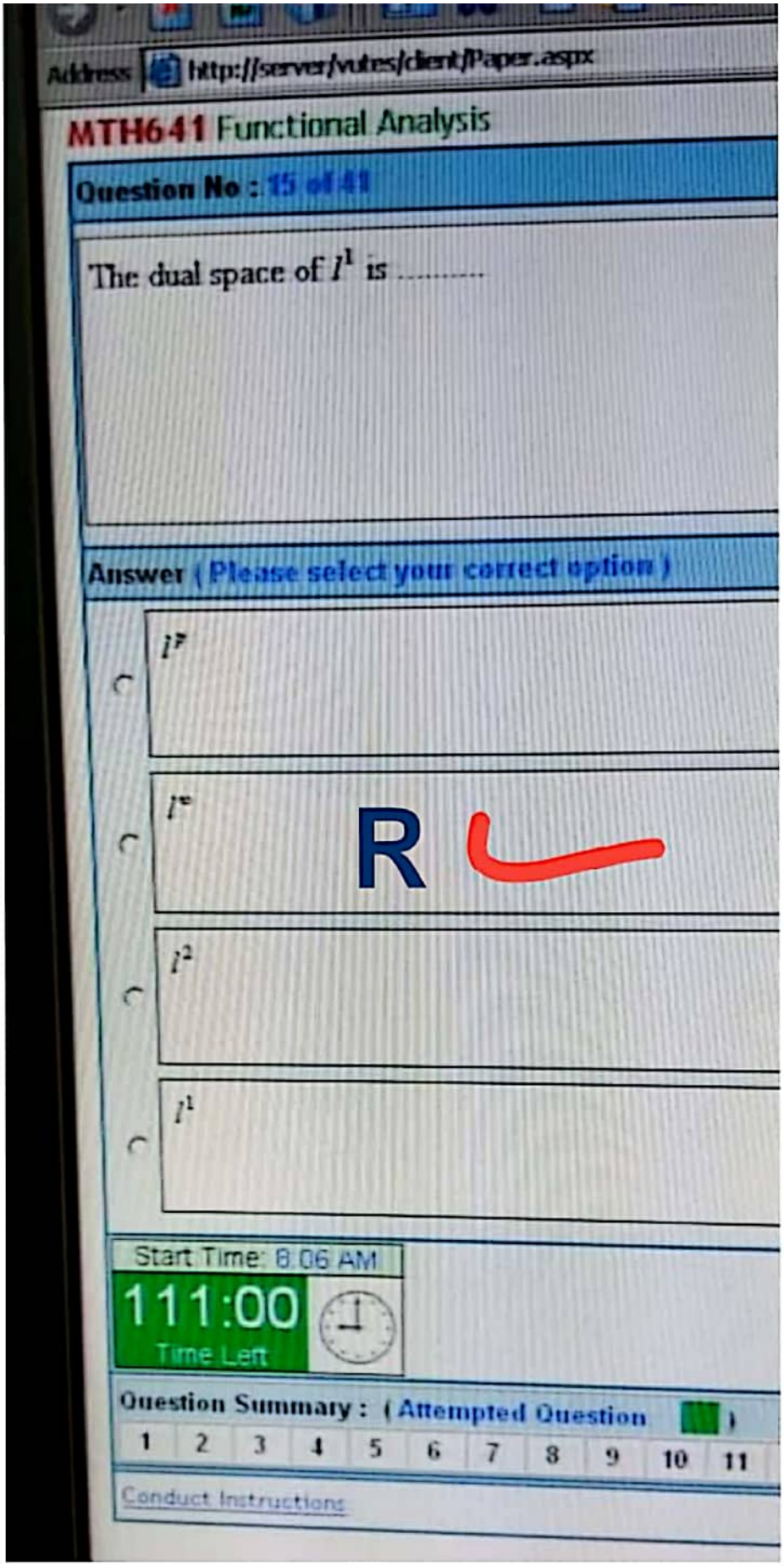
A mapping $T: X \to Y$ is said to be continuous if it is continuous at ..... point of X

Answer (Please select your correct option)

- Every
- atmost one
- exactly one
- atleast one

R

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Space l^1

The dual space of l^n is l^∞ means that it is bijective, it is linear and it preserve norm. After defining the map we shall prove these properties one by one.

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Question No : 6 of 41

Which one of the following axioms hold for any vectors u, v, w in vector space X and k, a, b in field K ?

Answer (Please select your correct option)

- $u + (-u) = (-u) + u$, for each $u \in X$
- $u + w = w - u$
- $k(u + v) = ku - kv$
- $(ab)u = a(bu)$

R

Start Time: 8:06 AM

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Question No : 30 of 41

An orthonormal set M is an orthogonal set in an inner product space whose elements have norm ....

Answer (Please select your correct option)

- equal to 1
- >1
- <1
- None of these

R

Start Time: 8:06 AM

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MTH641 Functional Analysis

Question No : 28 of 41

Inner product is the generalization of .....

Quiz: Complete inner product space is called a Hilbert Space.
In inner product the generalization is dot product.

Answer (Please select your correct option)

- Cross product
- Dot product
- Multiplication
- Distance function

R

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MTH641 Functional Analysis

Question No : 24 of 41

Counter example2: **Space** l^p

l^p Collection of all bounded sequences,

$P > 1, P \neq 2$ if p=2 then it will give inner product space

$$\{x_i\}, \quad \|x\| = \sqrt[p]{\sum_{i=1}^{\infty} |x_i|^p}$$

We will see that $< x, x >= \|x\|^2$ is an inner product space or not. We will check this if it satisfied the parallelogram or not.

Which of the following is not true for the space l^p with $p \neq 2$?

Answer (Please select your correct option)

- It is a complete metric space
- It is a complete norm space
- It is a complete inner product space
- None of these

R

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Question No : 29 of 45

Any pair of vectors in an orthogonal system is _____ to each other

Answer (Please select your correct option)

- Parallel
- Equal
- None of these
- Perpendicular

R

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Question No : 10 of 41

Let $T: D(T) \to Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then T is continuous if and only if T is _____

Answer (Please select your

- continuous
- linear
- bounded
- none of these

Start Time: 8:06 AM

OPERATORS ON FINITE DIMENSIONAL SPACES:

Remarks:

As in the case of linear operators on a finite dimensional normed space, every linear functional defined on a finite dimensional normed space is bounded and hence continuous.

R

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Question No : 12 of 41

Two operators T_1 *and* T_2 are defined to be equal, written $T_1 = T_2$. if _____ and if $T_1 x = T_2 x$ for all_____

Answer (Please select your correct option)

- $D(T_1) = D(T_2)$, $x \in D(T_1) = D(T_2)$
- $D(T_1) \subset D(T_2)$, $x \in D(T_1) = D(T_2)$
- $D(T_1) \supseteq D(T_2)$, $x \in D(T_1) = D(T_2)$
- $D(T_1) \supset D(T_2)$, $x \in D(T_1) = D(T_2)$

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MTH641 Functional Analysis

Question No : 20 of 41

Every complete inner product space is _____ space.

Hilbert Space

Quiz: Complete inner product space is called a Hilbert Space.

Answer (Please select your correct option)

- Hilbert
- Banach
- Unitary
- Euclidean

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Question Summary : (Attempted Question)

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Question No : 22 of 41

Which of the following is an example of an inner product space.

Examples of Inner product spaces

- **SPACE** $\mathbb{R}^n$
- **SPACE** $\mathbb{C}^n$
- **SPACE** $\mathbb{C}[a,b]$
- **SPACE** l^n
- **SPACE** P_n (Collection of all polynomials of degree n)

Answer (Please select your co

- (a,b)
- $[a,b]$
- Space R^n
- None of these

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 19 of 41

Which of the following is true about the inner product space?

Inner product define a norm and under this norm

Every inner product space is a norm space.

Every norm space is not an inner product space. This is not true always.

If a space is inner product then it satisfied the parallelogram law otherwise it is not an inner product space.

- Every normed space is an inner product space.
- Every inner product space is a normed space.
- Every inner product space is Hilbert space.
- None of these is true.

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 11 of 41

The dual space of a normed space X contains all ..... on X.

Since for linear funcionals range is either $\mathbb{R}$ or $\mathbb{C}$, which are complete. So X^* as the space of all bounded linear functionals defined on X, is also complete and hence is Banach space. This is true even if X is not a Banach space.

"Algebraic Dual Space of X": set of all linear funcionals defined on X.

"Dual or Conjugate Space of X": X^* set of all continuous or bounded linear functionals defined on X.

Answer

- Bounded linear functional
- Unbounded linear functional
- Bounded linear operator
- Unbounded linear operator

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Question No : 17 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

- $\langle x,y\rangle = \overline{\langle x,y\rangle}$
- $\langle ax,y\rangle = \langle x,ay\rangle$
- $\langle ax+y,z\rangle = \langle x,z\rangle + a\langle y,z\rangle$
- $\langle x,y\rangle = \overline{\langle y,x\rangle}$

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For any non empty subset A of (X,d), the diameter $\delta(A)$ of A is defined as

We call a nonempty subset $M \subset X$ a bounded set if its diameter $\delta(M) = \sup_{x,y \in M} d(x,y)$ is finite.

Answer (Please select your correct option)

- $\delta(A) = \inf_{x,y \in A} d(x,y)$
- $\delta(A) = d(x,y)$
- $\delta(A) = \sup_{x,y \in A} d(x,y)$ R ✓
- $\delta(A) = d(x,y)$

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Question No : 16 of 41

Which of the following is a condition of an inner product space?

Let $x, y, z \in V$; $\alpha \in F$ where α may be real or complex.

i. $< x, x > \geq 0; \quad < x, x >= 0 \quad \Leftrightarrow x = 0$

ii. $< \alpha x, y > = \alpha < x, y >$; but not true for second value as $< x, \alpha y > \neq \alpha < x, y >$

iii. $< x + y, z > = < x, z > + < y, z >$

iv. $< x, y > = \overline{< y, x >}$

Answer (Please select your correct option)

- $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$ **R**
- $\langle \alpha x + y, z \rangle = \langle x, z \rangle + \alpha \langle y, z \rangle$
- $\langle \alpha x, y \rangle = \langle x, \alpha y \rangle$
- $\langle x, y \rangle = \overline{\langle x, y \rangle}$

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MTH641 Functional Analysis

Marks: 1 (Budgeted Tim

Question No : 7 of 41

If $S = \{v_1, v_2, v_3, \ldots, v_n\}$ is a set of vectors in a finite dimensional vector space X, then S is called a basis for V if _____ and S is linearly independent.

Answer (Please select your correct option)

- S does not span V
- S is linearly dependent
- S is set of scalars
- S spans V

⇒ that basis would also be a basis for X, as element in basis are same, they span and linearly independent.

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Question No : 25 of 41

Isomorphism of an inner product space onto an inner product space over the same field is a ..... linear operator that preserves inner product.

Answer (Please select your correct option)

- Bijective
- Injective
- Surjective
- None of these

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Isomorphism:

It is one-one and onto map.

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MTH641 Functional Analysis

Question No : 23 of 41

Which of the following is not true for the space $C[a,b]$ with norm defined by $\|x\| = \max_{t\in[a,b]} |x(t)|$?

Answer (Please select your correct op

- It is a complete metric space
- It is a complete norm space
- It is a complete inner product space
- It is a vector space

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- $\mathbb{C}[a,b]$

  Here we prove that $\mathbb{C}[a,b]$ is complete metric space

Example:

The function space $\mathbb{C}[a,b]$ is complete; here $[a,b]$ is any given closed interval on $\mathbb{R}$.

Let (x_m) be any Cauchy sequence in $\mathbb{C}[a,b]$.

The metric space in $\mathbb{C}[a,b]$ is

$$d(x,y) = \max_{t\in[a,b]} |x(t) - y(t)| \quad , \quad \text{where [a,b]=J}$$

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Question No : 5 of 41

A _____ Y of a Banach space X is _____ if and only if the set Y is closed in X

A subspace Y of a Banach space X is complete if and only if the set Y is closed in X.

Answer (Please select your correct option)

- subspace , complete
- space, complete
- subspace, open
- space, open

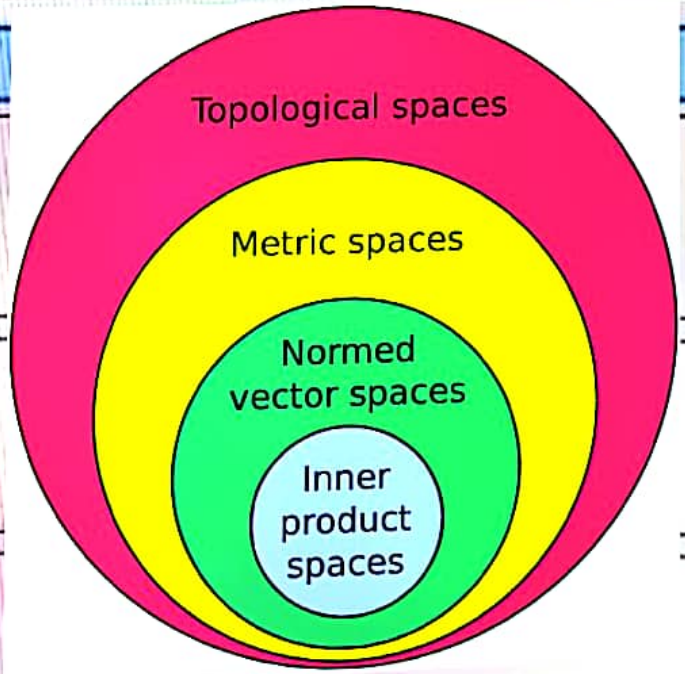
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Every metric space is a not _____ space.

Answer (Please select your correct option)

- a normed
- a topological
- both of these
- None of these



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MTH641 Functional Analysis

Question No : 21 of 41

The space ..... is not a Hilbert space

$8=4\times 2^{\frac{2}{p}}$ as $p>1, p\neq 2$

The values on both sides are also not equal so this does not satisfied the parallelogram law. Contradict to our supposition. So norm space is not an inner product space.

Answ

- l^p with $p \neq 2$
- Euclidean space R^n
- Unitary Space C^n
- l^p with $p = 2$

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MTH641 Functional Analysis

Question No : 10 of 41

A linear functional f with domain $D(f)$ in a normed space is continuous if and only if f is _____.

As in the case of linear operators on a finite dimensional normed space, every linear functional defined on a finite dimensional normed space is bounded and hence continuous.

Answer (Please select your correct option)

- continuous
- bounded
- linear
- non linear

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Question Summary : (Attempted Question)

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Question No : 3 of 41

Given a point $x_0 \in X$ and a real number $r > 0$, which of the following defines a closed ball ?

Answer (Please select your correct option)

- $\bar{B}(x_0;r)=\{x\in X \mid d(x,x_0) < r\}$
- $\bar{B}(x_0,r)=\{x\in X \mid d(x,x_0) > r\}$
- $\bar{B}(x_0;r)=\{x\in X \mid d(x,x_0) \le r\}$
- $\bar{B}(x_0,r)=\{x\in X \mid d(x,x_0) = r\}$

Ball and Sphere (General Form)

Open Ball:	$B(x_o;r)=\{x\in X \mid d(x,x_o)<r\}$
Closed Ball:	$\tilde{B}(x_o;r)=\{x\in X \mid d(x,x_o)\le r\}$
Sphere:	$S(x_o:r)=\{x\in X \mid d(x,x_o)=r\}$

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MTH641 Functional Analysis

Question No : 11 of 41

The space $C[a,b]$ is _____

3. $\mathbb{C}[a,b]$ be the space of all continuous function defined on [a, b].

Answer (Please select your correct option)

- collection of all functions defined on $[a,b]$.
- collection of all continuous functions defined on (a,b)
- collection of all continuous functions defined on $[a,b]$ **R**
- collection of all functions defined on (a,b)

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Question Summary : (Attempted Question)

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Question No : 12 of 41

Let X be an n-dimensional vector space and X^* be its dual space. Then _____.

Theorem:

Let X be an n-dimensional vector space and X^* be its dual space. Then

$$\dim X^* = \dim X = n.$$

X^* is collection of linear functions or linear operator while X may be any space.

Answer (Pleas

- $\dim X^* = \dim X = n$
- $\dim X = n$
- $\dim X^* = n$
- $\dim X^* = \dim X = m$

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MTH641 Functional Analysis

Question No : 2 of 41

For any three points x,y and z in a metric space (X,d), the following property is called .....

$d(x,z) \leq d(x,y) + d(y,z)$

Answer (Please select your correct option)

- Triangular inequality
- Rectangle inequality
- Holder's inequality
- Minkowski's inequality

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And in triangular inequality the distance between x and y is always less than the sum of distances of zy and zy.

Equality: $d(x,y) = d(x,z) + d(z,y)$

Inequality: $d(x,y) \leq d(x,z) + d(z,y)$

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Question Summary

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Question No : 7 of 41

A metric space X is said to be compact if every sequence in X has a _____ subsequence.

