

MTH642 Current Papers

Subjective

by MATHEMATIC2

Q: 1 Define non-dimensional variable? Give an example of non-dimensional variable?

Answer:

Non-dimensional (or dimensionless) variables are defined as quantities that change or vary in the problem, but have no dimensions.

Example is angle of rotation, measured in degrees or radians which are dimensionless units.

Q: 2 write the differential form of Newton Second law of motion in terms of linear momentum. What does this expression states?

Answer:

Newton's second law states that the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.

Newton's second law: $F = ma$

$$= m \, dv / dt$$
$$= d(mv) / dt$$

Newton second law is also decision as "time rate of change of liner momentum".

Q: 3 Expand $\frac{\partial p}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$ into the Cartesian coordinates where $V = (u, v, w)$.

Answer:

As we know that rate of change of mass within CV

Rate of change of mass within CV:

$$\int_{CV} \frac{\partial \rho}{\partial t} dV \cong \frac{\partial \rho}{\partial t} dx dy dz$$

Net mass flow rate out of CV:

$$\sum_{out} \dot{m} \cong \underbrace{\left(\rho u + \frac{\partial(\rho u)}{\partial x} \frac{dx}{2} \right) dy dz}_{\text{right face}} + \underbrace{\left(\rho v + \frac{\partial(\rho v)}{\partial y} \frac{dy}{2} \right) dx dz}_{\text{top face}} + \underbrace{\left(\rho w + \frac{\partial(\rho w)}{\partial z} \frac{dz}{2} \right) dx dy}_{\text{front face}}$$

Q:4 Write the continuity equation for compressible flow in cylindrical coordinates.

Answer:

In cylindrical coordinates:

$$\frac{1}{r} \frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$$

Q: 5 The equation of motion $\frac{d^2 z}{dt^2} = -g$ discharging the derivation z of an object falling by gravity through a result in a $z = z_0 + w_0 t - \frac{1}{2} g t^2$ after integrating and applying the initial condition. Find the non dimensional form of the given result.

Answer:

Equation of motion:

$$\frac{d^2 z}{dt^2} = -g$$

Dimensional result:

$$z = z_0 + w_0 t - \frac{1}{2} g t^2$$

Primary dimensions of all parameters:

$$\{z\} = \{L\} \quad \{t\} = \{t\} \quad \{z_0\} = \{L\} \quad \{w_0\} = \{L/t\} \quad \{g\} = \{L/t^2\}$$

Non-dimensionalized variables:

$$z^* = \frac{z}{z_0} \quad t^* = \frac{w_0 t}{z_0}$$

$$\frac{d^2 z}{dt^2} = \frac{d^2(z_0 z^*)}{d(z_0 t^*/w_0)^2} = \frac{w_0^2}{z_0} \frac{d^2 z^*}{dt^{*2}} = -g \quad \rightarrow \quad \frac{w_0^2}{gz_0} \frac{d^2 z^*}{dt^{*2}} = -1$$

Froude number:

$$Fr = \frac{w_0}{\sqrt{gz_0}}$$

Nondimensionalized equation of motion:

$$\frac{d^2 z^*}{dt^{*2}} = -\frac{1}{Fr^2}$$

Non-dimensional result:

$$z^* = 1 + t^* - \frac{1}{2Fr^2} t^{*2}$$

Q: 6 Water is flowing from a house attaches to a water main. A child place his thumb to corner most of house outlet, causing a thin jet of high-speed water to emerge attaining the water height will be maximum under the stated assumption. Height of 20m if the house is held upward, what could be pressure gage in the water main?

Answer: //////////////////////////////////////

Q: 7 Expand the material derivative $\frac{Dp}{Dt}$.

Answer:

We expand $\frac{\partial p}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0$

- We expand above equation by using the product rule on the divergence term,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = \underbrace{\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{\nabla} \rho}_{\text{Material derivative of } \rho} + \rho \vec{\nabla} \cdot \vec{V} = 0$$

Q:8 What is the Equation of law of conservation of mass .state the law.

Answer:

Conservation of mass for a CV:

$$0 = \int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} \rho \vec{V} \cdot \vec{n} dA$$
$$\int_{CV} \frac{\partial \rho}{\partial t} dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

- The net rate of change of mass within the control volume is equal to the rate at which mass flows into the control volume minus the rate at which mass flows out of the control volume.

