

The Lebesgue outer measure of Cantor set constructed from $[0,1]$, is _____

Answer☐ 1☒ 0☐ $\frac{1}{2}$ ☐ $\frac{1}{3}$

01

05

09

13

17

21

25

29

33

4 / 5

Attempt

Flagged

Next Question Total

If $\{E_n\}$ be an increasing sequence of measurable sets then _____

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Answer

☐ $E_1 - E_1 \subseteq E_1 - E_2 \subseteq E_1 - E_3 \subseteq \dots E_1 - E_n \subseteq \dots$

☐ $E_1 \neq E_2 \neq E_3 \neq \dots$

☒ $E_1 \subseteq E_2 \subseteq E_3 \subseteq \dots$

☐ $E_1 \supseteq E_2 \supseteq E_3 \supseteq \dots$

If (E_n) be a sequence of measurable sets such that $E_1 = E_2 = E_3 = \dots$, then (E_n) is an example of _____ sequence.

Answer

- ☐ (I) decreasing
- ☐ (II) increasing
- ☒ (III) neither increasing nor decreasing
- ☐ (IV) Both (I) and (II).



A (an) _____ of Lebesgue measure m to the class of Borel sets $\mathcal{B}$ is said to be Borel measure space

Answer☐ bijection☐ kernel☒ restriction☐ extension

Consider the sum modulo 1 for the interval $[0,1[$ then $x \hat{+} y$ of $x = y = \frac{1}{2} =$ _____

Answer



1



2



0



-1



Question No 10 of 51

Those countable sets which are Borel have _____ measure.

Answer

☒

zero

☐

positive

☐

infinite

☐

no

Question No. 12 of 51

Consider the sum modulo 1 for the interval $[0,1[$ then $x \frown y$ of $x = \frac{3}{2}$ and $y = \frac{1}{2}$ = _____

Answer

 $\frac{1}{2}$ 

1



3



Not defined at all

Question No. 14 of 51

Let $\mathcal{M}$ be a measurable space (a sigma algebra of measurable sets) then a function $f: E \rightarrow \mathbb{R} \cup \{\pm\infty\}$ is measurable if the set $\{x \in E: f(x) > a\}$ is measurable for all _____

Answer



$a \in \mathbb{R}$



$a \in \mathbb{R} \cup \{\pm\infty\}$



$x \in \mathbb{R}$



$x \in \mathbb{R} \cup \{\pm\infty\}$

Question No : 15 of 51

Suppose that f is a real valued identically constant function $f(x) = \beta$ then
 $\{x: f(x) > a\} = \underline{\hspace{2cm}}$ if $\beta = a$.

Answer

☒

$\emptyset$

☐

$\mathbb{R}$

☐

$\{\beta\}$

☐

$\{a\}$

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Question No : 16 of 51

Suppose that f is a real valued identically constant function $f(x) = \beta$ and $\{x: f(x) > a\} = \mathbb{R}$. If $\beta > a$ then _____.

Answer

- ☐ f is bijective
- ☐ f is 1 - 1
- ☒ f is measurable
- ☐ f is not measurable

Question No : 17 of 51

Suppose that f is a real valued identically constant function $f(x) = \beta$ then $\{x: f(x) > a\} = \underline{\hspace{2cm}}$ if $\beta < a$.

Answer

 $\emptyset$ Domain of f  $\{\beta\}$  $\{a\}$

Question No : 13 of 51

Let $\mathcal{M}$ be a measurable space (a sigma algebra of measurable sets) then a function $f: E \rightarrow \mathbb{R}$ is Lebesgue measurable function on $\emptyset \neq E \in \mathcal{M}$ if _____.

Answer

☐ $\{x \in E: f(x) > x\}$

☐ $\{a \in E: f(x) > a\}$

☒ $\{x \in E: f(x) > a\}$

☐ $\{x \in E: f(x) < x\}$

Next Question

Question No 18 of 51

Suppose that f is a measurable function then the set $\{x \in D: f(x) = a\}$ is _____.

Answer

- ☐ countable
- ☐ uncountable
- ☐ non – measurable
- ☒ measurable



On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the positive part of f is $f^+ =$ _____ if $f(x) \geq 0$.

Answer

☒ $f(x)$

☐ $-f(x)$

☐ 0

☐ $\max\{0, -f(x)\}$

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Question No. 20 of 51

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the negative part of f is $f^- =$ _____ if $f(x) \geq 0$.

Answer

- ☐ $f(x)$
- ☐ $-f(x)$
- ☒ 0
- ☐ $\max\{0, f(x)\}$

Let $X \neq \emptyset$ and $A \subseteq X$ then the characteristic function relative to A is $\chi_A: X \rightarrow \mathbb{R}$ is defined and given by _____.

Answer

☐ $\chi_A = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \in X \end{cases}$

☐ $\chi_A = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{if } x \in A \end{cases}$

☒ $\chi_A = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \in A^c \end{cases}$

☐ $\chi_A = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{if } x \in X \end{cases}$

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the positive part of f is $f^+ =$ _____

Answer

☐ $\min\{f(x), 0\}$

☒ $\max\{f(x), 0\}$

☐ $\min\{-f(x), 0\}$

☐ $\max\{-f(x), 0\}$

Let $\emptyset \neq X$, and $A \subseteq X$ then the characteristic function $\chi_A: X \rightarrow \mathbb{R}$ for $A = X$ is $\chi_X =$ _____

Answer

☐ 0

☒ 1

☐ 2

☐ -2

Question No. 24 of 51

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ and for $f(x) \geq 0$, $f^+ + f^- =$ _____

Answer

☐ $2f(x)$ ☐ 0 ☒ $f(x)$ ☐ $-f(x)$

Question No : 27 of 51

If f is an extended real valued and measurable function then the translation $k + f$ is also measurable _____.

Answer

☒ $\forall k \in \mathbb{R}$ ☐ $k > 0$ ☐ $\forall k \in \mathbb{N}$ ☐ $\forall k \in \mathbb{Z}$

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ and for $f(x) \geq 0$, $f^+ - f^- =$ _____

Answer

☒ $f(x)$

☐ $2f(x)$

☐ 0

☐ $-f(x)$

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On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ and for $f(x) < 0$ then $\frac{f + |f|}{2} =$ _____

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Answer



f



f^-



f^+



$|f|$

Let $(f_n)_{n=1}^{\infty}$ be a sequence of real valued measurable functions then $\sup_{k \geq n}(f_k)$ is measurable for _____

Answer

☐ $n \in \mathbb{N}$

☒ all $n \in \mathbb{N}$

☐ some $n \in \mathbb{N}$

☐ all $k \in \mathbb{Z}$

Question No : 25 of 51

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the positive part of f is $f^+ =$ _____ if $f(x) < 0$.

Answer

- ☐ $f(x)$
- ☒ $-f(x)$
- ☐ 0
- ☐ $\max\{-f(x), f(x)\}$

Question No 38 of 51

Let $(f_n)_{n=1}^{\infty}$ be a sequence of real valued measurable functions then _____ is also measurable.

Answer

- ☐ (I) $\sup_{n \in \mathbb{N}} \{f_n\}$
- ☐ (II) $\inf_{n \in \mathbb{N}} \{f_n\}$
- ☒ (III) Both (I) and (II).
- ☐ (IV) Neither (I) nor (II).