Answer (Plea A metric space X is said to be compact if every sequence in X has a convergent subsequence.

- Non decreasing
- Divergent
- Non increasing
- Convergent

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Question No : 14 of 41

If $T: XY$ is an Isometric Isomorphism between two norm spaces X and Y, then it implies that

Isometric Isomorphism

A linear operator $\phi: X \to Y$. X, Y normed spaces, is said to be Isometric Isomorphism if

ϕ isbijective.

ϕ preserve norms.

Answer (Please select you

- T is bijective and norm preserving
- T is injective and norm preserving
- T is surjective and norm preserving
- T is norm preserving

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MTH641 Functional Analysis

Question No : 21 of 41

Every complete normed space is _____ space.

Complete norm spaces are called Banach spaces.

Answer (Please select your correct option)

- Hilbert
- Banach
- Unitary
- Euclidean

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Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then _____

Lemma(Zero Vector):

Let X be a finite deimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$ then $x_0 = 0$.

Answer (Please select your correct option)

- $x_0 = 0$
- $x_0 = 0$
- $x_0 = 1$
- $x_0 = 1$

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TH641 Functional Analysis

Question No : 27 of 41

For orthogonal elements x,y we have <x,y>=0 and the Pythagorean relation is .....

As x and y are perpendicular so $< x, y >= 0, < y, x >= 0$

$$\|x+y\|^2 =< x, x > + < y, y >= \|x\|^2 + \|y\|^2$$

Answer (Please select your correct option)

- $\|x+y\|^2=2\|x\|^2+2\|y\|^2$
- $\|x+y\|^2=\|x\|^2-\|y\|^2$
- $\|x-y\|^2=\|x\|^2+\|y\|^2$
- $\|x+y\|^2=\|x\|^2+\|y\|^2$

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MTH641 Functional Analysis

Question No : 13 of 41

A finite dimensional normed space is .....to its second dual.

Theorem:
A finite dimensional vector space is reflexive.
Equivalently, A finite dimensional normed space is isomorphic space is isomorphic to its second dual.

Answer (Please select your correct option)

- isometric
- isomorphic
- homomorphic
- none of these

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MTH641 Functional Analysis

Question No : 6 of 41 **Marks: 1 (Budg**

Let $\{x_1, \ldots, x_n\}$ be ____ set of vectors in a normed space X. Then there is a number $c > 0$ such that for every choice of scalars $\alpha_1, \ldots, \alpha_n$, we have

$\|\alpha_1 x_1 + \ldots + \alpha_n x_n\| \geq c(|\alpha_1| + \ldots + |\alpha_n|)$

Let $\{x_1, \ldots, x_n\}$ be a linearly independent set of vectors in a normed space X (of any dimension).

Then there is a number $c>0$ such that for every choice of scalars $\alpha_1, \ldots, \alpha_n$ we have

$$\|\alpha_1 x_1 + \ldots + \alpha_n x_n\| \geq c(|\alpha_1| + \ldots + |\alpha_n|)$$

- linearly independent
- linearly dependent
- orthogonal
- orthonormal

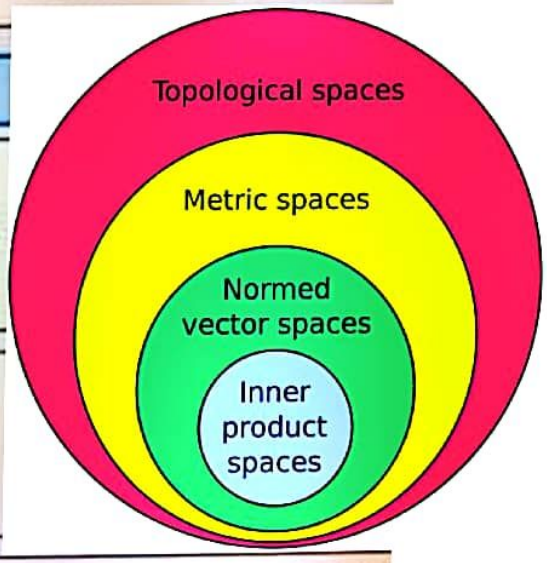
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Question No : 19 of 41

Every inner product space is a _____ space.

Answer (Please select your correct option)

- Metric
- Normed
- Topological
- All of above



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MTH641 Functional Analysis

Question No : 26 of 41

$\| x + y \| \leq \| x \| + \| y \|$ is called.....

iv): $\|x+y\| \leq \|x\| + \|y\|$ (N4) triangular inequality

Answer (Please select your correct option)

- Triangle inequality
- Minkowski's inequality
- Holder's inequality
- Schwarz inequality

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Question No : 9 of 41

If a normed space X is finite dimensional, then every linear operator on X is _____

As in the case of linear operators on a finite dimensional normed space, every linear functional defined on a finite dimensional normed space is bounded and hence continuous.

Answer (Please select your correct option)

- finite
- unbounded
- bounded
- none of these

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Question No : 8 of 41

If a normed space X is _____ dimensional, then every linear operator on X is bounded.

Answer (Please select your correct option)

infinite

As in the case of linear operators on a finite dimensional normed space, every linear functional defined on a finite dimensional normed space is bounded and hence continuous.

finite

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A sequence (x_n) in a metric space is said to be non-decreasing if for all n,

Answer (Please select your correct option)

- $x_n \geq x_{n+1}$
- $x_n \leq x_{n+1}$
- $x_n < x_{n+1}$
- $x_n > x_{n+1}$

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MTH641 Functional Analysis

Question No : 1 of 41

If d is a metric on X, Then for all $x, y \in X$, $d(x,y)$ is....

Answer (Please select your correct option)

- A non negative real number
- A negative real number
- Not a rational number
- A complex number

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MTH641 Functional Analysis

Question No : 11 of 41

Let T be a bounded linear operator. Then $x_n \to x$ (where $x_n, x \in D(T)$) implies _____

Answer (Please select your correct option)

- $x_n \to x$
- $Tx_n \to Tx$
- $Tx \to Tx_n$
- None of these

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Question No : 27 of 41

For subsets A and B of an inner product space X, A is orthogonal to B if .....

Answer (Please select your correct option)

- a is orthogonal to b for all $a \in A and b \in B$
- a is orthogonal to b for a fixed $a \in A and$ a fixed $b \in B$
- a is orthogonal to b for a fixed $a \in A and b \in B$
- a is orthogonal to b for all $a \in A and$ a fixed $b \in B$

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Question No : 26 of 41

For a subset A of an inner product space X, x is orthogonal to A if .....

Answer (Please select your correct option)

- x is orthogonal to a for all $a \in A$
- x is orthogonal to any two elements a and b $\in A$
- x is orthogonal to a for a fixed $a \in A$
- None of these

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Question No : 4 of 41

In a metric space (X,d) every convergent sequence is

Answer (Please select your correct option)

- Unbounded sequence
- Not a Constant sequence
- Oscillatory sequence
- A Cauchy Sequence

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In an inner product space V, the function $\|\ \| : V \to R^+$ given by _____ defines a _____ in V

Answer (Please select your correct option)

- $|x| = \sqrt{\langle x, y \rangle}$, $x, y \in V$, norm
- $|x| = \sqrt{\langle x, x \rangle}$, $x \in V$, norm
- $|x| = \sqrt{\langle x, x \rangle}$, $x \in V$, metric
- None of these

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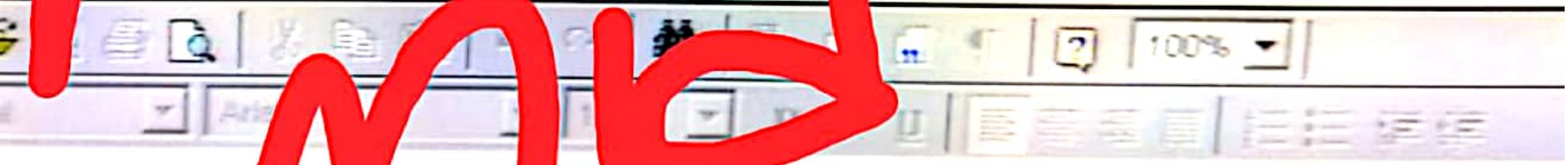
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MTH641 Functional Analysis

Question No : 35 of 41

Prove that if a set M is closed in X then it forms a complete subspace of X

Answer (ease ick here t Ans)



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b): Prove that if a set M Is closed in x then it forms a complete Subspace of X

M is closed, if and only if the situation $x_n \in M$, $x_n \to x$.

$$\Rightarrow \quad x \in M$$

M is closed if and only if $\bar{M} = M$,

Now we have to prove that $\bar{M} = M$ for this we prove that $\bar{M} \subseteq M$, $M \subseteq \bar{M}$

i): $\bar{M} \subseteq M$

by definition M contains M and its limit point so this condition is fulfilled.

ii): $M \subseteq \bar{M}$

Now we prove that $M \subseteq \bar{M}$

Let $x \in \bar{M}$, we will show that $x \in M$.

Now if we take $x$belongs to $\bar{M}$ then from above "a" part of this theorem, we have a sequence x_n in M such that $x_n \to x$ this implies $x \in M$.

That means $M \subseteq \bar{M}$.

Hence $\bar{M} = M$

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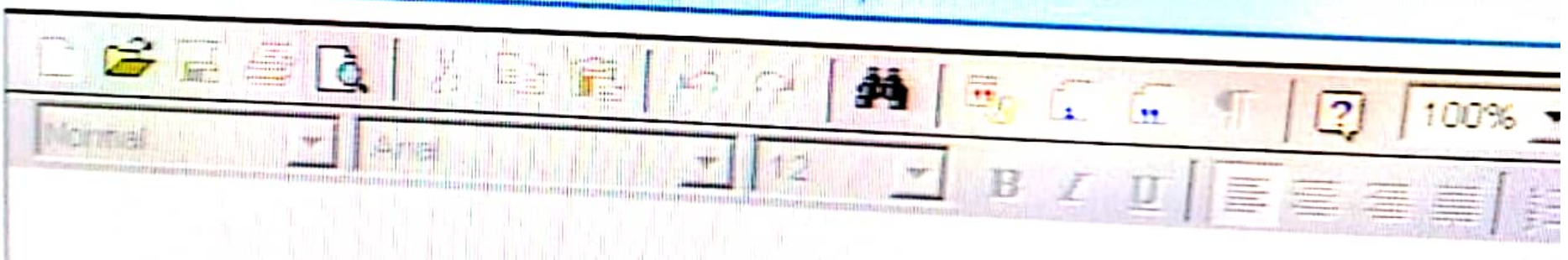
MTH641 Functional Analysis

Question No : 39 of 41

Show that canonical mapping is linear.

p=8 final

Answer (Please click here to Add Answer)



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Conical Mapping: Show that canonical mapping is linear

$C : X \to X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as

$x \mapsto g_x$.

$$C(\alpha x + \beta y)(f) = g_{\alpha x + \beta y}(f)$$

$$= f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) = \alpha g_x(f) + \beta g_{y0}(f)$$

$$= \alpha(Cx)(f) + \beta(Cy)(f)$$

So, this is a linear function as well. Canonical mapping is a relation between X and X^{**}.

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MTH641 Functional Analysis

Question No : 32 of 41

Define linear functional

Answer (Please click here to Add Answer)

module

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Question No : 41 of 41 Marks: 5 (Budgeted Time Min)

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $(ST)^* = T^*S^*$ (assuming $H_2 = H_1$).

Answer (Please click here to Add Answer)

module 106

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Question Summary : (Attempted Question)

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Module No. 106

Theorem
(Properties of Hilbert-adjoint Operators)

Let H_1 and H_2 be Hilbert spaces

$S : H_1 \longrightarrow H_2$ and

$T : H_1 \longrightarrow H_2$ .. bounded linear operators

and α any scalar. Then we have

a) $\langle T^* y, x \rangle = \langle y, Tx \rangle$

$x \in H_1, y \in H_2$

b) $(S+T)^* = S^* + T^*$

c) $(\alpha T)^* = \bar{\alpha} T^*$

d) $(T^*)^* = T$

e) $\|T^* T\| = \|T T^*\| = \|T\|^2$

f) $T^* T = 0 \iff T = 0$

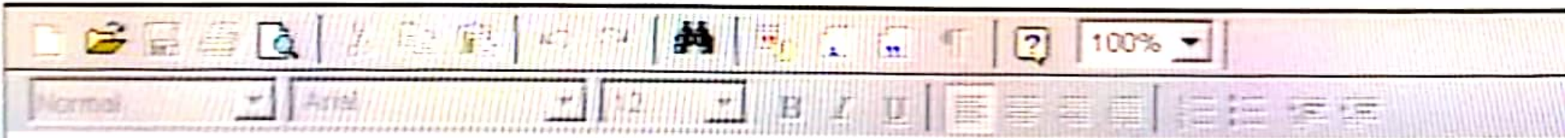
g) $(ST)^* = T^* S^*$ assuming $H_2 = H_1$)

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Question No : 34 of 41

Define unitary operator.

Answer (Please click here to Add Answer)



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MTH641 Functional Analysis

Question No : 36 of 41 **Marks: 3 (Budgete**

Let $T: D(T) \to Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then show that if T is continuous at a single point, it is continuous.

Answer (Please click here to Add Answer)

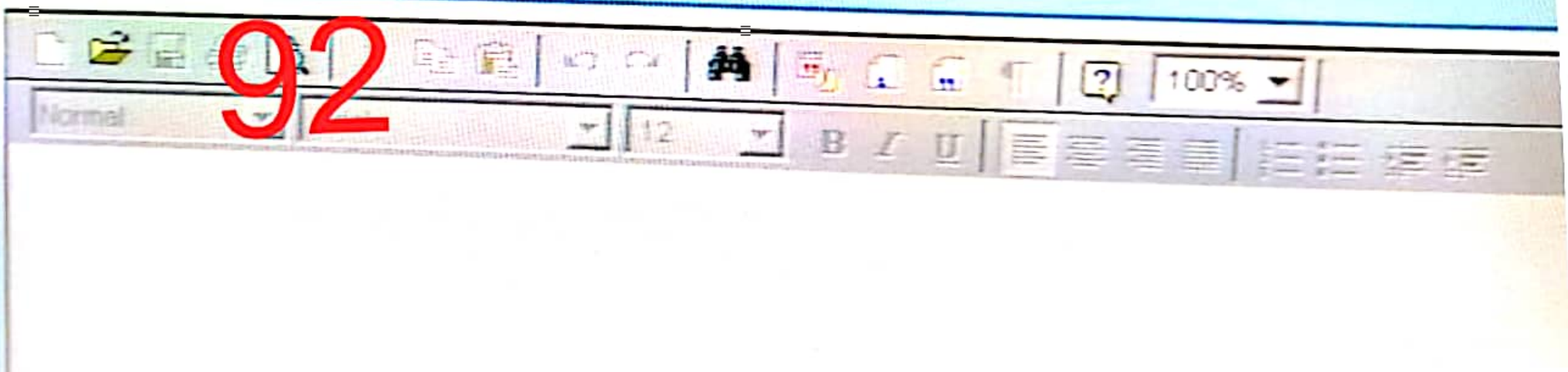
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Question No : 33 of 41

Write generalized form of orthogonal system.

module
92

Answer (Please click here to Add Answer)



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Generalized form:

$\{x_1, x_2, \ldots\ldots x_n\}$ be nonzero vectors in V inner product space such that

$< x_i, x_j >= 0 \quad , \quad i \neq j$

This system $\{x_1, x_2, \ldots\ldots x_n\}$ is called orthogonal system as all vectors inside it are perpendicular to each other.

The generalized statement is $\|x_1 + x_2 + \ldots\ldots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \ldots\ldots + \|x_n\|^2$

The idea of proof is

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \left\langle \left\|\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i\right\| \right\rangle$$

$$= < x_1 + \ldots\ldots + x_n, x_1 + \ldots\ldots + x_n >$$

$$= < x_1, x_1 + \ldots\ldots + x_n > + \ldots\ldots + < x_n, x_1 + \ldots\ldots + x_n >$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} < x_i, x_j >$$

$< x_i, x_j >= \|x_i\|^2$, if $i \neq j$ $< x_i, x_j >= 0$ and for $i = j$ then $< x_i, x_j >= \|x_i\|^2$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

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MTH641 Functional Analysis

Question No : 31 of 41

Show that the sequence $(x_n) = (1 + \frac{1}{n})$ is a Cauchy sequence with the usual metric defined on the set of real numbers.

Answer (Please click here to Add Answer)

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MTH641 Functional Analysis

Question No : 25 of 41 **Marks: 1 (Budg**

Isomorphism of an inner product space onto an inner product space over the same field is a ..... linear operator that preserves inner product.

Answer (Please select your correct option)

- Bijective
- Injective
- Surjective
- None of these

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MTH641 Functional Analysis

Question No : 38 of 41

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that

$\langle T^* y, x \rangle = \langle y, Tx \rangle$ $(x \in H_1, y \in H_2)$.