Question: 9

Consider the velocity field of Example 4–5—an unsteady, two-dimensional velocity field given by $\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + [1.5 + 2.5 \sin(\omega t) - 0.8y]\vec{j}$, where angular frequency ω is equal to 2π rad/s (a physical frequency of 1 Hz). Verify that this flow field can be approximated as incompressible.

Answer:

SOLUTION We are to verify that a given velocity field is incompressible.

Assumptions 1 The flow is two-dimensional, implying no z-component of velocity and no variation of u or v with z .

Analysis The components of velocity in the x - and y -directions, respectively, are

$$u = 0.5 + 0.8x \quad \text{and} \quad v = 1.5 + 2.5 \sin(\omega t) - 0.8y$$

If the flow is incompressible, Eq. 9–16 must apply. More specifically, in Cartesian coordinates Eq. 9–17 must apply. Let's check:

$$\underbrace{\frac{\partial u}{\partial x}}_{0.8} + \underbrace{\frac{\partial v}{\partial y}}_{-0.8} + \underbrace{\frac{\partial w}{\partial z}}_{0 \text{ since 2-D}} = 0 \quad \rightarrow \quad 0.8 - 0.8 = 0$$

So we see that the incompressible continuity equation is indeed satisfied at any instant in time, and **this flow field may be approximated as incompressible.**

Discussion Although there is an unsteady term in v , it has no y -derivative and drops out of the continuity equation.

Question: 10 Write the formulas of Cartesian and cylindrical coordinates in steady compressible flow.

Answer:

Special Case 1: Steady Compressible Flow

$$\vec{\nabla} \cdot (\rho \vec{V}) = 0$$

In Cartesian coordinates:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

In cylindrical coordinates:

$$\frac{1}{r} \frac{\partial(r \rho u_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$$

Question: 11 State the angular momentum equation. Write its expression about x-axis and y-axis.

Answer:

“Rate of change of angular Momentum”

Angular momentum equation:

$$\vec{M} = I \vec{\alpha} = I \frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

$$M_x = I_x \frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

About y-axis

$$M_y = I_y \frac{d\omega_y}{dt} = \frac{dH_y}{dt}$$

Question: 12 what is Rotational motion and Quantities.

Answer:

- Rotational motion: a motion during which all points in the body move in circles about the axis of rotation.
- Rotational motion is described with angular quantities such as the angular distance θ , angular velocity ω , and angular acceleration α .

$$\omega = \frac{d\theta}{dt} = \frac{d(l/r)}{dt} = \frac{1}{r} \frac{dl}{dt} = \frac{V}{r} \quad \text{and} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \frac{1}{r} \frac{dV}{dt} = \frac{a_t}{r}$$

$$V = r\omega \quad \text{and} \quad a_t = r\alpha$$

Question: 13 Condition of model and a prototype.

Answer:

- The principle of similarity: There are three necessary conditions for complete similarity between a model and a prototype
- The first condition is geometric similarity—the model must be the same shape as the prototype, but may be scaled by some constant scale factor.
- The second condition is kinematic similarity, which means that the velocity at any point in the model flow must be proportional (by a constant scale factor) to the velocity at the corresponding point in the prototype flow.
- The third and most restrictive similarity condition is that of dynamic similarity. Dynamic similarity is achieved when all forces in the model flow scale by a constant factor to corresponding forces in the prototype flow (force-scale equivalence).

Question: 14 write the expression for conservation of mass relation for a control volume. What does the expression satisfies?

Answer:

- Conservation of mass for a CV:

$$0 = \int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} \rho \vec{V} \cdot \vec{n} dA$$

$$\int_{CV} \frac{\partial \rho}{\partial t} dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

- The net rate of change of mass within the control volume is equal to the rate at which mass flows into the control volume minus the rate at which mass flows out of the control volume.

Question: 15 what is the difference between a dimension and a unit.

Answer: A dimension is a measure of a physical quantity (without numerical values).

- ♣ A unit is a way to assign a number to that dimension.
- ♣ There are seven primary dimensions (also called fundamental or basic dimensions): Mass, Length, Time, Temperature, Electric current, Amount of Light, and Amount of Matter.

Question: 16 what is Buckingham Pi Theorem.

