

MTH641 Functional Analysis

Question No : 28 of 41

Inner product is the generalization of

Answer (Please select your correct option)

☐ Cross product

☒ Dot product

☐ Multiplication

☐ Distance function

Start Time: 8:06 AM

109:00

Time Left



Question Summary : 1 of 41

A mapping $T: X \rightarrow Y$ is said to be continuous if it is continuous at point of X

Answer : Please select your correct option(s)

☒ Every

☐ atmost one

☐ exactly one

☐ atleast one

MTH641 Functional Analysis

Question No : 15 of 41

The dual space of l^1 is

Answer (Please select your correct option)

☐ l^p

☒ l^∞

☐ l^2

☐ l^1

Start Time: 8:05 AM

111:00

Time Left



Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11

Conduct Instructions

Question No : 13 of 41

A finite dimensional normed space is to its second dual.

Answer (Please select your correct option)

☐ isometric

☒ isomorphic

☐ homomorphic

☐ none of these

Question No: 5 of 41

Which one of the following axioms hold for any vectors u, v, w in vector space X and k, a, b in field K ?

Answer (Please select your correct option)

☐ $u + (-u) = (-u) + u$, for each $u \in X$

☐ $u + w = w - u$

☒ $k(u + v) = ku + kv$

☐ $(ab)u = a(bu)$

Start Time: 8:06 AM

Question No : 30 of 41

An orthonormal set M is an orthogonal set in an inner product space whose elements have norm

Answer (Please select your correct option)

☒ equal to 1

☐ >1

☐ <1

☐ None of these

Start Time: 8:06 AM

30

Which of the following is not true for the space l^p with $p \neq 2$?

Answer (Please select your correct option)

☐ It is a complete metric space

☐ It is a complete norm space

☒ It is a complete inner product space

☐ None of these

Start Time: 8:06 AM

110:00

Time Left



Question Summary: (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14

Conduct Instructions

Any pair of vectors in an orthogonal system is to each other.

Answer (Please select your correct option)

☐ Parallel

☐ Equal

☐ None of these

☒ Perpendicular



Question No: 10 of 41

Let $T: D(T) \rightarrow Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then T is continuous if and only if T is _____.

Answer (Please select your correct option)

☐ continuous

☐ linear

☒ bounded

☐ none of these

Start Time: 8:00 AM

10

Two operators T_1 and T_2 are defined to be equal, written $T_1 = T_2$, if _____ and if $T_1x = T_2x$ for all _____.

Answer : Please select your correct option)

☒ $D(T_1) = D(T_2), \quad x \in D(T_1) = D(T_2)$

☐ $D(T_1) \subset D(T_2), \quad x \in D(T_1) = D(T_2)$

☐ $D(T_1) \supset D(T_2), \quad x \in D(T_1) = D(T_2)$

☐ $D(T_1) \supset D(T_2), \quad x \in D(T_1) \neq D(T_2)$

Start Time: 8:05 AM

112:00

Question No : 20 of 41

Every complete inner product space is _____ space.

Answer (Please select your correct option)

☒ Hilbert

☐ Banach

☐ Unitary

☐ Euclidean

Start Time: 8:06 AM

111:00

Time Left



Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

[Conduct instructions](#)

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

☐ $\{a, b\}$

☐ $[a, b]$

☒ Space $\mathbb{R}^n$

☐ None of these

Start Time: 8:06 AM

110:00

Time Left



Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Conduct Interview

Which of the following is true about the inner product space?

Answer (Please select your correct option)

☐ Every normed space is an inner product space.

☒ Every inner product space is a normed space. ✓

☐ Every inner product space is Hilbert space.

☐ None of these is true.

Start Time: 8:06 AM

111:00

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Question Summary : (Attempted Question)

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Conduct Instructions

The dual space of a normed space X contains all on X .

Answer (Please select your correct option)

☒ Bounded linear functional

☐ Unbounded linear functional

☐ Bounded linear operator

☐ Unbounded linear operator

Start Time : 9:06 AM

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Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

☐ $\langle x, y \rangle = \overline{\langle x, y \rangle}$

☐ $\langle \alpha x, y \rangle = \langle x, \alpha y \rangle$

☐ $\langle \alpha x + y, z \rangle = \langle x, z \rangle + \alpha \langle y, z \rangle$

☒ $\langle x, y \rangle = \overline{\langle y, x \rangle}$

Start Time 8:06 AM

111:00

Time Left



Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Conduct Instructions

In an inner product space V , the function $\| \cdot \| : V \rightarrow \mathbb{R}^+$ given by _____ defines a _____ in V .

Answer (Please select your correct option)

☐ $\|x\| = \sqrt{\langle x, y \rangle}, x, y \in V$, norm

☐ $\|x\| = \sqrt{\langle x, x \rangle}, x \in V$, norm

☐ $\|x\| = \sqrt{\langle x, x \rangle}, x \in V$, metric

☐ None of these

Start Time: 8:16 AM

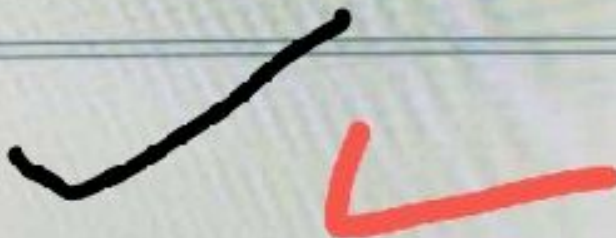
Question No : 8 of 41

If a normed space X is _____ dimensional, then every linear operator on X is bounded.

Answer (Please select your correct option)

☐ infinite

☒ finite



Question No : 26 of 41

For a subset A of an inner product space X , x is orthogonal to A if

Answer (Please select your correct option)

☒ x is orthogonal to a for all $a \in A$

☐ x is orthogonal to any two elements a and $b \in A$

☐ x is orthogonal to a for a fixed $a \in A$

☐ None of these

For any non empty subset A of (X,d) , the diameter $\delta(A)$ of A is defined as

Answer (Please select your correct option)

☐ $\delta(A) = \inf_{x,y \in A} d(x,y)$

☐ $\delta(A) = d(x,y)$

☒ $\delta(A) = \sup_{x,y \in A} d(x,y)$

☐ $\delta(A) = d(x,y)$



Start Time: 11:40 AM

11:40

A sequence (x_n) in a metric space is said to be non-decreasing if for all n ,

Answer (Please select your correct option)

☐ $x_n \geq x_{n+1}$

☒ $x_n \leq x_{n+1}$

☐ $x_n < x_{n+1}$

☐ $x_n > x_{n+1}$

Question No : 1 of 41

If d is a metric on X , Then for all $x, y \in X$, $d(x, y)$ is....

Answer (Please select your correct option)

☒ A non negative real number

☐ A negative real number

☐ Not a rational number

☐ A complex number

Question No : 16 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

☒ $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$

☐ $\langle \alpha x + y, z \rangle = \langle x, z \rangle + \alpha \langle y, z \rangle$

☐ $\langle \alpha x, y \rangle = \langle x, \alpha y \rangle$

☐ $\langle x, y \rangle = \overline{\langle x, y \rangle}$

Start Time: 8:06 AM

111:00

Time Left



Question Summary: (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14

Conduct Instructions

Address: <http://server/notes/dest/1/paper.aspx> Marks: 1 (Budgeted Time)

MTH641 Functional Analysis

Question No : 1 of 41

If $S = \{v_1, v_2, v_3, \dots, v_n\}$ is a set of vectors in a finite dimensional vector space X , then S is called a basis for V if _____ and S is linearly independent.

Answer : (Please select your correct option)

- ☐ S does not span V
- ☐ S is linearly dependent
- ☐ S is set of scalars
- ☒ S spans V

Time Taken: 0:05:34

If a normed space X is finite dimensional, then every linear operator on X is _____.

Answer (Please select your correct option)

☐ finite

☐ unbounded

☒ bounded

☐ none of these

Start Time: 9:06 AM

113:00

Time Left



Every complete normed space is _____ space.

Answer (Please select your correct option)

☐ Hilbert

☒ Banach

☐ Unitary

☐ Euclidean

Start Time: 8:06 AM

11:00

Time Left



Question Summary : (Attempted Question ☒)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

[Conduct Instructions](#)

Question No : 27 of 41

For subsets A and B of an inner product space X , A is orthogonal to B if

Answer (Please select your correct option.)

☒ a is orthogonal to b for all $a \in A$ and $b \in B$

☐ a is orthogonal to b for a fixed $a \in A$ and a fixed $b \in B$

☐ a is orthogonal to b for a fixed $a \in A$ and $b \in B$

☐ a is orthogonal to b for all $a \in A$ and a fixed $b \in B$

Question No : 4 of 41

In a metric space (X,d) every convergent sequence is

Answer (Please select your correct option)

☐ Unbounded sequence

☐ Not a Constant sequence

☐ Oscillating sequence

☒ A Cauchy Sequence

Start Time : 8:06 AM

114:00

Question No : 25 of 41

Isomorphism of an inner product space onto an inner product space over the same field is a linear operator that preserves inner product

Answer : Please select your correct option

☒ Bijective

☐ Injective

☐ Surjective

☐ None of these

Start Time : 10:06 AM

11:00

25

Question No.: 11 of 41

Let T be a bounded linear operator. Then $x_n \rightarrow x$ where $x_n, x \in D(T)$ implies _____

Answer: (Please select your correct option)

☐ $x_n \rightarrow x$

☒ $Tx_n \rightarrow Tx$

☐ $Tx \rightarrow Tx_n$

☐ None of these

Start Time: 9:05 AM

112:00



11

MTH641 Functional Analysis

Question No : 28 of 41

P_n is the collection of all

Answer (Please select your correct option)

☐ Polynomials of degree greater then equal to 'n'

☐ Polynomials of degree less then equal to 'n'

☒ Polynomials of degree equal to 'n'

☐ Polynomials of degree less then 'n'

Start Time 11:20 AM

110:00

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MTH641 Functional Analysis

Question No : 29 of 41

The pair $(V, \langle \cdot, \cdot \rangle)$ is called space

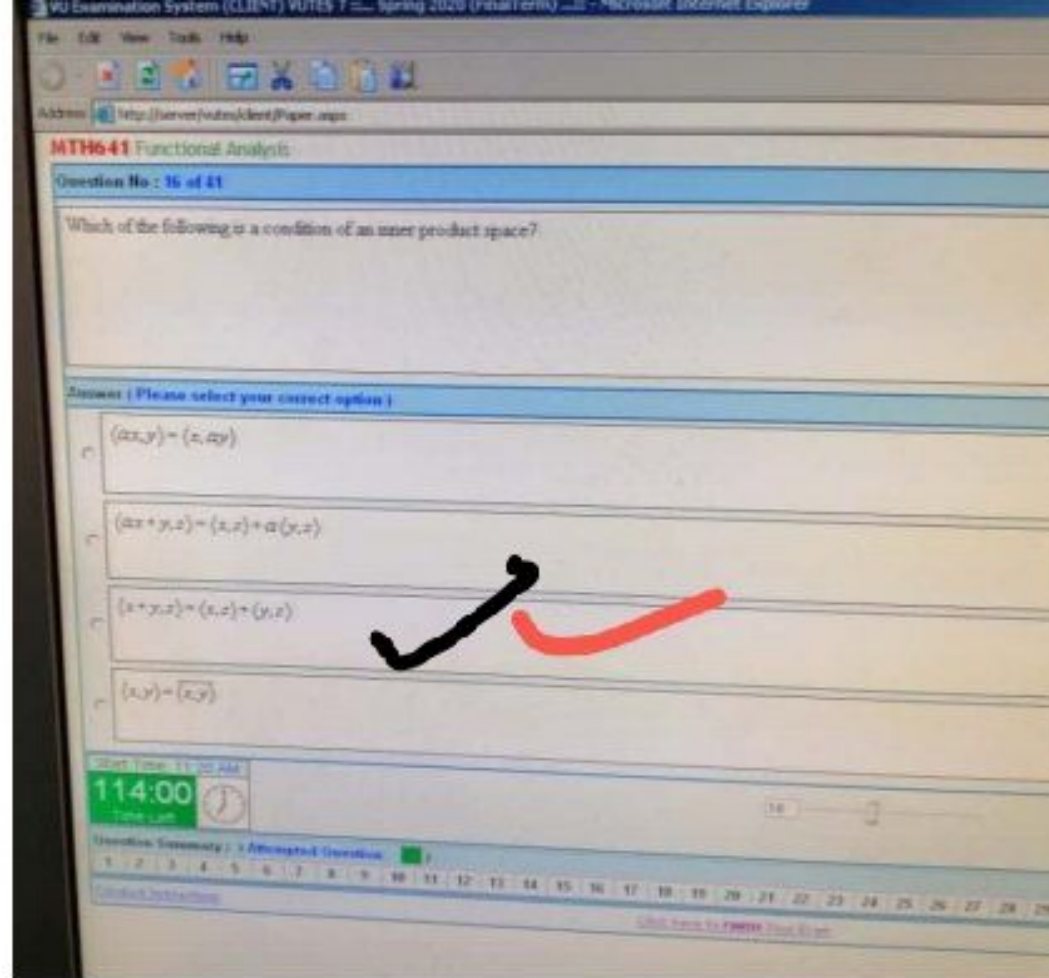
Answer (Please select your correct option)

☐ Norm

☐ Metric

☒ Inner product

☐ Vector



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Question No : 23 of 41

Which of the following is not true for the space $C[a, b]$ with norm defined by $\|x\| = \max_{t \in [a, b]} |x(t)|$?