Question No. 37 of 37
Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions and $f = \inf_{n \in \mathbb{N}} f_n$ is also measurable then for $x \in D$, _____.

Answer

- ☒ $f(x) \leq f_n(x)$, for all $n \in \mathbb{N}$ and $a \in \mathbb{R}$
- ☐ $f(x) \geq f_n(x)$, for all $n \in \mathbb{N}$ and $a \in \mathbb{R}$
- ☐ $f(x) \leq f_n(x)$, for some $n \in \mathbb{N}$ and for all $a \in \mathbb{R}$
- ☐ $f(x) \geq f_n(x)$, for some $n \in \mathbb{N}$ and for all $a \in \mathbb{R}$

Question No : 36 of 51

Let $f: X \rightarrow Y$ and $g: A \rightarrow Y$ be two functions. If $A \subseteq X$ and $f(x) = g(x)$ then g is called the _____ of f to $A \subseteq X$.

Answer

- ☒ restriction
- ☐ extension
- ☐ Lebesgue measure
- ☐ Lebesgue outer measure

Question No. 35 of 3

Let $f: X \rightarrow Y$ be a function and $A \subseteq X$ then $g: A \rightarrow Y$ is called the restriction of f to $A \subseteq X$, if _____

Answer

☐ $g(x) > f(x), \forall x \in A$

☐ $f(x) > g(x), \forall x \in A$

☒ $f(x) = g(x), \forall x \in A$

☐ Both f and g are measurable.

Question No : 34 of 51

If f is a real valued and measurable function then $\{x: f^2(x) > a, \forall a \in \mathbb{R}\} = \underline{\hspace{2cm}}$.

Answer

☐ $\{x: f(x) > \sqrt{a}\} \cup \{x: f(x) > -\sqrt{a}\}$

☐ $\{x: f(x) > \sqrt{a}\} \cap \{x: f(x) > -\sqrt{a}\}$

☐ $\{x: f(x) > \sqrt{a}\} \Delta \{x: f(x) > -\sqrt{a}\}$

☐ Domain of definition of f



If f is a real valued and measurable function then $\{x: |f(x)| > a, \forall a > 0\} = \underline{\hspace{2cm}}$

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Answer

☐ $\{x: f(x) > a\} \cap \{x: f(x) < -a\}$

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☒ $\{x: f(x) > a\} \cup \{x: f(x) < -a\}$



☐ $\{x: f(x) > a\} - \{x: f(x) < -a\}$

☐ Domain of definition of f

The real valued function $f.g$ is measurable even if _____.

Answer

☐ f is measurable but g is not measurable

☒ both f and g are measurable

☐ g is measurable but f is not measurable

☐ both f and g are not measurable



The real valued function $f - g$ is measurable even if _____

Answer

- ☐ f is measurable but g is not measurable
- ☐ g is measurable but f is not measurable
- ☐ both f and g are not measurable
- ☐ both f and g are measurable



Question No : 30 of 51

For $f(x) = \cos x, -\pi < x \leq \pi, f^+ =$ _____.

Answer



$$\sin x, -\frac{\pi}{2} < x < \frac{\pi}{2}$$



$$\sin x, -\frac{\pi}{2} < x \leq \frac{\pi}{2}$$



$$\cos x, -\frac{\pi}{2} \leq x < \pi$$



$$\cos x, -\frac{\pi}{2} < x < \frac{\pi}{2}$$



Question No : 29 of 51

For $f(x) = \sin x, -\pi < x \leq \pi, f^+ = \underline{\hspace{2cm}}$

Answer

☐ $\sin x, -\pi < x \leq 0$

☐ $\cos x, -\pi < x \leq 0$

☒ $\sin x, 0 < x \leq \pi$

☐ $\cos x, 0 < x \leq \pi$

Question No. 4
For a non empty set E in a measurable space $\mathcal{M}$, _____

Answer

☐ $m(E) > m^*(E)$

☐ $m(E) < m^*(E)$

☒ $m(E) = m^*(E)$

☐ $m(E) = 2m^*(E)$

For all $E \subseteq \mathbb{R}$, there is a G_δ set $G \supset E$, such that _____

Answer

- ☐ $m^*(E - G) > 0$
- ☐ $m^*(G - E) < 0$
- ☒ $m^*(G - E) = 0$ ✓
- ☐ $m^*(G - E)$ is not defined

For $\emptyset \neq E \subseteq \mathbb{R}$ and $\epsilon > 0$, there exists an open set O such that _____

Answer

- ☐ $E \subseteq O$
- ☒ $O \supseteq E$
- ☐ $E - O \neq \emptyset$
- ☐ $E = O$

An open set U in $\mathbb{R}$ is measurable because it can be expressed as _____ of open intervals.

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Answer

☐ countable intersection

☒ countable union

☐ arbitrary union

☐ arbitrary intersection

For $E \subseteq \mathbb{R}$, if there exists a closed set $F \subseteq E$ such that $m^*(E - F) = 0$ then show that $m^*(E^c - F) = 0$.

Answer



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U













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Date _____

$$m^*(E - F) = 0$$

here $E' \subseteq \mathbb{R}$

$$F \subseteq E'$$

Then

$$m^*(E' - F) = 0$$

Question No : 42 of 51

Exemplify the Borel set as an intersection of open intervals in $\mathbb{R}$.

Answer

✂ ✂ B I U ✓ L ; :: ✂ ✂ ✂ ✂ ✂ ✂ ✂ ✂ ✂ ✂ ✂ ✂
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For Solutions of Subjective Questions

See My Subjective Questions File.



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62 MB - 2022-08-18

Q.No.23) (2 mark)

Define Borel set.?

Sol:-

A set E is said to be a Borel set if it can be expressed as countable union or intersection of open or closed sets.

e.g

$$\{x\} = \bigcap_{n=1}^{\infty} \left(x - \frac{1}{n}, x + \frac{1}{n}\right)$$

Question No : 43 of 51

If $\emptyset \neq A, B \subseteq X$ and $A \subset B$ such that some members of B are **NOT** in A then show that $\chi_A \leq \chi_B$.

Answer

  | **B** *I* U |  *I_x* |   |   |     |   |

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Topic # 75

Theorem: Prove that if $A \subseteq B$, then $\chi_A \leq \chi_B$

Proof: Case I; If $x \in A$ and

$$A \subseteq B \Rightarrow x \in B$$

$$\Rightarrow \chi_A = 1 = \chi_B \Rightarrow \chi_A \leq \chi_B$$



Case II; If $x \in B$ but $x \notin A$

$$\Rightarrow \chi_A = 0 \text{ and } \chi_B = 1$$

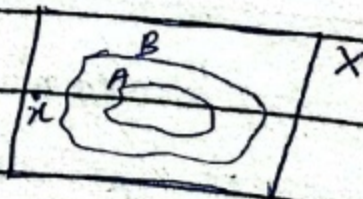
$$\Rightarrow \chi_A \leq \chi_B$$



Case III; If $x \notin A$ nor $x \in B$.