Answer (Please click here to Add Answer)

module 106

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Module No. 106

Theorem
(Properties of Hilbert-adjoint Operators)

Let H_1 and H_2 be Hilbert spaces

$S : H_1 \longrightarrow H_2$ and

$T : H_1 \longrightarrow H_2$ .. bounded linear operators

and α any scalar. Then we have

a) $\langle T^* y, x \rangle = \langle y, Tx \rangle$

$x \in H_1, y \in H_2$

b) $(S+T)^* = S^* + T^*$

c) $(\alpha T)^* = \bar{\alpha} T^*$

d) $(T^*)^* = T$

e) $\|T^* T\| = \|T T^*\| = \|T\|^2$

f) $T^* T = 0 \iff T = 0$

g) $(ST)^* = T^* S^*$ assuming $H_2 = H_1$)

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Question No : 40 of 41

Show that if $\{x_n\}, \{y_n\}$ are Cauchy sequences in V, then $\{x_n, y_n\}$ is a Cauchy sequence in F.

Answer (Please click here to Add Answer)

module 90

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MODULE NO.90

➢ THEOREM (CONTINUITY OF INNER PRODUCT)

Theorem:

Let V be any inner product space. For any sequences $\{x_n\}$ and $\{y_n\}$ in V

$$x_n \to x \ , \quad y_n \to y \ \text{ implies} < x_n, y_n > \ \to \ < x, y >$$

Proof:

$$|< x_n, y_n > - < x, y >|$$

$$= |< x_n, y_n > - < x_n, y > + < x_n, y > - < x, y >|$$

$$= |< x_n, y_n - y > + < x_n - x, y >|$$

$$\leq |< x_n, y_n - y >| + |< x_n - x, y >|$$

Now from Cauchy Swarzinequality

$$|x, y| \leq \|x\| \|y\|$$

$$\leq \|x_n\| \|y_n - y\| + \|x_n - x\| \|y\|$$

Given that $x_n \to x$, $y_n \to y$ so,

$$\|y_n - y\| = \|y - y\| = 0 \ , \quad \|x_n - x\| = \|x - x\| = 0 \quad \text{as } n \to \infty$$

As $n \to \infty$

$$|< x_n, y_n > - < x, y >| \leq 0$$

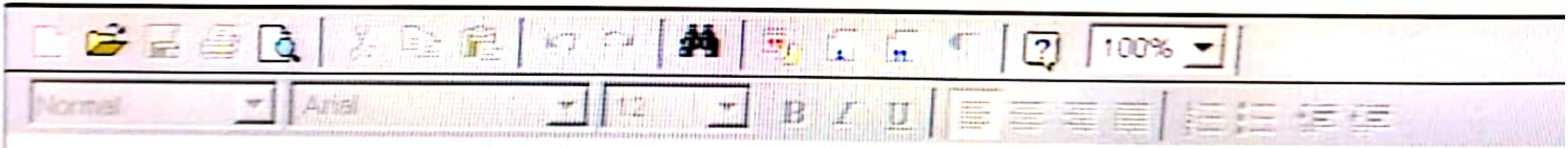
$$< x_n, y_n > \to < x, y > \quad \text{as } n \to \infty$$

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Question No : 37 of 41

Define norm and inner product for which the space R^n is an inner product space.

Answer (Please click here to Add Answer)



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S#	Statement	Answer
1.	The set of real numbers is the union of _____	Set of rational numbers Irrational numbers
2.	$x \in B(x_o; r) \Rightarrow$	$d(x, x_o) < r$
3.	$x \in \tilde{B}(x_o; r) \Rightarrow$	$d(x, x_o) \leq r$
4.	$x \in S(x_o : r) \Rightarrow$	$d(x, x_o) = r$
5.	A subset M of a metric space X is called a bounded set if its __ is finite.	Diameter
6.	Which of the following set has no limit points in R?	The set of integers
7.	$\delta(A) = \sup_{x,y \in A} d(x, y)$ Represents the ___ for a non-empty subset A of (X,d).	Diameter
8.	The closure of a set consists of all ____ of the set.	Limit points
9.	___ is collection of limit points and the set itself	Closure
10.	A set is called a _____ if all the limit points are present in that set.	Closed set
11.	If $\mathbb{Q}$ is dense in $\mathbb{R}$, then $\widetilde{\mathbb{Q}} =$	R
12.	If "d" is a discrete metric on R, the d(3,2)=___	1
13.	The set of all limit points of a set A is called _____ of A.	Closure
14.	A sequence (x_n) in a metric space $X = (X, d)$ is said to be ___ if for every $\varepsilon > 0$ there exist $N = N(\varepsilon)$ such that for all $d(x_m, x_n) < \varepsilon$.	Cauchy or fundamental sequence
15.	A sequence (x_n) in X is _____if the corresponding point set is a bounded subset of X.	bounded sequence
16.	A sequence (x_n) in a metric space $X = (X, d)$ is said to converge or to be convergent if there is an $x \in X$ such that ___	$\lim_{n \to \infty} d(x_n, x) = 0$
17.	The property d(x,y)≥ 0 of a metric space is called.	Non negativity
18.	The property $d(x, y) = 0$ *if and only if* ___	x=y
19.	The property $d(x, y) \leq d(x, z) + d(z, y)$	Triangle Inequality

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20.	For a metric space (X,d) for all x,y belongs to X $d(x,y)=d(y,x)$ is called __ property.	Symmetry
21.	C[a,b] shows the space of all _____ defined on [a,b]	Continuous function
22.	If T is typology on non-empty set X , then ___ intersection of member of bf belong to T.	Finite
23.	Space is called _____ for p=2	Hilbert Sequence space
24.	The inverse image of ____ set under a continuous mapping is open.	An open
25.	Given a point And a real number r>0, which of the following statement is true?	$S(x_0,\ r)=\tilde{B}(x_0,\ r)-B(x_0,\ r)$
26.	The real line R is a complete metric space having two ___ dense subsets.	Finite
27.	A subset M of a metric space X is said to be dense in X if	$\bar{M}=X$
28.	Closure set is a set along with its ___	Limit points
29.	A metric space X is said to be _____ if it has a countable subset which is dense in X	Separable
30.	In discrete metric space if elements are same then distance is __	0
31.	In discrete metric space if elements are different then distance is_____.	1
32.	How many limit points of a subset A of X exist if A is a finite subset of the metric space (X,d)?	0
33.	All those point which are on the boundary of this open ball are___.	accumulation points or limit points
34.	A subspace M of a complete metric X is itself ___ if and only if the set M closed in X.	Separable
35.	If "K" is a Negative real number, and d is a metric on X then for all x,y belongs to X.	Kd(x,y)≤ 0
36.	d(x,y)=|x-y| is a usual metric defined on ...	R
37.	It belongs to the space B(A), then it should be a___.	Functions having convergent sequence in A
38.	A ___sequence in a metric space is a Cauchy sequence.	Convergent
39.	The Centre of the sphere S(6;8) is;	$\frac{1}{2}\int_2^4 x^2dx$

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40.	A subset A of a metric space X is ___ in X iff A intersect every non empty open set.	Dense
41.	Discrete metric space on a set X is defined ad d(x,y)=	$\begin{cases}0, & x=y\\ 1, & x\neq y\end{cases} \quad \forall x,y\in X$
42.	If d is a metric on Y, then for all x,y∈Y, d(x,y) is _____	A non negative real number
43.	The following expression represents …… inequality $\frac{\lvert\xi_i-\eta_i\rvert}{1+\lvert\xi_i-\eta_i\rvert} \leq \frac{\lvert\xi_i-\alpha_i\rvert}{1+\lvert\xi_i-\alpha_i\rvert} + \frac{\lvert\alpha_i-\eta_i\rvert}{1+\lvert\alpha_i-\eta_i\rvert}$	Triangular
44.	Two sequences $\{x_n\}$ and $\{y_n\}$ in a metric space X are said to be equal if for all the difference x_n-y_n is equal to ____	Zero
45.	If d is a usual metric on R, then d(2,8)=	$2+\int_0^2 2xdx$
46.	In ____ an open neighborhood is an open interval.	R
47.	A discrete metric space X is separable iff X is ___	countable
48.	Given a point $x_o \in X$ and a real number r>0 which of the following defines an open ball?	$B(x_o;r)=\{x\in X \mid d(x,x_o)<r\}$
49.	Given a point $x_o \in X$ and a real number r>0 which of the following defines an close ball?	$\tilde{B}(x_o;r)=\{x\in X \mid d(x,x_o)\leq r\}$
50.	Given a point $x_o \in X$ and a real number r>0 which of the following defines an sphere?	$S(x_o:r)=\{x\in X \mid d(x,x_o)=r\}$
51.	For an inner product space $\|x+y\|^2 + \|x-y\|^2 =$ -----	$2(\|x\|^2+\|y\|^2)$
52.	Let B(X,Y) be the set of all …..... Operators from a normal space X to a normed space Y. If Y is a Banach spcace, then B(X,Y) is a Banach space.	Bounded linear
53.	Let B(X,Y) be the set of all bounded linear operators from a normal space X to a normed space Y. If Y is a Banach spcace, then B(X,Y) is _____.	Banach
54.	For an inner product space defined on a real vector space $\langle x,y\rangle =$	$\langle y,x\rangle$
55.	For an element x belongs to an inner product space $\langle x,x\rangle =$ … … …	0
56.	For all x,y belongs to an an inner product space. $\langle \alpha x,y\rangle =$ … … … …	$\alpha\langle x,y\rangle$

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57.	T is bijective and norm preserving then it means that T is a\an ..... between two norm space X and Y.	Isometric Isomorphism
58.	Which of the following is not a condition of an inner product space?	$\langle \alpha x + y, z\rangle = \langle x, z\rangle + \alpha\langle y, z\rangle$
59.	Which of the following is a condition of an inner product space?	$\langle x, x\rangle \geq 0$
60.	Which of the following space is an inner product space?	Euclidean space R^n
61.	Which of the following space is not an inner product space?	The space l^p $with$ $l^p \neq 2$
62.	The following expression $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|x\|^2)$ is called	Parallelogram equality
63.	Apollonius identity is :	$\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z-\frac{1}{2}(x+y)\right\|^2$
64.	Completer inner product space is called a _____	Hilbert space
65.	A Hilbert space is a/an	Completer inner product space
66.	Let $(V, \langle .,. \rangle)$ be an inner product space over a field F, then $\forall x, y, z \in V, \alpha, \beta \epsilon F$	$\langle x, \alpha y + \beta z\rangle = \bar{\alpha}\langle x, y\rangle + \bar{\beta}\langle x, z\rangle$
67.	Let $(V, \langle .,. \rangle)$ be an inner product space over a field F, then $\forall x, y, z \in V, \alpha, \beta \epsilon F$	$\langle \alpha x + \beta y, z\rangle = \alpha\langle x, z\rangle + \beta\langle y, z\rangle$
68.	The pair $(V, <\bullet,\bullet>)$ is called an inner product space. $<ax+by, z>$ $where$ $x, y, z \in V$, $a, b \in F$	$<ax+by, z> = <ax, z> + <by, z>$
69.	The pair $(V, <\bullet,\bullet>)$ is called an inner product space. $<ax+by, z>$ $where$ $x, y, z \in V$, $a, b \in F$	$a<x, z> + b<y, z>$
70.	The pair $(V, <\bullet,\bullet>)$ is called an inner product space. $<ax+by, z>$ $where$ $x, y, z \in V$, $a, b \in F$	$<0, z> = <0.x, z> = 0 <x, z> = 0$
71.	Let $(V, \langle .,. \rangle)$ be an inner product space over a field F, then $\forall x, y, z \in V, \alpha, \beta \epsilon F$	$\langle x, \alpha . y\rangle = \bar{\alpha}\langle x, y\rangle$
72.	Let $(V, \langle .,. \rangle)$ be an inner product space over a field F, then $\forall x, y, z \in V, \alpha, \beta \epsilon F$	$\langle x, \alpha . y\rangle = \bar{\alpha}\overline{\langle x, y\rangle}$
73.	Let $(V, \langle .,. \rangle)$ be an inner product space over a field F, then $\forall x, y, z \in V, \alpha, \beta \epsilon F$	$\langle x, \alpha . y\rangle = \overline{\langle \alpha y, x\rangle}$
74.	Let $(V, \langle .,. \rangle)$ be an inner product space over a field F, then $\forall x, y, z \in V, \alpha, \beta \epsilon F$	$\langle x, \alpha . y\rangle = \overline{\alpha\langle y, x\rangle}$
75.	A banach space is a/an	Completer norm space
76.	Complete norm space is called a	Banach space
77.	Complete inner product space is called a	Hilbert Space

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78.	For an inner product space <x+yz>=	<x,z>+<y,z>
79.	In inner product the generalization is _____.	Dot product
80.	$\overline{AC}^2 + \overline{BD}^2 = ------$	$2\left(\overline{AB}^2 + \overline{BC}^2\right)$
81.	Inner product defines a _____	Norm
82.	Every inner product space is a	norm space.
83.	Every norm space is not an inner product space.	This is not true always.
84.	If a space is inner product then it satisfied the _____ otherwise it is not an inner product space	parallelogram law
85.	Norm is a generalization of _____	distance function.
86.	For any two elements x,y is an inner product space V, $\lvert< x, y >\rvert \le \lVert x\rVert . \lVert y\rVert$, the define norm is	$\lVert x\rVert = \sqrt{< x, x >}$, x,y ∈ V
87.	As in the case of linear operators on a finite dimensional normed space, every linear functional defined on a finite dimensional normed space is bounded and hence continuous.	
88.	X^* as the space of all bounded linear functionals defined on X, is also complete and hence is Banach space	
89.	"Algebraic Dual Space of X": set of all linear funcionals defined on X.	
90.	"Dual or Conjugate Space of X": X^* set of all continuous or bounded linear functionals defined on X.	
91.	However there are Banach spaces which are not reflexive.	
92.	If X is an incomplete normed space even then X^* and X^{**} are Banach spaces. So in this case X cannot be a reflexive space.	
93.	A normed space X is said to algebraically reflexive if there is an isometric isomorphism between X and X^{**}.	
94.	Ordinarily a normed spacer may not be reflexive.	
95.	A finite dimensional vector space is reflexive.	
96.	Equivalently, A finite dimensional normed space is isomorphic space is isomorphic to its second dual.	
97.		

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SIR. WAQAS (13-03-2024) 6:30PM

MTH641 FINAL PAPER

Functional Analysis.

MCQS'

1) $\psi(x,y)$ is a bijective Linear operator which preserves norm, it means that every $x \in X$

* $\|\psi(x)\| = \|x\|$ * $\|\psi^{(x)}\| \leq \|x\|$ * $\|\psi^{(x)}\| < \|x\|$

* $\|\psi(x)\| > \|x\|$

2) $\langle x, y \rangle =$ _____

* $\langle x, y \rangle \leq \|x\| \, \|y\|$

3) Let T be a bounded Linear operator then _____ implies $Tx_n \to Tx$.

* $x_n \to x$

4) Pair $(V, \langle , \rangle)$ is _____

* inner product

5) $\|x+y\| \leq \|x\| + \|y\|$ is

* Triangular inequality.

6) A _____ dimesional vector space is reflexive.

* finite

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(7) Every metric space is not _____ space

* normed

(8) An orthonormal set M is an orthogonal set in an inner product space whose elements have norm _____

* equal to 1

(9) Inner product is the generalization of _____

* Dot product.

(10) inner product is the generalization of _____

* Dot product

{Note:- This mcq's was twicely given}

(11) Any pair of vectors in an orthogonal system is _____ to each other

* perpendicular

(12) Let $T: D(T) \longrightarrow Y$ be a Linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then T is _____ if and only if T is bounded.

* continuous.

(13) Which of the following is not true about the inner product space?

(a) Every Hilbert space is Inner product space

* Every inner product space is Hilbert space.

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* Every norm space is innerproduct space.

(14) Which of the following is an example of an inner product space.

* $\{a, b\}$ * $[a, b]$ * Space R^n * None

(15) The dual space of a normed space X contains all ____ on X

* Bounded Linear functional

(16) Which of the following is a condition of an inner product space?

* ~~$x < x, y >$~~

(17) If a normed space X is ____ dimensional, then every Linear operator on X is bounded.

* finite

(18) If a normed space X is finite dimensional, then every Linear operator on X is ____

* bounded

(19) For a subset A of an inner product space X, x is orthogonal to A if ____

* x is orthogonal to for all $a \in A$.

(20) Let X be an n-dimensional vector space and X^* be its dual space. Then ____

* $\dim X^* = \dim X = n$

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(21) Which of the following is not true for the space $C[a,b]$ with norm defined by $\|x\| = \max_{c[a,b]} |x(t)|$?

