MTH642 Mega File Final Term

Lec (23-45)

Topic (76-143) page# (77-158)

Objective Files (2)

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- MTH642 Quiz #3 All in 1 File 9March2022 pg30

Subjective Files (9)

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Mth642 Quiz#2

Lec (28-31), Topic (89-102)

Due Date (10Feb-12Feb) 2022

- **MTH642 Quiz2 7aug2021 pg10**
- **MTH642 Quiz#2 11feb2022 pg30**
- **MTH642 lec(28_31)mod(89-102)date(10-12)feb2022 pg31**

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 4 of 10 (Start time: 04:14:25 PM, 12 February 2022)

Which of the following is *Momentum flow rate across an inlet or outlet*?

Select the correct option

- ○ $\dot{m}\overline{V_{avg}}$ ✓
- ○ $\overline{\rho V_{avg}}$
- ○ $\overline{\rho V_{avg}} A_c$
- ○ $\dot{m}\overline{V_{avg}} A_c$

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 1 of 10 (Start time: 09:32:46 AM, 12 February 2022)

The radial flow devices are best analyzed with _____.

Select the correct option

○	linear momentum equation
◉	angular momentum equation 

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 2 of 10 (Start time: 09:33:07 AM, 12 February 2022)

Which of the following is the more general form of Newton's second law?

Select the correct option

◉	$\sum \overline{F} = d/dt \int_{sys} \rho \overline{V}\, dV$
○	$\sum \overline{F} = d/dt \int_{sys} \overline{V}\, dV$
○	$\sum \overline{F} = d/dt \int_{sys} m\overline{V}\, dV$
○	$\sum \overline{F} = \int_{sys} \rho \overline{V}\, dV$

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Question # 5 of 10 (Start time: 09:34:17 AM, 12 February 2022)

The linear momentum equation is obtained from Reynolds transport theorem by setting $b =$____ and $B =$_____.

Select the correct option

○	$b = m\overline{V}$ and $B = \overline{V}$
○	$b = \overline{V}$ and $B = \rho\overline{V}$
◉	$b = \overline{V}$ and $B = m\overline{V}$
○	$b = m\overline{V}$ and $B = m\overline{V}$

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 6 of 10 (Start time: 09:34:48 AM, 12 February 2022)

Which of the following is expression for rotational kinetic energy?

Select the correct option

◉	$1/2mr^2\omega^2$ ✓
○	$1/2mr\omega^2$
○	$1/2mr^2\omega$
○	$1/2\rho r^2\omega^2$

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 8 of 10 (Start time: 09:35:29 AM, 12 February 2022)

Total torque (M) is equal to _____, where α is angular acceleration and I is moment of inertia.

Select the correct option

○	$I\alpha^2$
○	$I^2\alpha$
◉	$I\alpha$ ✓
○	$\rho I\alpha$

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 2 of 10 (Start time: 09:39:51 AM, 12 February 2022)

The dimension of the Bernoulli constant is _____.

Select the correct option

○	$mL^{-2}t^{-2}$
○	$mL^{2}t^{-2}$
○	$mL^{-1}t^{-1}$
◉	$mL^{-1}t^{-2}$

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 10 of 10 (Start time: 09:35:59 AM, 12 February 2022)

Which of the following is expression for moment of a force?

Select the correct option

○	$\overline{V} \times \overline{F}$
○	$\overline{F} \times \overline{r}$
◉	$\overline{r} \times \overline{F}$ ✓
○	$\overline{r} . \overline{F}$

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 3 of 10 (Start time: 09:40:09 AM, 12 February 2022)

Meter is the unit for the dimension of _____ .

Select the correct option

○	time
○	amount of light
◉	length 
○	mass

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 4 of 10 (Start time: 09:40:31 AM, 12 February 2022)

Which of the following is dimension of force?

Select the correct option

○	mLt
○	mLt^2
○	mLt^{-1}
◉	mL/t^2 ✓

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 5 of 10 (Start time: 09:40:44 AM, 12 February 2022)

Which of the following is dimensionless quantity?

Select the correct option

◉	Angle ✓
○	force
○	energy
○	pressure

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Question # 6 of 10 (Start time: 09:41:11 AM, 12 February 2022)

Which of the relationship between linear acceleration in tangential direction (a_t) and angular acceleration (α)?

Select the correct option

○	$a_t = \omega\alpha/r$
○	$a_t = \omega r\alpha$
○	$a_t = \alpha/r$
◉	$a_t = r\alpha$ 

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 7 of 10 (Start time: 09:41:32 AM, 12 February 2022)

Which of the following has dimension equal to energy?

Select the correct option

○	pressure
○	force
○	power
◉	Work done ✓

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 8 of 10 (Start time: 09:41:48 AM, 12 February 2022)

____ is the dimension that is measured in candela (cd).

Select the correct option

○	Amount of matter
◉	Amount of light ✓
○	Electric current
○	Length

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 10 of 10 (Start time: 09:44:34 AM, 12 February 2022)

Which of the following is expression for moment of momentum?

Select the correct option

○	$\overline{r} \times \rho m \overline{V}$
◉	$\overline{r} \times m \overline{V}$ 
○	$\overline{r} \times \overline{V}$
○	$\overline{r} \times \rho \overline{V}$

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 3 of 10 (Start time: 09:57:58 AM, 12 February 2022)

The dimension of acceleration is _____.

Select the correct option

○	mLt^{-2}
○	mt^{-2}
○	Lt^{-1}
◉	Lt^{-2}

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 1 of 10 (Start time: 09:57:00 AM, 12 February 2022)

$\overline{r} \times \dot{m}\overline{V}$ represents _____.

Select the correct option

○	moment flow rate
○	linear momentum flow rate
◉	angular momentum flow rate ✓
○	moment flux

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 4 of 10 (Start time: 09:58:21 AM, 12 February 2022)

Mass flow rate across an inlet or outlet: $\dot{m} =$ _____.

Select the correct option

○	$V_{avg} A_c$
○	$m V_{avg} A_c$
○	$m \rho V_{avg} A_c$
◉	$\rho V_{avg} A_c$ ✓

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 5 of 10 (Start time: 09:58:38 AM, 12 February 2022)

Kilogram is the unit used to measure the dimension of _____.

Select the correct option

○	time
◉	mass 
○	amount of light
○	amount of matter

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MTH642 - Fluid Mechanics (Quiz 2)

Quiz Start Time: 09:57 AM, 12 February 2022

Question # 6 of 10 (Start time: 09:58:48 AM, 12 February 2022)

Total Marks: 1

The magnitude of torque M acting on a point mass m at normal distance r from the axis of rotation is equal to _____, where α is angular acceleration.

Select the correct option

Reload Math Equations

○	$mr\alpha^2$
○	$mr^2\alpha^2$
○	$mr\alpha$
◉	$mr^2\alpha$ ✓

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 10 of 10 (Start time: 10:00:02 AM, 12 February 2022)

mL^2t^{-2} is dimension of _____.

Select the correct option

◉	energy ✓
○	force
○	moment
○	linear momentum

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 2 of 10 (Start time: 10:04:10 AM, 12 February 2022)

Which of the following dimension is measured in unit ampere?

Select the correct option

○	Amount of matter
◉	Electric current ✓
○	Amount of light
○	Time

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 9 of 10 (Start time: 10:06:06 AM, 12 February 2022)

$\rho\overline{V}\,dV$ is called _____.

Select the correct option

○	moment
○	momentum flow rate
◉	momentum ✓
○	torque

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 2 of 10 (Start time: 10:20:30 AM, 12 February 2022)

Mole is the unit used to measure the dimension of _____.

Select the correct option

◉	amount of matter ✓
○	amount of light
○	electric current
○	mass

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MTH642 - Fluid Mechanics (Quiz 2)

Quiz Start Time: 10:38 AM, 12 February 2022

Question # 3 of 10 (Start time: 10:38:40 AM, 12 February 2022)

Total Marks: 1

All nonprimary dimensions can be formed by some combination of the _____ primary dimensions.

Select the correct option

Reload Math Equations

○	six
◉	seven
○	eight
○	nine

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 9 of 10 (Start time: 10:40:22 AM, 12 February 2022)

mL^2t^{-1} is dimension of _____.

Select the correct option

○	linear momentum
○	moment
○	force
◉	angular momentum ✓

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 2 of 10 (Start time: 10:44:00 AM, 12 February 2022)

$mL^{-1}t^{-2}$ is dimension of _____.

Select the correct option

○	momentum
○	energy
○	force
◉	pressure 

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MTH642 – Fluid Mechanics (Quiz 2)

Question # 4 of 10 (Start time: 10:44:41 AM, 12 February 2022)

The linear momentum equations is a____ equation.

Select the correct option

○	scalar
◉	vector ✓

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 6 of 10 (Start time: 10:45:08 AM, 12 February 2022)

Which of the following unit (s) is/are used to measure the dimension of temperature?

Select the correct option

◉	all of the given options ✓
○	kelvin
○	fahrenheit
○	rankine

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 7 of 10 (Start time: 10:45:20 AM, 12 February 2022)

The dimension of velocity is _____.

Select the correct option

○	Lt^{-2}
◉	Lt^{--1} 
○	mt^{-2}
○	mt^{-1}

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MTH642 - Fluid Mechanics (Quiz 2)

Question # 7 of 10 (Start time: 10:54:59 AM, 12 February 2022)

The magnitude of angular momentum (H) is equal to _____.

Select the correct option

◉	$I\omega$ ✓
○	$I\omega\alpha$
○	$I^2\omega\alpha$
○	$I\alpha$

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Mth642 Quiz#3

Lec (32-38), Module (103-124)

Due Date (7mar-9mar) 2022

- mth642 quiz3 26.08.2021 pg16
- MTH642 Quiz Solution with reference (26aug2021) pg21
- Raza file confirm Quiz#3 26aug2021 pg16
- Sunny file confirm Quiz#3 26aug2021 pg18
- M@l!k MTH642 quizz3 Solution-1 26aug2021 pg17
- M@l!k MTH642 quizz3 Solution 26aug2021 pg17
- Mth642 Quiz 3 confirmed old quiz pg13 by moeez
- Mth642,topic 100 to125 9mar2022 pg28
- 3 Groups quiz mth642 unlimited 9march2022

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MTH642:Quiz #3

Question # 7 of 10 (Start time: 04:02:08 PM, 26 August 2021)

Which of the following is used to define the stream function for two dimensional flows?

Select the correct option

- O the Reynolds Transport equation
- O the Cauchy equation
- O the continuity equation ✓ topic 132
- O the Navier-Stokes equation

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MTH642:Quiz #3

Question # 7 of 10 (Start time: 10:57:21 AM, 25 August 2021)

The differential equation representing the transport of mass and conservation of mass as well is called____.

Select the correct option

O	the Reynolds Transport equation
O	the continuity equation
O	the Navier-Stokes equation ✓
O	the Cauchy equation

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MTH642:Quiz 3

Question # 2 of 5 (Start time: 10:56:13 PM, 08 March 2022)

For the steady compressible flow, the continuity equation is given as _____.

Select the correct option

○	$\partial\rho/\partial t + \nabla.(\rho V) = 0$
○	$\partial\rho/\partial t + \nabla.V = 0$
○	$\nabla.V = 0$
●	$\nabla.(\rho V) = 0$ ✓ Topic 124

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Question # 3 of 5 (Start time: 10:56:50 PM, 08 March 2022)

Let n be total parameters and j equal to number of primary dimensions, then expected number of Π's (k) equal to____.

Select the correct option

- ○ $k = 2n - j$
- ● $k = n - j$ ✓ Topic 111, pg116
- ○ $k = n + j$
- ○ $k = j - n$

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MTH642:Quiz 3

Question # 4 of 5 (Start time: 10:57:05 PM, 08 March 2022)

The continuity equation is derived from the law of____.

Select the correct option

○	conservation of force
●	conservation of mass ✓  Topic 119, pg126
○	conservation of momentum
○	conservation of energy

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MTH642:Quiz 3

Question # 5 of 5 (Start time: 10:57:20 PM, 08 March 2022)

The Euler number (Eu) is equal to _____.

Select the correct option

○	$Eu = \frac{\rho V^2}{P_0 - P_\infty}$
○	$Eu = \frac{P_0 - P_\infty}{V^2}$
●	$Eu = \frac{P_0 - P_\infty}{\rho V^2}$ ✓ Topic 111, pg117
○	$Eu = \frac{\rho(P_0 - P_\infty)}{V^2}$

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MTH642:Quiz 3

Question # 1 of 5 (Start time: 12:03:08 AM, 09 March 2022)

_____ is the incompressible continuity equation in Cylindrical coordinates.

Select the correct option

- ● $\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial(ru_\theta)}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$ — Topic 124, pg132
- ○ $\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$
- ○ $\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial(ru_\theta)}{\partial \theta} + \frac{\partial(ru_z)}{\partial z} = 0$
- ○ $\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{\partial(ru_\theta)}{\partial \theta} + \frac{\partial(ru_z)}{\partial z} = 0$

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MTH642:Quiz 3

Question # 3 of 5 (Start time: 12:04:42 AM, 09 March 2022)

If $t = 3.01s, w_0 = 1.83m/s \; and \; z_0 = 2m$, then dimensionless time (t^*) is equal to_____.

Select the correct option

○	$t^* = 2.71$
○	$t^* = 2.79$
○	$t^* = 2.7$
●	$t^* = 2.75$

Topic 105, pg110

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MTH642:Quiz 3

Question # 4 of 5 (Start time: 12:04:55 AM, 09 March 2022)

The flow is appoximated as incompressible if _____.

Select the correct option

○	$\nabla \cdot V = 1$
●	$\nabla \cdot V = 0$ ✓
○	$\nabla \cdot V = -1$
○	$\nabla \cdot V = \infty$

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MTH642:Quiz 3

Question # 1 of 5 (Start time: 12:09:51 AM, 09 March 2022)

Which of the following is the incompressible continuity equation?

Select the correct option

○	$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho V) = 0$
○	$\rho \frac{DV}{Dt} = -\nabla P + \rho g + \mu \nabla^2 V$
○	$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + (V \bullet \nabla) V$
●	$\nabla \bullet V = 0$

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MTH642:Quiz 3

Question # 3 of 5 (Start time: 12:10:50 AM, 09 March 2022)

For the steady incompressible flow, the continuity equation is given as _____.

Select the correct option

○	$\partial\rho/\partial t + \nabla.(\rho V) = 0$
○	$\partial\rho/\partial t + \nabla.V = 0$
●	$\nabla.V = 0$ ✓
○	$\nabla.(\rho V) = 0$

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MTH642:Quiz 3

Question # 4 of 5 (Start time: 12:11:51 AM, 09 March 2022)

The method of repeating variables comprises on ___ steps.

Select the correct option

○	eight
○	five
●	seven
○	six ✓

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MTH642:Quiz 3

Question # 5 of 5 (Start time: 12:12:03 AM, 09 March 2022)

The Strouhal number (St) is equal to _____.

Select the correct option

○	$St = \frac{V}{fL}$
●	$St = \frac{fL}{V}$ ✓ Topic 106, pg110
○	$St = \frac{Vg}{fL}$
○	$St = \frac{fL}{Vg}$

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MTH642:Quiz 3

Question # 2 of 5 (Start time: 12:13:43 AM, 09 March 2022)

If $g = 1.63m/s^2, w_0 = 1.83m/s \; and \; z_0 = 2m$, then Froude number is equal to ____.

Select the correct option

●	$Fr = 1.01$
○	$Fr = 1.03$
○	$Fr = 1$
○	$Fr = 1.05$

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MTH642:Quiz 3

Question # 3 of 5 (Start time: 12:13:55 AM, 09 March 2022)

The material derivative $D\rho/Dt$ can be expanded as _____.

Select the correct option

●	$\frac{\partial \rho}{\partial t} + V \cdot \nabla \rho$ ✓ Topic 122, pg130
○	$\frac{\partial V}{\partial t} + \nabla . V\rho$
○	$\frac{\partial V}{\partial t} + V \cdot \nabla \rho$
○	$\frac{\partial \rho}{\partial t} + \nabla . V\rho$

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MTH642:Quiz 3

Question # 4 of 5 (Start time: 12:14:06 AM, 09 March 2022)

Which of the following is a Cartesian form of $\vec{\nabla}.(\rho\vec{V}) = 0$? Where $\vec{V} = (u, v, w)$.

Select the correct option

○	$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \frac{\partial w}{\partial z} = 0$
○	$\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z} = 0$
○	$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} = 0$
●	$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$ ✓

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MTH642:Quiz 3

Question # 1 of 5 (Start time: 12:15:56 AM, 09 March 2022)

The continuity equation for steady compressible flow in cylindrical coordinates is given as _____.

Select the correct option

●	$\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta}+\frac{\partial(\rho u_z)}{\partial z}=0$ ✓
○	$\frac{1}{r}\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta}+\frac{\partial(\rho u_z)}{\partial z}=0$
○	$\partial(\rho u)/\partial x+\partial(\rho v)/\partial y+\partial(\rho u)/\partial z=0$
○	$\frac{1}{r}\frac{\partial(r u_r)}{\partial r}+\frac{1}{r}\frac{\partial(u_\theta)}{\partial \theta}+\frac{\partial(u_z)}{\partial z}=0$

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MTH642:Quiz 3

Question # 2 of 5 (Start time: 12:16:48 AM, 09 March 2022)

_____ is the incompressible continuity equation in Cartesian coordinates.

Select the correct option

○	$\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$
○	$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$
●	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ ✓
○	$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$

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MTH642:Quiz 3

Question # 3 of 5 (Start time: 12:17:23 AM, 09 March 2022)

Which of the following is steady compressible form of continuity equation?

Select the correct option

○	$\vec{V}.(\rho\vec{\nabla}) = 0$
○	$\vec{V}.\vec{\nabla} = 0$
○	$\vec{\nabla}.\vec{V} = 0$
●	$\vec{\nabla}.(\rho\vec{V}) = 0$ ✓

Topic 124, pg132

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MTH642:Quiz 3

Question # 5 of 5 (Start time: 12:18:07 AM, 09 March 2022)

The Reynolds number (Re) is equal to _____.

Select the correct option

- ○ $Re = \frac{VL}{\mu}$
- ○ $Re = \frac{\mu}{\rho VL}$
- ● $Re = \frac{\rho VL}{\mu}$ ✓ Topic 111, pg117
- ○ $Re = \frac{VL}{\rho\mu}$

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Question # 3 of 5 (Start time: 12:26:37 AM, 09 March 2022)

Which of the following is a general form of continuity equation?

Select the correct option

○	$\frac{\partial \rho}{\partial t} + \vec{V} . (\rho \vec{\nabla}) = 0$
●	$\frac{\partial \rho}{\partial t} + \vec{\nabla} . (\rho \vec{V}) = 0$
○	$\vec{\nabla} . (\rho \vec{V}) = 0$
○	$\frac{\partial \rho}{\partial t} + \rho \vec{\nabla} . \vec{V} = 0$

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Question # 4 of 5 (Start time: 12:39:59 PM, 09 March 2022)

For an incompressible three dimensional flow, which of the following must be true?

Select the correct option

O	$\frac{\partial w}{\partial t} = 0$
O	$\frac{\partial v}{\partial t} = 0$
O	$\frac{\partial u}{\partial t} = 0$
O	$\frac{\partial \rho}{\partial t} = 0$ ✓

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MTH642:Quiz 3

Question # 1 of 5 (Start time: 12:46:08 AM, 09 March 2022)

Which of the following is the compressible continuity equation?