Answer:

- Step 1: List all the dimensional parameters involved
- Let n be the number of parameters
- Example: For drag on a sphere, F, V, D, ρ , μ and n = 5
- Step 2: Select a set of fundamental (primary) dimensions
- For example: M (kg) L (m) t (sec)
- Example: For drag on a sphere choose MLt
- Step 3: List the dimensions of all parameters in terms of primary dimensions

Step 4:

- Select a set of r dimensional parameters that includes all the primary dimensions.
- Example: For drag on a sphere ($m = r = 3$) select ρ, V, D

Step 5:

- Set up dimensionless groups π^s
- There will be $n - m$ equations
- Example: For drag on a sphere

Step 6:

- Check to see that each group obtained is dimensionless
- Example: For drag on a sphere

Question: 17 what is the general form of linear momentum equation for the fixed, moving or deforming control volume.

Answer:

General:

$$\sum \vec{F} = \frac{d}{dt} \int_{cv} \rho \vec{V} dV + \int_{cs} \rho \vec{V} (\vec{V}_r \cdot \vec{n}) dA$$

Fixed CV:

$$\sum \vec{F} = \frac{d}{dt} \int_{cv} \rho \vec{V} dV + \int_{cs} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA$$

Question:18

Two velocity components of a steady, incompressible, three-dimensional flow field are known, namely, $u = ax^2 + by^2 + cz^2$ and $w = axz + byz^2$, where a, b , and c are constants. The y velocity component is missing (Fig. 9–14). Generate an expression for v as a function of x, y , and z .

Answer:

SOLUTION We are to find the y -component of velocity, v , using given expressions for u and w .

Assumptions 1 The flow is steady. 2 The flow is incompressible.

Analysis Since the flow is steady and incompressible, and since we are working in Cartesian coordinates, we apply Eq. 9–17 to the flow field, Condition for incompressibility:

$$\frac{\partial v}{\partial y} = -\underbrace{\frac{\partial u}{\partial x}}_{2ax} - \underbrace{\frac{\partial w}{\partial z}}_{ax + 2byz} \rightarrow \frac{\partial v}{\partial y} = -3ax - 2byz$$

Next we integrate with respect to y . Since the integration is a *partial* integration, we add some arbitrary function of x and z instead of a simple constant of integration.

Solution: $v = -3axy - by^2z + f(x, z)$

Discussion Any function of x and z yields a v that satisfies the incompressible continuity equation, since there are no derivatives of v with respect to x or z in the continuity equation.

Question: 19

Consider a steady velocity field given by $\vec{V} = (u, v, w) = a(x^2y + y^2)\vec{i} + bxy^2\vec{j} + cx\vec{k}$, where a , b , and c are constants. Under what conditions is this flow field incompressible?

Answer:

SOLUTION We are to determine a relationship between constants a , b , and c that ensures incompressibility.

Assumptions 1 The flow is steady. 2 The flow is incompressible (under certain constraints to be determined).

Analysis We apply Eq. 9–17 to the given velocity field,

$$\underbrace{\frac{\partial u}{\partial x}}_{2axy} + \underbrace{\frac{\partial v}{\partial y}}_{2bxy} + \underbrace{\frac{\partial w}{\partial z}}_0 = 0 \quad \rightarrow \quad 2axy + 2bxy = 0$$

Thus to guarantee incompressibility, constants a and b must be equal in magnitude but opposite in sign.

Condition for incompressibility: $a = -b$

Discussion If a were not equal to $-b$, this might still be a valid flow field, but density would have to vary with location in the flow field. In other words, the flow would be *compressible*, and Eq. 9–14 would need to be satisfied in place of Eq. 9–17.

Question:20

The radial velocity component in an incompressible, two dimensional flow field ($v_z = 0$) is

$$v_r = 2r + 3r^2 \sin \theta$$

Determine the corresponding tangential velocity component, v_θ , required to satisfy conservation of mass.

Answer:

The continuity equation for incompressible steady flow in cylindrical coordinates is given by

$$\frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Since $v_z = 0$,

$$\frac{\partial v_\theta}{\partial \theta} = - \frac{\partial (rv_r)}{\partial r} \quad (1)$$

Question: 21 Derive Navier-stoks equation for incompressible ?