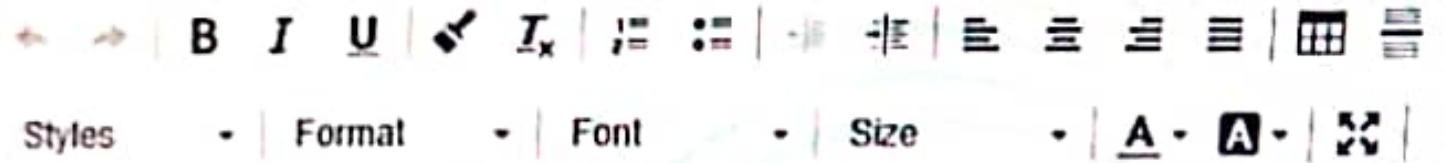
$$\Rightarrow \chi_A = 0 = \chi_B \Rightarrow \chi_A \leq \chi_B$$

(proved)...



If f and g are real valued and measurable functions then show that $f.g$ is also measurable.

Answer



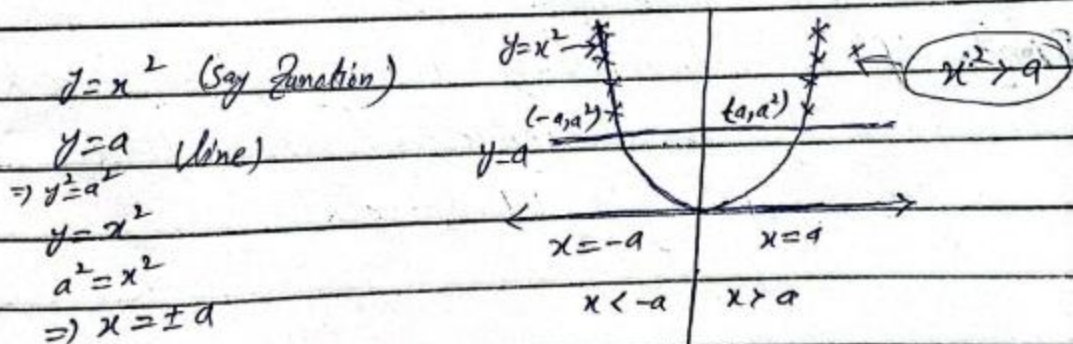


(iv) " fg " is measurable.

proof: $fg \equiv \frac{1}{4}[(f+g)^2 - (f-g)^2]$

Firstly we prove " f^2 " is measurable.

$\Rightarrow$ we need to prove $\{x: f^2(x) > a\} \in \mathcal{M}, \forall a \in \mathbb{R}$.



$$\begin{cases} f^2(x) > a \\ \Rightarrow f(x) > \sqrt{a}, \\ f(x) < -\sqrt{a} \end{cases}$$

$$\{x: f^2(x) > a\} = \begin{cases} \{x: f(x) > \sqrt{a}\} \cup \{x: f(x) < -\sqrt{a}\}, & a \geq 0 \\ \text{"All Domains" } (D), & a < 0 \end{cases}$$

$\therefore$ " f " is measurable $\Rightarrow \{x: f(x) > \sqrt{a}\} \in \mathcal{M}$
and $\{x: f(x) < -\sqrt{a}\} \in \mathcal{M}$

$\Rightarrow \{x: f(x) > \sqrt{a}\} \cup \{x: f(x) < -\sqrt{a}\} \in \mathcal{M}$
and "Domain" is given to be measurable i.e. $D \in \mathcal{M}$

$\therefore \{x: f^2(x) > a\} \in \mathcal{M} \Rightarrow "f^2" \text{ is measurable.}$

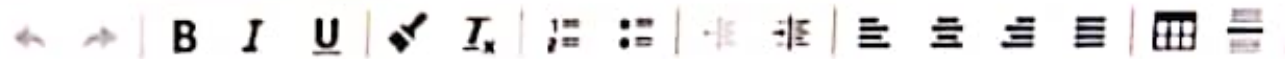
$\therefore f, g$ is measurable $\Rightarrow (f \pm g)$ is also measurable

$\Rightarrow (f \pm g)^2$ is measurable $\dots$

$\Rightarrow \frac{1}{4}[(f+g)^2 - (f-g)^2] = fg$ is also measurable.

For a non – empty set E in $\mathbb{R}$, if $m^*(E) = \infty$ then show that there exists an E_n such that $m^*(E_n) < \infty$.

Answer



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$$\text{if } m^*(E) = \infty$$

$$R = \bigcup_{n=1}^{\infty} I_n \quad \text{where } l(I_n) < \infty$$

$$\text{and } I_j \cap I_k = \emptyset, j \neq k.$$

$$\text{Put } E_n = E \cap I_n$$

$$\Rightarrow E_n \subseteq I_n \quad \forall n \in \mathbb{N}$$

$$m^*(E_n) \leq m^*(I_n) = l(I_n) < \infty$$

Hence

$$\boxed{m^*(E_n) < \infty.}$$

Justify your answer for the existence of $\mathbb{R}$ in Borel measurable space.

Answer

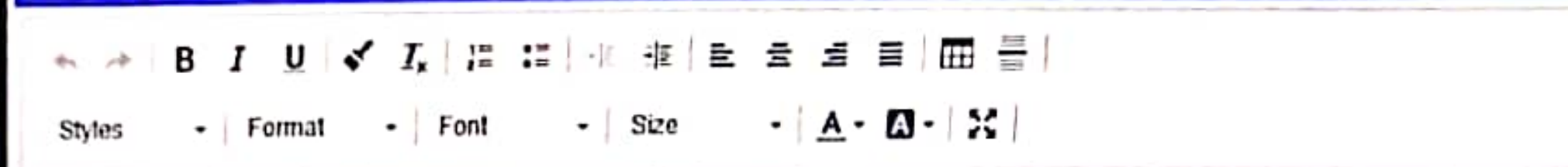
← → | **B** *I* U | ✓ $\mathcal{I}_x$ | $\int$ $\therefore$ | $\frac{1}{x}$ $\frac{1}{x^2}$ | $\frac{1}{x^3}$ $\frac{1}{x^4}$ | $\frac{1}{x^5}$ $\frac{1}{x^6}$ | $\frac{1}{x^7}$ $\frac{1}{x^8}$ | $\frac{1}{x^9}$ $\frac{1}{x^{10}}$ | $\frac{1}{x^{11}}$ $\frac{1}{x^{12}}$ |

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$\forall a \in \mathbb{R}$.

Restriction of Lebesgue
measure ~~to~~ m to the
class of Borel set B
is said to be Borel
measurable space i.e.
 $(\mathbb{R}, B, m)$. and here
 B may be described
as the smallest σ -algebra
that contains the class
of all intervals of $\mathbb{R}$.

Answer



Date _____

Day MTWTFS

Give that f is measurable
s.t. $\{x \in E : f(x) > a\}$, $a \in \mathbb{R}$

f is defined as $f: E \rightarrow \mathbb{R}$.

and we will show

that f is also measurable

on any subset B of

E s.t. $B \subseteq E$

Hence $g: B \rightarrow \mathbb{R}$

$\{y \in B : f(y) > a\}$

$\forall a \in \mathbb{R}$

Let f be an extended real valued measurable function on D and $\emptyset \neq A \subseteq D$ then show that $\{x \in A; f_A(x) > a\}$ is measurable, $\forall a \in \mathbb{R}$.

Answer

← → B I U ✓ L :: :: | | | | | | | | | |

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Theorem: Let " f " be an extended real valued measurable function on " D ". Let $A \subseteq D$. Then restriction of " f " to " A " is also measurable and $A \in \mathcal{M}$.