Select the correct option

●	$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho V) = 0$ ✓ Topic 119, pg126
○	$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + (V \bullet \nabla)V$
○	$\rho \frac{DV}{Dt} = -\nabla P + \rho g + \mu \nabla^2 V$
○	$\nabla \bullet V = 0$

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MTH642:Quiz 3

Question # 2 of 5 (Start time: 12:46:31 AM, 09 March 2022)

Which of the following is a polar form of $\vec{\nabla}.(\rho\vec{V}) = 0$? Where $\vec{V} = (u_r, u_\theta, u_z)$.

Select the correct option

○	$\frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$
●	$\frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho r u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$
○	$\frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{\partial(\rho r u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$
○	$\frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho r u_\theta)}{\partial \theta} + \frac{\partial(\rho r u_z)}{\partial z} = 0$

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MTH642:Quiz 3

Question # 1 of 5 (Start time: 10:05:46 AM, 09 March 2022)

The scaling parameters are based on _____.

Select the correct option

- (●) primary dimensions 
- ( ) both primary and secondary dimensions
- ( ) dimensionless quantities
- ( ) secondary dimensions

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MTH642:Quiz 3

Question # 4 of 5 (Start time: 10:06:45 AM, 09 March 2022)

For the steady compressible two dimensional flow, the continuity equation is given as _____.

Select the correct option

○	$\partial\rho/\partial t + \partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0$
○	$\partial\rho/\partial t + \rho\partial(u)/\partial x + \rho\partial(v)/\partial y = 0$
●	$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0$ ✓
○	$\partial(u)/\partial x + \partial(v)/\partial y = 0$

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MTH642:Quiz 3

Question # 4 of 5 (Start time: 09:27:47 PM, 09 March 2022)

The continuity equation for steady incompressible flow in cylindrical coordinates is given as _____.

Select the correct option

○	$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$
○	$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$
●	$\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$
○	$\frac{1}{r}\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$

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MTH642:Quiz 3

Question # 5 of 5 (Start time: 11:07:22 PM, 09 March 2022)

For the unsteady compressible three dimensional flow, the continuity equation is given as _____.

Select the correct option

○	$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$
○	$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho u)/\partial z = 0$
○	$\partial\rho/\partial t + \rho\partial(u)/\partial x + \rho\partial(\rho v)/\partial y + \rho\partial(u)/\partial z = 0$
●	$\partial\rho/\partial t + \partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z = 0$ 

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MTH642:Quiz 3

Question # 2 of 5 (Start time: 11:17:27 PM, 09 March 2022)

The Froude number (Fr) is equal to ____.

Select the correct option

- ○ $Fr = \frac{V}{\sqrt{gL}}$ ✓ Topic 111, pg117
- ● $Fr = \frac{V}{gL}$
- ○ $Fr = \frac{gL}{V}$
- ○ $Fr = \frac{\sqrt{gL}}{V}$

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MTH642 Mega File Final Term

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+ Husnain Ali Naqvi

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Topic No. 70

Bernoulli Equation

- **Bernoulli equation**: An approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

- Mathematically, we can be expressed as follows: We take the velocity V of a fluid particle to be a function of s and t. Taking the total differential of $V(s, t)$

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \qquad \text{and} \qquad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

In steady flow $\partial V / \partial t = 0$ and thus $V = V(s)$, and the acceleration in the s-direction becomes

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s} V = V\frac{dV}{ds}$$

Where $V = ds / dt$

Steady flow:

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a stream line)}$$

Steady, incompressible flow:

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a stream line)}$$

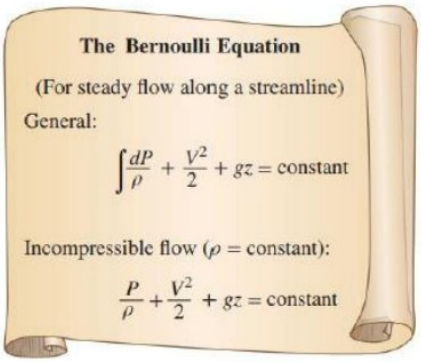
- The Bernoulli equation can also be written between any two points on the same streamline as

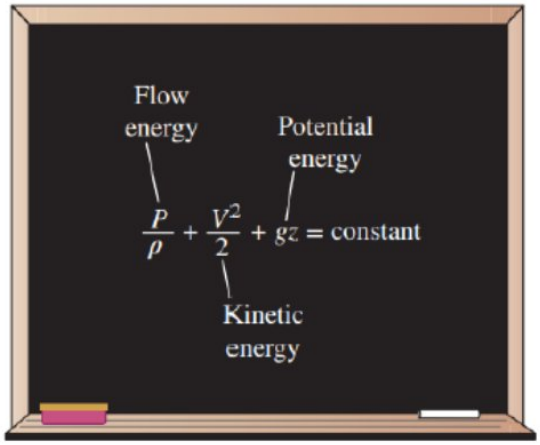
Steady, incompressible flow: $$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

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Topic No. 72

Derivation of Bernoulli Equation-2





Topic No. 73

Force Balance across Streamlines

- Force balance in the direction *n* normal to the streamline yields the following relation applicable *across* the streamlines for steady, incompressible flow:

$$\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{constant} \quad \text{(across streamlines)}$$

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Unsteady, Compressible Flow

- The Bernoulli equation for unsteady, compressible flow:

Unsteady compressible flow: $$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = \text{constant}$$

Topic No. 74

Static, Dynamic, and Stagnation Pressure

- The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density ρ.

$$\rho\left(\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a stream line)}\right)$$

$$\Rightarrow P + \rho\frac{V^2}{2} + \rho gz = \text{constant (along a streamline)}$$

- **Stagnation pressure:** The sum of the static and dynamic pressures is called the **stagnation pressure**. and it is expressed as

$$P_{stag} = P + \rho\frac{V^2}{2} \qquad \text{(kPa)}$$

- As the speed of a fluid increases, the pressure in the fluid decreases.

$$P \propto \frac{1}{v}$$

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- Bernoulli's Equation for any point along a flow tube or streamline

$$p+\frac{1}{2}\rho v^2+\rho gh=\text{constant}$$

Dimensions:

$$p[Pa]=\left[Nm^{-2}\right]=\left[Nmm^{-3}\right]=\left[Jm^{-3}\right]$$

$$\frac{1}{2}\rho v^2\left[kgm^{-3}m^2s^{-2}\right]=\left[kgm^{-1}s^{-2}\right]=\left[Nmm^{-3}\right]=\left[Jm^{-3}\right]$$

$$\rho gh\left[kgm^{-3}ms^{-2}m\right]=\left[kgms^{-2}mm^{-3}\right]=\left[Nmm^{-3}\right]=\left[Jm^{-3}\right]$$

Each term has the dimensions of energy/volume or energy density.

$\frac{1}{2}\rho v^2$ KE of bulk motion of fluid

ρgh GPE for location of fluid

p Pressure energy density arising from internal forces within moving fluid (similar to energy stored in a spring)

Topic No. 77

Applications of the Bernoulli's Equation-2

$$\frac{p}{\rho}+\frac{1}{2}v^2+gh=\text{constant}$$

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Assume liquid behaves as an ideal fluid and that Bernoulli's equation can be applied

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

A small hole is at level (2) and the water level (1) drops slowly $\Rightarrow v_1 = 0$

$$p_1 = p_{atm} \quad , \quad p_2 = p_{atm}$$

$$\rho g y_1 = \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$v_2^2 = 2g(y_1 - y_2) = 2gh \quad , \quad h = (y_1 - y_2)$$

$$v_2 = \sqrt{2gh} \qquad \text{Torricelli formula } (1608-1647)$$

This is the same velocity as a particle falling freely through a height h.

Topic No. 81

Applications of the Bernoulli's Equation-6

A large artery in a body has an inner radius of 4.00×10^{-3} m. Blood flows through the artery at the rate of 1.00×10^{-6} m^3s^{-1}. The blood has a viscosity of 2.084×10^{-3} Pa.s and a density of 1.06×10^{3} kgm^{-3}.

Calculate:

(i) The average blood velocity in the artery.

(ii) The pressure drop in a 0.100 m segment of the artery.

(iii) The Reynolds number for the blood flow.

Briefly discuss each of the following:

(iv) The velocity profile across the artery (diagram may be hopeful).

(v) The pressure drop along the segment of the artery.

(vi) The significance of the value of the Reynolds number calculated in part (iii).

Solution:

Radius R = 4.00×10^{-3} m

Volume flow rate Q = 1.00×10^{-6} m^3s^{-1}

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Viscosity of blood $\eta = 2.084\times10^{-3}$ Pa.s

Density of blood $\rho = 1.06\times10^{3}\,kgm^{-3}$



(i) Equation of continuity: $Q = Av$

$$A = \pi R^2 = \pi(4.00\times10^{-3})^2 = 5.03\times10^{-5}\,m^2$$

$$v = \frac{Q}{A} = \frac{1.00\times10^{-6}}{5.03\times10^{-5}} = 1.99\times10^{-2}\,ms^{-1}$$

Topic No. 82

Applications of the Bernoulli's Equation-7

(ii) Poiseuille's Equation:

$$Q = \frac{\Delta P \pi R^4}{8\eta L} \qquad L = 0.100\,m$$

$$\Delta P = \frac{8\eta L Q}{\pi R^4}$$

$$\Delta P = \frac{(8)(2.084\times10^{-3})(0.1)(1.00\times10^{-6})}{(\pi)(4.00\times10^{-3})^4}\,Pa$$

$$\Delta P = 2.07\,Pa$$

(iii) Reynolds Number:

$$R_e = \frac{\rho v L}{\eta} \quad \text{where } L = 2R \text{ (diameter of artery)}$$

$$R_e = \frac{(1.060\times10^{3})(1.99\times10^{-3})(2)(4.00\times10^{-3})}{2.084\times10^{-3}} = 8.1$$

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Topic No. 84

Newton's Laws of Motion-2

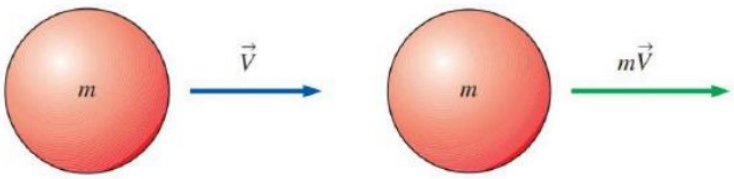
- Newton's laws:
- Relations between motions of bodies and the forces acting on them.
- Newton's first law: A body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.
- Therefore, a body tends to preserve its state of inertia.
- Newton's second law:
- The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.
- Newton's third law:
- When a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.

$$\text{Newton's second law:} \quad \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d(m\vec{V})}{dt}$$

Topic No. 85

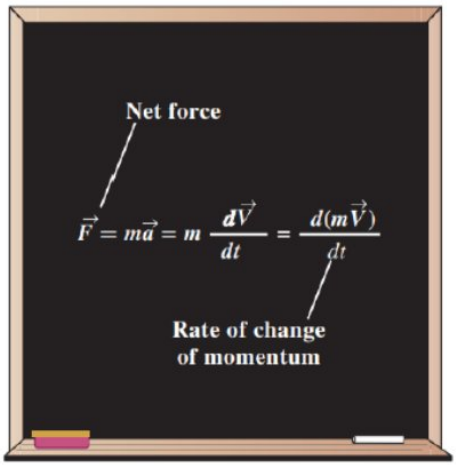
Newton's Laws of Motion-3

- Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero.



- Linear momentum is the product of mass and velocity, and its direction is the direction of velocity.

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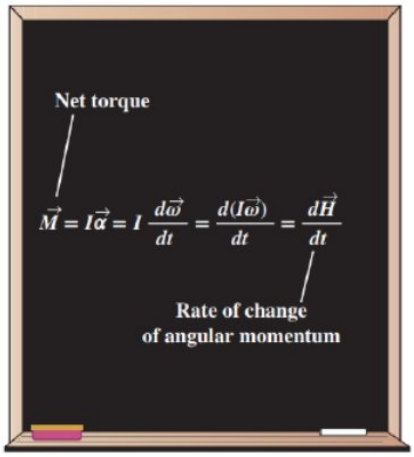
- Newton's second law is also expressed as the rate of change of the momentum of a body is equal to the net force acting on it.

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

Angular momentum about x-axis: $$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

The conservation of angular momentum principle: The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved.

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- The rate of change of the angular momentum of a body is equal to the net torque acting on it.

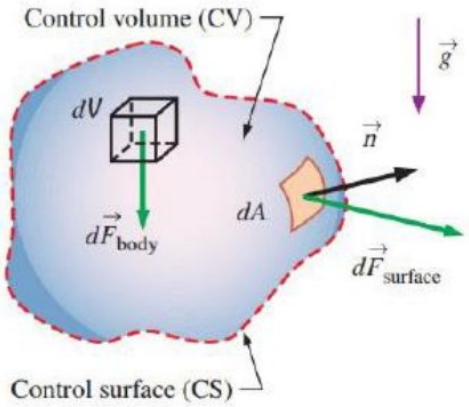
Topic No. 86

Forces Acting On a Control Volume-1

- The forces acting on a control volume consist of **body forces** that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and **surface forces** that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact).

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Total force acting on control volume: $\sum \vec{F} = \sum \vec{F}_{\text{body}} + \sum \vec{F}_{\text{surface}}$



The total force acting on a control volume is composed of body forces and surface forces; body force is shown on a differential volume element, and surface force is shown on a differential surface element.

The most common body force is that of **gravity**, which exerts a downward force on every differential element of the control volume.

Gravitational force acting on a fluid element: $d\vec{F}_{\text{gravity}} = \rho \vec{g}\, dV$

Gravitational vector in Cartesian coordinates: $\vec{g} = -g\vec{k}$

Total body force acting on control volume: $\sum \vec{F}_{\text{body}} = \int_{\text{CV}} \rho \vec{g}\, dV = m_{\text{CV}} \vec{g}$

Total force: $\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface forces}}$

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- **Normal stresses** are composed of pressure (which always acts inwardly normal) and viscous stresses.

Shear stresses are composed entirely of viscous stresses.

Total force: $$\underbrace{\sum \vec{F}}_{\text{total force}} = \underbrace{\sum \vec{F}_{\text{gravity}}}_{\text{body force}} + \underbrace{\sum \vec{F}_{\text{pressure}} + \sum \vec{F}_{\text{viscous}} + \sum \vec{F}_{\text{other}}}_{\text{surface forces}}$$

The Linear Momentum Equations

Newton's second law for a system of mass m subjected to net force $\Sigma\vec{F}$ is expressed as

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V})$$

where $m\vec{V}$ is the **linear momentum** of the system. Noting that both the density and velocity may change from point to point within the system, Newton's second law can be expressed more generally as

$$\sum \vec{F} = \frac{d}{dt}\int_{\text{sys}} \rho\vec{V}\, dV$$

- Newton's second law can be stated as the sum of all external forces acting on a system is equal to the time rate of change of linear momentum of the system.

The Reynolds transport theorem is expressed for linear momentum as

$$\frac{d(m\vec{V})_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \rho\vec{V}\, dV + \int_{\text{CS}} \rho\vec{V}(\vec{V}_r\cdot\vec{n})\, dA$$

General: $$\sum \vec{F} = \frac{d}{dt}\int_{\text{CV}} \rho\vec{V}\, dV + \int_{\text{CS}} \rho\vec{V}(\vec{V}_r\cdot\vec{n})\, dA$$

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Special Cases

Steady flow: $$\sum \vec{F} = \int_{CS} \rho \vec{V} (\vec{V}_r \cdot \vec{n})\, dA$$

Mass flow rate across an inlet or outlet: $$\dot{m} = \int_{A_c} \rho (\vec{V} \cdot \vec{n})\, dA_c = \rho V_{avg} A_c$$

- The rotational motion: A motion during which all points in the body move in circles about the axis of rotation.
- Rotational motion is described with angular quantities such as angular distance θ, angular velocity $\vec{\omega}$, and angular acceleration $\vec{\alpha}$.

- Newton's second law requires that there must be a force acting in the tangential direction to cause angular acceleration.
- The strength of the rotating effect, called the moment or torque, is proportional to the magnitude of the force and its distance from the axis of rotation.
- The perpendicular distance from the axis of rotation to the line of action of the force is called the moment arm, and the magnitude of torque M acting on a point mass m at normal distance r from the axis of rotation is expressed as
- Torque

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

Angular velocity versus rpm: $$\omega = 2\pi\dot{n}\ (\text{rad/min}) = \frac{2\pi\dot{n}}{60}\ (\text{rad/s})$$

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Shaft power: $\dot{W}_{\text{shaft}} = \omega M = 2\pi \dot{n} M$

Rotational kinetic energy: $\text{KE}_r = \frac{1}{2} I\omega^2$

- An important class of fluid devices, called turbo machines, which include centrifugal pumps, turbines, and fans, is analyzed by the angular momentum equation.
- A force whose line of action passes through point O produces zero moment about point O.

Radial-Flow Devices

- Radial-flow devices: Many rotary-flow devices such as centrifugal pumps and fans involve flow in the radial direction normal to the axis of rotation.
- Axial-flow devices are easily analyzed using the linear momentum equation.
- Radial-flow devices involve large changes in angular momentum of the fluid and are best analyzed with the help of the angular momentum equation.

Dimensions and Units

- **Dimension:** A measure of a physical quantity (without numerical values).
- **Unit:** A way to assign a *number* to that dimension.
- There are seven **primary dimensions** (also called **fundamental** or **basic dimensions**) mass, length, time, temperature, electric current, amount of light, and amount of matter.
- All nonprimary dimensions can be formed by some combination of the seven primary dimensions.

Dimensions of force: $\{\text{Force}\} = \left\{ \text{Mass} \frac{\text{Length}}{\text{Time}^2} \right\} = \{\text{mL/t}^2\}$

length is a dimension, but centimeter is a unit.

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EXAMPLE 7–2 **Dimensional Homogeneity of the Bernoulli Equation**

Bernoulli equation:
$$P + \frac{1}{2}\rho V^2 + \rho g z = C \quad (1)$$

Analysis (*a*) Each term is written in terms of primary dimensions,

$$\{P\} = \{\text{Pressure}\} = \left\{\frac{\text{Force}}{\text{Area}}\right\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\,\frac{1}{\text{Length}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

$$\left\{\frac{1}{2}\rho V^2\right\} = \left\{\frac{\text{Mass}}{\text{Volume}}\left(\frac{\text{Length}}{\text{Time}}\right)^2\right\} = \left\{\frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

$$\{\rho g z\} = \left\{\frac{\text{Mass}}{\text{Volume}}\,\frac{\text{Length}}{\text{Time}^2}\,\text{Length}\right\} = \left\{\frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

Primary dimensions of the Bernoulli constant:
$$\{C\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

Normalized Equation:

- If the nondimensional terms in the equation are of order unity, the equation is called **normalized**.
- Each term in a nondimensional equation is dimensionless.

Equation of motion:
$$\frac{d^2z}{dt^2} = -g$$

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Dimensional variables:

- Dimensional quantities that change or vary in the problem. Examples: z (dimension of length) and t (dimension of time).

Nondimensional (or dimensionless) variables

- Quantities that change or vary in the problem, but have no dimensions. Example: Angle of rotation, measured in degrees or radians which are dimensionless units.

Froude number: $$\mathrm{Fr} = \frac{w_0}{\sqrt{gz_0}}$$

Dimensional Analysis and Similarity-1

- In most experiments, to save time and money, tests are performed on a geometrically scaled **model**, rather than on the full-scale **prototype**.

There are three necessary conditions for complete similarity between a model and a prototype.