Answer:

Viscous stress tensor for an incompressible Newtonian fluid with constant properties:

$$\tau_{ij} = 2\mu\epsilon_{ij}$$

$$\tau_{ij} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} 2\mu \frac{\partial u}{\partial x} & \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & 2\mu \frac{\partial v}{\partial y} & \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & 2\mu \frac{\partial w}{\partial z} \end{pmatrix}$$

$$\sigma_{ij} = \begin{pmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -P \end{pmatrix} + \begin{pmatrix} 2\mu \frac{\partial u}{\partial x} & \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & 2\mu \frac{\partial v}{\partial y} & \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & 2\mu \frac{\partial w}{\partial z} \end{pmatrix}$$

Now we substitute Eq into the three Cartesian components of Cauchy's equation. Let's consider the x-component first

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial}{\partial y} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) + \mu \frac{\partial}{\partial z} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)$$

After some clever rearrangement of the viscous terms in Eq. (3)

$$\begin{aligned} \rho \frac{Du}{Dt} &= -\frac{\partial P}{\partial x} + \rho g_x + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial x} \frac{\partial w}{\partial z} + \frac{\partial^2 u}{\partial z^2} \right] \\ &= -\frac{\partial P}{\partial x} + \rho g_x + \mu \left[\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \end{aligned}$$

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \mu \nabla^2 u$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \rho g_y + \mu \nabla^2 v$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial P}{\partial z} + \rho g_z + \mu \nabla^2 w$$

Incompressible Navier–Stokes equation:

$$\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla}P + \rho \vec{g} + \mu \nabla^2 \vec{V}$$

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \mu \nabla^2 u$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \rho g_y + \mu \nabla^2 v$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial P}{\partial z} + \rho g_z + \mu \nabla^2 w$$

Question: 22 Define Normalized Equation.

Answer: Normalized equation: if the non-dimensional terms in the equation are of order unity, the equation is called normalized.

Question: 23 what is incompressible flow of continuity equation in Cartesian coordinates, cylindrical coordinates?

Answer:

Special Case 2: Incompressible Flow

$$\vec{\nabla} \cdot \vec{V} = 0 \quad (16) \quad \text{Since} \quad \partial \rho / \partial t \cong 0$$

Incompressible continuity equation in Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible continuity equation in cylindrical coordinates:

$$\frac{1}{r} \frac{\partial(ru_r)}{\partial r} + \frac{1}{r} \frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$$

Question: 24 consider a steady velocity field given by

$\vec{V} = (u, v, w) = a(x^2y + y^2)\vec{i} + bxy^2\vec{j} + cx\vec{k}$ (a, b, c) are constant. What condition is this field is incompressible?

Answer:

Formula: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ $\frac{\partial w}{\partial z} = 0$

$$2axy + 2bxy = 0$$

$$2xy(a + b) = 0$$

$$a + b = 0$$

$$a = -b \quad \text{condition for incompressibility.}$$

Question:25 Consider a Two dimensional incompressible flow in cylindrical

coordinates velocity is $u_\theta = \frac{k}{r}$ u is constant. Generate expression for other velocity component u_r .

Answer:

Incompressible continuity equation in cylindrical coordinates:

$$\frac{1}{r} \frac{\partial(ru_r)}{\partial r} + \frac{1}{r} \frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$$

$$\frac{\partial u_z}{\partial z} = 0$$

$$\frac{1}{r} \left[\frac{\partial(ru_r)}{\partial r} + \frac{\partial u_\theta}{\partial \theta} \right] = 0$$

$$\frac{\partial(ru_r)}{\partial r} = -\frac{\partial u_\theta}{\partial \theta} \dots\dots\dots(1)$$

u_θ is not a function, so (1) reduce to

$$\frac{\partial(ru_r)}{\partial r} = 0$$

$$ru_r = f(\theta, t)$$

$$u_r = \frac{f(\theta, t)}{r}$$

Question: 26 Two Dimensional velocity $V = (u, v) = (0.5+0.8x)i + [1.5+2.5\sin\omega t - 0.8y]j$ Verify by incompressible?