Proof: $\because$ " f " is measurable $\Rightarrow \{x \in D : f(x) > a\} \in \mathcal{M} \forall a \in \mathbb{R}$

For any $a \in \mathbb{R}$, To prove " f_A " is measurable we have $\{x \in A : f_A(x) > a\} \in \mathcal{M}$

Note: $\{x \in A : f_A(x) > a\} = \{x \in D : f(x) > a\} \cap A$

On R.H.S both the sets are measurable.

$\Rightarrow \{x \in A : f_A(x) > a\} \in \mathcal{M}$

$\therefore$ Intersection of two measurable sets is measurable.

$\Rightarrow$ " f_A " is measurable.

Question No : 49 of 51

In $\mathbb{R}$, if the interval (a, ∞) is measurable then show that bounded interval (a, b) is also measurable.

Answer

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Topic #48

Theorem: Open interval $(a, b) \subseteq \mathbb{R}$ is measurable.

proof: $\because (a, \infty)$ is measurable

$\Rightarrow (a, \infty)^c = (-\infty, a]$ is also measurable.

And $(-\infty, b - \frac{1}{n}]$ is also measurable $\forall n \in \mathbb{N}$

and $(-\infty, b) = \bigcup_{n=1}^{\infty} (-\infty, b - \frac{1}{n}]$ is also measurable $n \rightarrow \infty$

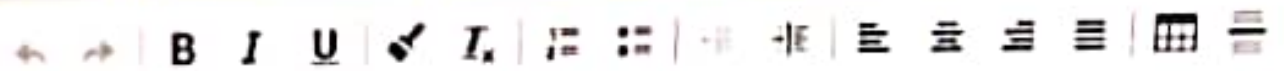
because being countable union of measurable set

$\Rightarrow (a, \infty) \cap (-\infty, b) = (a, b)$ i.e also measurable.

being intersection of two measurable set

Let $\emptyset \neq E$ be a measurable set and $f: E \rightarrow \mathbb{R} \cup \{\pm\infty\}$ then show that
 $\{x \in E: f(x) = \infty\} = \bigcap_{n=1}^{\infty} \{x \in E: f(x) > n\} \forall n \in \mathbb{N}.$

Answer



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Case III: If $a = -\infty$, then $\{x \in E : f(x) = -\infty\} = \bigcap_{n=1}^{\infty} \{x \in E : f(x) < -n\}$

Let $z \in \{x \in E : f(x) = -\infty\} \Rightarrow z \in \{x \in E : f(x) < -n\}$
 $\Rightarrow z \in \bigcap_{n=1}^{\infty} \{x \in E : f(x) < -n\}, \forall n \in \mathbb{N}$

$\Rightarrow \{x \in E : f(x) = -\infty\} \subseteq \bigcap_{n=1}^{\infty} \{x \in E : f(x) < -n\}$

Now let $s \in \bigcap_{n=1}^{\infty} \{x \in E : f(x) < -n\}, \forall n \in \mathbb{N}$

$\Rightarrow s \in \{x \in E : f(x) < -n\}, \forall n \in \mathbb{N}$

$\Rightarrow f(s) < -n, \forall n \in \mathbb{N}$

$\Rightarrow s \in \{x \in E : f(x) = -\infty\}$

$\Rightarrow \bigcap_{n=1}^{\infty} \{x \in E : f(x) < -n\} \subseteq \{x \in E : f(x) = -\infty\} \rightarrow \textcircled{b}$

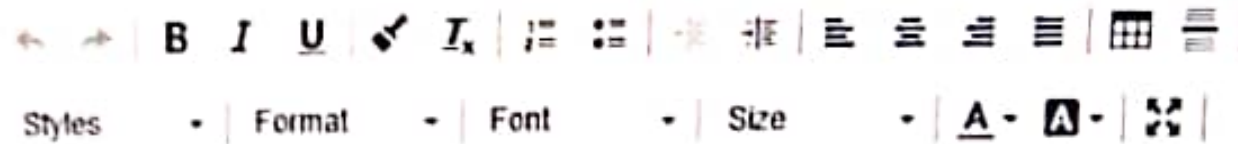
From $\textcircled{a}$ and $\textcircled{b}$

So, $\{x \in E : f(x) = -\infty\} = \bigcap_{n=1}^{\infty} \{x \in E : f(x) < -n\}$

Question No. 51 of 51

Let f be an extended real valued function defined on a set E of zero measure then show that f is measurable.

Answer





Topic # 89

Theorem: An extended real valued function defined on a set of measure zero is measurable.

Proof: Let $\phi \neq E \subseteq \mathbb{R}$, such that $m^*(E) = 0$ and $f: E \rightarrow \mathbb{R} \cup \{\pm\infty\}$. To prove " f " is measurable, we need to show that

$$\{x \in E: f(x) > a\} \in \mathcal{M}, \forall a \in \mathbb{R}.$$

$$\therefore \{x \in E: f(x) > a\} \subseteq E$$

$$\Rightarrow m^*\{x \in E: f(x) > a\} \leq m^*(E)$$

$\because$ By monotonicity of m^* .

$$\Rightarrow m^*\{x \in E: f(x) > a\} \leq 0 \quad \dots \text{(given)}$$

But by non-negativity of " m^* "

$$\Rightarrow m^*\{x \in E: f(x) > a\} \geq 0$$

$$\Rightarrow m^*\{x \in E: f(x) > a\} = 0, \forall a \in \mathbb{R}.$$

$$\Rightarrow \{x \in E: f(x) > a\} \in \mathcal{M}, \forall a \in \mathbb{R}$$

So " f " is measurable.

Question No : 37 of 51

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions and $f = \inf_{n \in \mathbb{N}} \{f_n\}$ is also measurable then for $x \in D$,

Answer

☒ $f(x) \leq f_n(x)$, for all $n \in \mathbb{N}$ and $a \in \mathbb{R}$ 

☐ $f(x) \geq f_n(x)$, for all $n \in \mathbb{N}$ and $a \in \mathbb{R}$

☐ $f(x) \leq f_n(x)$, for some $n \in \mathbb{N}$ and for all $a \in \mathbb{R}$

☐ $f(x) \geq f_n(x)$, for some $n \in \mathbb{N}$ and for all $a \in \mathbb{R}$

Question No : 23 of 51

Let $\emptyset \neq X$ and $A \subseteq X$ then the characteristic function $\chi_A: X \rightarrow \mathbb{R}$ for $A = \emptyset$ is $\chi_\emptyset =$ _____.

Answer

☒ 0☐ 1☐ ∞ ☐ $-\infty$

Question No : 39 of 51

Let $(f_n)_{n=1}^{\infty}$ be a sequence of real valued measurable functions then $\lim_{n \rightarrow \infty} f_n =$ _____.

Answer

☐

(I) $\overline{\lim}_{n \rightarrow \infty} f_n$

☐

(II) $\underline{\lim}_{n \rightarrow \infty} f_n$

☐

(III) Both (I) and (II).

☐

(IV) Neither of (I) nor (II).