- The first condition is **geometric similarity**, the model must be the same shape as the prototype, but may be scaled by some constant scale factor.
- The second condition is **kinematic similarity**, which means that the velocity at any point in the model flow must be proportional (by a constant scale factor) to the velocity at the corresponding point in the prototype flow.
- The third and most restrictive similarity condition is **dynamic similarity**, when all forces in the model flow scale by a constant factor to corresponding forces in the prototype flow (force-scale equivalence).

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Buckingham Pi Theorem

Step 1:

- List all the dimensional parameters involved
- Let n be the number of parameters
- Example: For drag on a sphere, F, V, D, ρ, μ & n = 5

Step 2:

- Select a set of fundamental (primary) dimensions.
- For Example M(kg), L(m), t(sec).
- Example: For drag on a sphere choose MLt

Step 3:

- List the dimensions of all parameters
- Let r be the number of primary dimensions.
- Example: For drag on a sphere r = 3

Step 4:

- Select a set of r dimensional parameters that includes all the primary dimensions.
- Example: For drag on a sphere (m = r = 3) select ρ, V, D

Step 5:

- Set up dimensionless groups π^s
- There will be n - m equations
- Example: For drag on a sphere

Step 6:

- Check to see that each group obtained is dimensionless
- Example: For drag on a sphere

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Significant Dimensionless Groups in Fluid Mechanics

Reynolds Number

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

Mach Number

$$M = \frac{V}{c}$$

Froude Number

$$Fr = \frac{V}{\sqrt{gL}}$$

Weber Number

$$We = \frac{\rho V^2 L}{\sigma}$$

Euler Number

$$Eu = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

Cavitation Number

$$Ca = \frac{p - p_v}{\frac{1}{2}\rho V^2}$$

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Divergence theorem:

$$\int_V \vec{\nabla}\cdot\vec{G}\,dV = \oint_A \vec{G}\cdot\vec{n}\,dA$$

$$0 = \int_{CV} \frac{\partial \rho}{\partial t}\,dV + \int_{CV} \vec{\nabla}\cdot(\rho\vec{V})\,dV$$

$$\int_{CV} \left[\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V})\right] dV = 0$$

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$$

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The divergence operation in Cartesian and cylindrical coordinates.

We expand equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$$

by using the product rule on the divergence term,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = \underbrace{\frac{\partial \rho}{\partial t} + \vec{V}\cdot\vec{\nabla}\rho}_{\text{Material derivative of } \rho} + \rho\vec{\nabla}\cdot\vec{V} = 0$$

Alternative form of the continuity equation:

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \vec{\nabla}\cdot\vec{V} = 0$$

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Topic No. 124

THE CONTINUITY EQUATION: Special Cases of the Continuity Equation

Special Case 1: Steady Compressible Flow

Steady continuity equation: $\vec{\nabla}\cdot(\rho\vec{V}) = 0$

In Cartesian coordinates, this reduces to

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

In cylindrical coordinates, it reduces to

$$\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$$

Special Case 2: Incompressible Flow

Incompressible continuity equation: $\vec{\nabla}\cdot\vec{V} = 0$

Since $\partial\rho / \partial t \cong 0$

Incompressible continuity equation in Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible continuity equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$$

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Topic No. 127

Examples of Application of the Continuity Equation

Example: Incompressibility of an Unsteady Two-Dimensional Flow

Consider the velocity field of Example 4–5—an unsteady, two-dimensional velocity field given by $\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + [1.5 + 2.5 \sin(\omega t) - 0.8y]\vec{j}$, where angular frequency ω is equal to 2π rad/s (a physical frequency of 1 Hz). Verify that this flow field can be approximated as incompressible.

SOLUTION We are to verify that a given velocity field is incompressible.

Assumptions **1** The flow is two-dimensional, implying no z-component of velocity and no variation of u or v with z.

Analysis The components of velocity in the x- and y-directions, respectively, are

$$u = 0.5 + 0.8x \quad \text{and} \quad v = 1.5 + 2.5 \sin(\omega t) - 0.8y$$

If the flow is incompressible, Eq. 9–16 must apply. More specifically, in Cartesian coordinates Eq. 9–17 must apply. Let's check:

$$\underbrace{\frac{\partial u}{\partial x}}_{0.8} + \underbrace{\frac{\partial v}{\partial y}}_{-0.8} + \underbrace{\frac{\partial \cancel{w}}{\cancel{\partial z}}}_{\text{0 since 2-D}} = 0 \quad \rightarrow \quad 0.8 - 0.8 = 0$$

So we see that the incompressible continuity equation is indeed satisfied at any instant in time, and **this flow field may be approximated as incompressible.**

Discussion Although there is an unsteady term in v, it has no y-derivative and drops out of the continuity equation.

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Topic No. 128

Examples of Application of the Continuity Equation

Three-Dimensional Flow

Example: Finding a Missing Velocity Component

Two velocity components of a steady, incompressible, three-dimensional flow field are known, namely, $u = ax^2 + by^2 + cz^2$ and $w = axz + byz^2$, where a, b, and c are constants. The y velocity component is missing (Fig. 9–14). Generate an expression for v as a function of x, y, and z.

SOLUTION We are to find the y-component of velocity, v, using given expressions for u and w.

Assumptions **1** The flow is steady. **2** The flow is incompressible.

Analysis Since the flow is steady and incompressible, and since we are working in Cartesian coordinates, we apply Eq. 9–17 to the flow field,

Condition for incompressibility:

$$\frac{\partial v}{\partial y} = -\underbrace{\frac{\partial u}{\partial x}}_{2ax} - \underbrace{\frac{\partial w}{\partial z}}_{ax + 2byz} \rightarrow \frac{\partial v}{\partial y} = -3ax - 2byz$$

Next we integrate with respect to y. Since the integration is a *partial* integration, we add some arbitrary function of x and z instead of a simple constant of integration.

Solution: $$v = -3axy - by^2z + f(x,z)$$

Discussion Any function $f(x,z)$ yields a v that satisfies the incompressible continuity equation, since there are no derivatives of v with respect to x or z in the continuity equation.

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Topic No. 129

Examples of Application of the Continuity Equation

Example: Two-Dimensional, Incompressible, Vertical Flow

Consider a two-dimensional, incompressible flow in cylindrical coordinates; the tangential velocity component is $u_\theta = K/r$, where K is a constant. This represents a class of vortical flows. Generate an expression for the other velocity component, u_r.

SOLUTION For a given tangential velocity component, we are to generate an expression for the radial velocity component.

Assumptions **1** The flow is two-dimensional in the xy- ($r\theta$-) plane (velocity is not a function of z, and $u_z = 0$ everywhere). **2** The flow is incompressible.

Analysis The incompressible continuity equation (Eq. 9–18) for this two-dimensional case simplifies to

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \underbrace{\frac{\partial u_z}{\partial z}}_{0\ (2\text{-D})} = 0 \quad \rightarrow \quad \frac{\partial(ru_r)}{\partial r} = -\frac{\partial u_\theta}{\partial \theta} \tag{1}$$

The given expression for u_θ is not a function of θ, and therefore Eq. 1 reduces to

$$\frac{\partial(ru_r)}{\partial r} = 0 \quad \rightarrow \quad ru_r = f(\theta, t) \tag{2}$$

where we have introduced an arbitrary function of θ and t instead of a constant of integration, since we performed a *partial* integration with respect to r. Solving for u_r,

$$u_r = \frac{f(\theta, t)}{r} \tag{3}$$

Thus, *any radial velocity component of the form given by Eq. 3 yields a two-dimensional, incompressible velocity field that satisfies the continuity equation.*

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Topic No. 130

Examples of Application of the Continuity Equation

Example: Conditions for Incompressible Flow

Consider a steady velocity field given by $\vec{V} = (u, v, w) = a(x^2y + y^2)\vec{i} + bxy^2\vec{j} + cx\vec{k}$, where a, b, and c are constants. Under what conditions is this flow field incompressible?

SOLUTION We are to determine a relationship between constants a, b, and c that ensures incompressibility.

Assumptions **1** The flow is steady. **2** The flow is incompressible (under certain constraints to be determined).

Analysis We apply Eq. 9–17 to the given velocity field,

$$\underbrace{\frac{\partial u}{\partial x}}_{2axy} + \underbrace{\frac{\partial v}{\partial y}}_{2bxy} + \underbrace{\frac{\partial w}{\partial z}}_{0} = 0 \quad \rightarrow \quad 2axy + 2bxy = 0$$

Thus to guarantee incompressibility, constants a and b must be equal in magnitude but opposite in sign.

Condition for incompressibility: $\boldsymbol{a = -b}$

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Topic No. 131

The Stream Function

The Stream Function in Cartesian Coordinates

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

A clever variable transformation enables us to rewrite above equation in terms of *one* dependent variable (ψ) instead of *two* dependent variables (u and v). We define the **stream function** ψ

Incompressible, two-dimensional stream function in Cartesian coordinates:

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

The stream function and the corresponding velocity potential function were first introduced by the Italian mathematician Joseph Louis Lagrange (1736–1813). Substitution of above equations yields

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = \frac{\partial^2 \psi}{\partial x\, \partial y} - \frac{\partial^2 \psi}{\partial y\, \partial x} = 0$$

which is identically satisfied for any smooth function $\psi(x, y)$, because the order of differentiation (y then x versus x then y) is irrelevant.

Along a streamline: $\quad \frac{dy}{dx} = \frac{v}{u} \quad \rightarrow \quad \underbrace{-v}_{\partial\psi/\partial x}\, dx + \underbrace{u}_{\partial\psi/\partial y}\, dy = 0$

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The Stream Function

Example: Calculation of the Velocity Field from the Stream Function

A steady, two-dimensional, incompressible flow field in the xy-plane has a stream function given by $\psi = ax^3 + by + cx$, where a, b, and c are constants: $a = 0.50\ (\text{m}\cdot\text{s})^{-1}$, $b = -2.0$ m/s, and $c = -1.5$ m/s. (a) Obtain expressions for velocity components u and v. (b) Verify that the flow field satisfies the incompressible continuity equation. (c) Plot several streamlines of the flow in the upper-right quadrant.

SOLUTION For a given stream function, we are to calculate the velocity components, verify incompressibility, and plot flow streamlines.

Assumptions **1** The flow is steady. **2** The flow is incompressible (this assumption is to be verified). 3 The flow is two-dimensional in the xy-plane, implying that $w = 0$ and neither u nor v depend on z.

Analysis (a) We use Eq. 9–20 to obtain expressions for u and v by differentiating the stream function,

$$u = \frac{\partial \psi}{\partial y} = b \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} = -3ax^2 - c$$

Equation for a streamline:

$$y = \frac{\psi - ax^3 - cx}{b}$$

The Stream Function

Example: Calculation of Stream Function for a Known Velocity Field

Consider a steady, two-dimensional, incompressible velocity field with $u = ax + b$ and $v = -ay + cx$, where a, b, and c are constants: $a = 0.50\ s^{-1}$, $b = 1.5$ m/s, and $c = 0.35\ s^{-1}$. Generate an expression for the stream function and plot some streamlines of the flow in the upper-right quadrant.

SOLUTION For a given velocity field we are to generate an expression for ψ and plot several streamlines for given values of constants a, b, and c.

$$\frac{\partial \psi}{\partial y} = u = ax + b$$

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$$\psi = axy + by + g(x)$$

$$v = -\frac{\partial \psi}{\partial x} = -ay - g'(x)$$

Topic No. 134

The Stream Function in Cylindrical Coordinates

$$\frac{\partial (ru_r)}{\partial r} + \frac{\partial (u_\theta)}{\partial \theta} = 0$$

Incompressible, planar stream function in cylindrical coordinates:

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{and} \quad u_\theta = -\frac{\partial \psi}{\partial r}$$

$$\frac{1}{r}\frac{\partial (ru_r)}{\partial r} + \frac{\partial (u_z)}{\partial z} = 0$$

Incompressible, axisymmetric stream function in cylindrical coordinates:

$$u_r = -\frac{1}{r}\frac{\partial \psi}{\partial z} \quad \text{and} \quad u_z = \frac{1}{r}\frac{\partial \psi}{\partial r}$$

Example: Stream Function in Cylindrical Coordinates

Consider a line vortex, defined as steady, planar, incompressible flow in which the velocity components are $u_r = 0$ and $u_\theta = K/r$, where K is a constant. This flow is represented in Fig. 9–15*a*. Derive an expression for the stream function $\psi(r, \theta)$, and prove that the streamlines are circles.

SOLUTION For a given velocity field in cylindrical coordinates, we are to derive an expression for the stream function and show that the streamlines are circular.

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$$\frac{\partial \psi}{\partial r} = -u_\theta = -\frac{K}{r} \quad \rightarrow \quad \psi = -K \ln r + f(\theta) \tag{1}$$

Now we use the other component of Eq. 9–27,

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r} f'(\theta) \tag{2}$$

$$f'(\theta) = 0 \quad \rightarrow \quad f(\theta) = C$$

$$\boldsymbol{\psi = -K \ln r + C} \tag{3}$$

Topic No. 137

The Navier-Stokes Equation: Introduction

the stress tensor σ_{ij} contains nine components, six of which are independent (because of symmetry).

fluid is at rest, the only stress acting at any surface of any fluid element is pressure P, which always acts inward and normal to the surface.

Fluid at rest:
$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -P \end{pmatrix}$$

Moving fluids:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -P \end{pmatrix} + \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

τ_{ij}, called the **viscous stress tensor** or the **deviatoric stress tensor**.

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Mechanical pressure is the mean normal stress acting inwardly on a fluid element. It is therefore also called **mean pressure.**

Mechanical pressure: $$P_m = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

Topic No. 138

Newtonian versus Non-Newtonian Fluids

- The study of the deformation of flowing fluids is called **rheology.**
- **Newtonian fluids**, defined as fluids for which the stress tensor is linearly proportional to the strain rate tensor.
- Fluids for which the stress tensor is not linearly related to the strain rate tensor are called **non-Newtonian fluids.**
- A fluid that returns (partially) to its original shape after the applied stress is released is called **viscoelastic.**

Some non-Newtonian fluids are called **shear thinning fluids** or **pseudo plastic fluids**, because the more the fluid is sheared, the less viscous it becomes.
Plastic fluids are those in which the shear thinning effect is extreme.
In some fluids a finite stress called the **yield stress** is required before the fluid begins to flow at all; such fluids are called **Bingham plastic fluids**.
Shear thickening fluids or **dilatant fluids**; the more the fluid is sheared, the more viscous it becomes.

Topic No. 143

What is a Boundary Layer?

- A **boundary layer** is a layer of flowing fluid near a boundary where viscosity keeps the flow velocity close to that of the boundary.

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Mcqz

1 Bernoulli's Equation Dimensions $\left\{\frac{m}{t^2 L}\right\}$ is k 2 mcqz thy topic 98,99

2 Linear momentum or just the momentum of the body: The product of the of a body mass and

the velocity topic 84

3 The total force acting topic 86

4 $\bar{\nabla}.\bar{v} = 0$ Steady and ncompressible Flow topic 124

5 2 quiz linear momentum pr

6 mass number Dimensions {0}

7 the momentum of force F about a point O is the vector product topic 91

1 The Angular Momentum Equation topic 91 short

2 5 number question : equation of motion $\frac{d^2z}{dt^2} = -g$ and dimensional result

$z = z_0 + w_0 t - \frac{1}{2} g t^2$ define intial condition ? topic 101

3 short question Reynolds Number ? topic 111

4 short continuity equation? Topic 119

5 5 number question : Two-Dimensional, Incompressible and : three-Dimensional, Incompressible, V example?

Topic 127, 128, 129, 130

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Example 8

- The radial velocity component in an incompressible, two dimensional flow field $(v_z = 0)$ is

$$v_r = 2r + 3r^2 \sin\theta$$

- Determine the corresponding tangential velocity component, v_θ, required to satisfy conservation of mass.

Solution.

- The continuity equation for incompressible steady flow in cylindrical coordinates is given by

$$\frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Since $v_z = 0$,

$$\frac{\partial v_\theta}{\partial \theta} = -\frac{\partial (r v_r)}{\partial r}$$

30

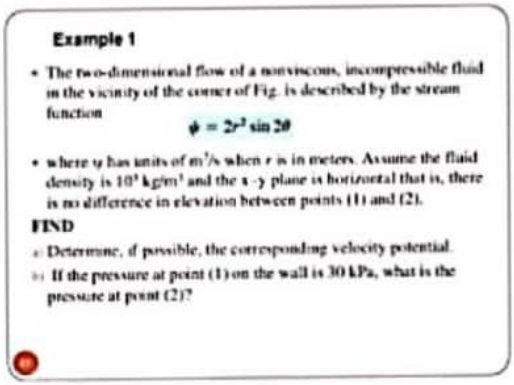
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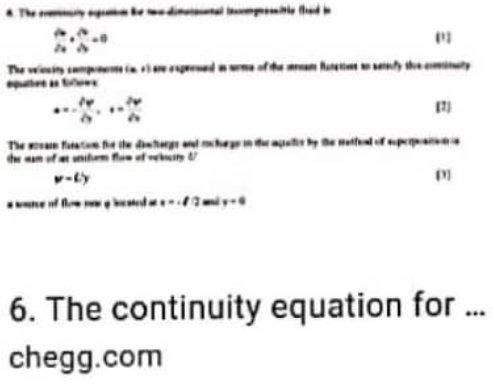
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$$u = \frac{\partial}{\partial x}\left[K\left(x^2 - y^2\right) + C\ln\left(x^2 + y^2\right)\right]$$

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Example 4

- Consider the following steady, three-dimensional velocity field in Cartesian coordinates:

$$\vec{V} = (u, v, w) = (axy^2 - b)\vec{i} + cy^3\vec{j} + dxy\vec{k}$$

where *a, b, c, and d* are constants. Under what conditions is this flow field incompressible?

Solution

Condition for incompressibility:

$$\underbrace{\frac{\partial u}{\partial x}}_{ay^2} + \underbrace{\frac{\partial v}{\partial y}}_{3cy^2} + \underbrace{\frac{\partial w}{\partial z}}_{0} = 0 \qquad ay^2 + 3cy^2 = 0$$

- Thus to guarantee incompressibility, constants *a and c* must satisfy the following relationship:

$$a = -3c$$

25

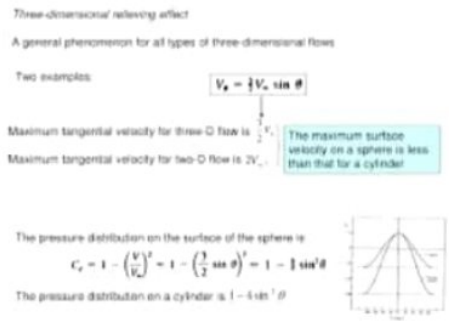
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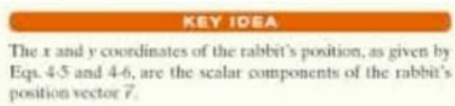


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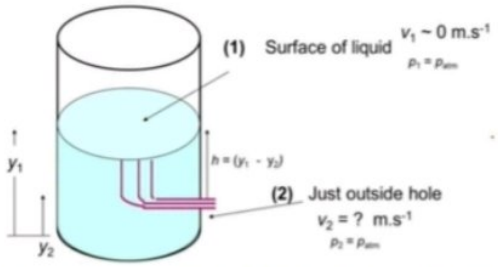




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- Viscosity ⇒ frictional forces ⇒ energy lost by heating fluid ⇒ increase in interval energy of fluid ⇒ increase in temperature of fluid.
- Where does the energy come from?
- Equation of continuity still applies ⇒ energy cannot come from KE of fluid ⇒ energy must be provided from the energy of fluid ⇒ pressure drop along tube.