Answer (Please select your correct option)

☐ It is a complete metric space

☒ It is a complete norm space

☐ It is a complete inner product space

☐ It is a vector space

Start Time: 11:20 AM

11:00

Time Left

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10

A _____ Y of a Banach space X is _____ if and only if the set Y is closed in X .

Answer (Please select your correct option)

☒ subspace , complete

☐ space, complete

☐ subspace, open

☐ space, open

Start Time : 11:20 AM

115:00

Time Left



Question Summary : (Attempted : 1)

Every metric space is a not _____ space.

Answer (Please select your correct option)

☒ a normed

☐ a topological

☐ both of these

☐ None of these

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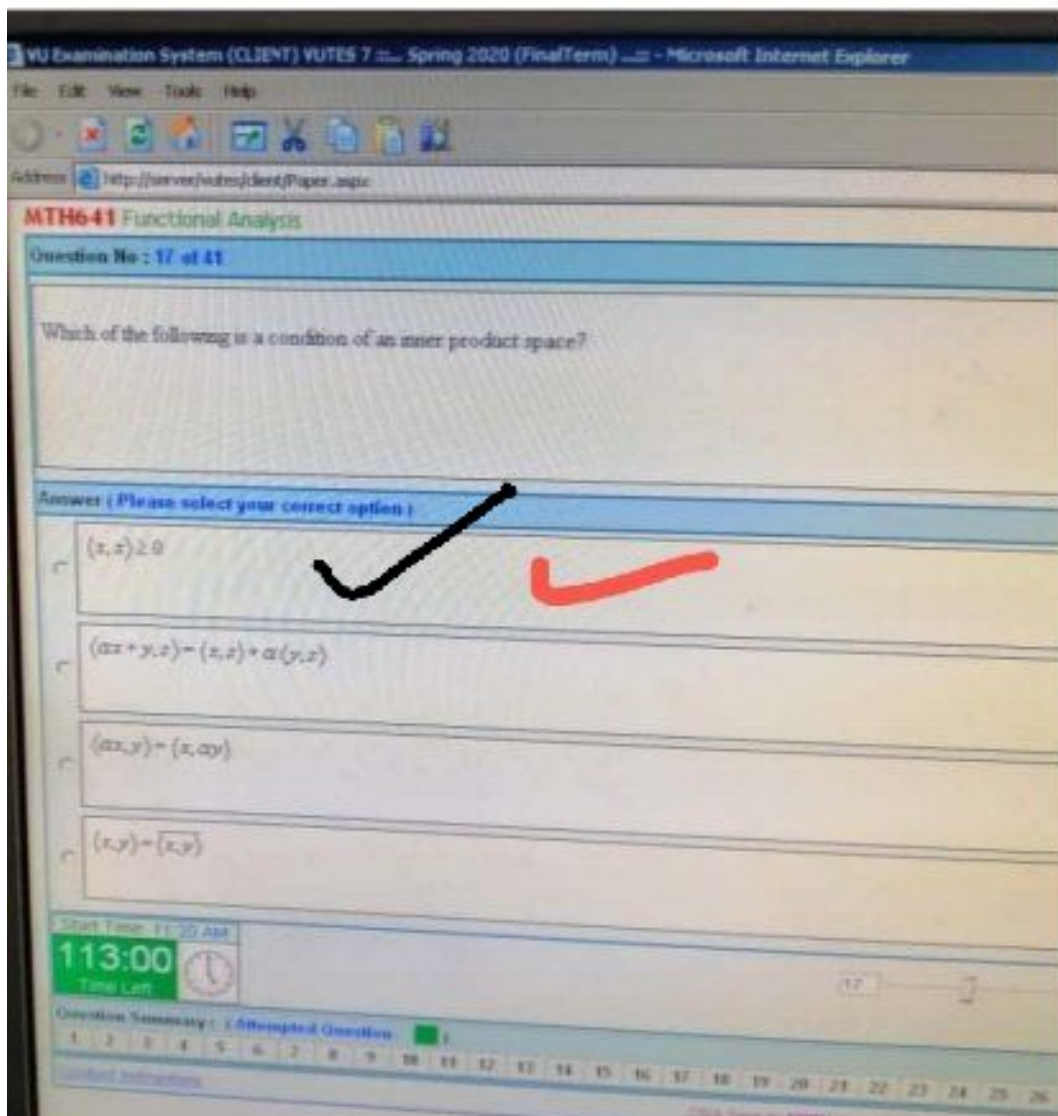
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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10



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MTH641 Functional Analysis

Question No : 24 of 43

The space is not a Hilbert space

Answer (Please select your correct option)

☒ l^p with $p \neq 2$

☐ Euclidean space R^n

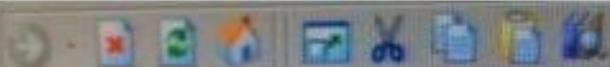
☐ Unitary Space C^n

☐ l^p with $p = 2$

Start Time: 11:00 AM

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MTH641 Functional Analysis

Question No : 30 of 41

Inner product is the generalization of _____

Answer (Please select your correct option)

☐ Cross product

☒ Dot product

☐ Multiplication

☐ Distance function

Start Time: 11:39 AM

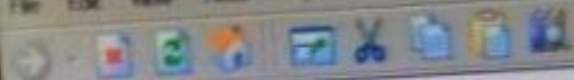
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Question Summary : 1 Attempted Question

1 2 3 4 5 6 7



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MTH641 Functional Analysis

Question No : 18 of 41

Which of the following is true about the inner product space?

Answer (Please select your correct option)

☐ Every normed space is an inner product space.

☒ Every inner product space is a normed space.

☐ Every inner product space is Hilbert space.

☐ None of these is true.

Start Time: 11:20 AM

113:00

Time Left



Question Summary : (Attempted Question 18)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

[Go back to previous question](#)

[Click here to go to next question](#)

MTH641 Functional Analysis

Question No : 3 of 41

Given a point $x_0 \in X$ and a real number $r > 0$, which of the following defines a closed ball?

Answer (Please select your correct option)

☐ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) < r\}$

☐ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) > r\}$

☒ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) \leq r\}$

☐ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) = r\}$

Start Time: 11:20 AM

116:00

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Question Summary : (Attempted Question)

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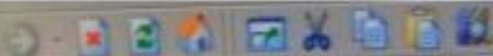
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MTH641 Functional Analysis

Question No : 11 of 11

The space $C[a, b]$ is _____

Answer (Please select your correct option)

- ☐ collection of all functions defined on $[a, b]$.
- ☐ collection of all continuous functions defined on (a, b)
- ☒ collection of all continuous functions defined on $[a, b]$
- ☐ collection of all functions defined on (a, b)

Start Time: 11:20 AM

115:00

Time Left



Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

Go back to question

MTH641 Functional Analysis

Question No: 12 of 41

Let X be an n -dimensional vector space and X^* be its dual space. Then _____

Answer (Please select your correct option)

☒ $\dim X^* = \dim X = n$

☐ $\dim X = n$

☐ $\dim X^* = n$

☐ $\dim X^* = \dim X = m$

Start Time: 11:00 AM

114:00

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Question Summary: 1 Attempted Question

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Go back to previous question

Click here to submit your answer

MTH641 Functional Analysis

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

☐ (a, b)

☐ $[a, b]$

☒ Space $\mathbb{R}^n$

☐ None of these

Start Time: 11:20 AM

111:00

MTH641 Functional Analysis

Question No : 2 of 11

For any three points x, y and z in a metric space (X, d) , the following property is called

$$d(x, z) \leq d(x, y) + d(y, z)$$

Answer (Please select your correct option)

☒ Triangular inequality☐ Rectangle inequality☐ Holder's inequality☐ Minkowski's inequality

Start Time: 11:20 AM

116:00

Time Left

Question Summary : 1 Answered

Address: <http://server/vats/client/Paper.aspx>

MTH641 Functional Analysis

Question No : 7 of 41

A metric space X is said to be compact if every sequence in X has a _____ subsequence.

Answer (Please select your correct option)

☐ Non decreasing

☐ Divergent

☐ Non increasing

☒ Convergent

Start Time: 11:20 AM

115:00 Time Left

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Unattempted Questions

Click here to FORUM Your Query

Address <http://server/votes/client/Paper.aspx>

MTH641 Functional Analysis

Question No : 9 of 41

Let T be a bounded linear operator. Then the _____ is closed.

Answer (Please select your correct option)

☐ Banach space

☐ null space $N(T)$

☒ Hilbert space

☐ None of these

Start Time: 11:20 AM

115:00

Time Left

Question Summary

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If $T: X \rightarrow Y$ is an Isometric Isomorphism between two norm spaces X and Y , then it implies that

Answer (Please select your correct option)

☒ T is bijective and norm preserving

☐ T is surjective and norm preserving

☐ T is injective and norm preserving

☐ T is norm preserving

Start Time: 11:20 AM

114:00

Time Left

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28

[Jump to Question](#)

[Click here to FINISH Your Exam](#)

Let $\{x_1, \dots, x_n\}$ be _____ set of vectors in a normed space X . Then there is a number $c > 0$ such that for every choice of scalars $\alpha_1, \dots, \alpha_n$, we have $\|\alpha_1 x_1 + \dots + \alpha_n x_n\| \geq c(|\alpha_1| + \dots + |\alpha_n|)$

Answer (Please select your correct option.)

☒ linearly independent

☐ linearly dependent

☐ orthogonal

☐ orthonormal

Start Time: 11:20 AM

115:00

Time Left

Question Summary: 1 Attempted Question

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Go back to question

Click here to FINISH Your Exam

MTH641 Functional Analysis

Question No : 21 of 41

Every complete normed space is _____ space.

Answer (Please select your correct option)

☐ Hilbert

☒ Banach

☐ Unitary

☐ Euclidean

Start Time: 11:20 AM

111:00

System (CLIENT) VOTES 7 ... Spring 2020 (FinalTerm) ... - Microsoft Internet Explorer

Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then _____

Answer (Please select your correct option)

☒ $x_0 = 0$

☐ $x_0 \neq 0$

☐ $x_0 = 1$

☐ $x_0 \neq 1$

Start Time: 11:20 AM

114:00

Time Left



Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Conduct Mock Test

[Click here to finish your Exam](#)

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The dual space of a normed space X contains all on X .

Answer (Please select your correct option)

☒ Bounded linear functional

☐ Unbounded linear functional

☐ Bounded linear operator

☐ Unbounded linear operator

Start Time: 11:20 AM

114:00

Time Left



Question Summary : (Attempted Question)

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Conduct subscribe

Every inner product space is a _____ space.

Answer (Please select your correct option)

☐ Metric

☐ Normed

☐ Topological

☒ All of above

Start Time: 11:20 AM

Question No : 1 of 41

The property $d(x, y) \geq 0$ of a metric space is called

Answer (Please select your correct option)

☒ Non negativity

☐ Reflexive property

☐ Symmetric property

☐ Triangle property

Start Time: 11:30 AM

116:00

Time Left



Question Summary : | Attempted Question

Address <http://server/notes/client/Paper.aspx>

MTH641 Functional Analysis

Question No : 4 of 41

Two metric spaces (X, d) and $(\hat{X}, \hat{d})$ are said to be isometric if there exists isometry of X onto $\hat{X}$

Answer (Please select your correct option)

- ☒ An injective
- ☐ A bijective
- ☐ A surjective
- ☐ none of these

Start Time: 11:20 AM

115:00
Time Left

Question Summary : (Attempted Question)

MTH641 Functional Analysis

Question No : 26 of 41

$\|x+y\| \leq \|x\| + \|y\|$ is called.....