Question No : 35 of 51

Let $f: X \rightarrow Y$ and $g: A \rightarrow Y$ be two functions and $A \subseteq X$ then the function g called the restriction of f if _____

Answer

- ☐ $A \supset X$
- ☒ $A \subseteq X$ 
- ☐ $A = X$
- ☐ $m^*(A) = m^*(X)$

Question No : 34 of 51

If f is a real valued and measurable function then $\{x: f^2(x) > a, \forall a \in \mathbb{R}\}$

Answer

☐ $\{x: f(x) > \sqrt{a}\} \cup \{x: f(x) > -\sqrt{a}\}$

☐ $\{x: f(x) > \sqrt{a}\} \cap \{x: f(x) > -\sqrt{a}\}$

☐ $\{x: f(x) > \sqrt{a}\} \Delta \{x: f(x) > -\sqrt{a}\}$

☐ Domain of definition of f ✓

If f is a real valued and measurable function then $\{x: |f(x)| > a, \forall a > 0\} =$ _____.

Answer

☐ $\{x: f(x) > a\} \cap \{x: f(x) < -a\}$

☒ $\{x: f(x) > a\} \cup \{x: f(x) < -a\}$ ✓

☐ $\{x: f(x) > a\} - \{x: f(x) < -a\}$

☐ Domain of definition of f

Question No : 17 of 51

Suppose that f is a real valued identically constant function $f(x) = \beta$ then $\{x: f(x) > a\} = ______$ if $\beta < a$.

Answer

 $\emptyset$ Domain of f  $\{\beta\}$  $\{a\}$

Question No : 13 of 51

Let $\mathcal{M}$ be a measurable space (a sigma algebra of measurable sets) then a function $f: E \rightarrow \mathbb{R}$ is Lebesgue measurable function on $\emptyset \neq E \in \mathcal{M}$ if _____.

Answer

☐ $\{x \in E: f(a) > x\}$

☐ $\{a \in E: f(x) > a\}$

☒ $\{x \in E: f(x) > a\}$ 

☐ $\{x \in E: f(a) < x\}$

Question No : 2 of 51

For $\emptyset \neq E \subseteq \mathbb{R}$ and $\epsilon > 0$, there exists a closed set F such that _____

Answer

☐ $F \supseteq E$

☒ $F \subseteq E$ 

☐ $F - E \neq \emptyset$

☐ $E = F$

Question No : 22 of 51

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{+\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the positive part of f is $f^+ =$ _____.

Answer

☐

$\min\{f(x), 0\}$

☒

$\max\{f(x), 0\}$

☐

$\min\{-f(x), 0\}$

☐

$\max\{-f(x), 0\}$

Question No : 14 of 51

Let $\mathcal{M}$ be a measurable space (a sigma algebra of measurable sets) then a function $f: E \rightarrow \mathbb{R}$ on $\emptyset \neq E \in \mathcal{M}$ is measurable if the set $\{x \in E: f(x) > a\}$ is measurable for all _____.

Answer

☒

$a \in \mathbb{R}$

☐

$a \in \mathbb{R} \cup \{\pm\infty\}$

☐

$x \in \mathbb{R}$

☐

$x \in \mathbb{R} \cap \{\pm\infty\}$

Question No : 6 of 51

In the 2nd iteration for constructing Cantor Set from $[0,1]$, we drop a subintervals of length _____.

Answer

☐ $\frac{1}{3^6}$

☐ $\frac{1}{3^1}$

☒ $\frac{1}{3^2}$ ✓

☐ $\frac{1}{3^3}$

Question No : 4 of 51

Let $\emptyset \neq A, B \in \mathcal{M}$: the measurable space such that $B \subseteq A$ then _____.

Answer

☐ $m(B) \geq m(A)$

☐ $m(B - A) = m(A)$

☒ $m(B) \leq m(A)$ 

☐ $m(B) = m(A)$

Question No : 1 of 51

If the interval $(-\infty, a)$ is measurable in $\mathbb{R}$, and $A \subseteq \mathbb{R}$, is a test set, then $m^*(A \cap (a, \infty)) + m^*(A \cap (-\infty, a))$ _____

Answer

- ☒ $= m^*(A)$ ✓
- ☐ $= m^*(-\infty, a)$
- ☐ $= m^*(a, \infty)$
- ☐ $\leq m^*(-\infty, a)$

If f is an extended real valued and measurable function then kf is also measurable _____

Answer (Please select your correct option)

☐ $\forall k \in \mathbb{Z}$

☒ $\forall k \in \mathbb{R}$

☐ $\forall k \in \mathbb{N}$

☐ $k > 0$

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the positive part of f is $f^+ =$ _____ if $f(x) \geq 0$.

Answer (Please select your correct option)

☐ $-f(x)$

☐ $\max\{0, -f(x)\}$

☒ $f(x)$

☐ 0

Suppose that f is a measurable function then the set $\{x \in D: f(x) = a\}$ is _____.

Answer (Please select your correct option)

☒ measurable

☐ countable

☐ non - measurable

☐ uncountable

Suppose that f is a real valued identically constant function $f(x) = \beta$ then $\{x: f(x) > a\} = \underline{\hspace{2cm}}$ if $\beta < a$.

Answer (Please select your correct option)

☒ $\emptyset$ ☐ $\{a\}$ ☐ $\{\beta\}$ ☐ Domain of f

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ and for $f(x) \geq 0$, $f^+ - f^- =$ _____

Answer (Please select your correct option)

☒ $f(x)$ 

☐ $2f(x)$

☐ $-f(x)$

☐ 0

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the positive part of f is $f^+ =$ _____

Answer (Please select your correct option)

☐ $\min\{f(x), 0\}$

☐ $\min\{-f(x), 0\}$

☐ $\max\{-f(x), 0\}$

☒ $\max\{f(x), 0\}$



If f is an extended real valued and measurable function then the translation $k + f$ is also measurable _____

Answer (Please select your correct option)

☐ $k > 0$

☐ $\forall k \in \mathbb{Z}$

☒ $\forall k \in \mathbb{R}$

☐ $\forall k \in \mathbb{N}$

Suppose that f is a real valued identically constant function $f(x) = \beta$ and $\{x: f(x) > a\} = \mathbb{R}$. If $\beta > a$ then

Answer (Please select your correct option)

☐ f is not measurable

☐ f is 1 - 1

☐ f is bijective

☒ f is measurable



Suppose that f is a real valued identically constant function $f(x) = \beta$ then $\{x: f(x) > a\} = \underline{\hspace{2cm}}$ if $\beta > a$.

Answer (Please select your correct option)

☒ $\mathbb{R}$

☐ $\{\beta\}$

☐ $\{a\}$

☐ $\emptyset$

On $\mathbb{R}$, if $f: A \rightarrow \mathbb{R} \cup \{\pm\infty\}$ and $A \neq \emptyset$ then $\forall x \in A$ the negative part of f is $f^- =$ _____ if $f(x) \geq 0$.

Answer (Please select your correct option)

☐ $-f(x)$

☐ $f(x)$

☒ 0

☐ $\max\{0, f(x)\}$

Let $\mathcal{M}$ be a measurable space (a sigma algebra of measurable sets) then a function $f: E \rightarrow \mathbb{R}$ on $\emptyset \neq E \in \mathcal{M}$ given as $\{x \in E: f(x) > a\}$ is measurable if $\forall a$, _____ under f is a measurable set.