Assume liquid behaves as an ideal fluid and that Bernoulli's equation can be applied

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

A small hole is at level (2) and the water level (1) drops slowly ⇒ $v_1 = 0$

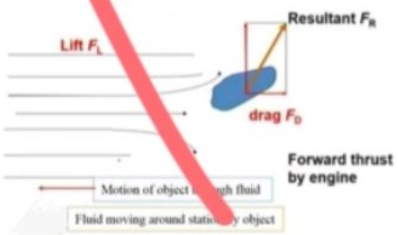
$$p_1 = p_{atm} \quad , \quad p_2 = p_{atm}$$
$$\rho g y_1 = \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$
$$v_2^2 = 2g(y_1 - y_2) = 2gh \quad , \quad h = (y_1 - y_2)$$
$$v_2 = \sqrt{2gh} \qquad \text{Torricelli formula (1608–1647)}$$

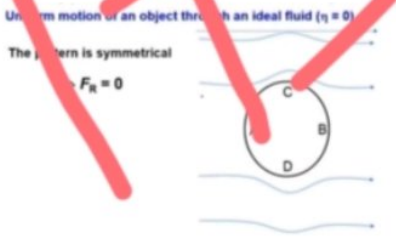
This is the same velocity as a particle falling freely through a height h

Topic No. 79

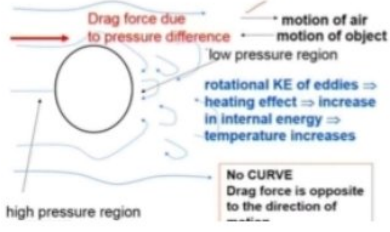
Applications of the Bernoulli's Equation-4

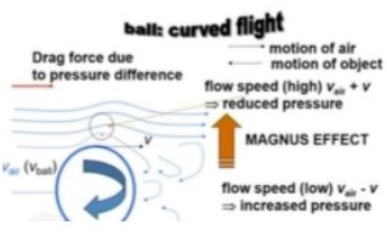
FORCES ACTING ON AN OBJECT MOVING THROUGH FLUID





- Why does a cricket ball swing or a baseball curve?
- Drag force
  - Frictional drag (viscosity)
  - Pressure drag (eddies – lower pressure)





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- Use control volume analysis to determine the moments caused by fluid flow and the torque transmitted

Topic No. 84

Newton's Laws of Motion-2

- Newton's laws:
- Relations between motions of bodies and the forces acting on them.
- Newton's first law: A body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.
- Therefore, a body tends to preserve its state of inertia.
- Newton's second law:
- The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.
- Newton's third law:
- When a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.
- Therefore, the direction of an exposed reaction force depends on the body taken as the system.

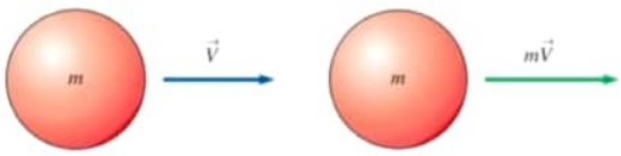
Newton's second law: $\vec{F} = m\vec{a} = m\dfrac{d\vec{V}}{dt} = \dfrac{d(m\vec{V})}{dt}$

- Linear momentum or just the momentum of the body: The product of the mass and the velocity of a body.
- Newton's second law is usually referred to as the linear momentum equation.

Topic No. 85

Newton's Laws of Motion-3

- Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero.



- Linear momentum is the product of mass and velocity, and its direction is the direction of velocity.

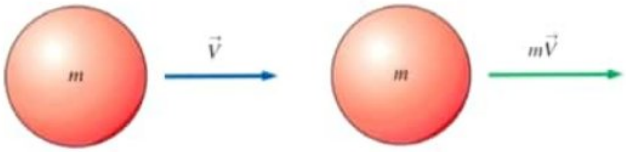
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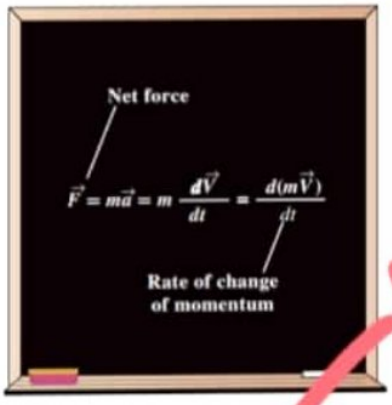
Topic No. 85

Newton's Laws of Motion-3

- Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero.



- Linear momentum is the product of mass and velocity, and its direction is the direction of velocity.



- Newton's second law is also expressed as the rate of change of the momentum of a body is equal to the net force acting on it.

The counterpart of Newton's second law for rotating rigid bodies is expressed as $\vec{M} = I\vec{\alpha}$, where $\vec{M}$ is the net moment or torque applied on the body, I is the moment of inertia of the body about the axis of rotation, and $\vec{\alpha}$ is the angular acceleration. It can also be expressed in terms of the rate of change of angular momentum $d\vec{H}/dt$ as

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

Angular momentum about x-axis: $$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

- The conservation of angular momentum principle: The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved.

MTH642 – Fluid Mechanics, Written by Mahar Afaq Safdar Muhammadi (MSc Math)



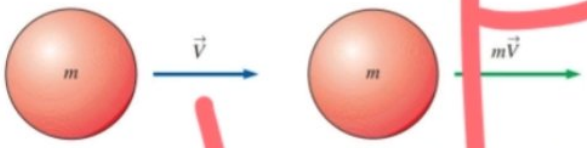
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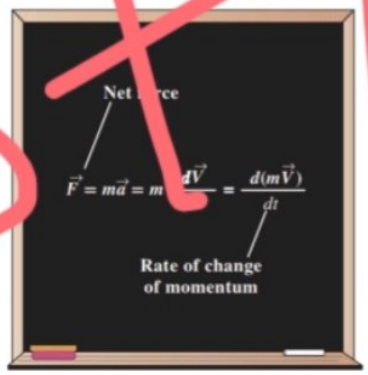
Topic No. 85

Newton's Laws of Motion-3

- Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero.



- Linear momentum is the product of mass and velocity, and its direction is the direction of velocity.



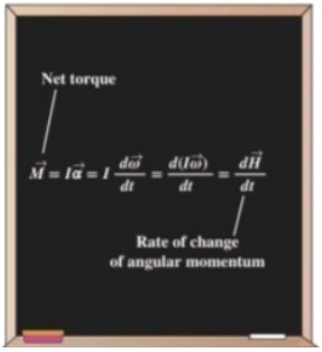
- Newton's second law is also expressed as the rate of change of the momentum of a body is equal to the net force acting on it.

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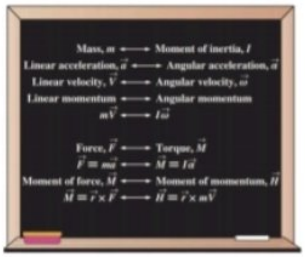
$$\textit{Angular momentum equation:} \qquad \vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

$$\textit{Angular momentum about x-axis:} \qquad M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

- The conservation of angular momentum principle: The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved.



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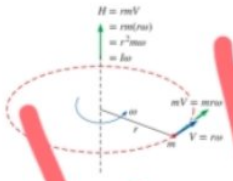
Analogy between corresponding linear and angular quantities.

Magnitude of angular momentum: $H = \int_{\text{mass}} r^2 \omega \, \delta m = \left[\int_{\text{mass}} r^2 \, \delta m\right] \omega = I\omega$

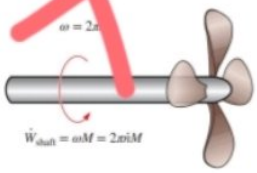
$$\vec{H} = I\vec{\omega}$$

Angular momentum equation: $\vec{M} = I\vec{\alpha} = I\dfrac{d\vec{\omega}}{dt} = \dfrac{d(I\vec{\omega})}{dt} = \dfrac{d\vec{H}}{dt}$

Angular velocity versus rpm: $\omega = 2\pi\dot{n}\ \text{(rad/min)} = \dfrac{2\pi\dot{n}}{60}\ \text{(rad/s)}$



Angular momentum of point mass rotating at angular velocity ω at distance r from the axis of rotation.



The relations between angular velocity, rpm, and the power transmitted through a rotating shaft.

$$\dot{W}_{\text{shaft}} = FV = Fr\omega = M\omega$$

Shaft power: $\dot{W}_{\text{shaft}} = \omega M = 2\pi\dot{n}M$

Rotational kinetic energy: $\text{KE}_r = \dfrac{1}{2} I\omega^2$

Topic No. 91

The Angular Momentum Equation-1

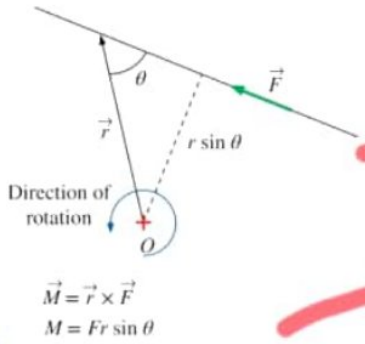
- Many engineering problems involve the moment of the linear momentum of flow streams, and the rotational effects caused by them.
- Such problems are best analyzed by the angular momentum equation, also called the moment of momentum equation.
- An important class of fluid devices, called turbo machines, which include centrifugal pumps, turbines, and fans, is analyzed by the angular momentum

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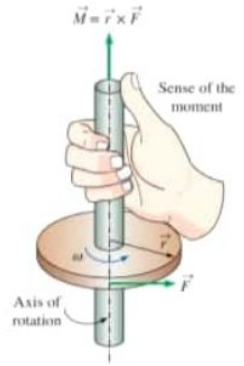
Topic No. 91

The Angular Momentum Equation-1

- Many engineering problems involve the moment of the linear momentum of flow streams, and the rotational effects caused by them.
- Such problems are best analyzed by the angular momentum equation, also called the moment of momentum equation.
- An important class of fluid devices, called turbo machines, which include centrifugal pumps, turbines, and fans, is analyzed by the angular momentum equation.
- A force whose line of action passes through point O produces zero moment about point O.



The moment of a force $\vec{F}$ about a point O is the vector product of the position vector $\vec{r}$ and $\vec{F}$.



The determination of the direction of the moment by the right-hand rule.

Moment of momentum: $\vec{H} = \vec{r} \times m\vec{V}$

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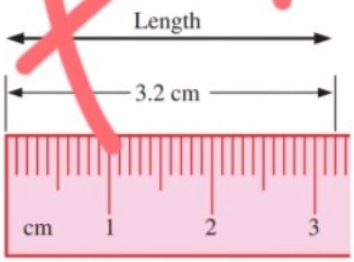
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Topic No. 97

Dimensions and Units

- **Dimension:** A measure of a physical quantity (without numerical values).
- **Unit:** A way to assign a *number* to that dimension.
- There are seven **primary dimensions** (also called **fundamental** or **basic dimensions**) mass, length, time, temperature, electric current, amount of light, and amount of matter.
- All nonprimary dimensions can be formed by some combination of the seven primary dimensions.

Dimensions of force: $\{\text{Force}\} = \left\{\text{Mass} \frac{\text{Length}}{\text{Time}^2}\right\} = \{\text{mL/t}^2\}$



A *dimension* is a measure of a physical quantity without numerical values, while a *unit* is a way to assign a number to the dimension. For example, length is a dimension, but centimeter is a unit.

Primary dimensions and their associated primary SI and English units

Dimension	Symbol*	SI Unit	English Unit
Mass	m	kg (kilogram)	lbm (pound-mass)
Length	L	m (meter)	ft (foot)
Time†	t	s (second)	s (second)
Temperature	T	K (kelvin)	R (rankine)
Electric current	I	A (ampere)	A (ampere)
Amount of light	C	cd (candela)	cd (candela)
Amount of matter	N	mol (mole)	mol (mole)

Topic No. 98 & 99

Dimensional Homogeneity

EXAMPLE 7–2 **Dimensional Homogeneity of the Bernoulli Equation**

Probably the most well-known (and most misused) equation in fluid mechanics is

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Topic No. 98 & 99

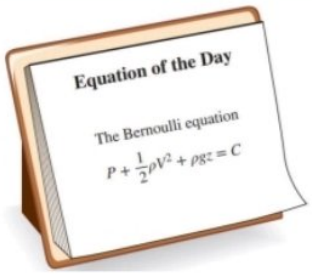
Dimensional Homogeneity

EXAMPLE 7–2 Dimensional Homogeneity of the Bernoulli Equation

Probably the most well-known (and most misused) equation in fluid m[illegible]s is the Bernoulli equation (Fig. 7–6), discussed in Chap. 5. One standard [illegible] of the Bernoulli equation for incompressible irrotational fluid flow is

Bernoulli equation: $$P + \frac{1}{2}\rho V^2 + \rho g z = C$$

(*a*) Verify that each additive term in the Bernoulli [illegible]uation has the same dimensions. (*b*) What are the dimensions of the constant C?



The Bernoulli equation is a good example of a dimensionally homogeneous equation. All additive terms, including the constant, have the *same* dimensions, namely that of pressure. In terms of primary dimensions, each term has dimensions $\{m/(t^2L)\}$.

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SOLUTION We are to verify that the primary dimensions of each additive term in Eq. 1 are the same, and we are to determine the dimensions of constant C.

Analysis (*a*) Each term is written in terms of primary dimensions.

$$\{P\} = \{\text{Pressure}\} = \left\{\frac{\text{Force}}{\text{Area}}\right\} = \left\{\text{Mass}\ \frac{\text{Length}}{\text{Time}^2}\ \frac{1}{\text{Length}^2}\right\} = \left\{\frac{m}{t^2 L}\right\}$$

$$\left\{\frac{1}{2}\rho V^2\right\} = \left\{\frac{\text{Mass}}{\text{Volume}}\left(\frac{\text{Length}}{\text{Time}}\right)^2\right\} = \left\{\frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2}\right\} = \left\{\frac{m}{t^2 L}\right\}$$

$$\{\rho g z\} = \left\{\frac{\text{Mass}}{\text{Volume}}\ \frac{\text{Length}}{\text{Time}^2}\ \text{Length}\right\} = \left\{\frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2}\right\} = \left\{\frac{m}{t^2 L}\right\}$$

Indeed, **all three additive terms have the same dimensions.**

(*b*) From the law of dimensional homogeneity, the constant must have the same dimensions as the other additive terms in the equation. Thus,

Primary dimensions of the Bernoulli constant: $$\{C\} = \left\{\frac{m}{t^2 L}\right\}$$

Discussion If the dimensions of any of the terms were different from the others, it would indicate that an error was made somewhere in the analysis.

Topic No. 100

Non Dimensionalization of Equations-1

Normalized Equation:

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Topic No. 100

Non Dimensionalization of Equations-1

Normalized Equation:

- If the nondimensional terms in the equation are of order unity, the equation is called **normalized**.
- Each term in a nondimensional equation is dimensionless.

Nondimensional Parameters:

- In the process of nondimensionalizing an equation of motion, **nondimensional parameters** often appear—most of which are named after a notable scientist or engineer (e.g., the Reynolds number and the Froude number).
- This process is referred to by some authors as **inspectional analysis**.

Topic No. 101

Non Dimensionalization of Equations-2

The nondimensionalized Bernoulli equation

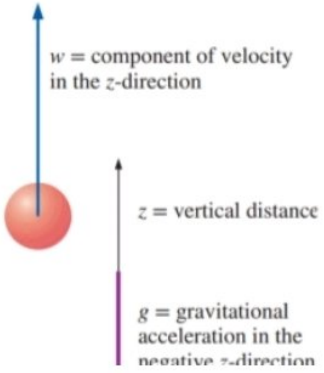
$$\underset{\{1\}}{\frac{P}{P_\infty}} + \underset{\{1\}}{\frac{\rho V^2}{2P_\infty}} + \underset{\{1\}}{\frac{\rho g z}{P_\infty}} = \underset{\{1\}}{\frac{C}{P_\infty}}$$

A nondimensionalized form of the Bernoulli equation is formed by dividing each additive term by a pressure (here we use P_∞). Each resulting term is dimensionless (dimensions of {1}).

Equation of motion:
$$\frac{d^2z}{dt^2} = -g$$

Dimensional result:
$$z = z_0 + w_0 t - \frac{1}{2} g t^2$$

- Object falling in a vacuum. Vertical velocity is shown positively so $w < 0$ for a falling object.



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- In such cases, care must be taken to properly scale the results. We introduce here a powerful technique called **dimensional analysis**.

The three primary purposes of dimensional analysis are

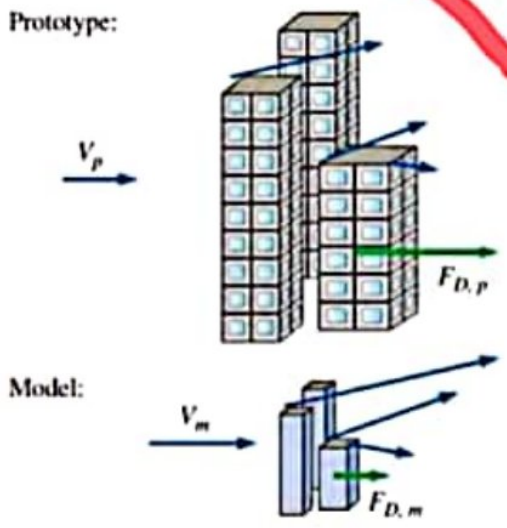
- To generate nondimensional parameters that help in the design of experiments (physical and/or numerical) and in the reporting of experimental results
- To obtain scaling laws so that prototype performance can be predicted from model performance
- To (sometimes) predict trends in the relationship between parameters

There are three necessary conditions for complete similarity between a model and a prototype.

- The first condition is **geometric similarity**, the model must be the same shape as the prototype, but may be scaled by some constant scale factor.
- The second condition is **kinematic similarity**, which means that the velocity at any point in the model flow must be proportional (by a constant scale factor) to the velocity at the corresponding point in the prototype flow.
- The third and most restrictive similarity condition is **dynamic similarity**, when all forces in the model flow scale by a constant factor to corresponding forces in the prototype flow (force-scale equivalence).

Topic No. 108

Dimensional Analysis and Similarity-2



Kinematic similarity is achieved when, at all locations, the speed in the model flow is proportional to that at corresponding locations in the prototype flow, and points in the same direction.

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Significant Dimensionless Groups in Fluid Mechanics

Reynolds Number

$$Re = \frac{\rho VL}{\mu} = \frac{VL}{\nu}$$

Mach Number

$$M = \frac{V}{c}$$

Froude Number

$$Fr = \frac{V}{\sqrt{gL}}$$

Weber Number

$$We = \frac{\rho V^2 L}{\sigma}$$

Euler Number

$$Eu = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

Cavitation Number

$$Ca = \frac{p - p_v}{\frac{1}{2}\rho V^2}$$

Topic No. 112

Differential Analysis of Fluid Flow; Introduction

- Continuity Equation
- Momentum Equation
- Euler's Equations
- Navier Stokes Equations
- Applications of the above equations
- Special cases
- Equations of Fluid Dynamics, Physical Meaning of the term
- Equations are based on the following physical principles:
- Mass is conserved
- Newton's Second Law: $\bar{F} = m\bar{a}$
- Control Volume Analysis
- The governing equations can be obtained in the integral form by choosing a

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$$\int_{CV} \frac{\partial \rho}{\partial t}\, dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

The net rate of change of mass within the control volume is equal to the rate at which mass flows into the control volume minus the rate at which mass flows out of the control volume.