Answer:

$$U = 0.5 + 0.8x$$

$$V = 1.5 + 2.5 \sin\omega(t) - 0.8y$$

Formula for incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\left[\frac{\partial w}{\partial z} = 0 \right]$$

$$0.8 - 0.8 = 0$$

$$0 = 0$$

Question: 27 Find a missing velocity component incompressible three dimensional flow where $u = ax^2 + by^2 + cz^2$, $w = axz + byz^2$ the y velocity component is missing?

Answer:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

We know that incompressible formula:

$$\frac{\partial}{\partial x}(ax^2 + by^2 + cz^2) + \frac{\partial v}{\partial x} + \frac{\partial}{\partial z}(axz + byz^2) = 0$$

$$2ax + 2b \frac{\partial y}{\partial y} + 2cz + az + 2bz = 0$$

Combine like term

$$2ax + 2b + 2cz + az + 2bz = 0$$

Isolate the missing y Velocity component (v):

$$\frac{\partial v}{\partial y} = -2b - az - 2bz$$

So, the missing y component is given by:

$$v = -2ax - 2by^2z + f(x, z)$$

Question: 28 Verify that each additive term in the Bernoulli's equation have same dimension? What is the dimension of C in Bernoulli equation?

Answer: we know that Bernoulli's equation is

$$P + \frac{1}{2} \rho V^2 + \rho gz = C$$

$$\begin{aligned} \{P\} &= \{\text{Pressure}\} = \left\{ \frac{\text{Force}}{\text{Area}} \right\} = \left\{ \frac{\text{Mass}}{\text{Time}^2} \frac{\text{Length}}{\text{Length}^2} \right\} = \left\{ \frac{\text{m}}{\text{t}^2 \text{L}} \right\} \\ \left\{ \frac{1}{2} \rho V^2 \right\} &= \left\{ \frac{\text{Mass}}{\text{Volume}} \left(\frac{\text{Length}}{\text{Time}} \right)^2 \right\} = \left\{ \frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2} \right\} = \left\{ \frac{\text{m}}{\text{t}^2 \text{L}} \right\} \\ \{\rho gz\} &= \left\{ \frac{\text{Mass}}{\text{Volume}} \frac{\text{Length}}{\text{Time}^2} \text{Length} \right\} = \left\{ \frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2} \right\} = \left\{ \frac{\text{m}}{\text{t}^2 \text{L}} \right\} \end{aligned}$$

From the homogeneity the constant has same dimension

So, $C = \left\{ \frac{\text{m}}{\text{t}^2 \text{L}} \right\}$

Question: 29 State Divergence theorem.

Answer:

The quickest and most straightforward way to derive the differential form of conservation of mass is to apply the divergence theorem. The divergence theorem is also called Gauss's theorem, named after the German mathematician Johann Carl Friedrich Gauss

Divergence theorem:
$$\int_V \vec{\nabla} \cdot \vec{G} dV = \oint_A \vec{G} \cdot \vec{n} dA$$

Question: 30 Prove that $v=r\omega$ $at=r\alpha$.

Answer:

As we know that

$$\omega = \frac{d\theta}{dt} \quad \because \theta = \frac{l}{r}$$

$$\omega = \frac{d\left(\frac{l}{r}\right)}{dt} \Rightarrow \omega = \frac{1}{r} \frac{dl}{dt}$$

$$\omega = \frac{v}{r} \Rightarrow v = r\omega$$

b) As we know that

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha = \frac{d}{dt} \frac{d\theta}{dt} \Rightarrow \alpha = \frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d^2}{dt^2} \left(\frac{l}{r} \right) \Rightarrow \alpha = \frac{1}{r} \frac{d}{dt} \left(\frac{dl}{dr} \right)$$

$$\alpha = \frac{1}{r} \frac{d}{dt} (v) \Rightarrow \alpha = \frac{1}{r} at$$

$at = r\alpha$

Question: 31 Write formulas of Froude number?

Answer:

Froude Number $Fr = \frac{V}{\sqrt{gL}}$

Reynolds Number $Re = \frac{\rho VL}{\mu} = \frac{VL}{\nu}$

Also asked for:

Mach Number $M = \frac{V}{c}$

Weber Number $We = \frac{\rho V^2 L}{\sigma}$

Euler Number $Eu = \frac{\Delta p}{\frac{1}{2} \rho V^2}$

Cavitation Number $Ca = \frac{p - p_v}{\frac{1}{2} \rho V^2}$

Question: 32 The velocity component in along x-axis of a steady incompressible two dimensional flow is $u = 2.5y + 0.9x^2$ component v is missing generate expression for v as a function of x.