* It is a complete metric space
* It is a " norm "
* " " " " inner product "
* " " " vector space

(22) The dual space of R^n is ____

* R^n * C^n * R^2 * R^3

(23) If an inner product space $x_n \to x$ & $y_n \to y$, then

* $\langle x_n, y_n \rangle \longrightarrow$ ($\langle x, y \rangle$

(24) Which of the following true for the space $C[a,b]$ with norm defined by $\|x\| = \max_{c[a,b]} |\bar{x}(\epsilon)|$?

* It is not Hilbert space.

(25) An orthonormal set is Linearly
* Independent

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(Q1) Examples of Linear functional. (2 marks)

(Q2) U^{-1} is unitary. (2 marks)

(Q3) Prove $(S+T)^* = S^* + T^*$ (3 marks)

(Q4) Let H_1 and H_2 be Hilbert spaces $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded Linear operator and α any scalar. Then

$$\langle T^* y, x \rangle = \langle y, Tx \rangle$$ (2 marks)

(Q5) In an inner product space V, the function $\|.\|: V \to R^+$ given by (2 m)

$\|x\| = \sqrt{\langle x, y \rangle}$, $x \in V$ defines a norm in V

(Q6) Let $T: R^3 \to R^3$ defined by $(\xi_1, \xi_2, \xi_3) \mapsto (\xi_1, \xi_2, -\xi_1, -\xi_3)$ then find the basis of $R(T)$ and $N(T)$.

(Q7) Define isometric Isomorphism

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Question # 10 of 10 (Start time: 08:37:31 PM, 19 November 2018)

If for k>0 , d(3,9)= $\frac{1+kd_1(3,9)}{kd_1(3,9)}$ where d_1 represents the usual metric on R then

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Select the correct option

- (●) d(3,9)>1
- ( ) d(3,9)<1
- ( ) d(3,9)=1
- ( ) d(3,9)=0

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MTH641:Quiz # 1

Quiz Start Time: 09:14 PM, 19 November 2018

Question # 4 of 10 (Start time: 09:15:43 PM, 19 November 2018) **Total Marks: 1**

If 'k' is a Negative real number, and d is a metric on X then for all x,y belongs to X,

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Select the correct option

Reload Math Equations

- $kd(x, y) < 0$
- $kd(x, y) \leqslant 0$
- $kd(x, y) \geqslant 0$
- $kd(x, y) > 0$

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1.1-6 Sequence space l^∞. This example and the next one give a first impression of how surprisingly general the concept of a metric space is.

5 Q 2 Mark

As a set X we take the set of all bounded sequences of complex numbers; that is, every element of X is a complex sequence

$$x = (\xi_1, \xi_2, \cdots) \qquad \text{briefly} \qquad x = (\xi_j)$$

such that for all $j = 1, 2, \cdots$ we have

$$|\xi_j| \leqq c_x$$

where c_x is a real number which may depend on x, but does not depend on j. We choose the metric defined by

(9) $$d(x, y) = \sup_{j \in \mathbf{N}} |\xi_j - \eta_j|$$

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space $X=(X, d)$.

1.3-1 Definition (Ball and sphere). Given a poi$x_0 \in X$ and a real number $r>0$, we define[6] three types of sets:

(1)

(a) $B(x_0; r)=\{x \in X \mid d(x, x_0)<r\}$ **(Open ball)**

(b) $\tilde{B}(x_0; r)=\{x \in X \mid d(x, x_0) \leqq r\}$ **(Closed ball)**

(c) $S(x_0; r)=\{x \in X \mid d(x, x_0)=r\}$ **(Sphere)**

In all three cases, x_0 is called the *center* and r the *radius*. ∎

We see that an open ball of radius r is the set of all points in X whose distance from the center of the ball is less than r. Furthermore, the definition immediately implies that

(2) $$S(x_0; r)=\tilde{B}(x_0; r)-B(x_0; r).$$

Warning. In working with metric spaces, it is a great advantage that we use a terminology which is analogous to that of Euclidean

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Fig. 7. Notation in part (*a*) of the proof of Theorem 1.3-4

ε-neighborhood N of Tx_0, the inverse image N_0 of N is open, since N is open, and N_0 contains x_0. Hence N_0 also contains a δ-neighborhood of x_0, which is mapped into N because N_0 is mapped into N. Consequently, by the definition, T is continuous at x_0. Since $x_0 \in X$ was arbitrary, T is continuous. ∎

We shall now introduce two more concepts, which are related. Let M be a subset of a metric space X. Then a point x_0 of X (which may or may not be a point of M) is called an **accumulation point** *of* M (or *limit point of* M) if every neighborhood of x_0 contains at least one point $y \in M$ distinct from x_0. The set consisting of the points of M and the accumulation points of M is called the **closure** *of* M and is denoted by

$$\bar{M}.$$

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and only if X is countable. (Cf. 1.1-8.)

Proof. The kind of metric implies that no proper subset of X can be dense in X. Hence the only dense set in X is X itself, and the statement follows.

1.3-9 Space l^∞. *The space l^∞ is not separable.* (Cf. 1.1-6.)

Proof. Let $y = (\eta_1, \eta_2, \eta_3, \cdots)$ be a sequence of zeros and ones. Then $y \in l^\infty$. With y we associate the real number whose binary representation is

$$\frac{\eta_1}{2^1} + \frac{\eta_2}{2^2} + \frac{\eta_3}{2^3} + \cdots .$$

We now use the fact that the set of points in the interval $[0, 1]$ is

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1.4-2 Lemma (Boundedness, limit). *Let $X = (X, d)$ be a metric space. Then:*

(a) *A convergent sequence in X is bounded and its limit is unique.*

(b) *If $x_n \longrightarrow x$ and $y_n \longrightarrow y$ in X, then $d(x_n, y_n) \longrightarrow d(x, y)$.*

Proof. **(a)** Suppose that $x_n \longrightarrow x$. Then, taking $\varepsilon = 1$, we can find an N such that $d(x_n, x) < 1$ for all $n > N$. Hence by the triangle inequality (M4), Sec. 1.1, for all n we have $d(x_n, x) < 1 + a$ where

$$a = \max \{d(x_1, x), \cdots, d(x_N, x)\}.$$

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converge) is not a point of X. This also illustrates that the concept of convergence is not an intrinsic property of the sequence itself but also depends on the space in which the sequence lies. In other words, a convergent sequence is not convergent "on its own" but it must converge to some point in the space.

Although condition (1) is no longer sufficient for convergence, it is worth noting that it continues to be necessary for convergence. In fact, we readily obtain the following result.

1.4-5 Theorem (Convergent sequence). *Every convergent sequence in a metric space is a Cauchy sequence.*

Proof. If $x_n \longrightarrow x$, then for every $\varepsilon > 0$ there is N such that

ε

mapping.

1.4-8 Theorem (Continuous mapping). *A mapping $T: X \longrightarrow Y$ of a metric space (X, d) into a metric space $(Y, \tilde{d})$ is continuous at a point*

$x_0 \in X$ if and only if

$$x_n \longrightarrow x_0 \qquad \text{implies} \qquad Tx_n \longrightarrow Tx_0.$$

Proof. Assume T to be continuous at x_0; cf. Def. 1.3-3. Then for

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various other applications.

1.6-1 Definition (Isometric mapping, isometric spaces) Let $X = (X, d)$ and $\tilde{X} = (\tilde{X}, \tilde{d})$ be metric spaces. Then:

(a) A mapping T of X into $\tilde{X}$ is said to be *isometric* or an *isometry* if T preserves distances, that is, if for all $x, y \in X$

$$\tilde{d}(Tx, Ty) = d(x, y),$$

where Tx and Ty are the images of x and y, respectively.

(b) The space X is said to be *isometric* with the space $\tilde{X}$ if there exists a bijective[9] isometry of X onto $\tilde{X}$. The spaces X and $\tilde{X}$ are then

SHORT

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In analysis, infinite dimensional vector spaces are of greater interest than finite dimensional ones. For instance, $C[a, b]$ and l^2 are infinite dimensional, whereas $\mathbf{R}^n$ and $\mathbf{C}^n$ are n-dimensional.

If $\dim X = n$, any linearly independent n-tuple of vectors of X is called a **basis** for X (or a *basis in* X). If $\{e_1, \cdots, e_n\}$ is a basis for X, every $x \in X$ has a unique representation as a linear combination of the basis vectors:

$$\alpha_1 e_1 + \cdots + \alpha_n e_n.$$

For instance, a basis for $\mathbf{R}^n$ is

$$e_1 = (1, 0, 0, \cdots, 0),$$

$$e_2 = (0, 1, 0, \cdots, 0),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

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$$\|y_m - y\| = \left\| \sum_{j=1}^{n} (\alpha_j^{(m)} - \alpha_j) e_j \right\| \leqq \sum_{j=1}^{n} |\alpha_j^{(m)} - \alpha_j| \, \|e_j\|.$$

On the right, $\alpha_j^{(m)} \longrightarrow \alpha_j$. Hence $\|y_m - y\| \longrightarrow 0$, that is, $y_m \longrightarrow y$. This shows that (y_m) is convergent in Y. Since (y_m) was an arbitrary Cauchy sequence in Y, this proves that Y is complete. ∎

From this theorem and Theorem 1.4-7 we have

2.4-3 Theorem (Closedness). *Every finite dimensional subspace Y of a normed space X is closed in X.*

We shall need this theorem at several occasions in our further work.

Note that infinite dimensional subspaces need not be closed

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2.5-5 Theorem (Finite dimension). *If a normed space X has the property that the closed unit ball $M=\{x \mid \|x\| \leqq 1\}$ is compact, then X is finite dimensional.*

Proof. We assume that M is compact but $\dim X=\infty$, and show that this leads to a contradiction. We choose any x_1 of norm 1. This x_1 generates a one dimensional subspace X_1 of X, which is closed (cf. 2.4-3) and is a proper subspace of X since $\dim X=\infty$. By Riesz's lemma there is an $x_2 \in X$ of norm 1 such that

$$\|x_2-x_1\| \geqq \theta=\frac{1}{2}.$$

The elements x_1, x_2 generate a two dimensional proper closed subspace X_2 of X. By Riesz's lemma there is an x_3 of norm 1 such that for all $x \in X_2$ we have

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work.

2.6-9 Theorem (Range and null space). *Let T be a linear operator. Then:*

(a) *The range $\mathcal{R}(T)$ is a vector space.*

(b) *If $\dim \mathcal{D}(T) = n < \infty$, then $\dim \mathcal{R}(T) \leqq n$.*

(c) *The null space $\mathcal{N}(T)$ is a vector space.*

Proof. **(a)** We take any $y_1, y_2 \in \mathcal{R}(T)$ and show that $\alpha y_1 + \beta y_2 \in \mathcal{R}(T)$

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ijective linear operators, where X, Y, Z are vector spaces (s
21). Then the inverse $(ST)^{-1}: Z \longrightarrow X$ *of the product (the compos*
ST exists, and

$$(ST)^{-1} = T^{-1}S^{-1}.$$

Proof. The operator $ST: X \longrightarrow Z$ is bijective, so that (ST)
ts. We thus have

$$ST(ST)^{-1} = I_Z$$

re I_Z is the identity operator on Z. Applying S^{-1} and usin
$S = I_Y$ (the identity operator on Y), we obtain

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in the following basic definition.

2.7-1 Definition (Bounded linear operator). Let X and Y be normed spaces and $T: \mathcal{D}(T) \longrightarrow Y$ a linear operator, where $\mathcal{D}(T) \subset X$. The operator T is said to be *bounded* if there is a real number c such that for all $x \in \mathcal{D}(T)$,

(1) $$\|Tx\| \leqq c\|x\|.$$ ∎

In (1) the norm on the left is that on Y, and the norm on the right is that on X. For simplicity we have denoted both norms by the same symbol $\|\cdot\|$, without danger of confusion. Distinction by subscripts ($\|x\|_0$, $\|Tx\|_1$, etc.) seems unnecessary here. Formula (1) shows that a bounded linear operator maps bounded sets in $\mathcal{D}(T)$ onto bounded sets in Y. This motivates the term "bounded operator."

Warning. Note that our present use of the word "bounded" is different from that in calculus. where a bounded function is one whose

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MTH-641

Final Term Solution

Q#01:-

Define Extension of T.

Extension of T on a Set

$$M \supset D(T)$$
$$\bar{T} : M \to T$$
$$\bar{T}x = Tx \quad \forall x \in D(T)$$

Q#02:-

Prove That

$$\langle x, ay + bz \rangle = \bar{a}\langle x, y \rangle + \bar{b}\langle x, z \rangle$$

Solution:-

$$\langle x, ay + bz \rangle = \overline{\langle ay + bz, x \rangle}$$
$$= \overline{\langle ay, x \rangle} + \overline{\langle bz, x \rangle}$$
$$= \bar{a}\overline{\langle y, x \rangle} + \bar{b}\overline{\langle z, x \rangle}$$
$$= \bar{a}\langle x, y \rangle + \bar{b}\langle x, z \rangle$$

Proved

"Remember me in your Prayers"
"Jazak Allah"

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Q#03:-

Appollonius Identity:-

$x, y, z \in V$

$$\|z-x\|+\|z-y\| = \frac{1}{2}\|x-y\|^2 + 2\left\|z-\frac{1}{2}(x+y)\right\|^2$$

Q#04:-

Define bounded linear function:

A bounded linear function F is a bounded linear operator with range in a scalar field of the normed space X.

$$|F(x)| \leq c\|x\|$$

Q#05

Find basis of $N(T)$.

$$x = (\xi_1, \xi_2, \xi_3) \Rightarrow x \in N(T)$$
$$\Rightarrow (0, 0, \xi_3)$$
$$\Rightarrow \xi_3(0, 0, 1)$$

Q#6:-

Define self adjoint operator:

A bounded linear operator $T: H \to H$ on a Hilbert space

His said to be, self adjoint. or Hesmition if

$$T^* = T$$

Q#07

Prove that U is normal.

To show U is normal.

$$UU^* = U^*U$$

$$U^* = U^{-1}$$

$$UU^{-1} = U^{-1}U = I$$

Proved

Q#08

Give Two examples of orthogonal system.

Two examples of orthogonal system is

* Space R^n
* Space R^3

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Note

Q#9.- (9 marks / 3 marks / 5 marks ma aa re ha)

If $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3) \to (\mathcal{E}_1, \mathcal{E}_2, -\mathcal{E}_1 - \mathcal{E}_2)$ then find matrix representation.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \\ \mathcal{E}_3 \end{bmatrix} = \begin{bmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \\ -\mathcal{E}_1 - \mathcal{E}_2 \end{bmatrix}$$

Hence required matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$

Q#10

Prove that

$$(\alpha T)^* = \bar{\alpha} T^*$$

$$\langle x, (\alpha T)^* y \rangle = \langle (\alpha T)x, y \rangle$$

$$= \langle \alpha (Tx), y \rangle$$

$$= \bar{\alpha} \langle Tx, y \rangle$$

$$= \bar{\alpha} \langle x, T^* y \rangle$$

$$= \langle x, \bar{\alpha} T^* y \rangle$$

$$\langle x, (\alpha T)^* y \rangle = \langle x, \bar{\alpha} T^* y \rangle$$

$$\Rightarrow (\alpha T)^* = \bar{\alpha} T^*$$

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Q NO 11:-

Write to examples of linear functional.

1) space $C[a\ b]$

2) space l^2

Q NO 12:-

Orthonormal System Definition:-

orthonormal system are those function which are real and complex whose inner product with itself result in 1 and with other functions result in 0.

Q NO 13

Let $T: R^3 \to R^3$ define by

$(\xi_1, \xi_2, \xi_3) \longrightarrow (\xi_1, \xi_2, -\xi_1 - \xi_2)$

Then find basis of $R(T)$.

$$(\xi_1, \xi_2, -\xi_1 - \xi_2) = \xi_1 (1, 0, -1) + \xi_2 (0, 1, -1)$$

$$D \Rightarrow \{(1, 0, -1), (0, 1, -1)\}$$

Imp Q NO: 14

Show that $U^{-1} = U^*$ is unitary. OR $\|U\| = 1$

Since U is bijective $\Rightarrow U^{-1}$ is bijective

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$$(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}$$

$$(U^{-1})^* = (U^{-1})^{-1}$$

$\Rightarrow U^{-1}$ is Unitary.

Q NO:15.

Difference b/w Algebraic dual Space and Conjugate space.

"Algebraic dual space of X":

Set of all linear functionals defined on X.

"Dual or Conjugat Space of X":

X^* Set of all continuous or bounded linear functionals defined on X.

We take algebraic dual when there is no condition of Continuous or bounded linear functions.

Q NO:16

Show that $\langle x, \alpha y \rangle = \bar{\alpha} \langle x, y \rangle$

For all $x, y \in V \quad \alpha \in F$

$$\langle x, \alpha y \rangle = \overline{\langle \alpha y, x \rangle} = \overline{\alpha \langle y, x \rangle}$$

$$\bar{\alpha} \overline{\langle y, x \rangle} = \bar{\alpha} \langle x, y \rangle$$

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QNO17:- Topic 64

Prove that if a normed space X is finite dimensional then every linear operator on X is bounded.