THE CONTINUITY EQUATION: Derivation Using the Divergence Theorem

- The quickest and most straightforward way to derive the differential form of conservation of mass is to apply the divergence theorem (Gauss's theorem).

Divergence theorem:

$$\int_V \vec{\nabla}\cdot\vec{G}\, dV = \oint_A \vec{G}\cdot\vec{n}\, dA$$

$$0 = \int_{CV} \frac{\partial \rho}{\partial t}\, dV + \int_{CV} \vec{\nabla}\cdot(\rho\vec{V})\, dV$$

$$\int_{CV} \left[\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V})\right] dV = 0$$

Continuity equation:

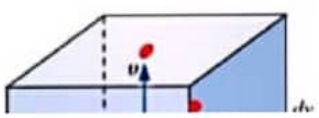
$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$$

- This equation is the compressible form of the continuity equation since we have not assumed incompressible flow. It is valid at any point in the flow domain.

Topic No. 120

THE CONTINUITY EQUATION: Derivation Using an Infinitesimal Control Volume

- A small box-shaped control volume centered at point *P* is used for derivation of the differential equation for conservation of mass in Cartesian c s; the red dots indicate the center of each face.



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Topic No. 124

THE CONTINUITY EQUATION: Special Cases of the Continuity Equation

Special Case 1: Steady Compressible Flow

Steady continuity equation:

$$\vec{\nabla}\cdot(\rho\vec{V}) = 0$$

In Cartesian coordinates, this reduces to

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

In cylindrical coordinates, it reduces to

$$\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0$$

Special Case 2: Incompressible Flow

Incompressible continuity equation:

$$\vec{\nabla}\cdot\vec{V} = 0$$

Since $\partial\rho / \partial t \equiv 0$

Incompressible continuity equation in Cartesian coordinates:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Incompressible continuity equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$$

Topic No. 125

THE CONTINUITY EQUATION: Examples of Application of the Continuity Equation

Example: Design of a compressible converging duct

A two-dimensional converging duct is being designed for a high-speed wind tu- nel. The bottom wall of the duct is to be flat and horizontal, and the top is to be curved in such a way that the axial wind speed u increases approximately linearly from $u_1 = 100$ m/s at section (1) to $u_2 = 300$ m/s at sec- (2) (Fig. 9–12). Meanwhile, the air density ρ is to decrease approximately lin-

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Topic No. 127

Examples of Application of the Continuity Equation

Example: Incompressibility of an Unsteady Two-Dimensional Flow

Consider the velocity field of Example 4–5—an unsteady, two-dimensional velocity field given by $\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + [1.5 + 2.5 \sin(\omega t) - 0.8y]\vec{j}$, where angular frequency ω is equal to 2π rad/s (a physical frequency of 1 Hz). Verify that this flow field can be approximated as incompressible.

SOLUTION We are to verify that a given velocity field is incompressible.

Assumptions **1** The flow is two-dimensional, implying no z-component of velocity and no variation of u or v with z.

Analysis The components of velocity in the x- and y-directions, respectively, are

$$u = 0.5 + 0.8x \quad \text{and} \quad v = 1.5 + 2.5 \sin(\omega t) - 0.8y$$

If the flow is incompressible, Eq. 9–[illegible] must apply. More specifically, in Cartesian coordinates Eq. 9–17 must apply. Let's check:

$$\underbrace{\frac{\partial u}{\partial x}}_{0.8} + \underbrace{\frac{\partial v}{\partial y}}_{-0.8} + \underbrace{\frac{\partial w}{\partial z}}_{0 \text{ since 2-D}} = 0 \quad \rightarrow \quad [illegible] - 0.8 = 0$$

So we see that the incompressible continuity equation is indeed satisfied at any instant in time, and **this flow field may be approximated as incompressible.**

Discussion Although the flow is unsteady term in v, it has no y-derivative and drops out of the continuity equation.

Topic No. 128

Examples of Application of the Continuity Equation

Three-Dimensional Flow

Example: Finding a Missing Velocity Component

Two velocity components of a steady, incompressible, three-dimensional flow field are known, namely, $u = ax^2 + by^2 + cz^2$ and $w = axz + byz^2$, where a, b, and c are constants. The y velocity component is missing (Fig. 9–14). Generate an expression for v as a function of x, y, and z.

SOLUTION We are to find the y-component of velocity, v, using given expressions for u and w.

Assumptions **1** The flow is steady. **2** The flow is incompressible.

Analysis Since the flow is steady and incompressible, and since we are working in Cartesian coordinates, we apply Eq. 9–17 to the flow field,

Condition for incompressibility:

$$\frac{\partial v}{\partial y} = -\underbrace{\frac{\partial u}{\partial x}}_{2ax} - \underbrace{\frac{\partial w}{\partial z}}_{ax + 2byz} \quad \rightarrow \quad \frac{\partial v}{\partial y} = -3ax - 2byz$$

Next we integrate with respect to y. Since the integration is a *partial* integration, we add some arbitrary function of x and z instead of a simple constant of integration.

Solution: $$v = -3axy - by^2z + f(x,z)$$

Discussion Any function $f(x,z)$ yields a v that satisfies the incompressible continuity equation, since there are no derivatives of v with respect to x or z in the continuity equation.

Topic No. 129

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Topic No. 129

Examples of Application of the Continuity Equation

Example: Two-Dimensional, Incompressible, Vertical Flow

Consider a two-dimensional, incompressible flow in cylindrical coordinates; the tangential velocity component is $u_\theta = K/r$, where K is a constant. This represents a class of vortical flows. Generate an expression for the other velocity component, u_r.

SOLUTION For a given tangential velocity component, we are to generate an expression for the radial velocity component.

Assumptions **1** The flow is two-dimensional in the xy- ($r\theta$-) plane (velocity is not a function of z and $u_z = 0$ everywhere). **2** The flow is incompressible.

Analysis The incompressible continuity equation (Eq. 9–18) for this two-dimensional case simplifies to

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \underbrace{\frac{\partial u_z}{\partial z}}_{0\ (2\text{-D})} = 0 \quad \rightarrow \quad \frac{\partial(ru_r)}{\partial r} = -\frac{\partial u_\theta}{\partial \theta} \tag{1}$$

The given expression for u_θ is not a function of θ, and therefore Eq. 1 reduces to

$$\frac{\partial(ru_r)}{\partial r} = 0 \quad \rightarrow \quad ru_r = f(\theta, t) \tag{2}$$

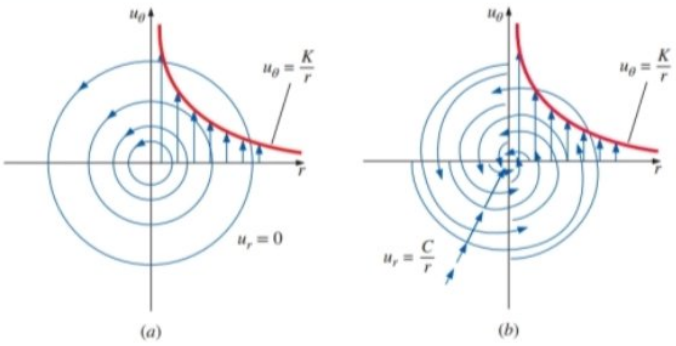
where we have introduced an arbitrary function of θ and t instead of a constant of integration, since we performed a *partial* integration with respect to r. Solving for u_r,

$$u_r = \frac{f(\theta, t)}{r} \tag{3}$$

Thus, *any radial velocity component of the form given by Eq. 3 yields a two-dimensional, incompressible velocity field that satisfies the continuity equation.*

We discuss some specific cases. The simplest case is when $f(\theta, t) = 0$ ($u_r = 0$, $u_\theta = K/r$). This yields the **line vortex** discussed in Chap. 4, as sketched in Fig. 9–15*a*. Another simple case is when $f(\theta, t) = C$, where C is a constant. This yields a radial velocity whose magnitude decays as $1/r$. For negative C, imagine a spiraling line vortex/sink flow, in which fluid elements not only revolve around the origin, but get sucked into a sink at the origin (actually a line sink along the z-axis). This is illustrated in Fig. 9–15*b*.

Discussion Other more complicated flows can be obtained by setting $f(\theta, t)$ equal to some other function. For any function $f(\theta, t)$, the flow satisfies the two-dimensional, incompressible continuity equation at a given instant in time.



Streamlines and velocity profiles for (*a*) a line vortex flow and (*b*) a spiraling line vortex/sink flow.

Topic No. 130

Examples of Application of the Continuity Equation

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Streamlines and velocity profiles for (*a*) a line vortex flow and (*b*) a spiraling line vortex/sink flow.

Topic No. 130

Examples of Application of the Continuity Equation

Example: Conditions for Incompressible Flow

Consider a steady velocity field given by $\vec{V} = (u, v, w) = a(x^2y + y^2)\vec{i} + bxy^2\vec{j} + cx\vec{k}$, where a, b, and c are constants. Under what conditions is this flow field incompressible?

SOLUTION We are to determine a relationship between constants a, b, and c that ensures incompressibility.

Assumptions **1** The flow is steady. **2** The flow is in[com]pressible (under certain constraints to be determined).

Analysis We apply [Eq.] 9–17 to the given velocity field

$$\underbrace{\frac{\partial u}{\partial x}}_{2axy} + \underbrace{\frac{\partial v}{\partial y}}_{2bxy} + \underbrace{\frac{\partial w}{\partial z}}_{0} = 0 \quad \rightarrow \quad 2axy + 2bxy = 0$$

Thus to guarantee incompressibility, constants a and b must be equal in magnitude but opposite in sign.

Condition for incompressibility: $\boldsymbol{a = -b}$

Discussion If a were not equal to $-b$, this might still be a valid flow field, but density would have to vary with location in the flow field. In other words, the flow would be *compressible*, and Eq. 9–14 would need to be satisfied in place of Eq. 9–17.

Topic No. 131

The Stream Function

The Stream Function in Cartesian Coordinates

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

A clever variable transformation enables us to rewrite above equation in terms of *one* dependent variable (ψ) instead of *two* dependent variables (u and v). We define the **stream function** ψ

Incompressible, two-dimensional stream function in Cartesian coordinates:

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

The stream function and the corresponding velocity potential function were first introduced by the Italian mathematician Joseph Louis Lagrange (1736–1813). Substitution of above equations yields

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = \frac{\partial^2 \psi}{\partial x\, \partial y} - \frac{\partial^2 \psi}{\partial y\, \partial x} = 0$$

which is identically satisfied for any smooth function $\psi(x, y)$, because the order of differentiation (y then x versus x then y) is irrelevant.

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Topic No. 136

Derivation Using the Divergence Theorem

Specifically, if we replace G_{ij} in the extended divergence theorem with the quantity $(\rho\vec{V})\vec{V}$, a second-order tensor, the last term in Eq. (135-a) becomes

$$\int_{CS} (\rho\vec{V})\vec{V}\cdot\vec{n}\,dA = \int_{CV} \vec{\nabla}\cdot(\rho\vec{V}\vec{V})\,dV \quad \text{___(135-c)}$$

Where $\vec{V}\vec{V}$ is a vector product called the outer product of the velocity vector with itself. (The outer product of two vectors is not the same as the inner or dot product, nor is it the same as the cross product of the two vectors.) Similarly, if we replace G_{ij} (in Divergence Theorem) by the stress tensor σ_{ij}, the second term on the left-hand side of Eq. (135-a) becomes

$$\int_{CS} \sigma_{ij}\cdot\vec{n}\,dA = \int_{CV} \vec{\nabla}\cdot\sigma_{ij}\,dV \quad \text{___(135-d)}$$

Thus, the two surface integrals of Eq. (135-a) become volume integrals by applying Eqs. (135-c) and (135-d). We combine and rearrange the terms, and rewrite Eq. (135-a) as

$$\int_{CV} \left[\frac{\partial}{\partial t}(\rho\vec{V}) + \vec{\nabla}\cdot(\rho\vec{V}\vec{V}) - \rho\vec{g} - \vec{\nabla}\cdot\sigma_{ij}\right] dV = 0 \quad \text{___(135-e)}$$

The above equation can only be zero if the [illegible] we have a general differential equation for linear momentum, known as **Cauchy's equation**,

Cauchy's equation: $$\frac{\partial}{\partial t}(\rho\vec{V}) + \vec{\nabla}\cdot(\rho\vec{V}\vec{V}) = \rho\vec{g} + \vec{\nabla}\cdot\sigma_{ij} \quad \text{___(135-f)}$$

Cauchy's equation is a differential form of the linear momentum equation. It applies to any type of fluid.

Equation (135-f) is named in honor of the French engineer and mathematician Augustin Louis de Cauchy (1789–1857). It is valid for compressible as well as incompressible flow since we have not made any assumptions about incompressibility. It is valid at any point in the flow domain. Note that Eq. (135-f) is a *vector* equation, and thus represents three scalar equations, one for each coordinate axis in three-dimensional problem

Alternative form of Cauchy's equation:

$$\rho\left[\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V}\right] = \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} + \vec{\nabla}\cdot\sigma_{ij} \quad \text{___(135-g)}$$

Topic No. 137

The Navier-Stokes Equation: Introduction

- Cauchy's equation (or its alternative form) is not very useful to us as is, because the stress tensor σ_{ij} contains nine components, six of which are independent (because of symmetry). Thus, in addition to density and the three velocity components, there are six additional unknowns, for a total of 10 unknowns. (In Cartesian coordinates the unknowns are [illegible]

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Topic No. 137

The Navier-Stokes Equation: Introduction

- Cauchy's equation (or its alternative form) is not very useful to us as is, because the stress tensor σ_{ij} contains nine components, six of which are independent (because of symmetry). Thus, in addition to density and the three velocity components, there are six additional unknowns, for a total of 10 unknowns. (In Cartesian coordinates the unknowns are ρ, u, v, w, σ_{xx}, σ_{xy}, σ_{xz}, σ_{yy}, σ_{yz}, and σ_{zz}.)
- We have discussed only four equations so far—continuity (one equation) and Cauchy's equation (three equations). We need six more equations which we obtain from components of the stress tensor in terms of the velocity field and pressure field.
- For this we separate the pressure stresses and the viscous stresses. When a fluid is at rest, the only stress acting at any surface of any fluid element is pressure P, which always acts inward and normal to the surface. Thus, regardless of the orientation of the coordinate axes, for a fluid at rest the stress tensor reduces to

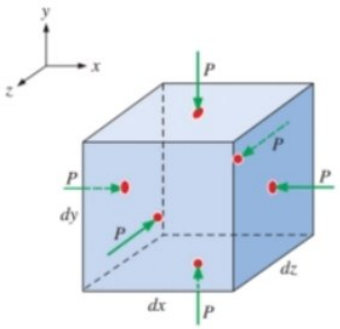
Fluid at rest:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -P \end{pmatrix}$$

Moving fluids:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -P \end{pmatrix} + \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

τ_{ij}, called the **viscous stress tensor** or the **deviatoric stress tensor**. [illegible] ve not helped the situation because we have replaced the six unknown compon[illegible] of σ_{ij} with six unknown components of τ_{ij}, and have added *another* unknown, pressure *P*. Fortunately; however, there are constitutive equations that express τ_{ij} in terms of the velocity field and measurable fluid properties such as viscosity.



For fluids at rest, the only stress on a fluid element is the hydrostatic pressure, which always acts inward and normal to any surface. Note that we are ignoring gravity in this case; otherwise pressure would increase in the direction of the gravitational acceleration.

Mechanical pressure is the mean normal stress acting inwardly on a fluid element. It is therefore also called **mean pressure.**

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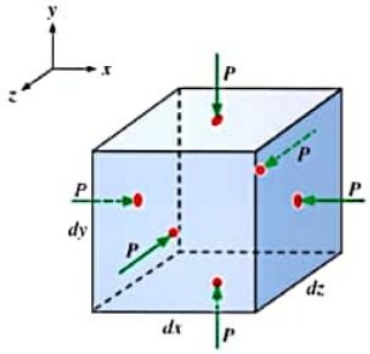
σ_{yy}, σ_{yz}, and σ_{zz}.)

- We have discussed only four equations so far—continuity (one equation) and Cauchy's equation (three equations). We need six more equations which we obtain from components of the stress tensor in terms of the velocity field and pressure field.
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Mechanical pressure is the mean normal stress acting inwardly on a fluid element. It is therefore also called **mean pressure.**

Mechanical pressure:
$$P_m = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

Topic No. 138

Newtonian versus Non-Newtonian Fluids

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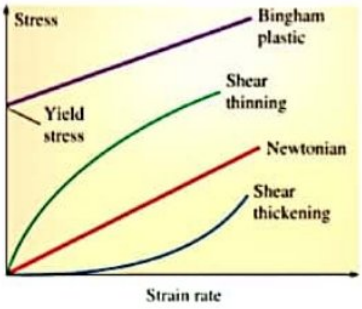
Mechanical pressure is the mean normal stress acting inwardly on a fluid element. It is therefore also called **mean pressure.**

Mechanical pressure: $P_m = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$

Topic No. 138

Newtonian versus Non-Newtonian Fluids

- The study of the deformation of flowing fluids is called **rheology.**
- **Newtonian fluids**, defined as fluids for which the stress tensor is linearly proportional to the strain rate tensor.
- Fluids for which the stress tensor is not linearly related to the strain rate tensor are called **non-Newtonian fluids.**
- A fluid that returns (partially) to its original shape after the applied stress is released is called **viscoelastic.**



Rheological behavior of fluids—stress as a function of strain rate.

- Some non-Newtonian fluids are called **shear thinning fluids** or **pseudo plastic fluids**, because the more the fluid is sheared, the less viscous it becomes.
- **Plastic fluids** are those in which the shear thinning effect is extreme.
- In some fluids a finite stress called the **yield stress** is required before the fluid begins to flow at all; such fluids are called **Bingham plastic fluids.**
- **Shear thickening fluids** or **dilatant fluids**; the more the fluid is sheared, the more viscous it becomes.



When someone falls into quicksand (a dilatant fluid), the faster he tries to move, the more viscous the fluid becomes.

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$$= -\frac{\partial P}{\partial x} + \rho g_x + \mu\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

The term in parentheses is zero because of the continuity equation for incompressible flow. We also recognize the last three terms as the **Laplacian** of velocity component u in Cartesian coordinates. Thus, we write the x-component of the momentum equation as

$$\rho\frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \mu\nabla^2 u$$

Similarly, the y- and z-components of the momentum equation reduce to

$$\rho\frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \rho g_y + \mu\nabla^2 v$$

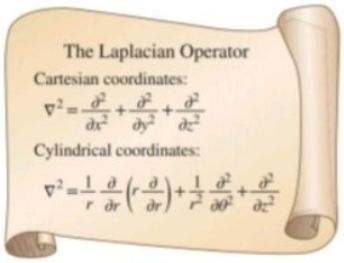
and

$$\rho\frac{Dw}{Dt} = -\frac{\partial P}{\partial z} + \rho g_z + \mu\nabla^2 w$$

respectively. Finally, we combine the three components into one vector equation; the result is the **Navier–Stokes equation** for incompressible flow with constant viscosity.

Incompressible Navier–Stokes equation:

$$\rho\frac{D\vec{V}}{Dt} = -\vec{\nabla}P + \rho\vec{g} + \mu\nabla^2\vec{V}$$

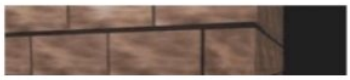


The Laplacian operator, shown here in both Cartesian and cylindrical coordinates, appears in the viscous term of the incompressible Navier–Stokes equation.