Answer (Please select your correct option)

☐ Triangle inequality

☐ Minkowski's inequality

☐ Holder's inequality

☒ Schwarz inequality

Start Time: 11:20 AM

110:00

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Conduct Instructions

For orthogonal elements x, y we have $\langle x, y \rangle = 0$ and the Pythagorean relation is

Answer (Please select your correct option)

☐ $\|x+y\|^2 = \|x\|^2 + \|y\|^2$

☐ $\|x+y\|^2 = \|x\|^2 - \|y\|^2$

☐ $\|x-y\|^2 = \|x\|^2 + \|y\|^2$

☒ $\|x-y\|^2 = \|x\|^2 - \|y\|^2$

Start Time: 11:30 AM

110:00

Time Left

Question Summary : 1 Attempted Question

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Conduct Instructions

A mapping $T: X \rightarrow Y$ is said to be continuous if it is continuous at point of X

Answer : (Please select your correct option)

☒ Every

☐ almost one

☐ exactly one

☐ atleast one

11:53 AM

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Question No : 13 of 41

A finite dimensional normed space is _____ to its second dual.

Answer (Please select your correct answer)

☒ isometric

☐ isomorphic

☐ homeomorphic

☐ none of these

11:53 AM ✓

Question No : 30 of 41

An orthonormal set M is an orthogonal set in an inner product space whose elements have norm

Answer (Please select your correct option)

☒ equal to 1

☐ >1

☐ <1

☐ None of these

Start Time: 6:06 AM

11:53 AM ✓

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

☐ $\{a, b\}$

☐ $[a, b]$

☒ Space $\mathbb{R}^n$

☐ None of these

Start Time: 8:06 AM

110:00

Time Left



Question Summary : (Attempted Question)

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11:53 AM



In an inner product space V , the function $\| \cdot \| : V \rightarrow \mathbb{R}^+$ given by _____ defines a _____ in V

Answer (Please select your correct option)

☐ $\|x\| = \sqrt{\langle x, y \rangle}, x, y \in V$, norm

☐ $\|x\| = \sqrt{\langle x, x \rangle}, x \in V$, norm

☐ $\|x\| = \sqrt{\langle x, x \rangle}, x \in V$, metric

☐ None of these

D

11:53 AM ✓

Start Time: 07:58 AM

A sequence (x_n) in a metric space is said to be non-decreasing if for all n ,

Answer : Please select your correct option :

☐ $x_n \geq x_{n+1}$

☒ $x_n \leq x_{n+1}$

☐ $x_n < x_{n+1}$

☐ $x_n > x_{n+1}$

11:53 AM ✓

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MTH641 Functional Analysis

Question No : 1 of 41

If d is a metric on X , Then for all $x, y \in X$, $d(x, y)$ is....

Answer (Please select your correct option)

☒ A non negative real number

☐ A negative real number

☐ Not a rational number

☐ A complex number

11:53 AM ✓

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Address: <http://server/notes/test/Paper.aspx> Marks: 1 (Budgeted)

MATH641 Functional Analysis

Question No : 7 of 41

If $S = \{v_1, v_2, v_3, \dots, v_n\}$ is a set of vectors in a finite dimensional vector space X , then S is called a basis for V if _____ and S is linearly independent.

Answer (Please select your correct option)

- ☐ S does not span V
- ☐ S is linearly dependent
- ☐ S is set of scalars
- ☒ S spans V

11:53 AM

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Question No : 13 of 41

A finite dimensional normed space is to its second dual.

Answer (Please select your correct option)

☐ isometric

☒ isomorphic

☐ homomorphic

☐ none of these

12:11 PM ✓

Every metric space is a not _____ space.

Answer (Please select your correct option)

☒ a normed

☐ a topological

☐ both of these

☐ None of these

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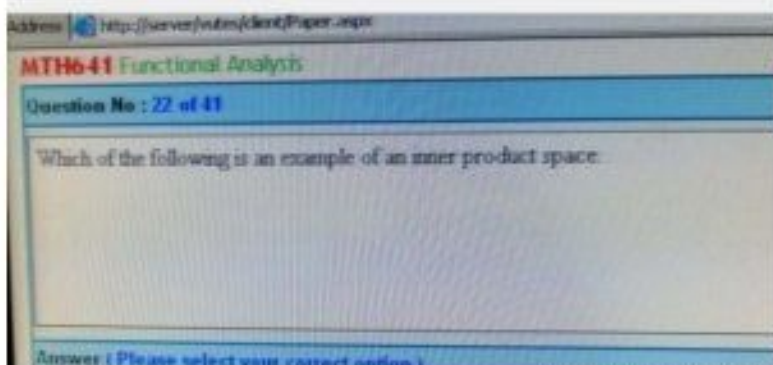
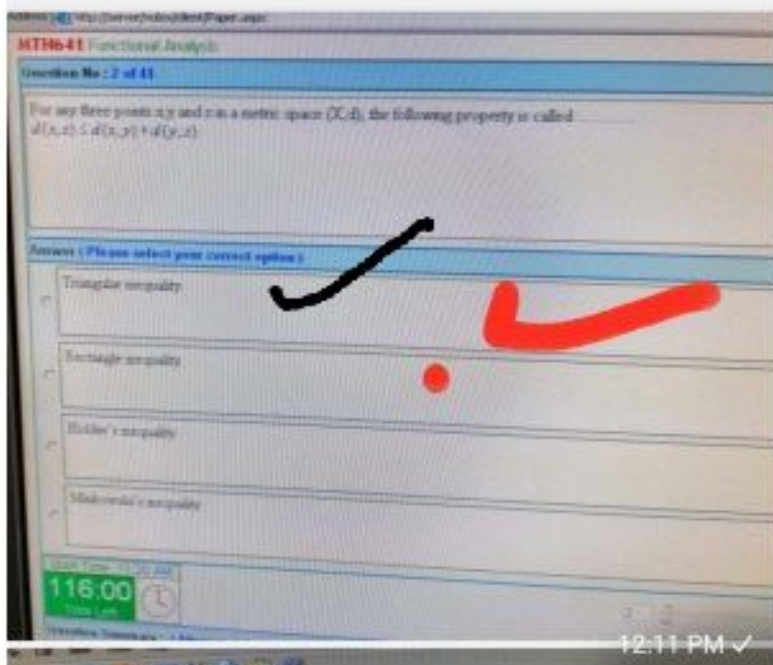
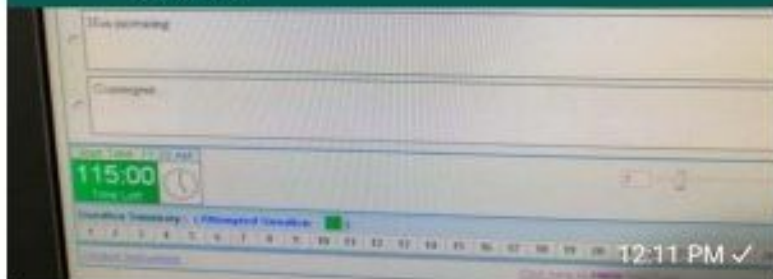
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MTH641 Functional Analysis

Question No : 7 of 41

A metric space X is said to be compact if every sequence in X has a _____ subsequence.

Answer (Please select your correct option)

☐ Non-decreasing

☐ Divergent

☐ Non-increasing

☒ Convergent

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

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MTH641 Functional Analysis

Question No : 6 of 11

Marks : 1 (Body)

Let $\{x_1, \dots, x_n\}$ be _____ set of vectors in a normed space X . Then there is a number $c > 0$ such that for every choice of scalars $\alpha_1, \dots, \alpha_n$, we have $\|\alpha_1 x_1 + \dots + \alpha_n x_n\| \geq c(\|\alpha_1\| + \dots + \|\alpha_n\|)$

Answer (Please select your correct option)

☒ linearly independent☐ linearly dependent☐ orthogonal☐ orthonormal

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115:00

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Unattempted Questions

[Click here to ENROLL your Exam](#)

12:11 PM ✓

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Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then _____

Answer (Please select your correct option)

☒ $x_0 = 0$

☐ $x_0 \neq 0$

☐ $x_0 = 1$

☐ $x_0 \neq 1$

Start Time: 11:20 AM

114:00

Time Left



13

Question Summary: (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

Correct Answer

Click here to view Question

12:11 PM ✓

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Question No : 19 of 41

Every inner product space is a _____ space.

Answer (Please select your correct option)

☒ Metric



☐ Normed

☐ Topological

☐ All of above



Start Time: 11:20 AM

12:11 PM ✓

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MTH641 Functional Analysis

Question No : 26 of 41

$\|x+y\| \leq \|x\| + \|y\|$ is called

Answer (Please select your correct option)

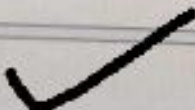
☐ Triangle inequality



☐ Minkowski's inequality

☐ Holder's inequality

☐ Schwarz inequality



Start Time: 11:20 AM

110:00

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Conduct Instructions

12:11 PM ✓

Question No : 7 of 11

If $S = \{v_1, v_2, v_3, \dots, v_n\}$ is a set of vectors in a finite-dimensional vector space V , then S is called a basis for V if _____ and S is linearly independent.

Answer: (Please select your correct option)

☐ S does not span V

☐ S is linearly dependent

☐ S is set of scalars

☒ S spans V



12:11 PM ✓

Question No: 11 of 41

Let T be a bounded linear operator. Then $x_n \rightarrow x$ where $x_n, x \in D(T)$ implies _____

Answer: (Please select your correct option)

☐ $x_n \rightarrow x$

☒ $Tx_n \rightarrow Tx$

☐ $Tx \rightarrow Tx_n$

☐ None of these

Start Time: 10:06 AM

112:00



12:11 PM ✓

Question No : 27 of 41

For subsets A and B of an inner product space X , A is orthogonal to B if

Answer (Please select your correct option)

☒ a is orthogonal to b for all $a \in A$ and $b \in B$

☐ a is orthogonal to b for a fixed $a \in A$ and a fixed $b \in B$

☐ a is orthogonal to b for a fixed $a \in A$ and $b \in B$

☐ a is orthogonal to b for all $a \in A$ and a fixed $b \in B$

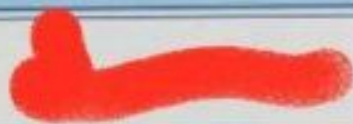
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Question No : 9 of 41

If a normed space X is finite dimensional, then every linear operator on X is _____.

Answer (Please select your correct option)

☐ finite



☐ unbounded



☐ bounded

☐ none of these

Start Time: 8:06 AM

113:00

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Question No : 12 of 41

Marks: 1 (Budgeted Time 00:00)

Two operators T_1 and T_2 are defined to be equal, written $T_1 = T_2$, if _____ and $T_1 x = T_2 x$ for all _____

Answer: (Please select your correct option)

☒ $D(T_1) = D(T_2), \quad x \in D(T_1) = D(T_2)$

☐ $D(T_1) \subset D(T_2), \quad x \in D(T_1) = D(T_2)$

☐ $D(T_1) \supset D(T_2), \quad x \in D(T_1) = D(T_2)$

☐ $D(T_1) \supset D(T_2), \quad x \in D(T_1) = D(T_2)$

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112:00

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Any pair of vectors in an orthogonal system is _____ to each other.

Answer (Please select your correct option)

☐ Parallel

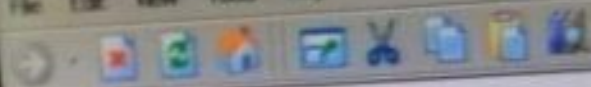
☐ Equal

☐ None of these

☒ Perpendicular



12:11 PM ✓

Address <http://server/votes/client/Paper.aspx>**MTH641** Functional Analysis

Question No : 18 of 41

Which of the following is true about the inner product space?

Answer (Please select your correct option)

☐ Every normed space is an inner product space.☒ Every inner product space is a normed space. ✓☐ Every inner product space is Hilbert space.☐ None of these is true.

Start Time: 11:20 AM

113:00

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

[Conduct Instructions](#)[Click here](#)

12:11 PM ✓

MTH641 Functional Analysis

Question No : 3 of 41

Given a point $x_0 \in X$ and a real number $r > 0$, which of the following defines a closed ball?

Answer (Please select your correct option)

☐ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) < r\}$

☐ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) > r\}$

☒ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) \leq r\}$

☐ $\tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) = r\}$

Start Time: 11:20 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10

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Address <http://server/vutes/client/Paper.aspx>**MTH641** Functional Analysis

Question No : 35 of 41

Prove that if a set M is closed in X then it forms a complete subspace of X

Answer ([Please click here to Add Answer](#))

Address <http://server/vites/client/Paper.aspx>

MTH641 Functional Analysis

Question No : 28 of 41

P_n is the collection of all.....