Proof:-

X is finite dimension.

⇒ every linear operator on X is bounded.

Let $\dim X = n \Rightarrow \{e_1, e_2, \ldots e_n\}$ is a basis for X

Let $x \in X$

$\Rightarrow x = \sum_{j=1}^{n} \xi_j e_j$

Let T be any linear operator on X.

$$\|Tx\| = \|T(\sum_{j=1}^{n} \xi_j e_j)\| = \|\sum_{j=1}^{n} \xi_j Te_j\|$$

(T linear)

$$\leq \sum_{j=1}^{n} |\xi_j| \|Te_j\| \leq N_3\{\|Te_1\|, \ldots \|Te_n\|\}$$

$$\leq \max \|Te_k\| \sum_{j=1}^{n} |\xi_j| \quad \text{---} (1)$$

$$\|\sum_{j=1}^{n} \xi_j e_j\| \geq c \left(\sum_{j=1}^{n} |\xi_j|\right)^k$$

$$\sum_{j=1}^{n} |\xi_j| \leq \frac{1}{c} \|\sum_{j=1}^{n} \xi_j e_j\| = \frac{1}{c}\|x\|$$

$$(1) \Rightarrow \|Tx\| \leq \max \|Te_k\| \frac{1}{c} \|x\|$$

$$\Rightarrow \|Tx\| \leq c' \|x\|$$

$\Rightarrow$ T is bounded.

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Q No 18:-

Show that $(S+T)^* = S^* + T^*$

Proof

$$\langle x, (S+T)^* y \rangle = \langle (S+T)x, y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle (Sx + Tx), y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle Sx, y \rangle + \langle Tx, y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, S^* y \rangle + \langle x, T^* y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, (S^* y + T^* y) \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, (S^* + T^*) y \rangle$$

Hence we have prove that

$$(S+T)^* = S^* + T^*$$

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Imp

Q NO: 19

Show that

$$\|x_1+x_2+x_3+\cdots+x_n\|^2 = \|x_1\|^2+\|x_2\|^2+\cdots+\|x_n\|^2$$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \left\langle \sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i \right\rangle$$

$$= \langle x_1, x_2+\cdots x_n, x_1+x_2+\cdots x_n\rangle$$

$$= \langle x_1, x_2+\cdots x_n\rangle + \cdots \langle x_1+x_2+\cdots x_n\rangle$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \langle x_i, x_j\rangle$$

$\because \langle x_i, x_j\rangle = \|x_i\|^2$

$\because \langle x_i, x_j\rangle = 0 \quad i \neq j$ So,

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$$\left\| \sum_{i=1}^{n} x_i \right\|^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \langle x_i, x_j \rangle$$

$$\langle x_i, x_j \rangle = 0$$

Q NO 20:-

Give an example of function which is not linear?

$\|\cdot\| : X \longrightarrow R$ Functional but not linear.

Q NO 21:-

Generalization of orthogonal System:-

$\{x_1, x_2, \ldots x_n\}$ be nonzero vectors in V inner product space such that

$\langle x_i, x_j \rangle = 0 \quad i \neq j$

This system $\{x_1, x_2, \ldots x_n\}$ is called as all vectors inside it are perpendicular to each other.

Q NO: 22

Isomorphsim of metric space:-

A function $f : (X, d) \longrightarrow (Y, \rho)$

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is an isomorphism if f is an isometry and f is surjective. Two metric space are called isomorphic if there is an isomorphism between them.

QNO 23:-
Define Canonical mapping

$C: X \to X^{**}$ this mapping is called Canonical mapping of X into X^{**} defined as $x \mapsto \hat{x}$.

made by baloch (PR+H)

QNO 24:-
Define Normal Operator:-

A bounded linear operator $T: H \to H$ on a Hilbert space H is said to be Unitary operator if T is bijective and $TT^* = T^*T$

Define equal operator:-

Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have same domain i.e $D(T_1) = D(T_2)$

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Q NO 28:-

Examples of Inner product Spaces:-

- ★ Space R^n
- ★ Space C^n
- ★ Space $C[a,b]$
- ★ Space l^2
- ★ Space P_n

Imp N.T

Q NO 29:-

Prove that null space is closed.

OR

$$\overline{N(T)} \subseteq N(T)$$

Solution:-

$$x \in N(T)$$

$\exists (x_n)$ such that $x_n \in N(T) \pm \underset{n}{x} \to x$

Therefore

$$Tx_n \to Tx$$

$$x_n \in N(T)$$

$$Tx_n = 0$$

$$Tn = 0$$

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$x \in N(T)$

$N(T) \subset N(T)$

$N(T) = N(T)$

$N(T)$ is closed.

Q No 30:-

Define Polarization Identity:-

For any x, y in complex inner product space

$$\langle x, y \rangle = \frac{1}{4}\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\}$$

We have prove this complex inner product space.

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Note:=

Find the Null space of $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $(\xi_1, \xi_2, \xi_3) \to (\xi_1, \xi_2, -\xi_1 - \xi_2)$

i) Basis of $R(T)$ PgNo: 5

2) Basis of $N(T)$ PgNo: 2

3) Matrix of representation PgNo: 4

Baqi Questions mein aa rahe hain.

Module No # 78

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Queestion 1



Show that $(\alpha T)^* = \alpha^{-1} T^*$

Solution:

$$< x, (\alpha T^*)^* y > = < (\alpha T)x, y >$$
$$< x, (\alpha T^*)^* y > = < \alpha(Tx), y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < Tx, y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < x, T^* y >$$
$$< x, (\alpha T^*)^* y > = < x, \bar{\alpha} T^* y >$$
$$(\alpha T^*)^* = \bar{\alpha} T^*$$

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Question 2



Prove that $(S+T)^* = S^* + T^*$

Prove:

$$< x, (S+T)^* y > = < (S+T)x, y >$$

$$< x, (S+T)^* y \geq < (Sx+Tx), y >$$

$$< x, (S+T)^* y > = < Sx, y > + < Tx, y >$$

$$< x, (S+T)^* y > = < x, S^* y > + < x, T^* y >$$

$$< x, (S+T)^* y > = < x, (S^* y + T^* y) >$$

$$< x, (S+T)^* y > = < x, (S^* + T^*) y >$$

Hence we have proved that

$$(S+T)^* = S^* + T^*$$

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Question 3



Find matrix multiplication if $(\xi_1, \xi_2, \xi_3) \leftrightarrow (\xi_1, \xi_2, -\xi_1 - \xi_2)$

Solution:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ -\xi_1, -\xi_2 \end{bmatrix}$$

Hence required matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

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Question 4

$$\|x_1 + x_2 + \ldots\ldots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \ldots\ldots + \|x_n\|^2$$

Prove:

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \langle \left\|\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i\right\| \rangle$$

$$= < x_1 + \ldots\ldots + x_n, x_1 + \ldots\ldots + x_n >$$

$$= < x_1, x_1 + \ldots\ldots + x_n > + \ldots + < x_n, x_1 + \ldots\ldots + x_n >$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} < x_i, x_j >$$

$< x_i, x_j >= \|x_i\|^2$, if $i \neq j$ $< x_i, x_j >= 0$ and for $i = j$ then $< x_i, x_j >= \|x_i\|^2$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

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Question 5

Prove that $(ST)^* = S^*T^*$

Prove:

$$< x, (ST)^* y >=< (ST)x, y >$$
$$< x, (ST)^* y \geq < (SxTx), y >$$
$$< x, (ST)^* y >=< Sx, y >< Tx, y >$$
$$< x, (ST)^* y >=< x, S^* y >< x, T^* y >$$
$$< x, (ST)^* y >=< x, (S^* + T^* y) >$$
$$< x, (ST)^* y >=< x, (S^* T^*) y >$$

Hence we have proved that

$$(ST)^* = S^* T^*$$

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Question 6

Show that $\overline{N(T)} < N(T)$

Prove:

$$x \in N(T)$$
$$\exists\ (x_n) such\ that\ x_n\ \in N(T) + x_n \to x$$

Therefore

$$Tx_n \to Tx$$
$$x_n\ \in N(T)$$
$$Tx_n = 0$$
$$T_n = 0$$
$$x\ \in N(T)$$
$$\overline{N(T)} < N(T)$$
$$\overline{N(T)} = N(T)$$
$$N(T) is\ closed.$$

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Question 7

Prove that $< x, ay + bz >= \bar{a} < x, y > + \bar{b} < x, z >$

Prove:

$$< x, ay + bz > = < \overline{ay + bz}, x >$$

$$< x, ay + bz > = < \overline{ay}, x > + < \overline{bz}, x >$$

$$< x, ay + bz > = \bar{a} < \overline{y}, x > + \bar{b} < \overline{z}, x >$$

$$< x, ay + bz > = \bar{a} < x, y > + \bar{b} < x, z >$$

Hence proved.

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Question 8



Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $(T^*)^* = T$.

Prove:

$$< x, (T^*)^* y \geq < xT^*, y >$$

$$< x, (T^*)^* y \geq < x, Ty >$$

It implies that

$(T^*)^* = T$ so its proved.

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Question 9

Prove that the product of two bounded self-adjoint linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute, $ST = TS$.

Prove:



$$< x, (ST)^{*}y >=< (ST)x, y >$$
$$< x, (ST)^{*}y \geq< (SxTx), y >$$
$$< x, (ST)^{*}y >=< Sx, y >< Tx, y >$$
$$< x, (ST)^{*}y >=< x, S^{*}y >< x, T^{*}y >$$
$$< x, (ST)^{*}y >=< x, (S^{*}+T^{*}y) >$$
$$< x, (ST)^{*}y >=< x, (S^{*}T^{*})y >$$

Hence we have proved that

$$(ST)^{*} = S^{*}T^{*}$$

$$(ST)^{*} = T^{*}S^{*} = TS.$$

Hence

$$ST = (ST)^{*} \iff ST = TS.$$

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Question 10

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then show that $\|T^*T\| = \|TT^*\| = \|T\|^2$.

Prove:

$$\|Tx\|^2 = \langle Tx, Tx\rangle = \langle T^*Tx, x\rangle \leq \|T^*Tx\|\,\|x\| \leq \|T^*T\|\,\|x\|^2.$$

Taking the supremum over all x of norm 1, we obtain $\|T\|^2 \leq \|T^*T\|$.

we have

$$\|T\|^2 \leq \|T^*T\| \leq \|T^*\|\,\|T\| = \|T\|^2.$$

Hence $\|T^*T\| = \|T\|^2$. Replacing T by T^* and we also have

$$\|T^{**}T^*\| = \|T^*\|^2 = \|T\|^2.$$

Here $T^{**} = T$

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Question 11

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that U is isometric; thus $\|Ux\| = \|x\|$ for all $x \in H$.

Prove:

$$\| U_x \|^2 = < U_x, U_x >$$
$$\| U_x \|^2 = < x, U^* U_x >$$
$$\| U_x \|^2 = < x, Ix >$$
$$\| U_x \|^2 = < x, x >$$
$$\| U_x \|^2 = \| x \|^2$$

It proved.

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Question 12

Show that $< T^*y, x > = < y, Tx >$

Prove:

$$< T^*y, x > = < \overline{y, T^*x} >$$

$$< T^*y, x > = < \overline{Tx, y} >$$

$$< T^*y, x > = < y, Tx >$$

Hence its proved.

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Question 13

Let X and Y be inner product spaces and $Q: X \to Y$ a bounded linear operator. Then Show that $Q=0$ if and only if $\langle Qx, y\rangle = 0$ for all $x \in X$ and $y \in Y$.

Prove:

$$Q = 0 \Longrightarrow Qx = 0 \; for \; all \; x \epsilon X$$

$$< Qx, y > = < 0, y >$$

$$< Qx, y > = < 0.w, y >$$

$$< Qx, y > = \; 0 < w, y >$$

$$< Qx, y > = \; 0. \quad for \; all \; x \epsilon X$$

It implies that Qx=0 and by definition Q=0. Its proved.

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Question 14

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that $\|U\| = 1$, provided $H \neq \{0\}$.

OR

Let the operators $U: H \to H$ and $V: H \to H$ be unitary; here, H is a Hilbert space. Then show that $U^{-1} (= U^*)$ is unitary.

Prove:

Since U is bijective it shows that U^{-1} is bijective

$$(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}.$$

Hence it implies that U^{-1} is unitary.

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Question 15

Let the operators $U : H \to H$ and $V : H \to H$ be unitary; here, H is a Hilbert space. Then show that U is normal.

Prove:

$$UU^* = U^*U$$

Since we know that

$$U^* = U^{-1}$$

So

$$UU^{-1} = U^{-1}U$$

$(UV)^* = (UV)^{-1} = I$ Hence its proved that U is normal.

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Question 16

In an inner product space V and x, y in V if $x \perp y$ then

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2$$

Prove:

$$\|x+y\|^2 = < x+y, x+y >$$

$$= < x, x > + < x, y > + < y, x > + < y, y >$$

As x and y are perpendicular so $< x, y >= 0, < y, x >= 0$

$$\|x+y\|^2 = < x, x > + < y, y > = \|x\|^2 + \|y\|^2$$

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Question 17

$$\|x+y\|^2+\|x-y\|^2=2\left(\|x\|^2+\|y\|^2\right) \quad \text{for all x,y } \in V$$

Prove:

$$\|x+y\|^2 =< x+y, x+y >$$

$$=< x,x > + < x,y > + \overline{< x,y >} + < y,y >$$

$$= \|x\|^2 + 2\,\mathrm{Re} < x,y > + \|y\|^2 \qquad \text{....(i}$$

Replace y=-y

$$\|x-y\|^2 =< x+y, x+y >$$

$$=< x,x > - < x,y > - \overline{< x,y >} + < y,y >$$

$$= \|x\|^2 - 2\,\mathrm{Re} < x,y > + \|y\|^2 \qquad \text{.....(ii}$$

Adding (i and (ii

$$\|x+y\|^2+\|x-y\|^2=2\|x\|^2+2\|y\|^2$$

That we have to prove.

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Question18

For any two elements x,y is an inner product space V,

$$|< x, y >| \leq \|x\|.\|y\|$$

Prove:

If x=y=0 then 0=0

Let at least one of x and y is not equal to zero

Let $|< x+\lambda y, x+\lambda y >| \geq 0$ by definition

$$< x, x+\lambda y > + < \lambda y, x+\lambda y >$$

$$< x, x+\lambda y > + y < y, x+\lambda y >$$

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Question 19

SHOW THAT T is uniquely determinal if the image $y_k = Te_k$ of n basis vectors $e_1, \ldots\ldots, e_n$ are prescribed.

Prove:

$$y=Tx \; ; y \in Y \quad \{b_1, \ldots\ldots, b_n\}$$

$$y=\eta_1 b_1 + \eta_2 b_2 + \ldots\ldots\ldots + \eta_r b_r$$

$$Te_k \in Y, \qquad Te_1 = \tau_{11} b_1 + \tau_{12} b_2 + \ldots\ldots\ldots + \tau_{1r} b_r$$

$$Te_k = \sum_{j=1}^{r} \tau_{kj} b_j$$

$$y=\sum_{j=1}^{r} \eta_j b_j = \sum_{k=1}^{n} \xi_k Te_k = \sum_{k=1}^{n} \xi_k \sum_{j=1}^{r} \tau_{kj} b_j$$

Comibinig these two summation

$$y = \sum_{j=1}^{r} \left(\sum_{k=1}^{n} \tau_{kj} \xi_k \right) b_j$$

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_k$$

The image $y=Tx=\sum \eta_j b_j$ of $x = \sum \xi_k Te_k$ can be obtained from

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_k$$

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Question 20

Sum and Scalar multipication of Dual.

Prove:

as follows. The SUM $f_1 + f_2$ of two **functionals** f_1 **and** f_2 is the functional s whose value **at every** $x \in X$ is

$$s(x) = (f_1 + f_2)(x) = f_1(x) + f_2(x);$$

the Product αf of a scalar α and **a functional** f is the functional p whose value **at** $x \in X$ is

$$p(x) = (\alpha f)(x) = \alpha f(x).$$

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Question 21

Define Inner Product Space:

The pair $(V, \langle \bullet, \bullet \rangle)$ is called an inner product space.

Define Appolonius Identity:

$$\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z-\frac{1}{2}(x+y)\right\|^2 \quad , \quad x, y, z \in V$$

Define Equal Operators:

Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have the same

domain i.e. $D(T_1) = D(T_2)$

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Question 22

Define Orthonormal System.

Orthonormal systems are those functions which are real or complex whose inner product with itself results in 1 and with other functions result in 0.

Define Polarization Identity.

For any x, y in complex inner product space

$$< x, y >= \frac{1}{4}\left\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\right\}$$

We have to prove this complex inner product space.