Topic No. 140

Derivation of the Navier–Stokes Equation for Incompressible, Isothermal Flow

- The Navier–Stokes equation is the cornerstone of fluid mechanics. It may look harmless enough, but it is an unsteady, nonlinear, second-order, partial differential equation.
- Above equation has four unknowns (three velocity components and pressure), yet it represents only three equations (three components since it is a vector equation). The fourth equation is the incompressible continuity equation.
- We need to choose a coordinate system and expand the equations in that coordinate system.



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Example 8

- The radial velocity component in an incompressible, two dimensional flow field $(v_z = 0)$ is

$$v_r = 2r + 3r^2 \sin\theta$$

- Determine the corresponding tangential velocity component, v_θ, required to satisfy conservation of mass.

Solution.

- The continuity equation for incompressible steady flow in cylindrical coordinates is given by

$$\frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Since $v_z = 0$,

$$\frac{\partial v_\theta}{\partial \theta} = -\frac{\partial (r v_r)}{\partial r}$$

30

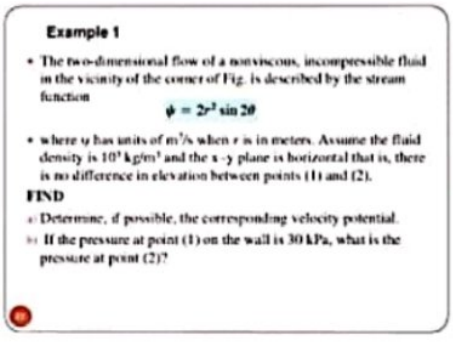
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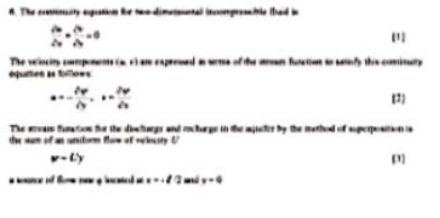
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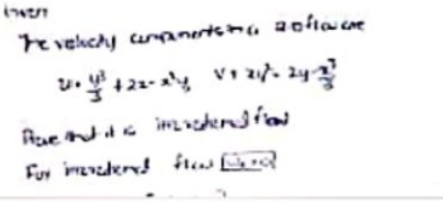


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$$u = \frac{\partial}{\partial x}\left[K\left(x^2 - y^2\right) + C\ln\left(x^2 + y^2\right)\right]$$



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mach number dimensions

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Mach number **has no dimensions** because it is the ratio of two velocities having the same dimensions.

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Mach Number: Definition, Significance, Types and Examples

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Example 4

- Consider the following steady, three-dimensional velocity field in Cartesian coordinates:

$$\vec{V} = (u, v, w) = (axy^2 - b)\vec{i} + cy^3\vec{j} + dxy\vec{k}$$

where *a, b, c, and d* are constants. Under what conditions is this flow field incompressible?

Solution

Condition for incompressibility:

$$\underbrace{\frac{\partial u}{\partial x}}_{ay^2} + \underbrace{\frac{\partial v}{\partial y}}_{3cy^2} + \underbrace{\frac{\partial w}{\partial z}}_{0} = 0 \qquad ay^2 + 3cy^2 = 0$$

- Thus to guarantee incompressibility, constants *a and c* must satisfy the following relationship:

$$a = -3c$$

25

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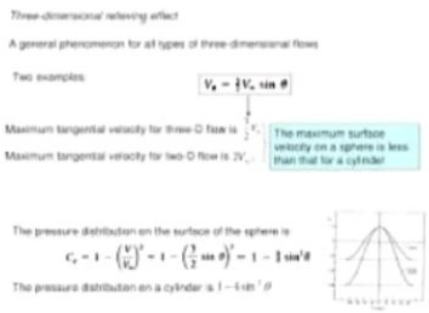
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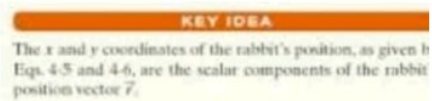
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Day: [S][M][T][W][T][F][S] ✓ Date: 6/9/2021

Time: 8:00 Am

MTH: 642

Q.No: 1 write the linear moment Equation in newton second Law. (3)

Q.No: 2 write the compressible continuity equation in cylindrical coordinates. (2)

Q.No: 3 Example Incompressible and Steady fluid flow (5)

Q.No: 4 Derive the Navier-Stokes Equation for incompressible (5)

Q.No: 5 Define normalized Equation. (2)

Q.No: 6 write the state of divergence theorem (3)

MCQs: Totally conceptual

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Write a linear momentum equation in newton's second law? 3marks

Answer:

Newton's second law:

$$F = ma$$

where F is the force acting on an object, m is the mass of the object, and a is the object's acceleration.

We can rewrite this using the definition $a \equiv \frac{\Delta v}{\Delta t}$, where Δv is the change in velocity of the object over some time duration Δt.

$$F = m\frac{\Delta v}{\Delta t}$$

which is equivalent to:

$$F = \frac{\Delta p}{\Delta t}$$

where $\Delta p \equiv m\Delta v$ is change in momentum.

Write the compressible continuity equation in cylindrical coordinates? 2marks

Answer:

Compressible

continuity equation: $\vec{\nabla}\cdot(\rho\vec{V}) = 0$

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In Cartesian coordinates, this reduces to

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

In cylindrical coordinates, it reduces to

$$\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta}+\frac{\partial(\rho u_z)}{\partial z}=0$$

Example of incompressible and study fluid flow? 5marks

Answer:

Compressible

continuity equation: $\vec{\nabla}\cdot(\rho\vec{V})=0$

In Cartesian coordinates, this reduces to

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

In cylindrical coordinates, it reduces to

$$\frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta}+\frac{\partial(\rho u_z)}{\partial z}=0$$

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Incompressible continuity equation: $\vec{\nabla}\cdot\vec{V}=0$

Since $\partial\rho/\partial t \cong 0$

Incompressible continuity equation in Cartesian coordinates:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Incompressible continuity equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial(u_\theta)}{\partial\theta}+\frac{\partial(u_z)}{\partial z}=0$$

Derive Navier–Stokes equation for incompressible? 5marks

Answer:

Incompressible Navier–Stokes equation:

$$\rho\frac{D\vec{V}}{Dt}=-\vec{\nabla}P+\rho\vec{g}+\mu\nabla^2\vec{V}$$

Define Normalized equation? 2marks

Answer:

Normalized Equation:

- If the nondimensional terms in the equation are of order unity, the equation is called **normalized**.

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State Divergence theorem? 2marks

Answer:

The quickest and most straightforward way to derive the differential form of conservation of mass is to apply the divergence theorem (Gauss's theorem).

Divergence theorem: $$\int_V \vec{\nabla}\cdot\vec{G}\,dV = \oint_A \vec{G}\cdot\vec{n}\,dA$$

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Day: [S] [M] [T] [W] [T] [F] [S]

Date: 6/9/2021

Time: 8:00 Am

MTH: 642

Q.No: 1

write the linear momentum Equation in newton second law. (3)

Q.No: 2

write the compressible continuity equation in cylindrical coordinates. (2)

Q.No: 3

Example: Incompressible and Steady fluid flow (5)

Q.No: 4

Topic 140 Derive the Navier-Stokes Equation for incompressible (5)

Q.No: 5 Define Normalized Equation. (2)

Q.No: 6 write the state of divergence theorem (3)

MCQs: Totally conceptual

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Q.No : 1

- Newton's second law:
- The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.
- Newton's third law:
- When a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.
- Therefore, the direction of an exposed reaction force depends on the body taken as the system.

Newton's second law: $\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d(m\vec{V})}{dt}$

- Linear momentum or just the momentum of the body: The product of the mass and the velocity of a body.
- Newton's second law is usually referred to as the linear momentum equation.

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Q No. 2

Compressible continuity equation in cylindrical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z}{\partial z}$$

Steady compressible follow of continuity ?

Answer:

$$\vec{\nabla} \cdot (\rho . \vec{V}) = 0$$

in Cartesion coordinates:

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z}$$

in cylindrical coordinates:

$$\frac{1}{r}\cdot\frac{\partial (r \rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0$$

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Q. No: 3

incompressible flow of continuity equation;

Answer: $\vec{\nabla} \cdot \vec{V} = 0$

incompressible continuity equation in cartesian coordinates?

Answer:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

incompressible continuity equation in cylindrical coordinates.

Answer:

$$\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial (u_\theta)}{r\,\partial \theta} + \frac{\partial (u_z)}{\partial z} = 0$$

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Q. No:5

Normalized Equation:

if the non-dimensional terms in the equation are of order unity, the equation is called normalized equation.

in most experiments, to save time and money, tests are performed on a geometrically scaled model, rather than on the full-scale prototype.

Compressible form of continuity equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0$$

Alternative form of continuity equation:

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \vec{\nabla} \cdot \vec{V} = 0$$

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Consider a steady velocity field given by $\vec{V} = (u, v, w) = a(x^2y + y^2)\vec{i} + bxy^2\vec{j} + cx\vec{k}$

where (a, b, c) are constant under what condition is this flow field is incompressible?

Answer

Formula: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \phi}{\partial z} =$

$\Rightarrow 2axy + 2bxy = 0$

$2xy(a+b) = 0$

$a + b = 0$

condition for incompressibility $a = -b$

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Consider a two dimensional incompressible flow in cylindrical coordinates velocity is $u_\theta = \frac{K}{r}$ K is constant. Generate expression for other velocity component u_r.

Solution:

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\cdot\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\frac{1}{r}\left[\frac{\partial(ru_r)}{\partial r} + \frac{\partial u_\theta}{\partial \theta}\right] = 0$$

$$\frac{\partial ru_r}{\partial r} = -\frac{\partial u_\theta}{\partial \theta} \rightarrow (2)$$

u_θ is not a function, so, (1) reduce to

$$\frac{\partial ru_r}{\partial r} = 0$$

$$ru_r = f(\theta, t)$$

$$u_r = \frac{f(\theta, t)}{r}$$

(3) mark V. imp.

Two dimensional velocity

$\vec{V} = (u, v) = (0.5 + 0.8x)i + [1.5 + 2.5 \sin\omega - 0.8y]\vec{j}$

Verify by incompressible?

Solution:

$u = 0.5 + 0.8x$

$v = 1.5 + 2.5 \sin\omega(t) - 0.8y$

Formula of incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$0.8 - 0.8 = 0$

$0 = 0$

Proved

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(3) marks

Write four dimensional Parameter

Answer:

1st E, 2nd V, 3rd D

4th ρ

Using product rule expand $\vec{\nabla}(\rho \vec{V})$

Answer:

$$\vec{\nabla}(\rho \vec{V}) = \vec{V} \cdot \nabla \rho + \rho \cdot \vec{\nabla} \cdot \vec{V}$$

pa

5 marks.

Expand $\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{V})$

Solution:

By using product rule on the divergence theorem

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{V})$$

$$= \frac{\partial \rho}{\partial t} + V \cdot \vec{\nabla}\rho + \rho \vec{\nabla} \vec{V} = 0$$

mark

Write conversation of mass relation

Answer:

Conversation of mas relation:

$$\frac{d\rho}{dt} \int_{CV} dv + \rho \int_{CV} (\vec{V} \cdot \vec{n}) \, dA = 0$$

The time rate chang within the control volume plus the net mass flow rate through the control sevice is equal to zero.

5marks.

Find a missing velocity componer incompressible three dimensional flow where $u = ax^2 + by^2 + cz^2$, $w = axz + byz^2$. the y velocity component is missing?

Solution:

We know incompressible formula

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}$$

$$\frac{\partial v}{\partial y} = -2ax - 2byz$$

$$\frac{v}{y} = -2ax - 2byz$$

$$v = -2axy - 2by^2z + f(x,z$$

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Varify that each aditive term in the Bernoulli's equation have same dimensions? What is the dimension of c in Bernoulli equations?

Answer:

We know that

$$P + \frac{1}{2}\rho V^2 + \rho g z = c$$

is a Bernoulli equation.

So, we check dimensions.

$$P = \left\{\frac{F}{A}\right\} = \frac{ma}{t^2} = \frac{mL}{t^2L^2} = \frac{m}{t^2L}$$

$$\frac{1}{2}\rho V^2 = \frac{mL^2}{t^2L^3} = \frac{m}{t^2L}$$

$$\rho g h = \frac{mL^2}{t^2L^3} = \frac{m}{t^2L}$$

From the law of homogeneity the constant have same dimension.

So, $c = \frac{m}{t^2L}$

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$\omega = \frac{d\theta}{dt}$, $a = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Rotational kinetic energy:

$$K.E_r = \frac{1}{2} I\omega^2$$

Dimension:

A measure of physical quantity without numerical value

Unit:

A way to show the number is that dimension. There are 7 primy dimension mass, lenght, time, temperature electric current, amount of light and amount of matter.

Dimension of force:

$$\text{Force} = \frac{\text{mass} \times \text{length}}{\text{time}^2} = \frac{ML}{t^2}$$

length is dimension but centimeter is a unit.

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Reynold's transport theorm is expressed as for linear momentem as:

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{v}\, d\vec{v} + \int_{CV} \rho \vec{v}(\vec{v}\cdot n)\, dA$$

Rotational motion:

A motion during which all points of a body moves in the circular path about the axis of rotation.

Rotational motion is described with angular quantities such that θ = distande and

w = velocity

α = acceleration.

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The conversation of angular momentum principle:

The total angular momentum of a rotating body remains constant when the net torque acting on it is zero and that's why angular momentum of such system is conserved.

Total force acting on control volume:

$$\Sigma F = \Sigma F_b + \Sigma F_s.$$

The total force acting on a control volume is composed of sum of body forces and sum of surface forces.

Total body force acting on a control volume:

$$\Sigma \vec{F}_{body} = \int_{CV} \rho \vec{g} dV = m_{cv} \vec{g}.$$

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TOPIC 85.

Newton's law of Motion:

Conversation of momentum principle:

The momentum of a system remains constant only when the net force acting on it and it will be zero.

Write or derive $\frac{d\vec{H}}{dt}$ as Angular momentum.

Solution:

Angular momentum equation:

$$\vec{M} = I\vec{a}$$

$$\therefore \vec{a} = \frac{d\vec{\omega}}{dt}$$

$$\vec{M} = I\vec{a} = I\frac{d\vec{\omega}}{dt}$$

$$\vec{M} = \frac{dI\vec{\omega}}{dt} \qquad \therefore I\vec{\omega} = \vec{H}$$

$$\vec{M} = \frac{d\vec{H}}{dt}$$

Angular momentum on x-axis.

$$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

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Newton's 1st law:

A body at rest remains at rest, and a body in a motion remains in motion at the same velocity in straight path when the net force acting on it is zero.

Newton's 2nd law:

The acceleration of a body is directly proportional to the net force acting on it and inversly proportional to its mass.

Newton's 3rd law:

When a body apply force on 2nd body it also apply same equal force on first body but in opposite direction

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Solution:

Radius $R = 4.00 \times 10^{-3}$ m.

Volume flow rate $Q = 1.00 \times 10^{-6}\ m^3 s^{-1}$

viscosity of blood $= \eta = 2.084 \times 10^{-3}$ Pa.s

Density of blood $= \rho = 1.06 \times 10^{3}\ kgm^{-3}$

Equation of continuity:

$$Q = AV$$

$$A = \pi R^2 = \pi (4.00 \times 10^{-2})^{+2} = 5.03 \times 10^{-5}\ m^2$$

$$V = \frac{Q}{A} = \frac{1.00 \times 10^{-6}}{5.03 \times 10^{-5}} = 1.99 \times 10^{-2}\ ms^{-1}$$

Newton's 2nd law:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

linear momentum:

The product of mass and the velocity of the body. Newton's 2nd law is usually refferred to as the linear momentem equation.

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Application of bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

as initial $V_1 = 0$

$$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$P_1 = P_{atom}$, $P_2 = P_{atom}$

$$\rho g h_1 = \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\rho g h_1 = \rho\left(\frac{1}{2}V_2^2 + g h_2\right)$$

$$g h_1 - g h_2 = \frac{1}{2}V_2^2$$

$$g(h_1 - h_2) = \frac{1}{2}V_2^2$$

$$2gh = V_2^2$$

$$V_2 = \sqrt{2gh}$$

which is known as torricelli formula

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MTH 642

Malik

Q.No.1:-

Expand $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V})$ in cartesian coordinates by using $V = (u, v, w)$.

Sol:-

As we know that

Rate of change of mass within CV.

$$\int_{CV} \frac{\partial \rho}{\partial t} dV \cong \frac{\partial \rho}{\partial t} dx dy dz$$

Net mass flow rate into CV

$$\sum_{in} \dot{m} \cong \left(\rho u - \frac{\partial (\rho u)}{\partial x} \frac{dx}{2}\right) dy dz +$$

$$\left(\rho v - \frac{\partial (\rho v)}{\partial y} \frac{dy}{2}\right) dx dz + \left(\rho w - \frac{\partial \rho w}{\partial z} \frac{dz}{2}\right) dx dy$$

Also

Net mass flow rate out of CV

$$\sum_{out} \dot{m} \cong \left(\rho u + \frac{\partial (\rho u)}{\partial x} \frac{dx}{2}\right) dy dz +$$

$$\left(\rho v + \frac{\partial \rho v}{\partial x} \frac{dy}{2}\right) dx dz + \left(\rho w + \frac{\partial (\rho w}{\partial z} \frac{dz}{2}\right) dx dy$$

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As we know that

$$\int_{CV} \frac{\partial \rho}{\partial t} dV = \sum_{out} \dot{m} - \sum_{in} \dot{m}$$

$$\frac{\partial \rho}{\partial t} dx dy dz = -\frac{\partial(\rho u)}{\partial x} dx dy dz - \frac{\partial(\rho v)}{\partial y} dx dy dz - \frac{\partial(\rho w)}{\partial z} dx dy dz$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}$$

which is eq. of continuity in cartesian coordinates.

Q. No. 2:-

Expand:- $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V})$ in cylindrical coordinates (r, θ, z).

Sol:-

[illegible] Prove [illegible] cartesian [illegible] Transformation

Put

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1} y/x$$

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$x = r\cos\theta$, $y = r\sin\theta$, $z = z$

then eqn. of continuity in cylindrical coordinates becomes

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r \rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0.$$

which is required.

Q. No. 3

What is the relation b/w torque, angular momentum angular velocity and angular acceleration.

Sol:-

As we know that Torque is equal to moment of inertial times the angular acceleration.

$$\vec{M} = I\vec{\alpha}$$

or torque is also defined as rate of change of

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angular momentum.

i.e.

$$\vec{M} = I\vec{\alpha}$$

$$= I\frac{d\vec{\omega}}{dt}$$

$$= \frac{d(I\vec{\omega})}{dt}$$

$$\vec{M} = \frac{d}{dt}\vec{H}$$

Angular momentum is defined as "The product of moment of inertia and angular velocity"

i.e.

$$\vec{H} = I\vec{\omega}$$

Q.No.4

State divergence theorem:

Sol:-

The divergence theorem is also known as Gauss's theorem. It allow us to convert the volume integral

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of the divergence of a vector into area integral over the surface that defines the volume.

For any vector $\vec{G}$, the divergence of $\vec{G}$ is defined as $\vec{\nabla} \cdot \vec{G}$ and divergence theorem is

$$\int_V \vec{\nabla} \cdot \vec{G}\, dV = \oint_A \vec{G} \cdot \hat{n}\, dA.$$

Q.No.5:-

write product rule of

$$\vec{\nabla} \cdot (\rho \vec{v}) = 0$$

Sol:-

$$\vec{\nabla} \cdot (\rho \vec{v}) = 0$$

Product rule is

$$\vec{v} \cdot \vec{\nabla}\rho + \rho\, \vec{\nabla} \cdot \vec{v} = 0.$$

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Q. No. 6:-

Why divers gets faster speed after a curly jump?