Answer (Please select your correct option)

☐ Polynomials of degree greater then equal to 'n'

☐ Polynomials of degree less then equal to 'n'

☒ Polynomials of degree equal to 'n'

☐ Polynomials of degree less then 'n'

Start Time: 11:20 AM

110:00
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Question Summary : 1 Attempted

MTH641 Functional Analysis

Question No : 29 of 41

The pair $(V, \langle \cdot, \cdot \rangle)$ is called space

Answer (Please select your correct option)

☐ Norm

☐ Metric

☒ Inner product

☐ Vector

Start Time: 11:20 AM

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Address <http://server/vutes/client/Paper.aspx>**MTH641 Functional Analysis****Question No : 16 of 41**

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

☐ $\langle \alpha x, y \rangle = \langle x, \alpha y \rangle$

☐ $\langle \alpha x + y, z \rangle = \langle x, z \rangle + \alpha \langle y, z \rangle$

☒ $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$

☐ $\langle x, y \rangle = \overline{\langle x, y \rangle}$

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16

Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 23 of 41

Which of the following is not true for the space $C[a, b]$ with norm defined by $\|x\| = \max_{t \in [a, b]} |x(t)|$?

Answer (Please select your correct option)

☐ It is a complete metric space☐ It is a complete norm space☐ It is a complete inner product space☐ It is a vector space

Start Time: 11:20 AM

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Question Summary : (Attempted Question)

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Address <http://server/vutes/client/Paper.aspx>

MTH641 Functional Analysis

Question No : 5 of 41

A _____ Y of a Banach space X is _____ if and only if the set Y is closed in X .

Answer (Please select your correct option)

☒ subspace , complete

☐ space, complete

☐ subspace, open

☐ space, open

Start Time: 11:20 AM

115:00

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Question Summary : (Attempted Questions)

MTH641 Functional Analysis

Question No : 20 of 41

Every metric space is a not _____ space.

Answer (Please select your correct option)

☒ a normed

☐ a topological

☐ both of these

☐ None of these

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Question Summary : (Attempted Question)

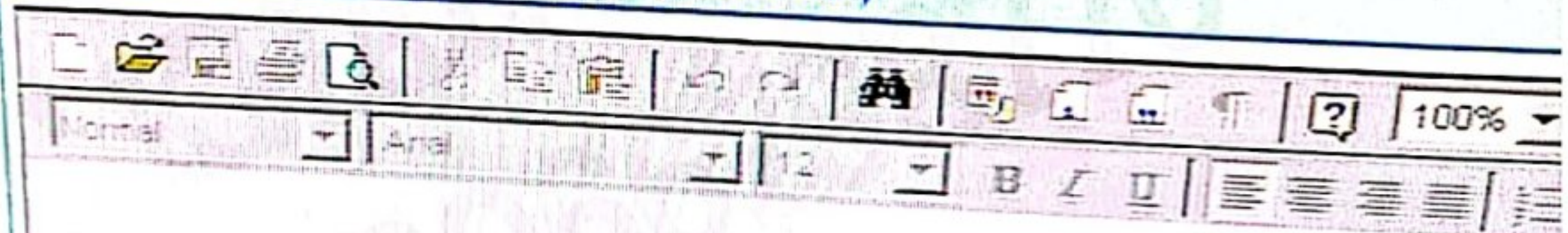
1 2 3 4 5 6 7 8 9 10

Address  http://server/vutes/client/Paper.aspx**MTH641** Functional Analysis

Question No : 39 of 41

Show that canonical mapping is linear.

Answer (Please [click here](#) to Add Answer)



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Address <http://server/vutes/client/Paper.aspx>**MTH641 Functional Analysis**

Question No : 17 of 41

Which of the following is a condition of an inner product space?

Answer (Please select your correct option)

☐ $\langle x, x \rangle \geq 0$

☐ $\langle \alpha x + y, z \rangle = \langle x, z \rangle + \alpha \langle y, z \rangle$

☐ $\langle \alpha x, y \rangle = \langle x, \alpha y \rangle$

☐ $\langle x, y \rangle = \overline{\langle x, y \rangle}$

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Question Summary : (Attempted Question)

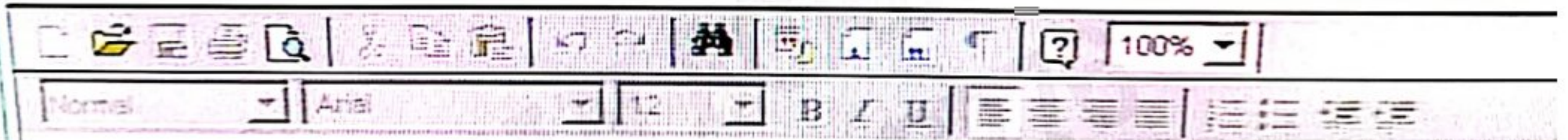
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Question No : 37 of 41

Define norm and inner product for which the space $\mathbb{R}^n$ is an inner product space.

Answer ([Please click here to Add Answer](#))



Address <http://server/vutes/client/Paper.aspx>**MTH641** Functional Analysis

Question No : 24 of 41

The space is not a Hilbert space

Answer (Please select your correct option)

☒ l^p with $p \neq 2$ ☐ Euclidean space R^n ☐ Unitary Space C^n ☐ l^p with $p = 2$

Start Time: 11:20 AM

111:00



Address <http://server/vutes/client/Paper.aspx>

MTH641 Functional Analysis

Question No : 30 of 41

Inner product is the generalization of

Answer (Please select your correct option)

☐ Cross product

☒ Dot product

☐ Multiplication

☐ Distance function

Start Time: 11:20 AM

109:00
Time Left

Question Summary : (Attempted Question)

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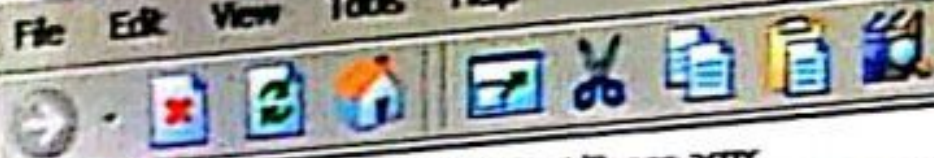
MTH641 Functional Analysis

Question No : 32 of 41

Define linear functional.

Answer ([Please click here to Add Answer](#))



**MTH641** Functional Analysis

Question No : 18 of 41

Which of the following is true about the inner product space?

Answer (Please select your correct option)

☐ Every normed space is an inner product space.

☒ Every inner product space is a normed space.

☐ Every inner product space is Hilbert space.

☐ None of these is true.

Start Time 11:20 AM

113:00

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18

Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

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**MTH641** Functional Analysis

Question No : 10 of 41

A linear functional f with domain $D(f)$ in a normed space is continuous if and only if f is _____.

Answer (Please select your correct option)

☐ continuous☒ bounded☐ linear☐ non linear

Start Time 11:20 AM

115:00

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10

Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
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MTH641 Functional Analysis

Question No : 3 of 41

Given a point $x_0 \in X$ and a real number $r > 0$, which of the following defines a closed ball?

Answer (Please select your correct option)

☐ $\bar{B}(x_0; r) = \{x \in X \mid d(x, x_0) < r\}$

☐ $\bar{B}(x_0; r) = \{x \in X \mid d(x, x_0) > r\}$

☒ $\bar{B}(x_0; r) = \{x \in X \mid d(x, x_0) \leq r\}$

☐ $\bar{B}(x_0; r) = \{x \in X \mid d(x, x_0) = r\}$



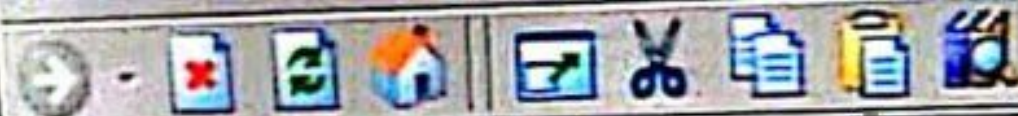
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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 11 of 41

The space $C[a, b]$ is _____.

Answer (Please select your correct option)

☐ collection of all functions defined on $[a, b]$.

☐ collection of all continuous functions defined on (a, b)

☒ collection of all continuous functions defined on $[a, b]$

☐ collection of all functions defined on (a, b)



Start Time 11:20 AM

115:00

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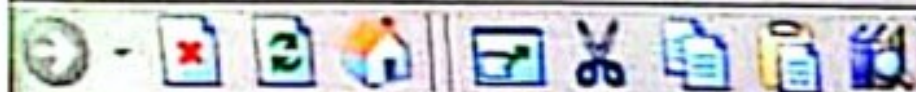


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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

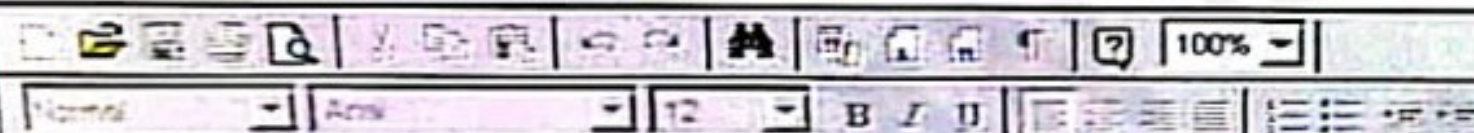
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Question No : 41 of 41

Marks: 5 (Budgeted Time Min)

Let H_1 and H_2 be Hilbert spaces. $S: H_1 \rightarrow H_2$ and $T: H_1 \rightarrow H_2$ be bounded linear operator and α any scalar. Then show that $(ST)^* = T^*S^*$ (assuming $H_2 = H_1$).

Answer ([Please click here to Add Answer](#))



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Question Summary : (Attempted Question)

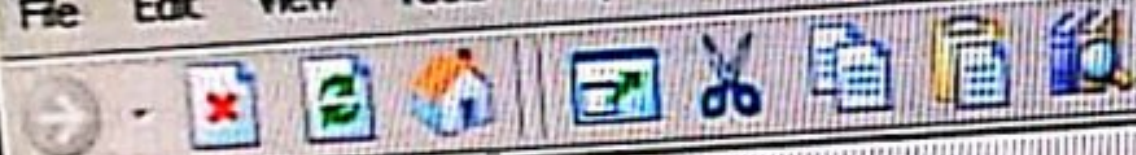
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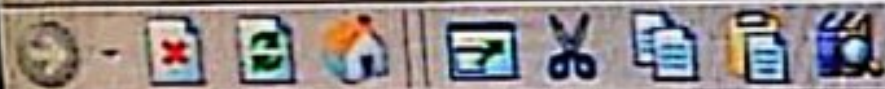
MTH641 Functional Analysis

Question No : 34 of 41

Define unitary operator.

Answer ([Please click here to Add Answer](#))





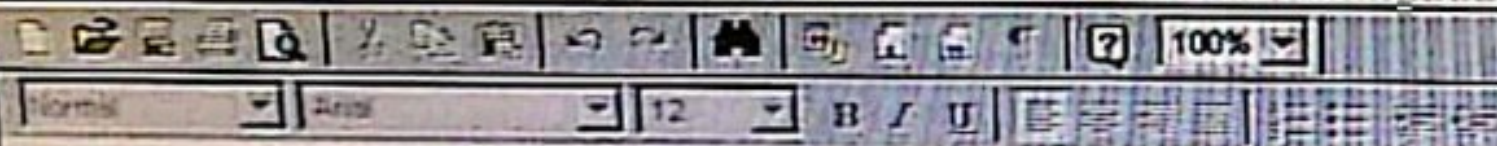
MTH641 Functional Analysis

Question No : 36 of 41

Marks: 3 (Budgeted)

Let $T: D(T) \rightarrow Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then show that if T is continuous at a single point, it is continuous.

Answer (Please click here to Add Answer)



MTH641 Functional Analysis

Question No : 12 of 41

Let X be an n -dimensional vector space and X^* be its dual space. Then _____.

Answer (Please select your correct option)

☒ $\dim X^* = \dim X = n$

☐ $\dim X = n$

☐ $\dim X^* = n$

☐ $\dim X^* = \dim X = m$

Start Time: 11:20 AM

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Question Summary : (Attempted Question)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
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MTH641 Functional Analysis

Question No : 22 of 41

Which of the following is an example of an inner product space.