Answer (Please select your correct option)

☐ image of (x, ∞)

☐ inverse image of (x, ∞)

☐ inverse image of (a, ∞)

☒ image of (a, ∞) 

Let $\mathcal{M}$ be a measurable space (a sigma algebra of measurable sets) then a function $f: E \rightarrow \mathbb{R}$ is Lebesgue measurable function on $\emptyset \neq E \in \mathcal{M}$ if _____

Answer: Please select your correct option(s)

☐ $\{x \in E: f(x) < x\}$

☐ $\{a \in E: f(x) > a\}$

☐ $\{x \in E: f(x) > x\}$

☒ $\{x \in E: f(x) > a\}$



If $\{E_n\}$ be a sequence of measurable sets such that $E_1 = E_2 = E_3 = \dots$, then $\{E_n\}$ is an example of _____ sequence.

Answer (Please select your correct option)

☐ (IV) Both (I) and (II).

☐ (I) decreasing

☐ (II) increasing

☒ (III) neither increasing nor decreasing



If $\{E_n\}$ be a decreasing sequence of measurable sets then _____.

Answer (Please select your correct option)

☒ $E_1 \supseteq E_2 \supseteq E_3 \supseteq \dots$



☐ $E_1 \subseteq E_2 \subseteq E_3 \subseteq \dots$

☐ $E_1 - E_1 \subseteq E_1 - E_2 \subseteq E_1 - E_3 \subseteq \dots E_1 - E_n \subseteq \dots$

☐ $E_1 \neq E_2 \neq E_3 \neq \dots$

Those countable sets which are Borel have _____ measure.

Answer (Please select your correct option)

☐ positive

☐ no

☒ zero

☐ infinite

MTH644 Measure Theory

Question No: 12 of 52

Consider the sum modulo 1 for $x, y \in [0, 1[$ then $x \oplus y$ of x and y is defined by: _____.

Answer (Please select your correct option)

☐ $\begin{cases} x + y, & \text{if } x + y \geq 1 \\ x + y - 1, & \text{if } x + y < 1 \end{cases}$

☐ $\begin{cases} x + y, & \text{if } x + y \leq 1 \\ x + y - 1, & \text{if } x + y > 1 \end{cases}$

☐ $x + y - 1$

☐ $\begin{cases} x + y, & \text{if } x + y < 1 \\ x + y - 1, & \text{if } x + y \geq 1 \end{cases}$ 

Consider the sum modulo 1 for the interval $[0,1[$ then $x \hat{+} y$ of $x = y = \frac{1}{2} =$ _____.

Answer (Please select your correct option)

☐ 2

☐ -1

☐ 1

☒ 0

For all $E \subseteq \mathbb{R}$, there is a G_δ set $G \supseteq E$, such that _____.

Answer (Please select your correct option)

☐ $m^*(G - E)$ is not defined

☒ $m^*(G - E) = 0$

☐ $m^*(G - E) < 0$

☐ $m^*(E - G) > 0$

For a non empty set E in a measurable space $\mathcal{M}$, _____.

Answer (Please select your correct option)

☐ $m(E) < m^*(E)$

☐ $m(E) = 2m^*(E)$

☐ $m(E) > m^*(E)$

☒ $m(E) = m^*(E)$ ✓

The Lebesgue outer measure of Cantor set constructed from $[0,1]$, is _____

Answer (Please select your correct option)

☐ $\frac{1}{3}$

☐ ∞

☒ 0

☐ 1

MTH644 Measure Theory

Question No : 6 of 62

In the first iteration for constructing Cantor Set from $[0,1]$, we drop a subinterval of length _____

Answer (Please select your correct option)

☐ 1☒ $\frac{1}{3}$ ☐ $\frac{1}{4}$ ☐ $\frac{1}{2}$

For $f(x) = \sin x$, $-\pi < x \leq \pi$, $f^+ =$ _____

Answer (Please select your correct option)

☒ $\sin x, 0 < x \leq \pi$

☐ $\sin x, -\pi < x \leq 0$

☐ $\cos x, -\pi < x \leq 0$

☐ $\cos x, 0 < x \leq \pi$

For $f(x) = \sin x, -\pi < x \leq \pi, f^{-1} =$ _____.

Answer (Please select your correct option)

☐ $\sin x, -\pi \leq x < 0$

☐ $\sin x, 0 \leq x < \pi$

☒ $-\sin x, -\pi \leq x < 0$

☐ $\cos x, 0 \leq x < \pi$

The real valued function $f + g$ is measurable even if _____.

Answer (Please select your correct option)

☐ f is measurable but g is not measurable

☒ both f and g are measurable

☐ g is measurable but f is not measurable

☐ both f and g are not measurable

The real valued function $f.g$ is measurable even if _____.

Answer (Please select your correct option)

☒ both f and g are measurable



☐ f is measurable but g is not measurable

☐ both f and g are not measurable

☐ g is measurable but f is not measurable

If f is a real valued and measurable function then $\{x: |f(x)| > a, \forall a < 0\} =$ _____

Answer: (Please select your correct option.)

☐ $\{x: f(x) > a\} - \{x: f(x) < -a\}$

☒ Domain of definition of f ✓

☐ $\{x: f(x) > a\} \cup \{x: f(x) < -a\}$

☐ $\{x: f(x) > a\} \cap \{x: f(x) < -a\}$

If f is a real valued and measurable function then $\{x: |f(x)| > a, \forall a > 0\} =$ _____

Answer (Please select your correct option)



$$\{x: f(x) > a\} - \{x: f(x) < -a\}$$



$$\{x: f(x) > a\} \cap \{x: f(x) < -a\}$$



Domain of definition of f



$$\{x: f(x) > a\} \cup \{x: f(x) < -a\}$$



Let $f: X \rightarrow Y$ and $g: A \rightarrow Y$ be two functions and $A \subseteq X$ then the function g called the restriction of f if

Answer (Please select your correct option)

☒ $A \subseteq X$

☐ $A \supseteq X$

☐ $A = X$

☐ $m^*(A) = m^*(X)$

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of real valued measurable functions then _____ is also measurable.

Answer: (Please select your correct option)

☒ (III) Both (I) and (II).

☐ (I) $\sup_{n \in \mathbb{N}} \{f_n\}$

☐ (II) $\inf_{n \in \mathbb{N}} \{f_n\}$

☐ (IV) Neither (I) nor (II).

Let $f: X \rightarrow Y$ and $g: A \rightarrow Y$ be two functions. If $A \subseteq X$ and $f(x) = g(x)$ then g is called the _____ of f to $A \subseteq X$.

Answer (Please select your correct option)

☐ extension

☐ Lebesgue outer measure

☒ restriction

☐ Lebesgue measure

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of real valued measurable functions then _____ is also measurable.

Answer: (Please select your correct option)

☒ (III) Both (I) and (II).

☐ (I) $\sup_{n \in \mathbb{N}} \{f_n\}$

☐ (II) $\inf_{n \in \mathbb{N}} \{f_n\}$

☐ (IV) Neither (I) nor (II).

MTH644 Measure Theory

Question No : 38 of 52

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of real valued measurable functions then $\lim_{n \rightarrow \infty} f_n =$ _____.

Answer (Please select your correct option)

☐ (I) $\overline{\lim}_{n \rightarrow \infty} f_n$

☒ (III) Both (I) and (II).

☐ (II) $\underline{\lim}_{n \rightarrow \infty} f_n$

☐ (IV) Neither of (I) nor (II).