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Question 23

Define Bounded Linear Function.

Let X and Y be normed spaces and $T : D(T) \to Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$\|Tx\| \leq c\|x\|$

Define Canonical Mapping.

$C : X \to X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

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Question 24

Define Unitary Operator.

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $T^* = T^{-1}$.

Define Normal Operator.

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $TT^* = T^*T$

Define Extension of T.

Extension of T on a set $M \supset D(T)$ is defined as $\tilde{T}: M \to Y$ or

$$\tilde{T}x = Tx \; for \; all \; x \in D(T)$$

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Question 25

Write examples of Bounded Linear Operators

Identity Operator, Zero Operator, Differentiation Operator, Integral Operator

Define Self Adjoint Operator or Hermitian Operator.

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Self Adjoint Operator or Hermitian Operator if $T^* = T$.

Give an example of function which is not linear.

$\| . \| : x \to R$ is functional but not linear.

Find basis of R(T).

$$(\xi_1, \xi_2, -\xi_1, -\xi_2) = \xi_1(1,0,-1) + \xi_2(0,1,-1)$$

So the basis of R(T)= $\{(1,0,-1), (0,1,-1)\}$

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Question 26

Give any two examples of Linear Functional.

Space C[a, b] and Space l^2

Define Linear Functional:

A linear functional f is a linear operator with domain in a vector space X and range in the scalar field K of X; thus $f: D(f) \to K$. where K=R if X is real and K=C if X is complex.

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Define Bounded Linear Extension:

$T: D(T) \to Y$ be a bounded linear operator, where D(T) lies in a normal space X and Y is a Banach Space then T has an extension $\tilde{T}: \overline{D(T)} \to Y$. Where $\tilde{T}$ is a bounded linear operator of norm.

$$\| \tilde{T} \| = \| T \|$$

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Bounded Linear Operator:

Let X and Y be normed spaces and $T : D(T) \to Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$$\|Tx\| \leq c\|x\|$$

Let T be a bounded linear operator as defined before then an alternate formula for the norm of T is

$$\|T\| = \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|Tx\|$$

The norm defined on T satisfies (N1) to (N4).

Examples of Bounded Linear Operators

Identity Operator
Zero Operator
Differentiation Operator
Integral Operator
Matrix

Linear Function:

Algebraic Dual Space
Second Algebraic Dual Space
Canonical Mapping

Algebraic Dual Space

Set of all linear function defined on a vector space X is itself a vector space and called Algebraic Dual Space and denoted by X^*

Conical Mapping:

$C : X \to X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

Linear Function (Examples):

Space $C[a \ \ b]$
Space l^2

Linear Operators and functional on finite Dimensional Spaces:

Finite dimensions mean basis which have finite many elements.

Let X and Y bef.gfinite dimension vector spaces over the same field.

Let $T : X \to Y$ be a linear operator. let $E = \{e_1, ..., e_n\}$ be the basis for X and $B = \{b_1, ..., b_n\}$ be the basis for Y.

$x \in X$, $\quad x = \xi_1 e_1 + \xi_2 e_2 + ... + \xi_n e_n$

$$y = Tx = T\left(\sum_{k=1}^{n} \xi_k e_k\right) = \sum_{k=1}^{n} T\left(\xi_k e_k\right) = \sum_{k=1}^{n} \xi_k T\left(e_k\right)$$

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T is uniquely determinal if the image $y_k = Te_k$ of n basis vectors $e_1, ..., e_n$ are prescribed.

y=Tx ; $y \in Y$ $\{b_1, ..., b_n\}$

$y = \eta_1 b_1 + \eta_2 b_2 + ... + \eta_r b_r$

$Te_k \in Y$, $\quad Te_1 = \tau_{11} b_1 + \tau_{12} b_2 + ... + \tau_{1r} b_r$

$$Te_k = \sum_{j=1}^{r} \tau_{kj} b_j$$

$$y = \sum_{j=1}^{r} \eta_j b_j = \sum_{k=1}^{n} \xi_k Te_k = \sum_{k=1}^{n} \xi_k \sum_{j=1}^{r} \tau_{kj} b_j$$

Comibinig these two summation

$$y = \sum_{j=1}^{r} \left(\sum_{k=1}^{n} \tau_{kj} \xi_k \right) b_j$$

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_k$$

The image $y = Tx = \sum \eta_j b_j$ of $x = \sum \xi_k Te_k$ can be obtained from

$$\eta_j = \sum_{k=1}^{n} \tau_{kj} \xi_k$$

Let X be an n-dimensional vector space and X^* be its dual space. Then

$$\dim X^* = \dim X = n.$$

X^* is collection of linear functions or linear operator while X may be any space.

Proof:

Let $\dim X = n.$

Let basis of X be $B = \{e_1, ..., e_2\}$

We define a function.

$$f_j(e_1) = \begin{cases} 1 & if \ i = j \\ 0 & if \ i \neq j : \ i, j = 1, ..., n \end{cases}$$

e.g. j=1 , $\quad f(e_1) = 1, f(e_2) = 0, f(e_3) = 0, ..., f(e_n) = 0$

j=2 , $f(e_1) = 0, f(e_2) = 1, f(e_3) = 0, ..., f(e_n) = 0$

but each n-tuples f_j in this case can be extended as linear functions on X.

Let X be a finite deimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$ then $x_0 = 0$.

B^* is the basis of X^*

$\{f_1, f_2, ..., f_n\}$

$$\Rightarrow f_j(e_i) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$= \delta_{ij}$$

Proof:

For all $x_0 = 0$,

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$$x_0 = \sum_{i=1}^{n} x_i e_i \quad ; \quad f \in X^* \quad ,$$

$$f(x_0) = 0 \quad \Rightarrow \quad \sum_{i=1}^{n} f\left(\sum_{i=1}^{n} x_i e_i\right) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{n} x_i f\left(e_i\right) = 0 \quad , \quad j=1,.....,n$$

$$\Rightarrow \quad x_j = 0 \quad , \quad \forall\ j=1,.....,n$$

$$x_0 = \sum_{i=1}^{n} x_i e_i = 0 \quad \Rightarrow \quad x_0 = \delta$$

Theroem

Let*B(X, Y)* be the set of all bounded linear operators form a normed space *X* to a normed space *Y*. If ***Y*** is a Banach space, then ***B(X, Y)*** is also a Banach.

Proof:

Let $\{T_n\}$ be an arbitrary Cauchy seq. in *B(X,Y).*

We will show that $\{T_n\}$converges to an operator *T* in *B(X, Y).*Since $\{T_n\}$is Cauchy for every

$\varepsilon > 0 \quad \exists$ N such that $\|T_n - T_m\| < \varepsilon \qquad$ (m,n>N)

For all $x \in X$ and (m,n>N) we have

$$\|T_n(x) - T_m(x)\| = \|(T_n - T_m)(x)\|$$
$$\leq \|T_n - T_m\|\|x\| < \varepsilon \|x\|$$

Thus for a fixed *x* and given $\bar{\varepsilon}$

Hence B(X, Y) is complete and Banach space.

Inner product Space

Let *V* be a vector space over a field F where *F*is $\mathbb{R}$ or $\mathbb{C}$.

An inner product in V is a function $<\bullet,\bullet>: V \times V \to F$ satisfying the following conditions:

Quiz:

Let $x, y, z \in V \quad ; \quad \alpha \in$ F where α may be real or complex.

i. $< x, x >\geq 0; \quad < x, x >= 0 \quad \Leftrightarrow x = 0$

ii. $< \alpha x, y >= \alpha < x, y >$; but not true for second value as $< x, \alpha y >\neq \alpha < x, y >$

iii. $< x + y, z >=< x, z > + < y, z >$

iv. $< x, y >= \overline{< y, x >}$

$<\bullet,\bullet>: V \times V \to F$ inner product.

Inner Product Space

The pair $(V, <\bullet,\bullet>)$ is called an inner product space.

a): $< ax + by, z > \qquad where \quad x, y, z \in V \ , \ a, b \in F$

Using (iii) property $< ax + by, z >=< ax, z > + < by, z >$

Using (ii) property $a < x, z > + b < y, z >$

$< 0, z >=< 0.x, z >= 0 < x, z >= 0$

b): ***Quiz:***

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for all $x, y \in V$, $a \in F$

$$< x, ay >= \overline{< ay, x >} = \overline{a < y, x >}$$

$$= \bar{a}\overline{< y, x >} = \bar{a} < x, y >$$

Theorem:

For any two elements *x,y* is an inner product space *V*,

$|< x, y >| \le \|x\|.\|y\|$, the define norm is $\|x\| = \sqrt{< x, x >}$, x,y ∈ V

Proof:

If x=y=0 then 0=0

Let at least one of x and y is not equal to zero

Let $|< x + \lambda y, x + \lambda y >| \ge 0$ by definition

$$< x, x + \lambda y > + < \lambda y, x + \lambda y >$$

$$< x, x + \lambda y > + y < y, x + \lambda y >$$

Theorem:

In an inner product space V, the function $\|\,.\,\| : V \to \mathbb{R}^+$ given by

$\|x\| = \sqrt{< x, y >}$, $x \in V$ defines a norm in V.

Proof:

N1: $\|x\| \ge 0$

For a $x \in V$, $\|x\| = \sqrt{< x, x >} \ge 0$ as $< x, x > \ge 0$

N2:

$\|x\| = 0 \Leftrightarrow \sqrt{< x, x >} = 0 \Leftrightarrow < x, x >= 0 \Leftrightarrow x = 0$

N3: $\|\alpha x\| = |\alpha|\|x\|$

now $\|\alpha x\| = \sqrt{< \alpha x, \alpha x >} \Rightarrow \|\alpha x\|^2 =< \alpha x, \alpha x >$

$\Rightarrow$ $\|\alpha x\|^2 = \alpha\bar{\alpha} < x, x >= |\alpha|^2 \|x\|^2$

N4: $\|x + y\| \le \|x\| + \|y\|$ ∀ x, y ∈ V

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$$\|x+y\|^2 = < x+y, x+y >$$
$$= < x, x+y > + < y, x+y >$$
$$= \overline{< x+y, x >} + \overline{< x+y, y >}$$
$$= \overline{< x, x > + < y, x >} + \overline{< x, y > + < y, y >}$$
$$= \overline{< x, x >} + \overline{< y, x >} + \overline{< x, y >} + \overline{< y, y >}$$

Now $= < x, x > + < x, y > + \overline{< x, y >} + < y, y >$

$$= \|x\|^2 + 2\,\mathrm{Re} < x, y > + \|y\|^2 \qquad \because \mathrm{Re}(z) \le |z|$$
$$\le \|x\|^2 + 2|< x, y >| + \|y\|^2$$
$$\le \|x\|^2 + 2\|x\|\|y\| + \|y\|^2 \qquad \because \ |< x, y >| \le \|x\|\|y\|$$
$$= \left(\|x\| + \|y\|\right)^2$$
$$\|x+y\|^2 \le \|x\| + \|y\|$$

Theorem:

$$\|x+y\|^2 + \|x-y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right) \qquad \text{for all x,y } \in \mathrm{V}$$

Proof:

$$\|x+y\|^2 = < x+y, x+y >$$
$$= < x, x > + < x, y > + \overline{< x, y >} + < y, y >$$
$$= \|x\|^2 + 2\,\mathrm{Re} < x, y > + \|y\|^2 \qquad \text{....(i}$$

Replace y=-y

$$\|x-y\|^2 = < x+y, x+y >$$
$$= < x, x > - < x, y > - \overline{< x, y >} + < y, y >$$
$$= \|x\|^2 - 2\,\mathrm{Re} < x, y > + \|y\|^2 \qquad \text{.....(ii}$$

Adding (i and (ii

$$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

That we have to prove.

Polarization Identity

For any x, y in complex inner product space

$$< x, y >= \frac{1}{4}\left\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\right\}$$

Appolonius Identity

$$\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z - \frac{1}{2}(x+y)\right\|^2 \ , \quad x, y, z \in V$$

Using parallelogram law

$$\|x'+y'\|^2 + \|x'-y'\|^2 = 2\|x'\|^2 + 2\|y'\|^2 \quad \text{put} \qquad \text{x′= z-x, y′=z-y}$$

Self-assignment

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Theorem:

Let V be any inner product space. For any sequences $\{x_n\}$ and $\{y_n\}$ in V

$x_n \to x$, $y_n \to y$ implies $< x_n, y_n > \to < x, y >$

Proof:

$$|< x_n, y_n > - < x, y >|$$
$$= |< x_n, y_n > - < x_n, y > + < x_n, y > - < x, y >|$$
$$= |< x_n, y_n - y > + < x_n - x, y >|$$
$$\leq |< x_n, y_n - y >| + |< x_n - x, y >|$$

Now from Cauchy Swarzinequality

$$|x, y| \leq \|x\|\|y\|$$
$$\leq \|x_n\|\|y_n - y\| + \|x_n - x\|\|y\|$$

Given that $x_n \to x$, $y_n \to y$ so,

$$\|y_n - y\| = \|y - y\| = 0 \quad , \quad \|x_n - x\| = \|x - x\| = 0 \quad \text{as } n \to \infty$$

As $n \to \infty$

$$|< x_n, y_n > - < x, y >| \leq 0$$
$$< x_n, y_n > \to < x, y > \quad \text{as } n \to \infty$$

Theorem:

If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in V, then the inner product $< x_n, y_n >$ is a Cauchy sequence in F.

Proof:

$\{x_n\}, \{y_n\}$ are Cauchy sequence

To show $< x_n, y_n >$ is also Cauchy Sequence.

$\Rightarrow$ $\|x_n - x_m\| \to 0$; $\|y_n - y_m\| \to 0$, m,n $\to \infty$

$$|< x_n, y_n > - < x_m, y_m >| = |< x_n, y_n > - < x_n, y_m > + < x_n, y_m > - < x_m, y_m >|$$
$$= |< x_n, y_n - y_m > + < x_n - x_m, y_m >|$$
$$\leq |< x_n, y_n - y_m >| + |< x_n - x_m, y_m >|$$
$$\leq \|x_n\|\|y_n - y_m\| + \|x_n - x_m\|\|y_m\|$$

$\Rightarrow$ $|< x_n, y_n > - < x_m, y_m >| \to 0$, as n,m $\to \infty$

$\Rightarrow$ $< x_n, y_n >$ is a Cauchy Sequence

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Theorem:

In an inner product space V and x, y in V if $x \perp y$ then

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2$$

Proof:

$$\|x+y\|^2 =< x+y, x+y >$$

$$= < x,x > + < x,y > + < y,x > + < y,y >$$

As x and y are perpendicular so $< x, y >= 0, < y, x >= 0$

$$\|x+y\|^2 =< x,x > + < y,y >= \|x\|^2 + \|y\|^2$$

Generalized form:

$\{x_1, x_2, ...x_n\}$ be nonzero vectors in V inner product space such that

$< x_i, x_j >= 0 \quad , \quad i \neq j$

This system $\{x_1, x_2, ...x_n\}$ is called orthogonal system as all vectors inside it are perpendicular to each other.

The generalized statement is $\|x_1 + x_2 + ... + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ... + \|x_n\|^2$

The idea of proof is

$$\left\| \sum_{i=1}^{n} x_i \right\|^2 = \langle \left\| \sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i \right\| \rangle$$

$$=< x_1 + ... + x_n, x_1 + ... + x_n >$$

$$=< x_1, x_1 + ... + x_n > + ... + < x_n, x_1 + ... + x_n >$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} < x_i, x_j >$$

$< x_i, x_j >= \|x_i\|^2$, if $i \neq j$ $< x_i, x_j >= 0$ and for $i = j$ then $< x_i, x_j >= \|x_i\|^2$

$$\left\| \sum_{i=1}^{n} x_i \right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

Any sequence $\{x_n\}$ of non-zero mutually orthogonal vectors in an inner product space V is linearly independent.

Proof: do it yourself

Let $x = (x_1, x_2, ...x_n)$ be the orthogonal sequence.

Remark:

If $< x, x_1 >= 0$, $\forall$ i=1,2,.....,n $\Rightarrow$ $\left\langle \sum_{i=0}^{n} \alpha_i x_i, x \right\rangle = 0$

$$\left\langle \sum_{i=0}^{n} a_i x_i, x \right\rangle = \langle a_1 x_1 + a_2 x_2 + ... + a_n x_n, x \rangle = a_1 \langle x_1, x \rangle + ... + a_n \langle x_n, x \rangle = 0$$

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IMPORTANT SUBJECTIVE FOR FINAL TERM

Q.No.01: Prove that $< x, ay + bz >= \bar{a} < x, y > + \bar{b} < x, z >$

Proof:

$$< x, ay + bz > = < \overline{ay + bz}, x >$$
$$< x, ay + bz > = < \overline{ay}, x > + < \overline{bz}, x >$$
$$< x, ay + bz > = \bar{a} < \overline{y}, x > + \bar{b} < \overline{z}, x >$$
$$< x, ay + bz > = \bar{a} < x, y > + \bar{b} < x, z >$$

Hence proved.