Answer (Please select your correct option)

☐ $\{a, b\}$

☐ $[a, b]$

☒ Space R^n

☐ None of these

Start Time: 11:20 AM

111:00

Time Left



Address <http://server/vutes/client/Paper.aspx>**MTH641** Functional Analysis

Question No : 2 of 41

For any three points x, y and z in a metric space (X, d) , the following property is called

$$d(x, z) \leq d(x, y) + d(y, z)$$

Answer (Please select your correct option)

☒ Triangular inequality☐ Rectangle inequality☐ Holder's inequality☐ Minkowski's inequality

Start Time: 11:20 AM

116:00

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Question Summary : (Answered)

MTH641 Functional Analysis

Question No : 7 of 41

A metric space X is said to be compact if every sequence in X has a _____ subsequence.

Answer (Please select your correct option)

☐ Non decreasing

☐ Divergent

☐ Non increasing

☒ Convergent

Start Time: 11:20 AM

115:00

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27

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MTH641 Functional Analysis

Question No : 9 of 41

Let T be a bounded linear operator. Then the _____ is closed.

Answer (Please select your correct option)

☐ Banach space

☐ null space $N(T)$

☒ Hilbert space

☐ None of these

Start Time: 11:20 AM

115:00

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Question Summary : (Attempted Question)

1 2 3 4 5

Address <http://server/vutes/client/Paper.aspx>**MTH641** Functional Analysis

Question No : 14 of 41

IFT: XY is an Isometric Isomorphism between two norm spaces X and Y , then it implies that.....

Answer (Please select your correct option)

☒ T is bijective and norm preserving☐ T is injective and norm preserving☐ T is surjective and norm preserving☐ T is norm preserving

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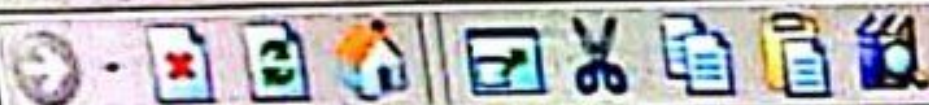
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Question Summary : (Attempted Question ☒)

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Address <http://server/vutes/client/Paper.aspx>**MTH641 Functional Analysis**

Question No : 6 of 41

Marks: 1 (Budgeted)

Let $\{x_1, \dots, x_n\}$ be set of vectors in a normed space X . Then there is a number $c > 0$ such that for every choice of scalars $\alpha_1, \dots, \alpha_n$, we have

$$|\alpha_1 x_1 + \dots + \alpha_n x_n| \geq c(|\alpha_1| + \dots + |\alpha_n|)$$

Answer (Please select your correct option)

☒ linearly independent☐ linearly dependent☐ orthogonal☐ orthonormal

Start Time: 11:20 AM

115:00
Time Left

6



Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

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MTH641 Functional Analysis

Question No : 21 of 41

Every complete normed space is _____ space.

Answer (Please select your correct option)

☒ Hilbert

☐ Banach

☐ Unitary

☐ Euclidean

Start Time: 11:20 AM

111:00



**MTH641** Functional Analysis

Question No : 13 of 41

Marks:

Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then _____

Answer (Please select your correct option)

☒ $x_0 = 0$

☐ $x_0 = 0$

☐ $x_0 = 1$

☐ $x_0 = 1$

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 15 of 41

The dual space of a normed space X contains all on X .

Answer (Please select your correct option)

☒ Bounded linear functional

☐ Unbounded linear functional

☐ Bounded linear operator

☐ Unbounded linear operator

Start Time: 11:20 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28

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MTH641 Functional Analysis

Question No : 19 of 41

Every inner product space is a _____ space.

Answer (Please select your correct option)

☐ Metric

☐ Normed

☐ Topological

☒ All of above

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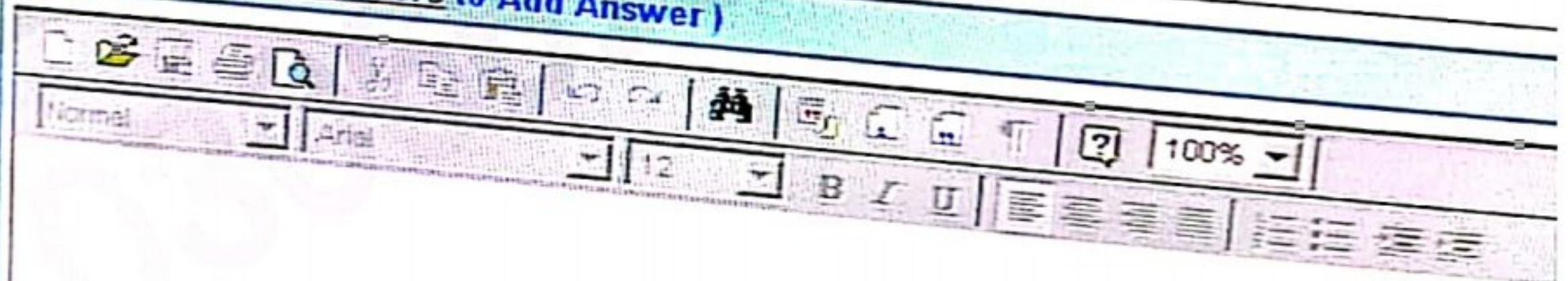
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MTH641 Functional Analysis

Question No : 33 of 41

Write generalized form of orthogonal system.

Answer ([Please click here to Add Answer](#))





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MTH641 Functional Analysis

Question No : 1 of 41

The property $d(x, y) \geq 0$ of a metric space is called.....

Answer (Please select your correct option)

☒ Non negativity

☐ Reflexive property

☐ Symmetric property

☐ Triangle property

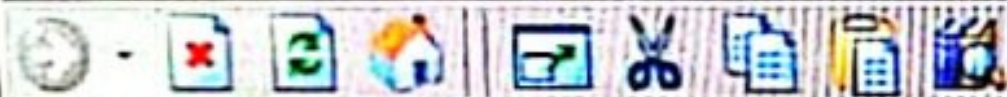
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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 31 of 41

Show that the sequence $(x_n) = (1 + \frac{1}{n})$ is a Cauchy sequence with the usual metric defined on the set of real numbers.

Answer ([Please click here to Add Answer](#))



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T and X, Y are normed spaces, then show that if T is continuous at a single point, then

100%



MTH641 Functional Analysis

Marks: 1 (Budge

Question No : 25 of 41

Isomorphism of an inner product space onto an inner product space over the same field is a linear operator that preserves inner product.

Answer (Please select your correct option)

☒ Bijective

☐ Injective

☐ Surjective

☐ None of these

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Question Summary : (Attempted Question)

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MTH641 Functional Analysis

Question No : 4 of 41

Two metric spaces (X, d) and $(\bar{X}, \bar{d})$ are said to be isometric if there exists isometry of X onto $\bar{X}$

Answer (Please select your correct option)

☐ An injective

☒ A bijective

☐ A surjective

☐ none of these

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Question Summary : (Attempted Question)



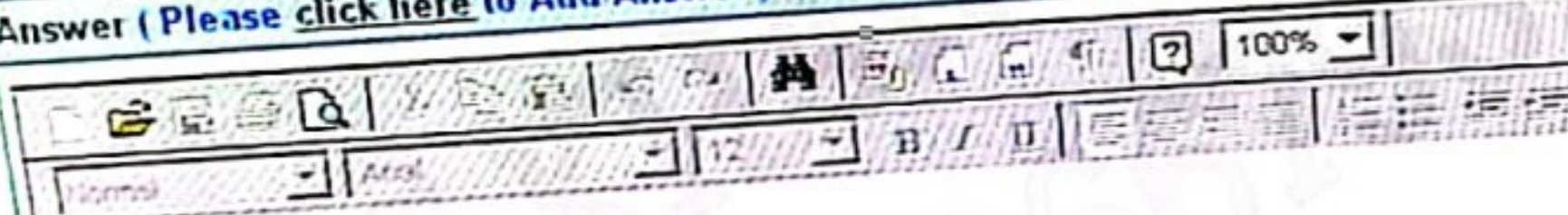
MTH641 Functional Analysis

Question No : 38 of 41

Let H_1 and H_2 be Hilbert spaces, $S: H_1 \rightarrow H_2$ and $T: H_1 \rightarrow H_2$ be bounded linear operator and α any scalar. Then show that

$$\langle T^* y, x \rangle = \langle y, Tx \rangle \quad (x \in H_1, y \in H_2).$$

Answer ([Please click here to Add Answer](#))



MTH641 Functional Analysis

Marks: 1 (Budgeted T

Question No : 8 of 41

Let $T : D(T) \rightarrow Y$ be a linear operator, where $D(T) \subseteq X$ and X, Y are normed spaces, then T is _____ if and only if T is bounded

Answer (Please select your correct option)

☐ continuous

☐ linear

☒ bounded

☐ none of these

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

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MTH641 Functional Analysis

Question No : 40 of 41

Show that if $\{x_n\}, \{y_n\}$ are Cauchy sequences in V , then $\{x_n, y_n\}$ is a Cauchy sequence in F .

Answer ([Please click here to Add Answer](#))



Address <http://server/vutes/client/Paper.aspx>

MTH641 Functional Analysis

Question No : 26 of 41

$\|x+y\| \leq \|x\| + \|y\|$ is called.....

Answer (Please select your correct option)

☐ Triangle inequality

☐ Minkowski's inequality

☐ Holder's inequality

☒ Schwarz inequality

Start Time: 11:20 AM

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Question Summary : (Attempted Question)

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Conduct Instructions

26

MTH641 Functional Analysis**Question No : 27 of 41**

For orthogonal elements x, y we have $\langle x, y \rangle = 0$ and the Pythagorean relation is

Answer (Please select your correct option)

☐ $\|x+y\|^2 = 2\|x\|^2 + 2\|y\|^2$

☐ $\|x+y\|^2 = \|x\|^2 - \|y\|^2$

☐ $\|x-y\|^2 = \|x\|^2 + \|y\|^2$

☒ $\|x+y\|^2 = \|x\|^2 + \|y\|^2$

Start Time: 11:20 AM

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Question Summary : (Attempted Question)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

[Conduct Instructions](#)

27

Bounded Linear Operator:

Let X and Y be normed spaces and $T : D(T) \rightarrow Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$$\|Tx\| \leq c\|x\|$$

Let T be a bounded linear operator as defined before then an alternate formula for the norm of T is

$$\|T\| = \sup_{\substack{x \in D(T) \\ \|x\|=1}} \|Tx\|$$

The norm defined on T satisfies (N1) to (N4).

Examples of Bounded Linear Operators

Identity Operator

Zero Operator

Differentiation Operator

Integral Operator

Matrix

Linear Function:

Algebraic Dual Space

Second Algebraic Dual Space

Canonical Mapping

Algebraic Dual Space

Set of all linear function defined on a vector space X is itself a vector space and called Algebraic Dual Space and denoted by X^*

Conical Mapping:

$C : X \rightarrow X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

Linear Function (Examples):

Space $C[a, b]$

Space l^2

Linear Operators and functional on finite Dimensional Spaces:

Finite dimensions mean basis which have finite many elements.

Let X and Y be finite dimension vector spaces over the same field.

Let $T : X \rightarrow Y$ be a linear operator. let $E = \{e_1, \dots, e_n\}$ be the basis for X and

$B = \{b_1, \dots, b_n\}$ be the basis for Y .

$x \in X$, $x = \xi_1 e_1 + \xi_2 e_2 + \dots + \xi_n e_n$

$$y = Tx = T\left(\sum_{k=1}^n \xi_k e_k\right) = \sum_{k=1}^n T(\xi_k e_k) = \sum_{k=1}^n \xi_k T(e_k)$$

T is uniquely determinal if the image $y_k = Te_k$ of n basis vectors $e_1, \dots, e_n$ are prescribed.

$$y = Tx ; y \in Y \quad \{b_1, \dots, b_n\}$$

$$y = \eta_1 b_1 + \eta_2 b_2 + \dots + \eta_r b_r$$

$$Te_k \in Y, \quad Te_1 = \tau_{11} b_1 + \tau_{12} b_2 + \dots + \tau_{1r} b_r$$

$$Te_k = \sum_{j=1}^r \tau_{kj} b_j$$

$$y = \sum_{j=1}^r \eta_j b_j = \sum_{k=1}^n \xi_k Te_k = \sum_{k=1}^n \xi_k \sum_{j=1}^r \tau_{kj} b_j$$

Comibinig these two summation

$$y = \sum_{j=1}^r \left(\sum_{k=1}^n \tau_{kj} \xi_k \right) b_j$$

$$\eta_j = \sum_{k=1}^n \tau_{kj} \xi_k$$

The image $y = Tx = \sum \eta_j b_j$ of $x = \sum \xi_k Te_k$ can be obtained from

$$\eta_j = \sum_{k=1}^n \tau_{kj} \xi_k$$

Let X be an n-dimensional vector space and X^* be its dual space. Then

$$\dim X^* = \dim X = n.$$

X^* is collection of linear functions or linear operator while X may be any space.