Answer:

As given that $u = 2.5y + 0.9x^2$

We have to find expression for component v: As flow field incompressible; we know that if flow field is steady incompressible then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \therefore \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \dots \dots \dots (1)$$

$$u = 2.5y + 0.9x^2$$

$$\frac{\partial u}{\partial x} = 0 + 2(0.9)x \Rightarrow \frac{\partial u}{\partial x} = 1.8x \quad \text{put in (1)}$$

$$\frac{\partial v}{\partial y} = -1.8x$$

Integrate w.r.t 'y'

$$v = -1.8xy + f(x)$$

where $f(x)$ is constant of integration.

Question: 33- Angular momentum for inlet and outlet for steady expression?

Answer:

Steady flow:

$$\sum \vec{M} = \int_{CS} (\vec{r} \times \vec{V}) \rho (\vec{V}_r \cdot \vec{n}) dA$$

- An approximate form of the angular momentum equation in terms of average properties at inlets and outlets becomes:

$$\sum \vec{M} = \frac{d}{dt} \int_{CV} (\vec{r} \times \vec{V}) \rho dV + \sum_{out} \vec{r} \times \dot{m} \vec{V} - \sum_{in} \vec{r} \times \dot{m} \vec{V}$$

Steady flow:

$$\sum \vec{M} = \sum_{out} \vec{r} \times \dot{m} \vec{V} - \sum_{in} \vec{r} \times \dot{m} \vec{V}$$

$$\sum M = \sum_{out} r \dot{m} V - \sum_{in} r \dot{m} V$$

Scalar angular form of angular momentum equation:

$$\sum M = \sum_{out} r \dot{m} V - \sum_{in} r \dot{m} V$$

Question:34 Show that the Froude number is a non dimensional parameter

where Froude number is given $Fr = \frac{w_o}{\sqrt{gz_o}}$.

Answer:

- Primary dimensions of all parameters:

$$\{z\} = \{L\} \quad \{t\} = \{t\} \quad \{z_o\} = \{L\} \quad \{w_o\} = \{L/t\} \quad \{g\} = \{L/t^2\}$$

- Non-dimensionalized variables:

$$z^* = \frac{z}{z_o} \quad t^* = \frac{w_o t}{z_o}$$

$$\frac{d^2 z}{dt^2} = \frac{d^2 (z_o z^*)}{d(z_o t^* / w_o)^2} = \frac{w_o^2}{z_o} \frac{d^2 z^*}{dt^{*2}} = -g \quad \rightarrow \quad \frac{w_o^2}{gz_o} \frac{d^2 z^*}{dt^{*2}} = -1$$

- Froude number:

$$Fr = \frac{w_o}{\sqrt{gz_o}}$$

- The Froude number also appears as a non-dimensional parameter in free surface flow.

Question:35 CONTINUITY AND NAVIER-STOKES EQUATIONS IN CYLINDRICAL COORDINATES?

Answer:

The Laplacian operator is shown in cylindrical coordinates of the incompressible Navier-Stokes equation.

Incompressible continuity equation:

$$\frac{1}{r} \frac{\partial(r u_r)}{\partial r} + \frac{1}{r} \frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0 \quad (62a)$$

r-component of the incompressible Navier-Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) \\ &= -\frac{\partial P}{\partial r} + \rho g_r + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] \end{aligned} \quad (62b)$$

θ-component of the incompressible Navier-Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z \frac{\partial u_\theta}{\partial z} \right) \\ &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right] \end{aligned} \quad (62c)$$

z-component of the incompressible Navier-Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) \\ &= -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \end{aligned} \quad (62d)$$

Question:36 WHAT IS A BOUNDARY LAYER?

Answer:

A boundary layer is a layer of flowing fluid near a boundary where viscosity keeps the flow velocity close to that of the boundary. If the boundary is not moving, then the normal and tangential flow velocities at the boundary vanish, and the normal and tangential flow velocities near the boundary are small compared to those of the Inviscid (potential) flow far from the boundary.

THE END

“Please keep me in your prayers”

BY MATHEMATICS

MATHEMATICS