Q.No.02: Prove that UV **is unitary OR** $(UV)^* = (UV)^{-1}$

Proof:

$$(UV)^* = U^* V^*$$

Since we know that

$$U^* = U^{-1} \text{ and } V^* = V^{-1}$$

So

$$(UV)^* = U^{-1} V^{-1}$$
$$(UV)^* = (UV)^{-1}$$

It shows that UV is unitary.

Q.No.03: Give any two examples of Linear Functional.

Solution:

$$\text{Space } C[a, b] \text{ and Space } l^2$$

Q.No.04: Prove that $(S + T)^* = S^* + T^*$

Proof:

$$< x, (S + T)^* y >=< (S + T)x, y >$$
$$< x, (S + T)^* y \geq < (Sx + Tx), y >$$
$$< x, (S + T)^* y >=< Sx, y > + < Tx, y >$$
$$< x, (S + T)^* y >=< x, S^* y > + < x, T^* y >$$
$$< x, (S + T)^* y >=< x, (S^* y + T^* y) >$$
$$< x, (S + T)^* y >=< x, (S^* + T^*) y >$$

Hence we have proved that

$$(S + T)^* = S^* + T^*$$

Q.No.05: Find matrix multiplication if $(\xi_1, \xi_2, \xi_3) \leftrightarrow (\xi_1, \xi_2, -\xi_1 - \xi_2)$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ -\xi_1, -\xi_2 \end{bmatrix}$$

$$\textit{Hence required matrix is } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Q.No.06: Show that

$$< x_i, x_j >= \|x_i\|^2 \text{ if } i \neq j$$

$$< x_i, x_j >= 0 \text{ if } i = j \text{ then } < x_i, x_j >= \|x_i\|^2$$

Proof:

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The generalized statement is $\|x_1 + x_2 + ... + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ... + \|x_n\|^2$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \langle \left\|\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i\right\| \rangle$$

$$=< x_1 + ... + x_n, x_1 + ... + x_n >$$

$$=< x_1, x_1 + ... + x_n > + ... + < x_n, x_1 + ... + x_n >$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} < x_i, x_j >$$

$< x_i, x_j >= \|x_i\|^2$, if $i \neq j$ $< x_i, x_j >= 0$ and for $i = j$ then $< x_i, x_j >= \|x_i\|^2$

$$\left\|\sum_{i=1}^{n} x_i\right\|^2 = \sum_{i=1}^{n} \|x_i\|^2$$

Q.No.07: Show that $\overline{N(T)} < N(T)$

Solution:

$$x \in N(T)$$
$$\exists\ (x_n) such\ that\ x_n\ \in N(T) + x_n \to x$$

Therefore

$$Tx_n \to Tx$$
$$x_n\ \in N(T)$$
$$Tx_n = 0$$
$$T_n = 0$$
$$x\ \in N(T)$$
$$\overline{N(T)} < N(T)$$
$$\overline{N(T)} = N(T)$$
$$N(T) is\ closed.$$

Q.No.08: Give an example of function which is not linear.

Solution: $\|\ .\ \|: x \to R$ is functional but not linear.

Q.No.09: Find basis of R(T).

Solution:

$$(\xi_1, \xi_2, -\xi_1, -\xi_2) = \xi_1(1,0,-1) + \xi_2(0,1,-1)$$

So the basis of R(T)= $\{(1,0,-1), (0,1,-1)\}$

Q.No.10: Generalization of Orthogonal

Solution: $\{x_1, x_2, ...x_n\}$ be nonzero vectors in V inner product space such that $< x_i, x_j >= 0$, $i \neq j$ This system $\{x_1, x_2, ...x_n\}$ is called orthogonal system as all vectors inside it are perpendicular to each other. The generalized statement is $\|x_1 + x_2 + ... + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ... + \|x_n\|^2$

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Q.No.11: Show that U is normal.

Solution:

$$UU^* = U^*U$$

Since we know that

$$U^* = U^{-1}$$

So

$$UU^{-1} = U^{-1}U$$

$(UV)^* = (UV)^{-1} = I$ Hence its proved that U is normal.

Q.No.12: Show that U^{-1} is Unitary or Prove that $U^* = U^{-1}$ or $\| U\| = 1$

Solution: Since U is bijective it shows that U^{-1} is bijective $(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}$. Hence it implies that U^{-1} is unitary.

Q.No.14: Show that U is Isometric.

Solution:

$$\| U_x\|^2 =< U_x, U_x >$$
$$\| U_x\|^2 =< x, U^*U_x >$$
$$\| U_x\|^2 =< x, Ix >$$
$$\| U_x\|^2 =< x, x >$$
$$\| U_x\|^2 = \| x\|^2 \text{ It proved.}$$

Q.No.15: Prove that $(ST)^* = S^*T^*$

Proof:

$$< x, (ST)^*y >=< (ST)x, y >$$
$$< x, (ST)^*y \geq< (SxTx), y >$$
$$< x, (ST)^*y >=< Sx, y >< Tx, y >$$
$$< x, (ST)^*y >=< x, S^*y >< x, T^*y >$$
$$< x, (ST)^*y >=< x, (S^*+T^*y) >$$
$$< x, (ST)^*y >=< x, (S^*T^*)y >$$

Hence we have proved that

$$(ST)^* = S^*T^*$$

Q.No.16: Show that $T^*T=0$ or $T=0$ is normal.

Solution:

$$T^*T = 0 \; iff \; T = 0$$
$$e = \|T^*T\| = \| T\|$$

(f) is true by e.

Q.No.17: Show that $(\alpha T)^* = \alpha^{-1}T^*$.

Solution:

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$$< x, (\alpha T^*)^* y > = < (\alpha T)x, y >$$
$$< x, (\alpha T^*)^* y > = < \alpha(Tx), y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < Tx, y >$$
$$< x, (\alpha T^*)^* y > = \bar{\alpha} < x, T^* y >$$
$$< x, (\alpha T^*)^* y > = < x, \bar{\alpha} T^* y >$$
$$(\alpha T^*)^* = \bar{\alpha} T^*$$

Q.No.18: Show that $(T^*)^* = T$

Solution:

$$< x, (T^*)^* y \geq < xT^*, y >$$
$$< x, (T^*)^* y \geq < x, Ty >$$

It implies that

$(T^*)^* = T$ so its proved.

Q.No.19: Show that $\|T^*T\| = \|TT^*\| = \|T\|^2$

Solution:

$$\| U_x \|^2 = < U_x, U_x >$$

Q.No.20: Show that $< T^* y, x > = < y, Tx >$

Solution:

$$< T^* y, x > = < \overline{y, T^* x} >$$
$$< T^* y, x > = < \overline{Tx, y} >$$
$$< T^* y, x > = < y, Tx >$$

Hence its proved.

Q.No.21: Show that $Q = 0$ if and only if $< Qx, y >= 0$

Solution:

$$Q = 0 \Longrightarrow Qx = 0 \; for \; all \; x \epsilon X$$
$$< Qx, y > = < 0, y >$$
$$< Qx, y > = < 0.w, y >$$
$$< Qx, y > = 0 < w, y >$$
$$< Qx, y > = 0. \quad for \; all \; x \epsilon X$$

It implies that Qx=0 and by definition Q=0. Its proved.

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Q.No.22: Write examples of Bounded Linear Operators

Solution: Identity Operator, Zero Operator, Differentiation Operator, Integral Operator

Q.No.23: Define Self Adjoint Operator or Hermitian Operator.

Solution:

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Self Adjoint Operator or Hermitian Operator if $T^* = T$.

Q.No.24: Define Self Adjoint Operator or Hermitian Operator.

Solution:

The Hilbert Adjoint Operator T^* of T is defined by $< Tx, y > =< x, T^* y >$ If T is self adjoint then it becomes $< Tx, y > =< x, Ty >$. If T is self adjoint or unitary then T is normal.

Q.No.25: Define Unitary Operator.

Solution:

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $T^* = T^{-1}$.

Q.No.26: Define Normal Operator.

Solution:

Abounded Linear Operator $T: H \to H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $TT^* = T^*T$

Q.No.27: Define Extension of T.

Solution: Extension of T on a set $M \supset D(T)$ is defined as $\tilde{T}: M \to Y$ or

$$\tilde{T}x = Tx \; for \; all \; x \in D(T)$$

Q.No.28: Define Polarization Identity.

Solution:

For any x,y in complex inner product space we have

$$< x, y >= \frac{1}{4}\left\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\right\}$$

Q.No.29: Define Bounded Linear Function.

Solution:

Let X and Y be normed spaces and $T: D(T) \to Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$\|Tx\| \leq c\|x\|$

Q.No.30: Define Canonical Mapping.

Solution:

$C: X \to X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

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Q.No.31: Define Orthonormal System.

Solution:

Orthonormal systems are those functions which are real or complex whose inner product with itself results in 1 and with other functions result in 0.

Q.No.32: Define Polarization Identity.

Solution:

For any x, y in complex inner product space

$$< x, y >= \frac{1}{4}\left\{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2\right\}$$

We have to prove this complex inner product space.

Q.No.33: Define Appolonius Identity:

Solution: $\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z-\frac{1}{2}(x+y)\right\|^2 \quad , \quad x, y, z \in V$

Q.No.34: Define Equal Operators:

Solution: Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have the same domain i.e. $D(T_1) = D(T_2)$

Q.No.35: Write examples of Inner Product Space:

Solution: Identity

Q.No.36: Define Inner Product Space:

Solution: The pair $(V, <\bullet,\bullet>)$ is called an inner product space.

Q.No.37: Define Linear Functional:

Solution: A linear functional f is a linear operator with domain in a vector space X and range in the scalar field K of X; thus $f: D(f) \to K$. where K=R if X is real and K=C if X is complex.

Q.No.38: Define Bounded Linear Extension:

Solution: $T: D(T) \to Y$ be a bounded linear operator, where D(T) lies in a normal space X and Y is a Banach Space then T has an extension $\tilde{T}: \overline{D(T)} \to Y$. Where $\tilde{T}$ is a bounded linear operator of norm.

$$\|\tilde{T}\| = \|T\|$$

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MTH 641 31-8-2019

Q.1 Define Extension of T. 2

2. Prove that
$(x, ay+bz) = a(x,y) + b(x,z)$ - 2

3. Appolloniys identity- 2

4- Define bounded linear 2
functional.

5- Find basis of $N(T)$- 3

6- Define Self adjoint operator- 2

7- Prove that U is normal- 3

8- Give Two examples of
orthogonal System- 3

9- Show that T is linear- 5

10- If $(\xi_1, \xi_2, \xi_3) \to (\xi_1, \xi_2, -\xi_1 - \xi_2)$ -5
then find matrix representation

11- Prove that $\alpha(T^*) = \bar{\alpha} T^*$ - 5

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"MTH-641" 30-8-2019

Q1: Write two examples of linear functional. (2)

Q2: Orthonormal system Definition. (2)

Q3: Let $T: R^3 \to R^3$ define by $(\xi_1, \xi_2, \xi_3) \mapsto (\xi_1, \xi_2, -\xi_1 - \xi_2)$ then find basis of $R(T)$? (2)

Q4: Show that $U^{-1} = U^*$ is unitary. (2)

Q5: difference b/w Algebraic dual space and conjugate space. (3)

Q6: Show that $\langle x, \alpha y \rangle = \bar{\alpha} \langle x, y \rangle$ (3)

Q7: Apollonius Identity. (3)

Q8: Show that $\|U\| = 1$ (3)

Q9: Prove that if a normed space X is finite dimensional then every linear operator on X is bounded. (5)

Q10: Show that $(S+T)^* = S^* + T^*$ (5)

Q11: Show that:

$$\|x_1 + x_2 + x_3 + \cdots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \cdots + \|x_n\|^2$$

(MCQs were very easy)

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Mth 641

Q:1 Give an example of function which is not linear?

Q:2 Write generalized from of orthognal system

Q#3.4 Let $R^3 \to R^3$ define by $(\xi_1, \xi_2, \xi_3) \to (\xi_1, \xi_2 - \xi_1, \xi_2)$ then find basis of $R(T)$ is ko $N(T)$ mai krna tha.

Q:5 self - adjoint operator

Q#6 Isomorphsim of metric space.

Q:7 U is normal?

Q:8 if $(x, x_i) = 0 \quad \forall i = 1, 2, 3 \cdots$ show $\left\langle \sum_{i=1}^{n} a_i x_i, x \right\rangle = 0$ hence $x \perp \sum a_i x_i$

Q:9 Theorem that $\|\tilde{T}\| = \|T\|$

Q:10 Inner product or show $\|x_1 + x_2 \cdots x_n\|^2 = \|x_1\|^2 + \cdots \|x\|^2$

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Define canonical mapping

Define apolagious identity

Generalization of orthogonal System

Define Isomorphism of metric spaces

Two example of orthogonal System

One example of linear function

Define vector dimensional Scalar multiplication.

Define normal operator

Define vector addition by using dual space

Define equal operator

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Diary No.	File No. and Date	SUBJECT	REMARKS
			(appollonius)
		MTH-641 (polarization)	or
Q.1	Define	appollonius identity	
Q2	write	generalized form of appol orthogonal system.	
Q.3	prove	that $(ST)^* = T^* S^*$	
Q.4	prove that	$\langle u, \alpha y + \beta \rangle = \bar{\alpha} \langle x, y \rangle + \beta \langle y, z \rangle$	
Q.5	Define	norms operator.	
Q.6	prove that	null space is closed.	
Q.7	prove that	U is normal	
Q.8	Define	Canonical mapping	
Q.9	If	$(\xi_1, \xi_2, \xi_3) \to (\xi_1, \xi_2, -\xi_1, -\xi_2)$ determine matrix representation	
	If	$(\xi_1, \xi_2, \xi_3) = (\xi_1, \xi_2, -\xi_1, -\xi_2)$	
Q.10	Determine	basis of $N(T)$.	
		solutions:-	
	Q.4	3.1 → 3(c) part	
	Q.9	2.9 → excercise problem.	
	Q.2	3.1 → (2)	
	Q.1	3.8 → 8(10) (polarization)	
		3.1 → 8(10)	
		(appolonius) or 3.1(5) exercise problem	

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Cardiac ... tional Cardiol... (Cardiology)

$\langle Tx, x \rangle$ is real for all $x \in H$ (5)

$T: R^3 \to R^3$ defined by $(\xi_1, \xi_2, \xi_3) \to (\xi_1, \xi_2 - \xi_3, -\xi_2)$ then find matrix representation. (5)

Show that UV is unitary (3)

Show that $U+V$ is unitary (2)

Define Apollonius identity (3)

Define Vector ... Scalar multiplication

Define ... of vector spaces (3)

find basis of $R(T)$ (3)

Two Example of orthogonal system (3)

One Example of linear functional (2)

Ph: 051-...

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1- Define appollogius identity 2/

2- Write generilized form of appolt orthogonal system 2.

3. Prove that $(ST)^* = T^* S^*$ 5/-

4- Prove that $\langle x, \alpha y + \beta z \rangle = \bar{\alpha} \langle x, y \rangle + \bar{\beta} \langle y, z \rangle$ 5/.

5- Define norm operator. 2.

6. Prove that null space is closed. 3-

7. Prove that U is normal. 3/-

8. Define canonical mapping 2/-

9. If $(\xi_1, \xi_2, \xi_3) = (\xi_1, \xi_2, -\xi_1 - \xi_2)$ Determine matrix representation. 5

10. If $(\xi_1, \xi_2, \xi_3) = (\xi_1, \xi_2, -\xi_1, -\xi_2)$ Determine basis of $N(T)$ 3/-

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Examples of Inner Product Spaces

Q

- Space $\mathbb{R}^n$
- Space $\mathbb{C}^n$
- Space $C[a, b]$
- Space l^2
- Space P_n

ans

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Let X and Y be finite dimensional vector spaces over the same field and $T: X \longrightarrow Y$ a linear operator. We choose a basis $E = \{e_1, \cdots, e_n\}$ for X and a basis $B = \{b_1, \cdots, b_r\}$ for Y, with the vectors arranged in a definite order which we keep fixed. Then every $x \in X$ has a unique representation

$$x = \xi_1 e_1 + \cdots + \xi_n e_n. \tag{1}$$

Since T is linear, x has the image

$$y = Tx = T\left(\sum_{k=1}^{n} \xi_k e_k\right) = \sum_{k=1}^{n} \xi_k T e_k. \tag{2}$$

Since the representation (1) is unique, we have our first result:

Show that

T is uniquely determined if the images $y_k = Te_k$ of the n basis vectors $e_1, \cdots, e_n$ are prescribed.