Proof:

Let $\dim X = n$.

Let basis of X be $B = \{e_1, \dots, e_n\}$

We define a function.

$$f_j(e_i) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j : i, j = 1, \dots, n \end{cases}$$

$$\text{e.g. } j=1, \quad f(e_1)=1, f(e_2)=0, f(e_3)=0, \dots, f(e_n)=0$$

$$j=2, \quad f(e_1)=0, f(e_2)=1, f(e_3)=0, \dots, f(e_n)=0$$

but each n-tuples f_j in this case can be extended as linear functions on X.

Let X be a finite deimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$

for all $f \in X^*$ then $x_0 = 0$.

B^* is the basis of X^*

$$\{f_1, f_2, \dots, f_n\}$$

$$\Rightarrow f_j(e_i) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \\ = \delta_{ij}$$

Proof:

For all $x_0 = 0$,

$$\begin{aligned}
x_0 &= \sum_{i=1}^n x_i e_i \quad ; \quad f \in X^* \quad , \\
f(x_0) &= 0 \quad \Rightarrow \quad \sum_{i=1}^n f\left(\sum_{i=1}^n x_i e_i\right) = 0 \\
\Rightarrow \quad \sum_{i=1}^n x_i f(e_i) &= 0 \quad , \quad j=1, \dots, n \\
\Rightarrow \quad x_j &= 0 \quad , \quad \forall j=1, \dots, n \\
x_0 &= \sum_{i=1}^n x_i e_i = 0 \quad \Rightarrow \quad x_0 = \delta
\end{aligned}$$

Theroem

Let $B(X, Y)$ be the set of all bounded linear operators from a normed space X to a normed space Y . If Y is a Banach space, then $B(X, Y)$ is also a Banach.

Proof:

Let $\{T_n\}$ be an arbitrary Cauchy seq. in $B(X, Y)$.

We will show that $\{T_n\}$ converges to an operator T in $B(X, Y)$. Since $\{T_n\}$ is Cauchy for every

$$\varepsilon > 0 \quad \exists \quad N \text{ such that } \|T_n - T_m\| < \varepsilon \quad (m, n > N)$$

For all $x \in X$ and $(m, n > N)$ we have

$$\begin{aligned}
\|T_n(x) - T_m(x)\| &= \|(T_n - T_m)(x)\| \\
&\leq \|T_n - T_m\| \|x\| < \varepsilon \|x\|
\end{aligned}$$

Thus for a fixed x and given $\bar{\varepsilon}$

Hence $B(X, Y)$ is complete and Banach space.

Inner product Space

Let V be a vector space over a field F where F is $\mathbb{R}$ or $\mathbb{C}$.

An inner product in V is a function $\langle \bullet, \bullet \rangle: V \times V \rightarrow F$ satisfying the following conditions:

Quiz:

Let $x, y, z \in V$; $\alpha \in F$ where α may be real or complex.

- i. $\langle x, x \rangle \geq 0$; $\langle x, x \rangle = 0 \Leftrightarrow x = 0$
- ii. $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$; but not true for second value as $\langle x, \alpha y \rangle \neq \alpha \langle x, y \rangle$
- iii. $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$
- iv. $\langle x, y \rangle = \overline{\langle y, x \rangle}$

$\langle \bullet, \bullet \rangle: V \times V \rightarrow F$ inner product.

Inner Product Space

The pair $(V, \langle \bullet, \bullet \rangle)$ is called an inner product space.

a): $\langle ax + by, z \rangle$ where $x, y, z \in V$, $a, b \in F$

Using (iii) property $\langle ax + by, z \rangle = \langle ax, z \rangle + \langle by, z \rangle$

Using (ii) property $a \langle x, z \rangle + b \langle y, z \rangle$

$$\langle 0, z \rangle = \langle 0, x \rangle = 0 \quad \langle x, z \rangle = 0$$

b): **Quiz:**

for all $x, y \in V$, $a \in F$

$$\begin{aligned} \langle x, ay \rangle &= \overline{\langle ay, x \rangle} = \overline{a \langle y, x \rangle} \\ &= \overline{a} \overline{\langle y, x \rangle} = \overline{a} \langle x, y \rangle \end{aligned}$$

Theorem:

For any two elements x, y in an inner product space V ,

$$|\langle x, y \rangle| \leq \|x\| \cdot \|y\|, \text{ the define norm is } \|x\| = \sqrt{\langle x, x \rangle}, \quad x, y \in V$$

Proof:

If $x=y=0$ then $0=0$

Let at least one of x and y is not equal to zero

Let $|\langle x + \lambda y, x + \lambda y \rangle| \geq 0$ by definition

$$\langle x, x + \lambda y \rangle + \langle \lambda y, x + \lambda y \rangle$$

$$\langle x, x + \lambda y \rangle + \lambda \langle y, x + \lambda y \rangle$$

Theorem:

In an inner product space V , the function $\|\cdot\|: V \rightarrow \mathbb{R}^+$ given by

$$\|x\| = \sqrt{\langle x, x \rangle}, \quad x \in V \text{ defines a norm in } V.$$

Proof:

$$N1: \quad \|x\| \geq 0$$

$$\text{For a } x \in V, \quad \|x\| = \sqrt{\langle x, x \rangle} \geq 0 \quad \text{as } \langle x, x \rangle \geq 0$$

N2:

$$\|x\| = 0 \Leftrightarrow \sqrt{\langle x, x \rangle} = 0 \Leftrightarrow \langle x, x \rangle = 0 \Leftrightarrow x = 0$$

$$N3: \quad \|\alpha x\| = |\alpha| \|x\|$$

$$\text{now } \|\alpha x\| = \sqrt{\langle \alpha x, \alpha x \rangle} \Rightarrow \|\alpha x\|^2 = \langle \alpha x, \alpha x \rangle$$

$$\Rightarrow \|\alpha x\|^2 = \alpha \overline{\alpha} \langle x, x \rangle = |\alpha|^2 \|x\|^2$$

$$N4: \quad \|x + y\| \leq \|x\| + \|y\| \quad \forall x, y \in V$$

$$\begin{aligned}
\|x+y\|^2 &= \langle x+y, x+y \rangle \\
&= \langle x, x+y \rangle + \langle y, x+y \rangle \\
&= \overline{\langle x+y, x \rangle} + \overline{\langle x+y, y \rangle} \\
&= \overline{\langle x, x \rangle + \langle y, x \rangle} + \overline{\langle x, y \rangle + \langle y, y \rangle} \\
&= \overline{\langle x, x \rangle} + \overline{\langle y, x \rangle} + \overline{\langle x, y \rangle} + \overline{\langle y, y \rangle}
\end{aligned}$$

Now

$$\begin{aligned}
&= \langle x, x \rangle + \langle x, y \rangle + \overline{\langle x, y \rangle} + \langle y, y \rangle \\
&= \|x\|^2 + 2 \operatorname{Re} \langle x, y \rangle + \|y\|^2 \quad \because \operatorname{Re}(z) \leq |z| \\
&\leq \|x\|^2 + 2|\langle x, y \rangle| + \|y\|^2 \\
&\leq \|x\|^2 + 2\|x\|\|y\| + \|y\|^2 \quad \because |\langle x, y \rangle| \leq \|x\|\|y\| \\
&= (\|x\| + \|y\|)^2 \\
\|x+y\|^2 &\leq (\|x\| + \|y\|)^2
\end{aligned}$$

Theorem:

$$\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2) \quad \text{for all } x, y \in V$$

Proof:

$$\begin{aligned}
\|x+y\|^2 &= \langle x+y, x+y \rangle \\
&= \langle x, x \rangle + \langle x, y \rangle + \overline{\langle x, y \rangle} + \langle y, y \rangle \\
&= \|x\|^2 + 2 \operatorname{Re} \langle x, y \rangle + \|y\|^2 \quad \dots(i)
\end{aligned}$$

Replace $y=-y$

$$\begin{aligned}
\|x-y\|^2 &= \langle x-y, x-y \rangle \\
&= \langle x, x \rangle - \langle x, y \rangle - \overline{\langle x, y \rangle} + \langle y, y \rangle \\
&= \|x\|^2 - 2 \operatorname{Re} \langle x, y \rangle + \|y\|^2 \quad \dots(ii)
\end{aligned}$$

Adding (i) and (ii)

$$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

That we have to prove.

Polarization Identity

For any x, y in complex inner product space

$$\langle x, y \rangle = \frac{1}{4} \left\{ \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 \right\}$$

Appolonius Identity

$$\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z - \frac{1}{2}(x+y)\right\|^2, \quad x, y, z \in V$$

Using parallelogram law

$$\|x'+y'\|^2 + \|x'-y'\|^2 = 2\|x'\|^2 + 2\|y'\|^2 \quad \text{put } x'=z-x, y'=z-y$$

Self-assignment

Theorem:

Let V be any inner product space. For any sequences $\{x_n\}$ and $\{y_n\}$ in V

$$x_n \rightarrow x, \quad y_n \rightarrow y \text{ implies } \langle x_n, y_n \rangle \rightarrow \langle x, y \rangle$$

Proof:

$$\begin{aligned} & |\langle x_n, y_n \rangle - \langle x, y \rangle| \\ &= |\langle x_n, y_n \rangle - \langle x_n, y \rangle + \langle x_n, y \rangle - \langle x, y \rangle| \\ &= |\langle x_n, y_n - y \rangle + \langle x_n - x, y \rangle| \\ &\leq |\langle x_n, y_n - y \rangle| + |\langle x_n - x, y \rangle| \end{aligned}$$

Now from Cauchy Swarzinequality

$$\begin{aligned} |\langle x, y \rangle| &\leq \|x\| \|y\| \\ &\leq \|x_n\| \|y_n - y\| + \|x_n - x\| \|y\| \end{aligned}$$

Given that $x_n \rightarrow x$, $y_n \rightarrow y$ so,

$$\|y_n - y\| = \|y - y\| = 0, \quad \|x_n - x\| = \|x - x\| = 0 \text{ as } n \rightarrow \infty$$

As $n \rightarrow \infty$

$$\begin{aligned} |\langle x_n, y_n \rangle - \langle x, y \rangle| &\leq 0 \\ \langle x_n, y_n \rangle &\rightarrow \langle x, y \rangle \text{ as } n \rightarrow \infty \end{aligned}$$

Theorem:

If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in V , then the inner product $\langle x_n, y_n \rangle$ is a Cauchy sequence in F .

Proof:

$\{x_n\}, \{y_n\}$ are Cauchy sequence

To show $\langle x_n, y_n \rangle$ is also Cauchy Sequence.

$$\begin{aligned} \Rightarrow & \|x_n - x_m\| \rightarrow 0 \quad ; \quad \|y_n - y_m\| \rightarrow 0, \quad m, n \rightarrow \infty \\ & |\langle x_n, y_n \rangle - \langle x_m, y_m \rangle| = |\langle x_n, y_n \rangle - \langle x_n, y_m \rangle + \langle x_n, y_m \rangle - \langle x_m, y_m \rangle| \\ &= |\langle x_n, y_n - y_m \rangle + \langle x_n - x_m, y_m \rangle| \\ &\leq |\langle x_n, y_n - y_m \rangle| + |\langle x_n - x_m, y_m \rangle| \\ &\leq \|x_n\| \|y_n - y_m\| + \|x_n - x_m\| \|y_m\| \\ \Rightarrow & |\langle x_n, y_n \rangle - \langle x_m, y_m \rangle| \rightarrow 0, \text{ as } n, m \rightarrow \infty \\ \Rightarrow & \langle x_n, y_n \rangle \text{ is a Cauchy Sequence} \end{aligned}$$

Theorem:

In an inner product space V and x, y in V if $x \perp y$ then

$$\|x + y\|^2 = \|x\|^2 + \|y\|^2$$

Proof:

$$\begin{aligned}\|x + y\|^2 &= \langle x + y, x + y \rangle \\ &= \langle x, x \rangle + \langle x, y \rangle + \langle y, x \rangle + \langle y, y \rangle\end{aligned}$$

As x and y are perpendicular so $\langle x, y \rangle = 0, \langle y, x \rangle = 0$

$$\|x + y\|^2 = \langle x, x \rangle + \langle y, y \rangle = \|x\|^2 + \|y\|^2$$

Generalized form:

$\{x_1, x_2, \dots, x_n\}$ be nonzero vectors in V inner product space such that

$$\langle x_i, x_j \rangle = 0, \quad i \neq j$$

This system $\{x_1, x_2, \dots, x_n\}$ is called orthogonal system as all vectors inside it are perpendicular to each other.