Sol:-

By conservation of angular momentum. There is this relationship that must be conserved as the diver does somersaults.

As we know that

Angular momentum = L x w

Suppose L is the length of body and w is how fast they are spinning.

Since angular momentum is conserved (it's a constant), then if L goes down, w goes up. If w goes down, L goes up.

So, divers get faster speed after a curly jump.

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Q.No. 7:-

write formulas of cartesian and cylindrical coordinates in steady compressible flow.

Sol:-

when fluid flow is stead compressible then continuity eq.

$$\vec{\nabla} \cdot (\rho \vec{V}) = 0$$

Cartesian coordinate

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Cylindrical coordinate

$$\frac{1}{r} \frac{\partial(r \rho u_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} =$$

Q.No. 8

write formulas of cartesian and cylindrical coordinate in steady incompressible flow.

Sol:-

when fluid flow is

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Steady incompressible then continuity eq. is

$$\vec{\nabla} \cdot \vec{V} = 0$$

Cartesian Coordinate

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Cylindrical coordinate

$$\frac{1}{r} \frac{\partial (r u_r)}{\partial r} + \frac{1}{r} \frac{\partial (u_\theta)}{\partial \theta} + \frac{\partial (u_z)}{\partial z} = 0.$$

Q. No. 9:-

Prove that

(a) $V = r\omega$ (b) $a_t = r\alpha$

Sol:-

(a)

As we know that

$$\omega = \frac{d\theta}{dt} \qquad \because \theta = \frac{l}{r}$$

$$= \frac{d(l/r)}{dt}$$

$$= \frac{1}{r} \frac{dl}{dt}$$

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$$\omega = \frac{V}{r}$$

$$\Rightarrow \boxed{V = r\omega}$$

(b)

As we know that

$$\alpha = \frac{d\omega}{dt}$$

$$= \frac{d}{dt}\left(\frac{d\theta}{dt}\right)$$

$$= \frac{d^2\theta}{dt^2}$$

$$= \frac{d^2}{dt^2}\left(\frac{l}{r}\right)$$

$$= \frac{1}{r}\frac{d}{dt}\left(\frac{dl}{dt}\right)$$

$$= \frac{1}{r}\frac{d}{dt}(V)$$

$$\alpha = \frac{1}{r}\, a_t$$

$$\Rightarrow \boxed{a_t = r\alpha}$$

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Q. No. 10:-

Write 4 type of Dimensionless parameter.

Sol:-

(1) Reynolds No. (2) Froude No.
(3) Strouhal No. (4) Euler No.

Reynolds No. is the most well known and useful dimensionless parameter in all of fluid mechanics.

Q. No. 11:-

Write Formula of Froude number.

Sol:-

$$F_r = \frac{V}{\sqrt{gL}}$$

Also asked for

Reynolds No. :- $Re = \frac{\rho VL}{\mu} = \frac{VL}{\nu}$

Mach No :- $M = \frac{V}{C}$

Weber No. $We = \frac{\rho V^2 L}{\sigma}$

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Euler No. $E_u = \frac{\Delta p}{\frac{1}{2}\rho V^2}$

Cavitation No. $C_a = \frac{p - p_0}{\frac{1}{2}\rho V^2}$

Q. No. 12

The velocity component in along x-axis of a steady incompressible two dimensional flow is $u = 2.5y + 0.9x^2$. Component v is missing. Generate expression for v as a function of x.

Sol:-

As given that

$u = 2.5y + 0.9x^2$

we have to find expression for component v.

As flow field is incompressible; we know that if flow field is steady incompressible then

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

But flow field is in two dimension. So

$$\frac{\partial w}{\partial z} = 0.$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \longrightarrow (1)$$

As

$$u = 2.5y + 0.9x^2$$

$$\frac{\partial u}{\partial x} = 0 + 2(0.9)x$$

$$= 1.8x$$

Put in (1)

$$\frac{\partial v}{\partial y} = -1.8x$$

Integrate w.r.t 'y'

$$V = -1.8xy + f(x)$$

where $f(x)$ is constant of integration.

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Q. No. 13

Write the expression for conservation of mass relation for a control volume. What does the expression state?

Sol:-

Conservation of mass for a control volume is given as

$$0 = \int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} \rho \vec{V} \cdot \vec{n} \, dA$$

$$\Rightarrow \int_{CV} \frac{\partial \rho}{\partial t} dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

The net rate of change of mass within the control volume is equal to the rate at which mass flows into the CV minus the rate at which mass flows out of CV.

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Q. No. 14

Name the three necessary condition for complete similarity b/w a model and a prototype.

Sol:-

There are three necessary conditions for complete similarity b/w a model and a prototype.

(1) Geometric similarity
(2) Kinematic Similarity
(3) Dynamic similarity.

Q. No. 15

Why the ice skaters spin faster when they bring their arms closer to their bodies? Explain in the light of principle of conservation of momentum?

Sol:-

As the angular momentum eqn is

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$\vec{M} = I\vec{\alpha}$

the ice skaters spinning faster when they bring their arms close to their bodies because the moment of inertia (I) decreased and thus the angular velocity ω is increased as the outer part of body are brought closer to the axis of rotation.

Same why divers get more speed after curly jump?

Q. No. 16

write the linear moment in newton 2nd' law?

Sol:-

As the Newton's 2nd law of motion is given by

$\vec{F} = m\vec{a}$

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where $\vec{a}$ is acceleration and defined as Time rate of change of velocity.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

So

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

Newton's 2nd law also describe as

"Time rate of change of Linear momentum".

Q. No. 17:-

Derive Navier-Stokes eqn. for incompressible?

Sol:-

As viscous stress Tensor for an incompressible newtonian fluid with constant properties is

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$$\tau_{ij} = 2\mu \varepsilon_{ij}$$

$$\tau_{ij} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} \begin{pmatrix} 2\mu\frac{\partial u}{\partial x} & \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & \mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \\ \mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) & 2\mu\frac{\partial v}{\partial y} & \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) \\ \mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right) & \mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right) & 2\mu\frac{\partial w}{\partial z} \end{pmatrix}$$

Now

$$\sigma_{ij} = \begin{pmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{pmatrix} \begin{pmatrix} 2\mu\frac{\partial u}{\partial x} & \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & \mu\left(\frac{\partial u}{\partial z}+\frac{\partial u}{\partial x}\right) \\ \mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) & 2\mu\frac{\partial v}{\partial y} & \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) \\ \mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right) & \mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right) & 2\mu\frac{\partial w}{\partial z} \end{pmatrix}$$

we substitute this eq. into 3 cartesian component of cauchy's eq.

For x - component.

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho g_x + 2\mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)$$

After Rearrangements

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial}{\partial x}\frac{\partial w}{\partial z} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

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$$= \frac{-\partial P}{\partial t} + \rho g_x + \mu \left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + \right.$$

$$\left. \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

For incompressible flow of continuity eq.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

and

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \nabla^2 u$$

So

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \mu \nabla^2 u$$

Similarly

For y- component

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \rho g_y + \mu \nabla^2 v$$

For z- component

$$\rho \frac{Dw}{Dt} = -\frac{\partial P}{\partial z} + \rho g_z + \mu \nabla^2 w$$

Combining three component we get Navier-stokes eq. for incompressible flow which is.

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$$\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla} P + \rho \vec{g} + \mu \nabla^2 \vec{V}.$$

Q. No. 18

Define Normalized eqn.

Solution:-

If the non-dimensional terms in the eqn. are of order unity, the eqn. is called normalized.

Each term in a non-dimensional eqn. is dimensionless.

Eqn. of motion:- $\frac{d^2z}{dt^2} = -g$

Q. No. 19

Consider a steady velocity field given by $\vec{V} = (u, v, w) = a(x^2 - y + y^2)\vec{i} + bxy^2\vec{j} + cx\vec{k}$, where a, b and c are constants. under what conditions is this flow field incompressible.

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Sol:-

Assumption:-

(1) Flow is steady.

(2) Flow is incompressible (under certain constraints to be determined).

Calculation:-

$$\vec{V} = (u, v, w) = a(x^2y + y^2)\vec{i} + bxy^2\vec{j} + cx\vec{k}$$

here

$$u = a(x^2y + y^2)$$

$$v = bxy^2$$

$$w = cx$$

now

$$\frac{\partial u}{\partial x} = 2axy$$

$$\frac{\partial v}{\partial y} = 2bxy$$

$$\frac{\partial w}{\partial z} = 0$$

If flow field is incompressible then $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$2axy + 2bxy + 0 = 0$$

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$$2ax y = -2by$$

$$\boxed{a = -b}$$

If $a = -b$ then flow is incompressible.

Q. No. 20

Describe the dimensions of non-dimensional eq. and prove that $\frac{d^2z}{dt^2} = -g$

Sol:-

$\frac{d^2z}{dt^2} = -g$ is eq. of motion.

Dimensional Result of eq. of motion is.

$$z = z_0 + w_0 t - \frac{1}{2} g t^2$$

Primary dimensions of all parameters are

$[z] = [L]$, $[t] = [t]$

$[z_0] = [L]$, $[w_0] = [L/t]$

$[g] = [L/t^2]$.

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Non-dimensionalized variable are

$$z^* = \frac{z}{z_0} \quad , \quad t^* = \frac{\omega_0 t}{z_0}$$

$$\Rightarrow z = z^* z_0 \quad , \Rightarrow t = \frac{t^* z_0}{\omega_0}$$

So eqn. of motion becomes

$$\frac{d^2 z}{dt^2} = -g$$

$$\frac{d^2(z_0 z^*)}{d(z_0 t^*/\omega_0)^2} = \frac{\omega_0^2 d^2 z^*}{z_0 dt^{*2}} = -g$$

$$\Rightarrow \frac{\omega_0^2}{g z_0} \frac{d^2 z^*}{dt^{*2}} = -1.$$

As we know that

Froude No.

$$F_r = \frac{\omega_0}{\sqrt{g z_0}}$$

Non-dimensionalize eqn. of motion is

$$\frac{d^2 z^*}{dt^{*2}} = -\frac{1}{Fr^2}$$

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Non-dimensional result

$$z^* = 1 + t^* - \frac{1}{2F_r^2} t^{*2}$$

Q.No.21

State angular momentum eq. write its expression about x-axis and y-axis.

Sol:-

Angular momentum eq. stated as

"Rate of change of angular momentum".

i/e

$$\vec{M} = I\vec{\alpha}$$

$$= I\frac{d\vec{\omega}}{dt}$$

$$= \frac{d(I\vec{\omega})}{dt}$$

$$\vec{M} = \frac{d\vec{H}}{dt}$$

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about x-axis

$$M_x = I_x \frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

about y-axis

$$M_y = I_y \frac{d\omega_y}{dt} = \frac{dH_y}{dt}$$

Q.No.22

Angular momentum for inlet and outlet for steady expression?

Sol:-

During steady flow, the amount of angular momentum within the control volume remains constant and thus time rate of change of angular momentum of contents of control volume is zero.

For steady flow

$$\sum \vec{M} = \int_{cs} (\vec{r} \times \vec{v}) \rho (\vec{v_r} \cdot \vec{n}) dA$$

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Now, an approximation form of angular momentum eqn. in terms of average properties at inlets and outlets becomes,

$$\Sigma \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V}) \rho dV + \sum_{out} \vec{r} \dot{m} \vec{V}$$

$$- \sum_{in} \vec{r} \times \dot{m} \vec{V}$$

$$= \sum_{out} \vec{r} \times \dot{m} \vec{V} - \sum_{in} \vec{r} \times \dot{m} \vec{V}$$

In scalar form

$$\Sigma M = \sum_{out} r \dot{m} V - \sum_{in} r \dot{m} V.$$

For unsteady flow:-

For unsteady flow angular momentum eqn. is

$$\Sigma \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} dV + \sum_{outH} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} V$$

$$\Sigma \vec{M} = \frac{d}{dt}\int_{CV} (\vec{r} \times \vec{V}) \rho dV + \sum_{out} \vec{r} \times \dot{m} \vec{V}$$

$$- \sum_{in} \vec{r} \times \dot{m} \vec{V}$$

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flow with no external moments:-

$$0 = \frac{d\vec{H}_{CV}}{dt} + \sum_{out} (\vec{r} \times \dot{m}\vec{v}) - \sum_{in} (\vec{r} \times \dot{m}\vec{v})$$

Q. No. 23:-

Difference b/w Dimensional Variable and Dimensional constant?

Sol:-

Dimensional Variable:-

dimensional quantities that changes or vary in problem are called dimensional variable.

For example z (dimension of length) t (dimension of time).

Dimensional constant:-

The dimensional quantities which remains constant are called dimensional constant.

Dimensional constant may changes from problem to problem. They are fixed for particular problem

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and thus ~~distiung~~ distinguished from dimensional variables.

For example gravitational constant g, initial location 3_0 and initial vector speed v_0.

Also asks for

Non-dimensional / dimensionless variable:-

The quantities that changes or vary in the problem but have no dimension is called dimensionless variable.

For example:-

Angle of rotation measured in radian and degrees have no dimension.

Parameter:-

The combined set of dimensional variable, non-dimensional variable and dimensional constant in the problem is called parameter.

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Q. No. 24

write expression for conversion of mass relation for a control volume?

Sol:-

Conversion of mass relation:-

The conversion of mass relation for closed system is expressed as m_{sys}

$$m_{sys} = \text{constant}$$

$$\frac{dm_{sys}}{dt} = 0$$

which state that the mass of system remains constant during a process.

The conversion of mass relation in control volume is

$$\sum_{in} \dot{m} - \sum_{out} \dot{m} = \frac{dm_{CV}}{dt}$$

where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the total mass flow

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into and out of the control volume and $\frac{dm_{CV}}{dt}$ is the rate of change of mass with the boundaries of control volume.

Conversion of mass relation in fixed control volume is expressed as

$$\frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho(\vec{V}\cdot\vec{n})dA = 0.$$

$$\frac{d}{dt}\int_{CV} \rho dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

$$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

Q. No. 25

what is dimensions and units:-

Sol:-

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Dimension:-

Any physical quantity can be characterized by dimension.

or

measure of physical quantity (without any numeric value) is called dimension.

Units:-

The magnitudes that are assign to dimension are called units.

Q. No. 26:-

what are primary and secondary units?

Sol:-

Primary units:-

Some basic dimension such as mass, length, temperature time, electric current and amount of light are selected as primary or fundamental units.

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There are 7 basic primary units.

Secondary units:-

All non-primary dimensions such as velocity energy and force can be formed by some combination of primary units are called secondary or derived dimensions.

Remember in prayers
Malik hafeez

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Final paper

Time 3 pm

MTH642

16-Sep-21

Some MCQs conceptual and some written below

1) The off-diagonal components of ~~shear~~ stress tensor σ_{ij} are named as
(a) Viscous stress (b) shear stress (c) Normed stress (d) turbulent stress

2) The angular distance traveled by any point in a rigid body during a complete rotation is
(a) π rad (b) $\frac{\pi}{2}$ rad (c) $\frac{2}{\pi}$ rad (d) 2π rad

3) The moment of force about a point O is ____ of the position vector and the force
(a) inner product (b) outer product (c) tensor product (d) vector product

4) The number of pimary dimension is a
(a) 6 (b) 7 (c) 8 (d) 5

5) Which is following not nondimensional parameter
(a) Froude ~~equation~~ number (b) Angle of rotation (c) Gravitational acceleration (d) Reynolds number

6) What could be dimesion of Mach number?
(a) $\frac{\text{length}}{\text{Time}}$ (b) 0 (c) 1 (d) $\frac{\text{Time}}{\text{length}}$

7) Divergence theorem also known as
(a) Gauss's theorem (b) Continous theorem

8) 3 MCQs about Bernoulli equation

9) 1 MCQs is about ~~co~~ true statement about continuity equation ← 2 MCQs

10) 1 MCQs about statement true about kinematic similarity

11) 1 MCQs is term of Navier-Stokes equation is nonlinear

12) The accelaration of a body is proportinal to the net force acting on it and is inversely proportional to it mass (a) Newton 1st law (b) Newton's 2nd law (c) Newton's 3rd law

13) If water discharges in the atmosphere from..

(1) What does angular momentum equation state 2M

(2) Express the divergence theorem in word 2 Mark

(3) Write differential form of Newton's second law of motion in terms of linear momentun. What does this express state. 3 Marks

(4) Write continuity equation for a compressible flow in cylindrical coordinates. 3 Marks

(5) Newton's 2nd law can also be stated as the net force acting on a body is equal to the time rate of change of momentum. Determine the primary dimension of the momentum 5 Marks

(6) Consider an unsteady two dimensional velocity field $\vec{V}=(u,v)=(2.5+0.8x^2)\,i + (1.5+2.5\sin(\omega t)-1.6xy)\,j$ where angular frequency (ω) is equal to 2π rad/s. Verify this flow is incompressible 5M

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FALL/SPRING MID/FINAL TERM 20__ (Year)

Campus Code:_____

Course Code:_____

Rough Sheet

Date: 1/9/21

Superintendent Signature:_____

Student Signature:_____

Student ID:_____

① Show that $a_t = \gamma\alpha$ where a_t is the linear acceleration α is the angular acceleration and γ is the radius?

② Euler $\frac{\partial \rho}{\partial t} + \vec{v}(\rho\vec{v}) = 0$ into the cartesian coordinates where $V = (u, v, w)$?

③ why the divers rotate faster when they curl the Jump? Explain it in the light of ~~conti~~ conservation of angular moment Principle?

④ consider a steady velocity field given by $\vec{V} = (u, v, w) = 2ax^2 i + (by + cz) + (az - 4ax) k$ where a, b and c are constant Under what condition is this flow field.?

⑤ Define Nondimensional variable?

⑥ In a fluid flow Problem in which the flow is three dimensional and incompressible what will be the minimum number of unknowns? None

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Ans

(1) Expand $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{V}$ in Cartesian Coordinates by using $(V = x, y, z)$ 3 marks

(2) write Expression for equation of continuity for unsteady compressible in Cartesian Coordinates.

(3) Difference b/w Dimensional variable and Dimensional Constant

(4) State and write mathematical form of Divergence theorem.

(5) write expression for Conversion of mass relation for a control volume.

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(2) Kinematic Similarity
(3) Dynamic Similarity.

Q. No. 15

Why the ice skaters spin faster when they bring their arms closer to their bodies? Explain in the light of principle of conservation of momentum?

Sol:-

As the angular momentum eqn is

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$\vec{M} = I\vec{\alpha}$

the ice skaters spinning faster when they brige their arms close to their bodies because the moment of inertia (I) decreased and thus the angular velocity ω is increased as the outer part of body are brought closer to the axis of rotation.

Same why divers get more speed after curly jump?

Q. No. 16

Write the linear moment in newton 2nd law?

Sol:-

As the Newton's 2nd law of motion is given by

$\vec{F} = m\vec{a}$

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Q. No. 16

Write the linear moment in newton 2nd law?

Sol:-

As the Newton's 2nd law of motion is given by

$\vec{F} = m\vec{a}$

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where $\vec{a}$ is acceleration and defined as Time rate of change of velocity.

$\vec{a} = \frac{d\vec{v}}{dt}$

So

$\vec{F} = m\frac{d\vec{v}}{dt}$

$\vec{F} = \frac{d}{dt}(m\vec{v})$

Newton's 2nd law also describe as

"Time rate of change of linear momentum".