Since y and $y_k = Te_k$ are in Y, they have unique representations of the form

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sum & scalar multiplication of Dual

as follows. The *sum* $f_1 + f_2$ of two functionals f_1 and f_2 is the functional s whose value at every $x \in X$ is

$$s(x) = (f_1 + f_2)(x) = f_1(x) + f_2(x);$$

the *product* αf of a scalar α and a functional f is the functional p whose value at $x \in X$ is

$$p(x) = (\alpha f)(x) = \alpha f(x).$$

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2.8-3 Theorem (Continuity and boundedness). *A linear functional f with domain $\mathcal{D}(f)$ in a normed space is continuous if and only if f is bounded.*

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2.8-1 Definition (Linear functional). A *linear functional* f is a linear operator with domain in a vector space X and range in the scalar field K of X; thus,

$$f: \mathcal{D}(f) \longrightarrow K,$$

where $K = \mathbf{R}$ if X is real and $K = \mathbf{C}$ if X is complex. ∎

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2.7-11 Theorem (Bounded linear extension). *Let*

$$T: \mathcal{D}(T) \longrightarrow Y$$

be a bounded linear operator, where $\mathcal{D}(T)$ lies in a normed space X and Y is a Banach space. Then T has an extension

$$\tilde{T}: \overline{\mathcal{D}(T)} \longrightarrow Y$$

where $\tilde{T}$ is a bounded linear operator of norm

$$\|\tilde{T}\| = \|T\|.$$

5 Marks

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2.7-9 Theorem (Continuity and boundedness). *Let T: $\mathcal{D}(T) \longrightarrow Y$ be a linear[7] operator, where $\mathcal{D}(T) \subset X$ and X, Y are normed spaces. Then:*

MCQ

(a) *T is continuous if and only if T is bounded.*

(b) *If T is continuous at a single point, it is continuous.*

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MTH641 16-09-2020 (11:00)

Q.1 Define metric or Euclidean Place (2
$\mathbb{R}^2$ and Three-dimensional Place $\mathbb{R}^3$?

Q.2 Define isometric metric space?

Q.3 Define dual space and double algebraic dual space

Q.4 Any sequence $\{x_n\}$ of a non-zero (5
mutually orthogonal vectors in an Inner Product Space 'V' is linearly Independent.

Q.5 Define Subspace of a metric Space? (3

Q.6 Prove that $U^* = U^{-1}$

Q.7 Show that $Q = 0$ iff $\langle Qx, y \rangle = 0$

Q.8 Example of linear functional dwight

Q.9 Show that $(\alpha T)^* = \bar{\alpha} T^*$

Q.10 Show the norm and Inner Product Space of ℓ^2.

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Q Define Canonical mapping

Q Define Unitary operator

Q Show UV is unitary

Q Show R^n is orthogonal

Q Show that $\|T\| = 1$

Q Define Norm & Inner product space.

C The function $\|\cdot\| : V \to R^+$ given by $\|x\| = \sqrt{\langle x, x \rangle}$ $x \in V$ depen upon norm in V that $\|x\| = 0$

Q write examples of Banach space.

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Q#1: Show that the zero operator $0: X \to Y$ on a normed space X is bounded and has norm $\|0\| = 0$

Q#2: Define equal operator.

Q#3: In an inner product space V, the function $\|\cdot\|: V \to \mathbb{R}^+$ given by $\|x\| = \sqrt{(x,x)}$, $x \in V$ defines a norm in V. Show that $\|x\| = 0 \iff x = 0$

Q#4: Define unitary operator.

Q#5: Give five examples of normed space?

Q#6: Define norm and inner product for which the space l^2 is an inner product space.

Q#7: Define the relation b/w X and X^{**}.

Q#8: Let the operators $U: H \to H$ and $V: H \to H$ be unitary, here, H is a Hilbert space. Then show that U is normal

Q#9: Show that $f: \mathbb{R}^3 \to \mathbb{R}$ defined $f(x) = x \cdot a = \xi_1 x_1 + \xi_2 x_2 + \xi_3 x_3$. (for a fixed a) is bounded.

Q#10: Prove that in an inner Product space $(V, \langle \cdot, \cdot \rangle)$, $\|x+y\| \leq \|x\| + \|y\|$, $\forall x, y \in V$

Q#11: Let X and Y be inner product spaces and $Q: X \to Y$ a bounded linear operator. Then show that if $Q: X \to X$, where X is complex and $\langle Qx, x \rangle = 0$ for all $x \in X$, then $Q = 0$

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MTH641.

1= Prove Q of real line R is not complete. (8)

2= Define Hilbert Adjoint of T^*. (2)

3_ Name the example of linear functional (2)

4. $\langle ax + by, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$ (3)

5. Define
Dense set (3)
Separable Space

6 Prove $\bar{\alpha} \langle x, y \rangle = \langle x, \alpha y \rangle$. (3)

7 Prove. $Q = 0$ if and only if $\langle Qx, y \rangle = 0 \quad \forall x \in X$ and $\forall y \in Y$ (3)

8 Prove $(T^*)^* = T$ (5)

9 Prove $\|x + y\|^2 = \|x\|^2 + \|y\|^2$. (5)

10.
Let $T: X \to Y$ be a linear operator
Prove
$y = Tx = T\left(\sum_{k=1}^{n} \xi_k e_k\right)$ is obtained
by $\eta_j \sum_{k=1}^{n} \tau_{kj} b_j$ (5)

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(1) U is norm preserving. (5)

(2) $\|x+y\|^2+\|x-y\|^2 = 2(\|x\|^2+\|y\|^2)$ (5)

(3) U is normal (5)

(4) $(T^*)^* = T$. (5)

(5) If $\|x\| = \sqrt{\langle x, y\rangle}$ Show that $\langle x, x\rangle \geq 0$. (2)

(6) bounded linear functional? (2)
examples of normed space? (2)

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Today My Paper MTh 641

MCQs was very easy.

1. Define equal operator? (2 marks).
2. example of four spaces are complete? (2 Marks)
3. Define norm and which is C^n innerproduct space? (2 Marks)
4. Define and x and x^{**} relation? (2 Marks)
5. Is an Inner product space V. ~~x∈V~~ The function $\|\cdot\| : V \to R^+$ given by $\|x\| = \sqrt{\langle x, y \rangle}$, $x \in V$ defines a norm in V. Show That $\|x\| = 0$ (3 Marks)
6. $\|x+y\| \leq \|x\| + \|y\|$ $\forall$ $x, y \in V$. (5 Marks)
7. Show That $ST = TS$ (3 Marks)
8. Show That U is Isometric. (3 Marks)
9. show That $(T^*)^* = T$ (5 marks)
10. $\varphi : X \to X^{**}$, i) φ is Linear Module #77 Theorem (proved) 5 Marks.

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MTh 401

13-09-2020

Samreen:

Qno:1: Prove that U is normal.

Qno:2: Define Orthogonal system.

Qno:3: Define linear functional.

Qno:4: Prove that every inner product space is metric space.

Qno:5 Prov $(ST)^* = T^* S^*$ S

Qno:6: Bounded linear Operator.

Qno:7 Prove

$$\|x+y\| \leq \|x\| + \|y\|$$

Qno:7. Find Matrix Multiplication $(\varepsilon_1, \varepsilon_2, \varepsilon_3) \rightarrow$

Qno:8: As $x_n \rightarrow x_0$ To prove $Tx_n \longrightarrow T_n$

Qno:9 Define normal operator.

Qno:10 Basis of finite dimension.

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Let $T: D(T) \to Y$ be a linear operator where $D(T) \subseteq X$ and X, Y are normed spaces then show that if T is bounded then T is continuous (3)

Give five examples of normed spaces (3)

$(U^{-1}) = (U^{*})$ is unitary (2)

Define orthonormal system (2)

an example of linear functional (2)

List the elements of sphere $S(3,2)$ where 3 is centre and 2 is radius

Sphere in case of a discrete metric space. Also justify your answer (2)

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Let H_1 and H_2 be Hilbert spaces $S: H_1 \to H_2$ be bounded linear operators and $T: H_1 \to H_2$ and α any scalar. Then show that $(T^*)^* = T$ (5)

$x \perp y$ $\|x+y\|^2 = \|x\|^2 + \|y\|^2$ (5)

Show that I^2 is bounded linear functional (5)

Let the operators $U: H \to H$ and $V: H \to H$ be unitary H is Hilbert space. Then show that UV is unitary (3)

Define Appollonius identity (3)

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641

(1) Defination unitary.

(2) prove $\|0\| = 0$

(3) write dual space of $X^{**} \longrightarrow X^*$

(4) Theoram

$$\|x+y\|^2 + \|x-y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$$

(5) Theorams

Let V be any inner product space. For any sequence $\{x_n\}$ and $\{y_n\}$ in

$$x_n \longrightarrow x \qquad y_n \longrightarrow y \quad \text{implies}$$

$$\langle x_n, y_n \rangle \longrightarrow \langle x, y \rangle$$

(6)

(1) Define normal Operator?

(2) Differentiate b/w Algebraic dual or conjugate Space of X?

(3) Define Extension of Operator?

(4) If $U: H \to H$ & $V: H \to H$ is unitary then prove U is normal.

(5) Theorem $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + \|y\|^2$

(6) If a normed Space X is finite dimensional then every linear Operator on X is bounded.

(7) prove $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$

(8) ℓ^∞ & ℓ^p What is Sequence Space

(9) Show R^n is orthogonal System

(10) be bounded linear

Q6 Show that Let H_1 and H_2 be Hilbert Space, $S: H_1 \to H_2$ and $T: H_1 \to H_2$ be bounded linear operator and α any scalar. Then Show that

$$\langle T^* y, x \rangle = \langle y, Tx \rangle : x \in H_1, y \in H_2$$

Ans:-

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10-09-2020.

MTH 641

Define the subspace of a metric?

Define Polarization identity?

Prove that $\langle Tx, y \rangle = \langle x, T^* y \rangle$

Prove that $U^* = U$ is unitary.

Prove $(\alpha T)^* = \bar{\alpha} T^*$

$(T^*)^* = T$.

Give two examples of linear function.

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Mth 641

(1) Show that Φ is Norm preserving.

(2) Show that
$f : R^3 \to R$ defined $f(x) = x \cdot a$
$= \xi_1 a_1 + \xi_2 a_2 + \xi_3 a_3$ is bounded

3. Define Appollonius identity.

4. Write norm on I^p for which it is a Banach Space.

5. Show that U is Normal.

6. Define operations on algebric dual Space.

7. prove Continous mapping of a Compact Subject M of a Metri space X into R assume a max and min at some points of M.

(8) Define Conical mapping.

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Sheikh

MTH 641

Q:- Define Norm and inner product for which the space l^2 is an inner product space

Q:- Prove that if a set M is closed in X then it forms a complete subspace of X

Q Give at least two example of infinte dimensional vector space.

Q. $\langle \alpha x + \beta y, z \rangle = \alpha \langle x, z \rangle + \beta \langle y, z \rangle$

Q. U is Normal

Q Basis of $R(t)$

Q T is sef - adjoint $\langle Tx, x \rangle$

Q. Show that the sequence $\{x_n\}$ of non-zero orthogonal vectors is an inner product space V is linearly independent.

Q $\varphi = X^* \to R^n$ is norm preserving.

Q $(\alpha T)^* = \bar{\alpha} T^*$

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Rough Sheet

FALL/SPRING MID/FINAL TERM 20__ (Year) Date: 10/09/2020 Superintendent Signature:

Student Signature:

Campus Code: PCKW02 Student ID: MC180401231

Course Code:

Q.No1 Write the norm on R^n for which it is a Banach Space

Q.No2 Define Equal operator

Q.No3 Write Generalized Form of orthogonal System

Q.No4 Define Unitary operator

Q.No5 Proof that a Continuous mapping, f of a Compact Subset M of a Metric Space X into R assumes a maximum and minimum at some point of M

Q.No6 Let V be a finite dimensional vector space. If $x_0 \in X$ has the Property that $f(x_0) = 0$ for all $f \in X^*$

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MTH 641 10-09-2020

① Define complete metric space

② Define equal operators. (Topic 67)

③ Find basis $N(T)$

$(\varepsilon_1, \varepsilon_2, \varepsilon_3) : (\varepsilon_1, \varepsilon_2, -\varepsilon_2 - \varepsilon_1)$

(Topic 78)

④ Prove that $\|Ux\| = \|x\|$ (Topic 113)

⑤ $\|U\| = 1$ (Topic 113)

⑥ $(ST)^* = T^* S^*$ (Topic 106) (5)

⑦ Write the generalized form of orthogonal system (Topic 92)

⑧ Define norm and inner product on R^n (Topic 91 + 41)

⑨ Define a metric on sets one has two elements and one has 1 element (mid practice question)

⑩ Theorem topic 64 (5 marks)

⑪

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Mth 641

(1) Show that φ is Norm preserving.

(2) Show that

$F : R^3 \to R$ defined $f(x) = x \cdot a$

$= \xi_1 a_1 + \xi_2 a_2 + \xi_3 a_3$ is bounded

3- Define Appollonius identity.

4. Write norm on l^p for which it is a Banach Space.

5- Show that U is Normal.

6. Define operations on algebric dual Space.

7- prove Continous mapping of a Compact Subject M of a Metric space X into R assume a max and min at some points of M.

(8) Define Conical mapping.

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Q. No 7 Prove that

$$x, y \in V,\ \|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

Q. No 8:- Let $U: H \to H$ and $V: H \to H$ be unitary, here H is a Hielbert Space - then show that UV is unitary

Q. No 1 Let $T: D(T) \to y$ be a linear operator where $D(T) \subseteq x$ and x, y are normed space, the show that if T is continuous then T is bounded

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① show that $\phi : X^* \to l^\infty$ is injective.

② show that l^2 is bounded linear function

③ Define norm and Inner product for which the space l^2 is an Inner product space

④ Every convergent sequence in a metric space is a Cauchy seq.

Q & A

unitary then prove normal.

(5) Theorem $\|x+y\|^2+\|x-y\|^2 = 2\|x\|^2+\|y\|$

(6) If a normed space X is finite dimensional then every linear Operator on X is bounded.

(7) prove $\langle \alpha x+\beta y, z\rangle = \alpha\langle x, z\rangle + \beta\langle y, z\rangle$

(8) What is Sequence Space?

(9) Show R^n is orthogonal System?

(10) ~~let $T: H \to H$ be bounded linear~~

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Show that the sequence $\{x_n\}$ of non zero mutually orthogonal vector in a inner product space V is linearly independent. (3)

Prove that if a normed space X is finite dimensional then every linear operator on X is bounded (5)

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(3) $\langle T^* y, x \rangle = \langle y, Tx \rangle$

If $\langle x, x_i \rangle = 0$; $\forall i = 1, 2, \ldots n$ then show that (7)

$\left\langle \sum_{i=1}^{n} a_i x_i, x \right\rangle = 0$ and hence $x \perp \sum_{i=1}^{n} a_i x_i$

Every convergent sequence in a metric space is a Cauchy sequence. (3)

Show that Real line R is seperable (2)

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MTH-641 (15-09-20 at 0500 PM) Date:

1- Define equivalent norms?

2- Give Two examples of linear functionals?

3- In ~~xxxx~~ Inner product space norm is define as

$\|x\| \sqrt{\langle x, x \rangle}$

Then show that $\|x\| > 0$

4- Define Unitary operator?

5- Define Cauchy sequence in Norm space?

6- Let T be a bounded linear operator. Then show that as $x_n \to x$ (where $x, x \in D(T)$) implies $Tx_n \to Tx$.

7- Show that inner product space.

$\langle x, \alpha y \rangle = \bar{\alpha} \langle x, y \rangle$.

8- Let H_1 and H_2 be Hilbert spaces, $S: H_1 \to H_2$. and $T: H_1 : H_2$ be bounded linear operator. Then show that $T^*T = 0 \iff T = 0$

9- If a norm space X is finite dimension then show that every linear operator on X is bounded.

10- In Inner product space, show that

$\|x+y\| \leq \|x\| + \|y\|$

11- Let the operators $U: H_1 \to H$ & $V: H \to H$ be unitary operators then show that U is normal.

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$\theta = 0$

$\langle \theta x, y \rangle =$

$\langle \theta x, y \rangle =$

(1) $\theta(aT)^* = \bar{a} T \theta$

(2) $\theta : x^* \to \int^\infty R^3 \to R$

3 $R^3 - R$

$f(x) = x \cdot a$

$f(x) = x \cdot a$

$\xi_1 x_1 + \cdots$

$= \|x\| \cdot$

$\|x \cdot a\| \leq \|$

$\geq$

4 $Q = 0$

$\theta x = 0$

5 $R^3 \to R^3$

$(\xi_1, \xi_2, -\xi_1 - \xi_2)$ $f(x) = \|f(x)\|$

6 $\|x+y\|^2 = 2\|x\|^2 + \|y\|^2$

$x = a$

$f(a) = \|f(a)\|$

$\frac{\|f(a)\|}{\|a\|}$

(7) norm space

Completion

8 Unitar

9 linear functional

Ex orthogonal

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