The generalized statement is $\|x_1 + x_2 + \dots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \dots + \|x_n\|^2$

The idea of proof is

$$\begin{aligned}\left\|\sum_{i=1}^n x_i\right\|^2 &= \left\langle \sum_{i=1}^n x_i, \sum_{i=1}^n x_i \right\rangle \\ &= \langle x_1 + \dots + x_n, x_1 + \dots + x_n \rangle \\ &= \langle x_1, x_1 + \dots + x_n \rangle + \dots + \langle x_n, x_1 + \dots + x_n \rangle \\ &= \sum_{i=1}^n \sum_{j=1}^n \langle x_i, x_j \rangle \\ \langle x_i, x_j \rangle &= \|x_i\|^2, \quad \text{if } i = j \quad \langle x_i, x_j \rangle = 0 \text{ and for } i \neq j \text{ then } \langle x_i, x_j \rangle = 0 \\ \left\|\sum_{i=1}^n x_i\right\|^2 &= \sum_{i=1}^n \|x_i\|^2\end{aligned}$$

Any sequence $\{x_n\}$ of non-zero mutually orthogonal vectors in an inner product space V is linearly independent.

Proof: do it yourself

Let $x = (x_1, x_2, \dots, x_n)$ be the orthogonal sequence.

Remark:

$$\text{If } \langle x, x_i \rangle = 0, \quad \forall i=1, 2, \dots, n \quad \Rightarrow \quad \left\langle \sum_{i=0}^n \alpha_i x_i, x \right\rangle = 0$$

$$\left\langle \sum_{i=0}^n \alpha_i x_i, x \right\rangle = \langle \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n, x \rangle = \alpha_1 \langle x_1, x \rangle + \dots + \alpha_n \langle x_n, x \rangle = 0$$

IMPORTANT SUBJECTIVE FOR FINAL TERM

Q.No.01: Prove that $\langle x, ay + bz \rangle = \bar{a} \langle x, y \rangle + \bar{b} \langle x, z \rangle$

Proof:

$$\begin{aligned}\langle x, ay + bz \rangle &= \overline{\langle ay + bz, x \rangle} \\ \langle x, ay + bz \rangle &= \overline{\langle ay, x \rangle} + \overline{\langle bz, x \rangle} \\ \langle x, ay + bz \rangle &= \bar{a} \overline{\langle y, x \rangle} + \bar{b} \overline{\langle z, x \rangle} \\ \langle x, ay + bz \rangle &= \bar{a} \langle x, y \rangle + \bar{b} \langle x, z \rangle\end{aligned}$$

Hence proved.

Q.No.02: Prove that UV is unitary OR $(UV)^* = (UV)^{-1}$

Proof:

$$(UV)^* = U^*V^*$$

Since we know that

$$U^* = U^{-1} \text{ and } V^* = V^{-1}$$

So

$$\begin{aligned}(UV)^* &= U^{-1}V^{-1} \\ (UV)^* &= (UV)^{-1}\end{aligned}$$

It shows that UV is unitary.

Q.No.03: Give any two examples of Linear Functional.

Solution:

Space $C[a, b]$ and Space l^2

Q.No.04: Prove that $(S + T)^* = S^* + T^*$

Proof:

$$\begin{aligned}\langle x, (S + T)^*y \rangle &= \langle (S + T)x, y \rangle \\ \langle x, (S + T)^*y \rangle &= \langle Sx + Tx, y \rangle \\ \langle x, (S + T)^*y \rangle &= \langle Sx, y \rangle + \langle Tx, y \rangle \\ \langle x, (S + T)^*y \rangle &= \langle x, S^*y \rangle + \langle x, T^*y \rangle \\ \langle x, (S + T)^*y \rangle &= \langle x, (S^*y + T^*y) \rangle \\ \langle x, (S + T)^*y \rangle &= \langle x, (S^* + T^*)y \rangle\end{aligned}$$

Hence we have proved that

$$(S + T)^* = S^* + T^*$$

Q.No.05: Find matrix multiplication if $(\xi_1, \xi_2, \xi_3) \leftrightarrow (\xi_1, \xi_2, -\xi_1 - \xi_2)$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ -\xi_1 - \xi_2 \end{bmatrix}$$

$$\text{Hence required matrix is } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Q.No.06: Show that

$$\langle x_i, x_j \rangle = \|x_i\|^2 \text{ if } i \neq j$$

$$\langle x_i, x_j \rangle = 0 \text{ if } i = j \text{ then } \langle x_i, x_j \rangle = \|x_i\|^2$$

Proof:

The generalized statement is $\|x_1 + x_2 + \dots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \dots + \|x_n\|^2$

$$\begin{aligned}\left\|\sum_{i=1}^n x_i\right\|^2 &= \left\langle \sum_{i=1}^n x_i, \sum_{i=1}^n x_i \right\rangle \\ &= \langle x_1 + \dots + x_n, x_1 + \dots + x_n \rangle \\ &= \langle x_1, x_1 + \dots + x_n \rangle + \dots + \langle x_n, x_1 + \dots + x_n \rangle \\ &= \sum_{i=1}^n \sum_{j=1}^n \langle x_i, x_j \rangle\end{aligned}$$

$\langle x_i, x_j \rangle = \|x_i\|^2$, if $i = j$ $\langle x_i, x_j \rangle = 0$ and for $i \neq j$ then $\langle x_i, x_j \rangle = 0$

$$\left\|\sum_{i=1}^n x_i\right\|^2 = \sum_{i=1}^n \|x_i\|^2$$

Q.No.07: Show that $\overline{N(T)} < N(T)$

Solution:

$$\begin{aligned}x &\in N(T) \\ \exists (x_n) \text{ such that } x_n &\in N(T) + x_n \rightarrow x\end{aligned}$$

Therefore

$$\begin{aligned}Tx_n &\rightarrow Tx \\ x_n &\in N(T) \\ Tx_n &= 0 \\ T_n &= 0 \\ x &\in N(T) \\ \overline{N(T)} &< N(T) \\ \overline{N(T)} &= N(T) \\ N(T) &\text{ is closed.}\end{aligned}$$

Q.No.08: Give an example of function which is not linear.

Solution: $\| \cdot \|: \mathcal{X} \rightarrow \mathbb{R}$ is functional but not linear.

Q.No.09: Find basis of $R(T)$.

Solution:

$$\begin{aligned}(\xi_1, \xi_2, -\xi_1, -\xi_2) &= \xi_1(1, 0, -1) + \xi_2(0, 1, -1) \\ \text{So the basis of } R(T) &= \{(1, 0, -1), (0, 1, -1)\}\end{aligned}$$

Q.No.10: Generalization of Orthogonal

Solution: $\{x_1, x_2, \dots, x_n\}$ be nonzero vectors in V inner product space such that $\langle x_i, x_j \rangle = 0$, $i \neq j$. This system $\{x_1, x_2, \dots, x_n\}$ is called orthogonal system as all vectors inside it are perpendicular to each other. The generalized statement is $\|x_1 + x_2 + \dots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \dots + \|x_n\|^2$

Q.No.11: Show that U is normal.

Solution:

$$UU^* = U^*U$$

Since we know that

$$U^* = U^{-1}$$

So

$$UU^{-1} = U^{-1}U$$

$$(UV)^* = (UV)^{-1} = I \text{ Hence its proved that U is normal.}$$

Q.No.12: Show that U^{-1} is Unitary or Prove that $U^* = U^{-1}$ or $\|U\| = 1$

Solution: Since U is bijective it shows that U^{-1} is bijective $(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}$. Hence it implies that U^{-1} is unitary.

Q.No.14: Show that U is Isometric.

Solution:

$$\|U_x\|^2 = \langle U_x, U_x \rangle$$

$$\|U_x\|^2 = \langle x, U^*U_x \rangle$$

$$\|U_x\|^2 = \langle x, Ix \rangle$$

$$\|U_x\|^2 = \langle x, x \rangle$$

$$\|U_x\|^2 = \|x\|^2 \text{ It proved.}$$

Q.No.15: Prove that $(ST)^* = S^*T^*$

Proof:

$$\langle x, (ST)^*y \rangle = \langle (ST)x, y \rangle$$

$$\langle x, (ST)^*y \rangle = \langle (SxTx), y \rangle$$

$$\langle x, (ST)^*y \rangle = \langle Sx, y \rangle \langle Tx, y \rangle$$

$$\langle x, (ST)^*y \rangle = \langle x, S^*y \rangle \langle x, T^*y \rangle$$

$$\langle x, (ST)^*y \rangle = \langle x, (S^*+T^*)y \rangle$$

$$\langle x, (ST)^*y \rangle = \langle x, (S^*T^*)y \rangle$$

Hence we have proved that

$$(ST)^* = S^*T^*$$

Q.No.16: Show that $T^*T=0$ or $T=0$ is normal.

Solution:

$$T^*T = 0 \text{ iff } T = 0$$

$$e = \|T^*T\| = \|T\|$$

(f) is true by e.

Q.No.17: Show that $(\alpha T)^* = \alpha^{-1}T^*$.

Solution:

$$\begin{aligned}
\langle x, (\alpha T^*)^* y \rangle &= \langle (\alpha T)x, y \rangle \\
\langle x, (\alpha T^*)^* y \rangle &= \langle \alpha(Tx), y \rangle \\
\langle x, (\alpha T^*)^* y \rangle &= \bar{\alpha} \langle Tx, y \rangle \\
\langle x, (\alpha T^*)^* y \rangle &= \bar{\alpha} \langle x, T^* y \rangle \\
\langle x, (\alpha T^*)^* y \rangle &= \langle x, \bar{\alpha} T^* y \rangle \\
(\alpha T^*)^* &= \bar{\alpha} T^*
\end{aligned}$$

Q.No.18: Show that $(T^*)^* = T$

Solution:

$$\begin{aligned}
\langle x, (T^*)^* y \rangle &\geq \langle x T^*, y \rangle \\
\langle x, (T^*)^* y \rangle &\geq \langle x, T y \rangle
\end{aligned}$$

It implies that

$$(T^*)^* = T \text{ so its proved.}$$

Q.No.19: Show that $\|T^*T\| = \|TT^*\| = \|T\|^2$

Solution:

$$\|U_x\|^2 = \langle U_x, U_x \rangle$$

Q.No.20: Show that $\langle T^*y, x \rangle = \langle y, Tx \rangle$

Solution:

$$\begin{aligned}
\langle T^*y, x \rangle &= \langle \overline{y, T^*x} \rangle \\
\langle T^*y, x \rangle &= \langle \overline{Tx}, y \rangle \\
\langle T^*y, x \rangle &= \langle y, Tx \rangle
\end{aligned}$$

Hence its proved.

Q.No.21: Show that $Q = 0$ if and only if $\langle Qx, y \rangle = 0$

Solution:

$$\begin{aligned}
Q = 0 &\Rightarrow Qx = 0 \text{ for all } x \in X \\
\langle Qx, y \rangle &= \langle 0, y \rangle \\
\langle Qx, y \rangle &= \langle 0, w, y \rangle \\
\langle Qx, y \rangle &= 0 \langle w, y \rangle \\
\langle Qx, y \rangle &= 0. \text{ for all } x \in X
\end{aligned}$$

It implies that $Qx=0$ and by definition $Q=0$. Its proved.

Q.No.22: Write examples of Bounded Linear Operators

Solution: Identity Operator, Zero Operator, Differentiation Operator, Integral Operator

Q.No.23: Define Self Adjoint Operator or Hermitian Operator.

Solution:

A bounded Linear Operator $T: H \rightarrow H$ on a Hilbert Space H is said to be Self Adjoint Operator or Hermitian Operator if $T^* = T$.

Q.No.24: Define Self Adjoint Operator or Hermitian Operator.

Solution:

The Hilbert Adjoint Operator T^* of T is defined by $\langle Tx, y \rangle = \langle x, T^*y \rangle$. If T is self adjoint then it becomes $\langle Tx, y \rangle = \langle x, Ty \rangle$. If T is self adjoint or unitary then T is normal.

Q.No.25: Define Unitary Operator.

Solution:

A bounded Linear Operator $T: H \rightarrow H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $T^* = T^{-1}$.

Q.No.26: Define Normal Operator.

Solution:

A bounded Linear Operator $T: H \rightarrow H$ on a Hilbert Space H is said to be Unitary Operator if T is bijective and $TT^* = T^*T$

Q.No.27: Define Extension of T.

Solution: Extension of T on a set $M \supset D(T)$ is defined as $\tilde{T}: M \rightarrow Y$ or

$$\tilde{T}x = Tx \text{ for all } x \in D(T)$$

Q.No.28: Define Polarization Identity.