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of real valued measurable functions then $\inf_{k \geq n} (f_k)$ is measurable for _____

Answer (Please select your correct option)

☒ all $k \in \mathbb{Z}$

☐ some $n \in \mathbb{N}$

☐ $n \notin \mathbb{N}$

☐ all $n \in \mathbb{N}$

$\sup_{n \in \mathbb{N}} (f_n)$ is also measurable then for $x \in D$, _____

Answer (Please select your correct option.)

$f(x) \leq f_n(x)$, for all $n \in \mathbb{N}$ and $a \in \mathbb{R}$



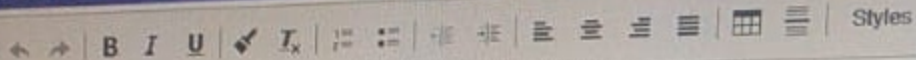
$f(x) \geq f_n(x)$, for some $n \in \mathbb{N}$ and for all $a \in \mathbb{R}$

$f(x) \geq f_n(x)$, for all $n \in \mathbb{N}$ and $a \in \mathbb{R}$

$f(x) \leq f_n(x)$, for some $n \in \mathbb{N}$ and for all $a \in \mathbb{R}$

Determine the 1st step construction of Cantor set from $[0,1]$.

Answer



Styles

(7) Determine the measure 1st step construction of cantor set from $[0,1]$

Sol: We know that

$$m(G) = m\left(\bigcup_{n=1}^{\infty} \bigcup_{m=1}^{2^{n-1}} I_{nm}\right)$$

For 1st step

$$\text{put } n=1 \Rightarrow m = 2^{n-1} = 2^{1-1} = 2^0 = 1$$

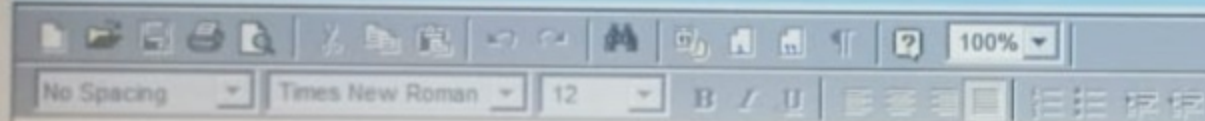
$$m(G) = m(I_{11}) = l(I_{11})$$

$$m(G) = m(I_{11}) - \cancel{m(I_{11})} = \frac{1}{3} = \frac{1}{3} \quad \because l(I_{nm}) = \frac{1}{3^n}$$

Ans

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions and $f = \max_{1 \leq i \leq n} \{f_i\}$ then show that $\bigcup_{j=1}^n \{x: f_j(x) > a\} \subseteq \{x: f(x) > a\}, \forall a \in \mathbb{R}.$

Answer ([Please click here to Edit Answer](#))



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$$y \in \bigcup_{i=1}^n \{x : f_i(x) > a\}$$

$$y \in \{x : f_k(x) > a\} \text{ for some } k = \{1, 2, \dots, n\}$$

$$f_k(y) > a, \text{ by def.}$$

$$f_i(y) \leq \max(f_i(y)) = f(y)$$

and

$$f_k(y) \leq f(y), \text{ } f_k(y) > a$$

$$y \in \{x : f(x) > a\}$$

hence

$$\bigcup_{i=1}^n \{x : f_i(x) > a\} \subseteq \{x : f(x) > a\}$$

Let f and g are the extended real valued functions defined on any set $A \neq \emptyset$, then prove that:

$$\min(f, g) = \frac{(f+g) - |f-g|}{2}.$$

Answer (Please [click here](#) to Edit Answer)



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Q. No. 15) (5 marks)

Prove that $\min(f, g)$ is measurable.
if f, g and $(f+g)$ are measurable.

$$\min(f, g) = \frac{(f+g) - (f-g)}{2}$$

Sol:-

As given that f, g and $f+g$ are measurable.
First we will show that $f-g$ is measurable.

For this

As g is measurable

$\Rightarrow -g$ is also measurable.

$\Rightarrow f + (-g)$ is measurable.

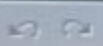
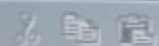
$\Rightarrow f-g$ is measurable.

As $f, g, f+g$ and $f-g$ are measurable
so,

$\min(f, g)$ is also measurable.

Let $\mathcal{M}$ be a measurable space (a sigma algebra of measurable sets) on $\emptyset \neq X$ and a given function, $f: A \rightarrow \mathbb{R}$ on $\emptyset \neq A \in \mathcal{M}$, defined as $f(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$. Then show that $\{x \in E: f(x) > a\}$ is measurable.

Answer (Please [click here](#) to Edit Answer)



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MTH644 Final...



23 MB - 2022-09-14

ϕ Example (2):

$$f(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

$$\{x : f(x) > a\} = \begin{cases} \phi & \\ A \neq X & \\ X = A & \end{cases}$$

$\phi, A, X \in \mathcal{M} \Rightarrow "f" \text{ is measurable.}$

For $\emptyset \neq E \subseteq \mathbb{R}$, if given that the F_σ set $F \subseteq E$ and $m^*(E - F) = 0$ then show that there exists a G_δ set $G \supseteq E$ and $m^*(G - E) = 0$.

Answer ([Please click here to Edit Answer](#))

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MTH644 Mega...



62 MB - 2022-08-18

(27) If $m^*(G-E)=0$, then show that $m^*(G-E^c)=0$ ②

Sol: $\because m^*(G-E)=0$, $\phi \neq E \subseteq \mathbb{R} \Rightarrow \phi \neq E' \subseteq \mathbb{R}$

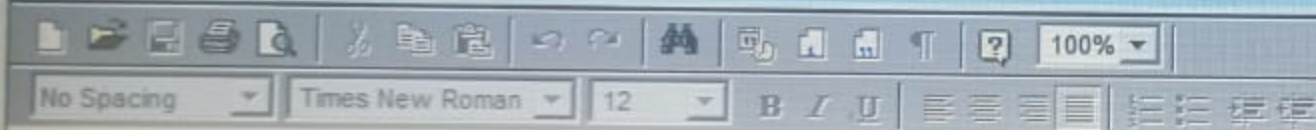
$\therefore$ for $\phi \neq E'$ gives $E' \subseteq G$

$\because E \subseteq G$

Hence $m^*(G-E^c)=0$ (proved)

Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions on D then construct a sequence of measurable functions from $\sup_{k \geq n} f_k$.

Answer ([Please click here to Edit Answer](#))



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$$\{x \mid f(x) > a\} = \{x \mid f(x) > a\}$$

$$\sup \{f_k, f_{k+1}, f_{k+2}, \dots\}$$

$$k = 1, 2, 3, \dots$$

$$\sup \{f_1, f_2, f_3, \dots\} = F_1$$

$$\sup \{f_2, f_3, \dots\} = F_2$$

Let $\emptyset \neq E$ be a measurable set and $f: E \rightarrow \mathbb{R} \cup \{\pm\infty\}$ then show that $\{x \in E: f(x) = a\}$ is also measurable $\forall a < \infty$.

Answer ([Please click here to Edit Answer](#))

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MTH644 Mega...