Q. No. 17:-

Derive Navier-Stokes eqn. for incompressible?

Sol:-

As viscous flow

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$$Z^* = 1 + \ell^* - \frac{1}{2F_\#^2}\ell^{*2}.$$

Q.No.21

State angular momentum eq. write its expression about x-axis and y-axis.

Sol:-

Angular momentum eq. stated as

"Rate of change of angular momentum".

i/e

$$\vec{M} = I\vec{\alpha}$$

$$= I\frac{d\vec{\omega}}{dt}$$

$$= \frac{d(I\vec{\omega})}{dt}$$

$$\vec{M} = \frac{d\vec{H}}{dt}$$

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about x-axis

$$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

about y-axis

$$M_y = I_y\frac{d\omega_y}{dt} = \frac{dH_y}{dt}$$

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Hence $\lambda t = 2 \times 7 = 14$

$$P(X = x) = \begin{cases} \frac{e^{-\lambda t}(\lambda t)^x}{x!} = \frac{e^{-14}(14)^x}{x!}, & x = \\ 0 & \text{elsewhere} \end{cases}$$

Solved By Mahar Afaq Safdar Muhamma

Now we require $X \leq 2$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-14}(14)^0}{0!} + \frac{e^{-14}(14)^1}{1!} + \frac{e^{-14}(14)^2}{2!}$$

$$= e^{-14} + 14e^{-14} + 98e^{-14}$$

$$= 113e^{-14}$$

$$= 0.0000\,9396$$

$$P(X \leq 2) = 9.396 \times 10^{-5}$$

We can say that the probability that at most 2 accidents will occur at a busy crossing in a week, this probability is almost 0, which is "Next to impossible".

Q No. 21: Write down the mean of the multinomial distribution.

Ans:

The mean of multinomial distribution is

$$E(X_i) = np_i$$

The variance of multinomial distribution is

$$Var(X_i) = np_i(1-p_i)$$

$$= np_iq_i$$

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as the product of moment of inertia and angular velocity

i.e. $\vec{H} = I\vec{\omega}$

Q.No.4

State divergence theorem:

Sol:-

The divergence theorem is also known as Gauss's theorem. It allow us to convert the volume integral

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of the divergence of a vector into area integral over the surface that defines the volume.

For any vector $\vec{G}$, the divergence of $\vec{G}$ is defined as $\vec{\nabla}\cdot\vec{G}$ and divergence theorem is

$$\int_V \vec{\nabla}\cdot\vec{G}\,dV = \oint_A \vec{G}\cdot\hat{n}\,dA.$$

Q.No.5:-

write product rule of $\vec{\nabla}\cdot(\rho\vec{v}) = 0$

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10:42
Edit
50
Easily add text in PDF
Add

Q.No 1:-

Expand $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V})$ in cartesian coordinates by using $V = (u, v, w)$.

Sol:-

As we know that

Rate of change of mass within CV

$$\int_{CV} \frac{\partial \rho}{\partial t} dV = \frac{\partial \rho}{\partial t} dx dy dz$$

Net mass flow rate into CV

$$\sum_{in} \dot{m} = \left(\rho u - \frac{\partial (\rho u)}{\partial x} \frac{dx}{2}\right) dy dz + \left(\rho v - \frac{\partial (\rho v)}{\partial y} \frac{dy}{2}\right) dx dz + \left(\rho w - \frac{\partial \rho w}{\partial z} \frac{dz}{2}\right) dx dy$$

Also

net mass flow rate out of CV

$$\sum_{out} \dot{m} = \left(\rho u + \frac{\partial (\rho u)}{\partial x} \frac{dx}{2}\right) dy dz + \left(\rho v + \frac{\partial \rho v}{\partial y} \frac{dy}{2}\right) dx dz + \left(\rho w + \frac{\partial (\rho w)}{\partial z} \frac{dz}{2}\right) dx dy$$

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As we know that

$$\int_{CV} \frac{\partial \rho}{\partial t} dV = \sum_{out} \dot{m} - \sum_{in} \dot{m}$$

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Q.No.21

State angular momentum eq. write its expression about x-axis and y-axis.

Sol:-

Angular momentum eq. stated as

"Rate of change of angular momentum".

i/e

$$\vec{M} = I\vec{\alpha}$$

$$= I\frac{d\vec{\omega}}{dt}$$

$$= \frac{d(I\vec{\omega})}{dt}$$

$$\vec{M} = \frac{d\vec{H}}{dt}$$

3

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The counterpart of Newton's second law for rotating rigid bodies is expressed as $\vec{M} = I\vec{\alpha}$, where $\vec{M}$ is the net moment or torque applied on the body, I is the moment of inertia of the body about the axis of rotation, and $\vec{\alpha}$ is the angular acceleration. It can also be expressed in terms of the rate of change of angular momentum $d\vec{H}/dt$ as

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

Angular momentum about x-axis: $$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

- The conservation of angular momentum principle: The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved.

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4:37

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$$= m_1 \times \frac{d}{dt}(r_1\omega)$$

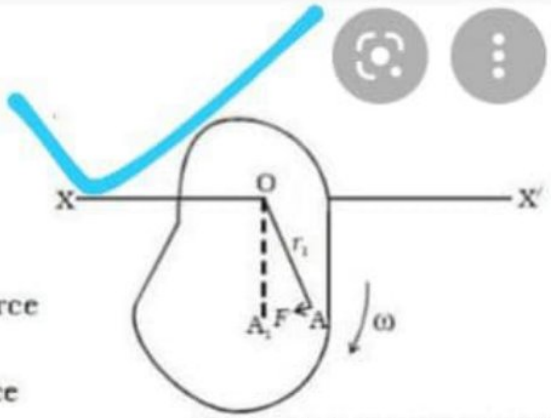
$$= m_1 r_1 \frac{d\omega}{dt}$$

$$= m_1 r_1 \frac{d^2\theta}{dt^2}$$

The moment of this force about the axis of rotation

= Force × perpendicular distance

$$= m_1 r_1 \frac{d^2\theta}{dt^2} \times r_1$$



Relation between torque and angular acceleration

Therefore, the total moment of all the forces acting on all the particles

$$= m_1 r_1^2 \frac{d^2\theta}{dt^2} + m_2 r_2^2 \frac{d^2\theta}{dt^2} + \ldots$$

(i.e) torque $= \sum_{i=1}^{n} m_i r_i^2 \times \frac{d^2\theta}{dt^2}$

or $\tau = I\alpha$

where $\sum_{i=1}^{n} m_i r_i^2$ = moment of inertia I of the rigid body and $\alpha = \frac{d^2\theta}{dt^2}$ angular acceleration.

Physics

Relation between torque and angular momentum -...

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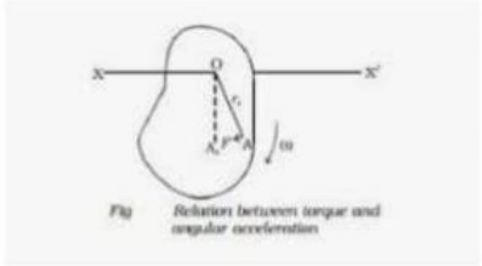
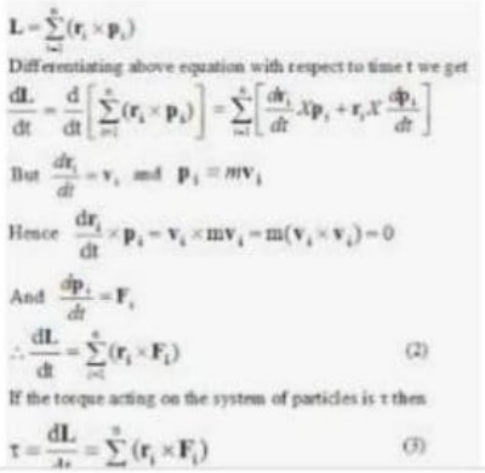


Fig Relation between torque and angular acceleration

Relation between torque and ...

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$$\mathbf{L} = \sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{p}_i)$$

Differentiating above equation with respect to time t we get

$$\frac{d\mathbf{L}}{dt} = \frac{d}{dt}\left[\sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{p}_i)\right] = \sum_{i=1}^{n}\left[\frac{d\mathbf{r}_i}{dt} \times \mathbf{p}_i + \mathbf{r}_i \times \frac{d\mathbf{p}_i}{dt}\right]$$

But $\frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i$ and $\mathbf{p}_i = m\mathbf{v}_i$

Hence $\frac{d\mathbf{r}_i}{dt} \times \mathbf{p}_i = \mathbf{v}_i \times m\mathbf{v}_i = m(\mathbf{v}_i \times \mathbf{v}_i) = 0$

And $\frac{d\mathbf{p}_i}{dt} = \mathbf{F}_i$

$$\therefore \frac{d\mathbf{L}}{dt} = \sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{F}_i) \quad (2)$$

If the torque acting on the system of particles is τ then

$$\tau = \frac{d\mathbf{L}}{dt} = \sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{F}_i) \quad (3)$$

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10:43

Edit

10.39: The names of 5 men and 5 women written on slips of paper and placed in a . four names are drawn. What is probability t 2 are men and 2 are women?

:

Let X denote the number of men.

en , $N = 5+5 = 10$ names to be drawn from.

$K = 5$ and $n = 4$

and here the possible values of x are

, 1, 2, 3, 4, i.e n.

Hence the hypergeomatric distribution is

$$h(x; N, n, K) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$h(x; 10, 4, 5) = \frac{\binom{5}{x}\binom{10-5}{4-x}}{\binom{10}{4}}$$

nd the required probability i.e $P(X=2)$ is

$$h(2; 10, 4, 5) = \frac{\binom{5}{2}\binom{5}{4-2}}{\binom{10}{4}} = \frac{\binom{5}{2}\binom{5}{2}}{\binom{10}{4}}$$

$$= \frac{(10)(10)}{210} = \frac{10}{21}$$ Ans

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Mth642 today paper 9.30am
Objective total conceptual thy past file se kuch ni aya
Subjective mey Angular momentum ka question tha
Froud number ka formula likhna tha
Compresible cylindrical coardinate ka formula likhna tha 3 number ka tha
Divergence ka ik question tha

12:27 pm

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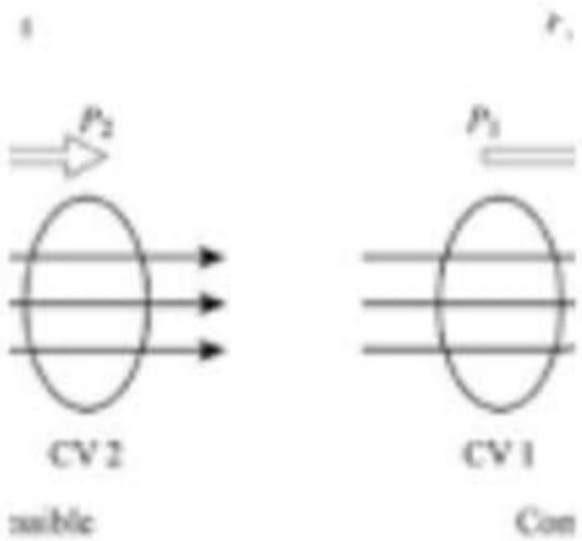
> +92 303 0254654
> **MTH 642** Done... Today 3:00 pm...
> **Alhamdu lillah**... Kuch nahi ata tha... Total theory & total conceptual... 😂

#Compressible & imcompressible flow ki continuity ka 3 dimensional & 2 dimensional formula...
#Why divers gets faster speed after a curly jump?
#Mcqs were totally theoritical & conceptual
#Angular momentum sy related bahut question aye
#X and Y axis pr compressibility flow ka ek numerical tha

★ 4:15 pm

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Incompressible flow refers to the fluid flow in which the fluid's density is constant. ... All the liquids at constant temperature are incompressible. Compressible flow means **a flow that undergoes a notable variation in density with trending pressure**.



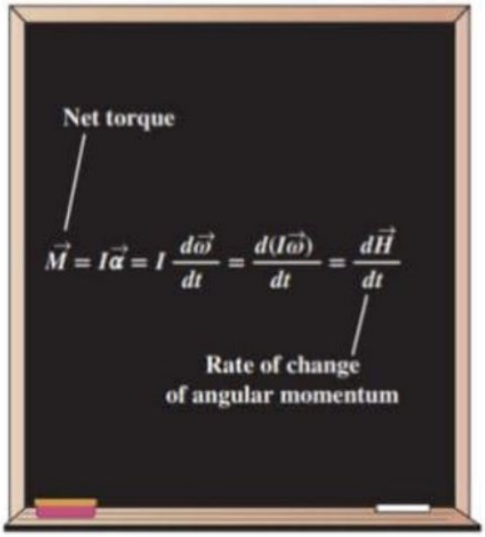
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The counterpart of Newton's second law for rotating rigid bodies is expressed as $\vec{M} = I\vec{\alpha}$, where $\vec{M}$ is the net moment or torque applied on the body, I is the moment of inertia of the body about the axis of rotation, and $\vec{\alpha}$ is the angular acceleration. It can also be expressed in terms of the rate of change of angular momentum $d\vec{H}/dt$ as

Angular momentum equation: $$\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{H}}{dt}$$

Angular momentum about x-axis: $$M_x = I_x\frac{d\omega_x}{dt} = \frac{dH_x}{dt}$$

- The conservation of angular momentum principle: The total angular momentum of a rotating body remains constant when the net torque acting on it is zero, and thus the angular momentum of such systems is conserved.



- The rate of change of the angular momentum of a body is equal to the net torque acting on it.

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What are examples of compressible fluids?

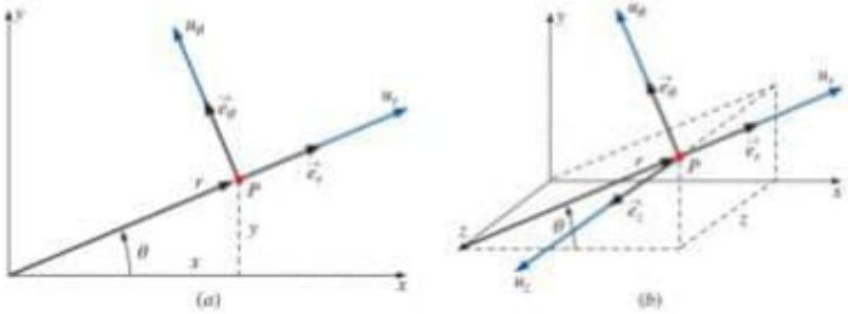
Vapors and Gases

The flow of compressible fluids such as **gas, vapor, steam, etc.**, is considered in general the same as for liquids or non-compressible fluids.

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and $\vec{e}_\theta$, are also shown in Fig. (a). In three dimensions, imagine sliding everything in Fig. (a) out of the page along the *z*-axis (normal to the *xy*-plane) by some distance *z*. We have attempted to draw this in Fig. (b). In three dimensions, we have a third velocity component, u_z, and a third unit vector, $\vec{e}_z$, also sketched in Fig. (b)



Velocity components and unit vectors in cylindrical coordinates:
(*a*) Two-dimensional flow in the *xy*- or $r\theta$-plane,
(*b*) Three-dimensional flow.

Coordinate transformations:

$$r = \sqrt{x^2 + y^2} \quad x = r\cos\theta \quad y = r\sin\theta \quad \theta = \tan^{-1}\frac{y}{x}$$

Coordinate *z* is the same in cylindrical and Cartesian coordinates.

Continuity equation in cylindrical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0$$

Topic No. 124

THE CONTINUITY EQUATION: Special Cases of the Continuity Equation

Special Case 1: Steady Compressible Flow

Steady continuity equation:

$$\vec{\nabla}\cdot(\rho\vec{V}) = 0$$

In Cartesian coordinates, this reduces to

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

In cylindrical coordinates, it reduces to

$$\frac{1}{r}\frac{\partial (r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0$$

Special Case 2: Incompressible Flow

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Ak example aie hui thiii us ma incompressible find krna tha 2.dimensionl flow wo be conceptial thi aur y component find krna rhA

8:53 am

Incompressible flow wli exmples dakh lyna ap log 2.dimensional aur 3.dimensional wli us sa related aye gaee

8:53 am

#Compressible & imcompressible flow ki continuity ka 3 dimensional & 2 dimensional formula...
#Why divers gets faster speed after a curly jump?
#Mcqs were totally theoritical & conceptual
#Angular momentum sy related bahut question aye
#X and Y axis pr compressibility flow ka ek numerical tha

8:53 am

Aoa myraa paper howaa mth642 ka ..jahn py likhaa thaa sir ny page ly lyaa..
Mcqs concptual saryyy exmples ma sy koi swl nhi ayaaa definition bhi nhi i koi question bhi concept py hi base karty thy saryy.....

8:53 am



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Rough Sheet

FALL/SPRING MID/FINAL TERM 20__ (Year)

Campus Code:_____

Course Code:_____

Date: 1/9/21

Superintendent Signature:_____

Student Signature:_____

Student ID:_____

① Show that $a_t = \gamma a$ where a_t is the linear acceleration α is the angular acceleration and γ is the radius?

② Euler $\frac{\partial \rho}{\partial t} + \vec{v}(\rho \vec{v}) = 0$ into the cartesian coordinates where $V = (u, v, w)$?

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④ consider a steady velocity field given by $\vec{V} = (u, v, w) = 2ax^2 i + (by + cz + (az - 4ax)k$ where a, b and c are constant Under what condition is this flow field.?

⑤ Define Nondimensional variable?

⑥ In a fluid flow Problem in which the flow is three dimensional and incompressible what will be the minimum number of unknowns? Name

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The velocity component in along x-axis of a steady, incompressable, two dimensional flow is given by $u = (2.5y + 0.9x^2)$ Component v is missing. Generate expression for v as a function of x (5M)

Expand $\frac{\partial \rho}{\partial t} + \bar{\nabla}(\rho \bar{V}) = 0$. (5M)

into a cartesian coordinates, where $\bar{V} = (u, v, w)$.

Write the expression for conservation of mass relation for a control volume. What does the expression state? (3M)

Name four dimensional parameters (3M)

Products of fluid flow.

Using Product rule expand $\bar{\nabla}(PV)$ (2M)

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MTH-642 01-09-2021

Q No (2) (3) marks

Write the algebraic expression of the divergence theorem and named the term is used

Q.No (4) (5) marks

Decribe the dimensions of nondimensional equation and prove that $\frac{d^2z}{dt^2} = -g$

Q.No (3) (3) mark

State angular momentum equation and write its expression about x-axis and z-axis 5 (mark)

Q No (4) example (independent vaible of r) incompressible

MCQ's (Totally Conceptial) (Quiz) mid mid

Write the algebraic

expression of the

divergence theorem and

named the term as

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FALL/SPRING MID/FINAL TERM 20__ (Circle)

Campus Code: _____

Course Code: EE3[illegible]

Roll# [illegible]

Date: [illegible]

Invigilator Signature: _____

Student Signature: _____

Student ID: _____

None Item

If a fluid flow passes in which the flow is three dimensional incompressible and why is it minimum and maximum? None the nondimensional

$$\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(2)

Write angular velocity and write the expression about y-axis

2-axis

$$\vec{F} = m\vec{a}$$

$$F = m\frac{d\vec{v}}{dt}$$

$$= d(mv)$$

$$= \left(\frac{d}{dt}\right)$$

Note: Submit this sheet to Invigilator before leaving the Examination Center

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$\frac{d^2z}{dt^2} = -g$ describe th

$z = z_0 + w_0 t - \frac{1}{2} g t$ after mayn
and app the initial

Find the non-dimensional form of the

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