Solution:

For any x, y in complex inner product space we have

$$\langle x, y \rangle = \frac{1}{4} \left\{ \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 \right\}$$

Q.No.29: Define Bounded Linear Function.

Solution:

Let X and Y be normed spaces and $T: D(T) \rightarrow Y$ a linear operator, where $D(T) \subset X$. The operator T is said to be bounded if there is a real number c such that for all $x \in D(T)$.

$$\|Tx\| \leq c\|x\|$$

Q.No.30: Define Canonical Mapping.

Solution:

$C: X \rightarrow X^{**}$ this mapping is called canonical mapping of X into X^{**} defined as $x \mapsto g_x$.

Q.No.31: Define Orthonormal System.

Solution:

Orthonormal systems are those functions which are real or complex whose inner product with itself results in 1 and with other functions result in 0.

Q.No.32: Define Polarization Identity.

Solution:

For any x, y in complex inner product space

$$\langle x, y \rangle = \frac{1}{4} \left\{ \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 \right\}$$

We have to prove this complex inner product space.

Q.No.33: Define Appolonius Identity:

$$\text{Solution: } \|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z - \frac{1}{2}(x+y)\right\|^2, \quad x, y, z \in V$$

Q.No.34: Define Equal Operators:

Solution: Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have the same domain i.e. $D(T_1) = D(T_2)$

Q.No.35: Write examples of Inner Product Space:

Solution: Identity

Q.No.36: Define Inner Product Space:

Solution: The pair $(V, \langle \cdot, \cdot \rangle)$ is called an inner product space.

Q.No.37: Define Linear Functional:

Solution: A linear functional f is a linear operator with domain in a vector space X and range in the scalar field K of X ; thus $f: D(f) \rightarrow K$. where $K=\mathbb{R}$ if X is real and $K=\mathbb{C}$ if X is complex.

Q.No.38: Define Bounded Linear Extension:

Solution: $T: D(T) \rightarrow Y$ be a bounded linear operator, where $D(T)$ lies in a normal space X and Y is a Banach Space then T has an extension $\tilde{T}: \overline{D(T)} \rightarrow Y$. Where $\tilde{T}$ is a bounded linear operator of norm.

$$\|\tilde{T}\| = \|T\|$$

MTH-641

Final Term Solution

Q#01:-

Define Extension of T .

Extension of T on a Set

$$M \supset D(T)$$

$$\bar{T} : M \rightarrow T$$

$$\bar{T}x = Tx \quad \forall x \in D(T)$$

Q#02:-

Prove That

$$\langle x, ay + bz \rangle = \bar{a} \langle x, y \rangle + \bar{b} \langle x, z \rangle$$

Solution:-

$$\begin{aligned} \langle x, ay + bz \rangle &= \langle ay + bz, x \rangle \\ &= \langle ay, x \rangle + \langle bz, x \rangle \\ &= \bar{a} \langle y, x \rangle + \bar{b} \langle z, x \rangle \\ &= \bar{a} \langle x, y \rangle + \bar{b} \langle x, z \rangle \end{aligned}$$

Proved

Remember me in your prayers

"Jazak Allah"

Q#03.-

Appollonius Identity:-

$x, y, z \in V$

$$\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\|z - \frac{1}{2}(x+y)\|^2$$

Q#04:-

Define bounded linear function:

A bounded linear function F is a bounded linear operator with range in a scalar field of the normed space X .

$$|F(x)| \leq c \|x\|$$

Q#05

Find basis of $N(T)$.

$$x = (e_1, e_2, e_3) \Rightarrow x \in N(T)$$

$$\Rightarrow (0, 0, e_3)$$

$$\Rightarrow e_3(0, 0, 1)$$

Q#6:-

Define self adjoint operator

A bounded linear operator

$T: H \rightarrow H$ on a Hilbert space

(3)

His said to be, self adjoint.
or Hermitian if

Q#07 $T^* = T$

Prove that U is normal.

To show U is normal.

$$UU^* = U^*U$$

$$U^* = U^{-1}$$

$$UU^{-1} = U^{-1}U = I$$

Proved

Q#08

Give Two examples of orthogonal system.

Two examples of orthogonal system is

* Space R^n

* Space R^3

(4)

Note

9 marks

3 marks

5 marks

ma a re ha)

Q#9.

$$T(E_1, E_2, E_3) \rightarrow (E_1, E_2, E_3)$$

then find matrix representation

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ -E_1 - E_2 \end{bmatrix}$$

Hence required matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

Q#10

Prove that

$$(\alpha T^*)^* = \bar{\alpha} T^*$$

$$\langle x, (\alpha T^*)^* y \rangle = \langle (\alpha T)x, y \rangle$$

$$= \langle \alpha(Tx), y \rangle$$

$$= \bar{\alpha} \langle Tx, y \rangle$$

$$= \bar{\alpha} \langle x, T^* y \rangle$$

$$= \langle x, \bar{\alpha} T^* y \rangle$$

$$\langle x, (\alpha T^*)^* y \rangle = \langle x, \bar{\alpha} T^* y \rangle$$

$$\Rightarrow (\alpha T)^* = \bar{\alpha} T^*$$

Q No 11:-

Write two examples of linear functional

- 1) space $C[a, b]$
- 2) space l^2

Q No 12:-

Orthonormal System Definition:-

Orthonormal system are those functions which are real and complex whose inner product with itself result in 1 and with other functions result in 0.

Q No 13

Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ define by
 $(e_1, e_2, e_3) \mapsto (e_1, e_2, -e_1 - e_2)$
 Then find basis of $R(T)$.

$$(e_1, e_2, -e_1 - e_2) = e_1(1, 0, -1) + e_2(0, 1, -1)$$

$$D \Rightarrow \{(1, 0, -1), (0, 1, -1)\}$$

Q No 14

Show that $U^{-1} = U^*$ is unitary. OR $\|U\| = 1$
 Since U is bijective $\Rightarrow U^{-1}$ is bijective

$$(U^{-1})^* = (U^*)^* = U = (U^{-1})^{-1}$$

$$(U^{-1})^* = (U^{-1})^{-1}$$

$\Rightarrow U^{-1}$ is unitary.

Q No: 15.

Difference b/w Algebraic dual space and Conjugate space.

"Algebraic dual space of X :"

Set of all linear functionals defined on X .

"Dual or Conjugate space of X :"

X^* Set of all continuous or bounded linear functionals defined on X .

We take algebraic dual when there is no condition of continuous or bounded linear functions.

Q No: 16

Show that $\langle x, \alpha y \rangle = \overline{\alpha} \langle x, y \rangle$

For all $x, y \in V$ $\alpha \in F$

$$\langle x, \alpha y \rangle = \overline{\langle \alpha y, x \rangle} = \overline{\alpha \langle y, x \rangle}$$

$$\overline{\alpha \langle y, x \rangle} = \overline{\alpha} \langle x, y \rangle$$

QNO17:- Topic 64

prove that if a normed space X is finite dimensional then every linear operator on X is bounded.

Proof:-

X is finite dimension.

$\Rightarrow$ every linear operator on X is bounded.

Let $\dim X = n \Rightarrow \{e_1, e_2, \dots, e_n\}$ is a basis for X

Let $x \in X$

$$\Rightarrow x = \sum_{j=1}^n \xi_j e_j$$

Let T be any linear operator on X .

$$\|Tx\| = \left\| T\left(\sum_{j=1}^n \xi_j e_j\right) \right\| = \left\| \sum_{j=1}^n \xi_j T e_j \right\|$$

$$\begin{aligned} \text{linear} &\leq \sum_{j=1}^n |\xi_j| \|T e_j\| \leq N_3 \{\|T e_1\|, \dots, \|T e_n\|\} \\ &\leq \max \|T e_k\| \sum_{j=1}^n |\xi_j| \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \left\| \sum_{j=1}^n \xi_j e_j \right\| &\geq c \left(\sum_{j=1}^n |\xi_j| \right)^k \\ \sum_{j=1}^n |\xi_j| &\leq \frac{1}{c} \left\| \sum_{j=1}^n \xi_j e_j \right\| = \frac{1}{c} \|x\| \end{aligned}$$

$$\textcircled{1} \Rightarrow \|Tx\| \leq \max \|T e_k\| \frac{1}{c} \|x\|$$

$$\Rightarrow \|Tx\| \leq c' \|x\|$$

$\Rightarrow T$ is bounded.

Qno 18:-

Show that $(S+T)^* = S^* + T^*$

Proof

$$\langle x, (S+T)^* y \rangle = \langle (S+T)x, y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle Sx + Tx, y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle Sx, y \rangle + \langle Tx, y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, S^* y \rangle + \langle x, T^* y \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, (S^* y + T^* y) \rangle$$

$$\langle x, (S+T)^* y \rangle = \langle x, (S^* + T^*) y \rangle$$

Hence we have prove that

$$(S+T)^* = S^* + T^*$$

Qmp
Q No: 19

Q No: 19

Show that

$$\|x_1 + x_2 + x_3 + \dots + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + \dots + \|x_n\|^2$$

$$\begin{aligned} \left\| \sum_{i=1}^n x_i \right\|^2 &= \left\langle \sum_{i=1}^n x_i, \sum_{i=1}^n x_i \right\rangle \\ &= \left\langle x_1, x_2 + \dots + x_n \right\rangle + \left\langle x_1 + x_2 + \dots + x_n, x_1 \right\rangle \\ &= \left\langle x_1, x_2 + \dots + x_n \right\rangle + \left\langle x_1 + x_2 + \dots + x_n, x_1 \right\rangle \\ &= \sum_{i=1}^n \sum_{j=1}^n \langle x_i, x_j \rangle \end{aligned}$$

$$\therefore \langle x_i, x_j \rangle = \|x_i\|^2$$

$$\therefore \langle x_i, x_j \rangle = 0 \quad i \neq j \quad \text{So,}$$

$$\| \sum_{i=1}^n x_i \|^2 = \sum_{i=1}^n \sum_{j=1}^n \langle x_i, x_j \rangle$$

$$\langle x_i, x_j \rangle = 0$$

Q NO 20:-

Give an example of function which is not linear.

$\|\cdot\|: X \rightarrow \mathbb{R}$ Functional but not linear.

Q NO 21:-

Generalization of orthogonal system:-

$\{x_1, x_2, \dots, x_n\}$ be non zero vectors in V inner product space such that $\langle x_i, x_j \rangle = 0 \quad i \neq j$

This system $\{x_1, x_2, \dots, x_n\}$ is called as all vectors inside it are perpendicular to each other.

Q NO: 22

Isomorphism of metric space:-

A function $f: (X, d) \rightarrow (Y, \rho)$

is an isomorphism if f is an isometry and f is surjective. Two metric space are called isomorphic if there is an isomorphism between them.

QNO 23:-

Define Canonical mapping

$C: X \rightarrow X^{**}$ this mapping is called Canonical mapping of X into X^{**} defined as $x \mapsto j_x$.
made by baloch (12+H)

QNO 24:-

Define Normal Operator:-

A bounded linear operator $T: H \rightarrow H$ on a Hilbert space H is said to be Unitary operator if T is bijective and $TT^* = T^*T$.

Define equal operator:-

Let T_1 and T_2 be two operators defined on equal i.e. $T_1 = T_2$ if they have same domain i.e. $D(T_1) = D(T_2)$.

Q NO 28:-

Examples of Inner product Spaces:-

* Space R^n

* Space C^n

* Space $C[a, b]$

* Space l^2

* Space P_n

Qmp
N.29

Q NO 29:-

Prove that null space is closed.

OR

$$\overline{N(T)} \subseteq N(T)$$

Solution:-

$$x \in N(T)$$

$$\exists (x_n) \text{ such that } x_n \in N(T) + x \rightarrow x$$

Therefore

$$Tx_n \rightarrow Tx$$

$$x_n \in N(T)$$

$$Tx_n = 0$$

$$Tx = 0$$

(13)

$$x \in N(T)$$

$$\overline{N(T)} \subset N(T)$$

$$N(T) = \overline{N(T)}$$

$N(T)$ is closed.

Qno 30:-

Define Polarization Identity:-

For any x, y in complex inner product space

$$\langle x, y \rangle = \frac{1}{4} \{ \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2 \}$$

We have prove this complex inner product space.

Note :-

Find the Null space of $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$
defined by $(\xi_1, \xi_2, \xi_3) \rightarrow (\xi_1, \xi_2, -\xi_1 + \xi_2)$

- 1) Basis of $R(T)$ PgNo: 5
- 2) Basis of $N(T)$ PgNo: 2
- 3) Matrix of representation PgNo: 4

پہلے کے اسی پر ہیں Questions

Module No # 78

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