62 MB - 2022-08-18

Q.No.6:- (5 marks)

Let $\emptyset \neq E \in \mathcal{M}$. $f: E \rightarrow \mathbb{R} \cup \{+\infty\}$,
Then

$$\{x \in E : f(x) = a\} \in \mathcal{M} \quad \forall a \in \mathbb{R}.$$

Sol:-

i) if $a < \infty$

Since $a \in \mathbb{R}$

$$\Rightarrow \{x : f(x) = a\} = \{x : f(x) \geq a\} \cap \{x : f(x) < a\}$$

As

$$\{x : f(x) \geq a\}, \{x : f(x) < a\} \in \mathcal{M}$$

Then

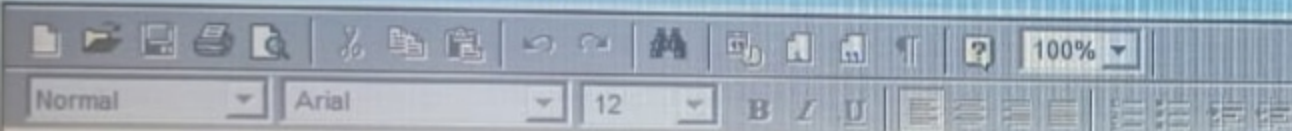
$$\{x : f(x) \geq a\} \cap \{x : f(x) < a\} \in \mathcal{M}$$

So,

$$\{x : f(x) = a\} \in \mathcal{M}.$$

Suppose that the triplet $(\mathbb{R}, \mathcal{B}, m)$ be the smallest sigma algebra of open sets in $\mathbb{R}$ then show that the identity function $f(x) = x$ on $\mathbb{R}$ is a measurable function.

Answer ([Please click here to Edit Answer](#))



A rich text editor toolbar with various icons for document manipulation. It includes icons for new, open, save, print, find, undo, redo, bold, italic, underline, link, unlink, list, and indent. A dropdown menu shows '100%' for zoom. Below the icons, there are dropdowns for 'Normal' style, 'Arial' font, and '12' size. Further right are buttons for bold (B), italic (I), underline (U), and alignment options (left, center, right, justified).

Example (3): $(\mathbb{R}, \mathcal{B}, m) \rightarrow (\text{Borel } \sigma\text{-algebra})$

$$f(x) = x$$

$$\Rightarrow \{x : f(x) > a\} = \{ (a, +\infty) \in \mathcal{B}$$

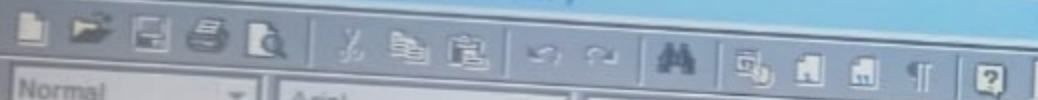
i.e. always measurable.

$\Rightarrow f$ is measurable.

MTH644 Measure Theory

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Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions and $f = \inf_{n \in \mathbb{N}} \{f_n\}$.
Further, if $\{x: f(x) > a\} = \bigcap_{n=1}^{\infty} \{x: f_n(x) > a\}$, $\forall a \in \mathbb{R}$ then show that f is measurable.

Answer (Please [click here to Add Answer](#))

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(proved)

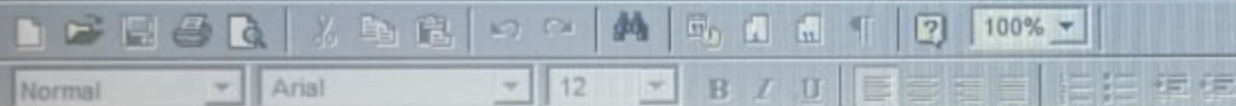
(49) let $\{f_n\}_{n=1}^{\infty}$ be a sequence of extended real valued measurable function on D . $f = \inf_{n \in \mathbb{N}} \{f_n\}$, further if $\{x: f(x) > a\} = \bigcap_{n=1}^{\infty} \{x: f_n(x) > a\}$, $\forall a \in \mathbb{R}$, then show that f is measurable. (2)

Sol: Given that $f = \inf_{n \in \mathbb{N}} \{f_n\}$ and $\{x: f(x) > a\} = \bigcap_{n=1}^{\infty} \{x: f_n(x) > a\}$, $\forall a \in \mathbb{R}$

The intersection of a countable number of measurable sets is measurable. So f must be measurable.

In $\mathbb{R}$, show that the Lebesgue measure m on a sequence $\{E_i\}_{i=1}^{\infty}$ of pairwise disjoint measurable sets in a measurable space $\mathcal{M}$ is countably additive.

Answer ([Please click here to Edit Answer](#))





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Q.No.11) (2 marks)

Show that Lebesgue measure 'm' of a sequence $\{E_i\}_{i=1}^{\infty}$ is countable subadditive.

Sol:-

Since All E_i 's are measurable

$\Rightarrow \bigcup_{i=1}^{\infty} E_i$ is also measurable.

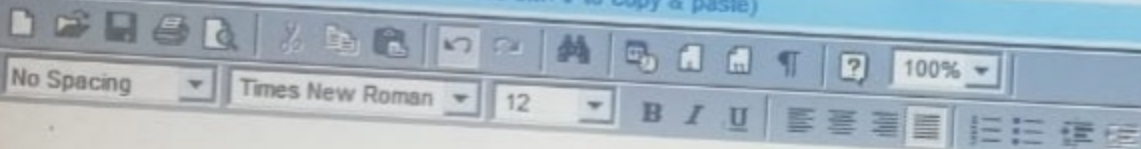
$\Rightarrow$ 'M' is space of Lebesgue measurable sets.

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m^*(E_i)$$

$$= \sum_{i=1}^{\infty} m(E_i)$$

If $f: \mathbb{R} \rightarrow \mathbb{R}$ defined and given by $f(x) = |x + 2|$ then find the measurable interval of positive part f^+ of f .

Answer (Please Edit Answer) (Use ctrl+C and ctrl+V to copy & paste)





Q.No.19) (2 Marks)

$$f(x) = |x+2|$$

find f^+ if $f(x) \geq 0$.

sol:-

As we know that

if $f(x) \geq 0$

$$f^+ = \frac{f + |f|}{2}$$

here

$$f(x) = |x+2| = x+2$$

$$|f(x)| = |x+2| \quad \because f(x) \geq 0$$

$$= x+2$$

So,

$$f^+ = \frac{(x+2) + (x+2)}{2}$$

$$= x+2$$

$$= |x+2|$$

$$= f(x) \quad \text{Ans}$$

Evaluate the following sum modulo 1 for the pairs of numbers from $[0,1[$:

(I) $\left\{\frac{1}{4}, \frac{3}{4}\right\}$

(II) $\left\{\frac{1}{4}, \frac{1}{2}\right\}$

Answer ([Please click here to Add Answer](#))



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Q17) Evaluate the following sum modulo 1 for the pairs of numbers from $[0, 1]$:

(I) $\{\frac{1}{4}, \frac{3}{4}\}$ (II) $\{\frac{1}{4}, \frac{1}{2}\}$

$$x + y \geq 1$$

$$\Rightarrow x + y - 1$$

$$\Rightarrow \frac{1}{4} + \frac{3}{4} - 1$$

$$\Rightarrow 1 - 1 = \boxed{0} \text{ Ans}$$

$$x + y < 1$$

$$\Rightarrow x + y$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2}$$

$$= \frac{3}{4}$$