

0310, 2875241

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MTH 645

* Final Term Solved Papers. *

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QNO:01 Define strictly monotone T-Norm?

Sol :- T-norm Δ is called strictly monotone t-norm if $x \Delta y \leq x \Delta z$ whenever $x \geq 0$ and $y \leq z$

QNO:02 :- $x \nabla y = x + y - xy$ Find ∇ -implication

Sol :- As we know that ∇ -implication can be define as

$$x \Rightarrow y = \eta(x) \nabla y \quad \because \eta(x) = (1-x) \\ = (1-x) \nabla y \quad \text{--- (1)}$$

But we have given that $x \nabla y = x + y - xy$
Apply on eq (1)

$$(1-x) \nabla y = (1-x) + y - (1-x)y$$

$$1-x + y - 1 + x + xy - 1 + x =$$

$$1-x + y - 1 + x + xy - 1 + x =$$

$$1-x + y - 1 + x + xy - 1 + x =$$

Ans

Qno:03 Define nilpotent t-norm?

Sol: A is called nilpotent t-norm if for $b \neq 1$ we get $b^{(n)} = 0$ for some positive integer "n" where 'n' depends on 'b'.

Qno:04: What is idempotent element?

Sol: Let $a \in (0,1)$ is called idempotent element if $a \Delta a = a$
for example $0 \Delta 0 = 0 \Rightarrow 0$ is idempotent element
 $1 \Delta 1 = 1 \Rightarrow 1$ is idempotent e.

Qno:05 Define picture fuzzy sets?

Sol: Let 'X' be a non empty set, $x \in X \neq \emptyset$ then picture fuzzy set "p" on "X" can be define as:

$$p = \{ \langle x, \mu_p(x), \tau_p(x), \beta_p(x) \rangle \mid x \in X \}$$

where $\mu_p(x), \tau_p(x), \beta_p(x) \in [0,1]$ and fulfill the condition $0 \leq \mu_p(x) + \tau_p(x) + \beta_p(x) \leq 1$

Qno:05 Write five (5) name of fuzzy Relation?

Sol:

- i) Reflexive relation
- ii) Symmetric relation
- iii) Antisymmetric relation
- iv) Transitive relation
- v) Equivalence relation

Qno:06 write five name of fuzzy set

Sol 1. i) intuitionistic fuzzy set.

ii) Picture fuzzy set.

iii) Hesitant fuzzy set.

iv) normal fuzzy set.

v) ~~to~~ empty fuzzy set.

Qno:07 $A = \{x, (0.4, 0.2, 0.3)\}$ is a
Picture fuzzy set on "X" than
find A^c ~~and~~ ~~find~~

Sol 1. Since given that $A = \{x, (0.4, 0.2, 0.3)\}$.

Here $\mu_A(x) = 0.4$, $\gamma_A(x) = 0.2$, $\beta_A(x) = 0.3$

we know that ~~the~~ complement
of picture fuzzy can be written as

$$A^c = \{x, (\beta_A(x), \gamma_A(x), \mu_A(x))\}$$

Now putting the value

$$A^c = \{x, (0.3, 0.2, 0.4)\} \quad \underline{\text{Ans}}$$

Qno:08

Qno:08 Let $X = \{x_i\}$ be universal set and $\lambda = 3$
 $h = \{x_i, (0.2, 0.8)\}$ is a HFS on "X"
 Evaluate λh ?

Sol: $h = \{x_i, (0.2, 0.8)\}$ and $\lambda = 3$.

As we know that

$$\lambda h = \{1 - (1 - x)^{\lambda} / x \in h(x)\}$$

$$\lambda h = \{1 - (1 - 0.2)^{\lambda}, 1 - (1 - 0.8)^{\lambda}\}$$

$$\because \lambda = 3$$

$$3h = \{1 - (0.8)^3, 1 - (0.2)^3\}$$

$$= \{1 - 0.51, 1 - 0.008\}$$

$$= \{0.49, 0.99\} \text{ Ans.}$$

Qno:09 Prove that Δ_0 is nilpotent t-norm.

Sol: Since we know that Δ_0 can be defined

$$x \Delta_0 y = \begin{cases} \min(x, y) & ; \max(x, y) = 1 \\ 0 & , \text{ otherwise} \end{cases}$$

$$\text{Let } \max(x, y) = 1$$

$$\text{Let } x \neq 1$$

$$\Rightarrow \max(x, x) \neq 1$$

$$\Rightarrow x \Delta_0 x = 0$$

$$\Rightarrow x^{[2]} = 0$$

Here we get true "n" such that

$$x^{[n]} = 0$$

$\Rightarrow \Delta_0$ is nilpotent t-norm

Q No, 10 If $A = (0.3, 0.4, 0.2)$ & $B = (0.3, 0.5, 0.1)$
 Then find (i) $A \subseteq B$ (ii) $A \cup B$ (iii) $A \cap B$.

Sol: (i) $A \subseteq B$ if $\frac{d_A(x)}{d_B(x)} \leq \frac{\gamma_A(x)}{\gamma_B(x)}$
 $\frac{B(x)}{A} \geq \frac{B(x)}{B}$

Here $0.3 \leq 0.3$, $0.4 \leq 0.5$, $0.2 \geq 0.1$
 Hence the condition is fulfilled

$\Rightarrow A \subseteq B$

$$(ii) A \cup B = \{x, \max(\frac{d_A(x)}{d_B(x)}, \frac{d_B(x)}{d_A(x)}), \min(\frac{\gamma_A(x)}{\gamma_B(x)}, \frac{\gamma_B(x)}{\gamma_A(x)})\}$$

$$A \cup B = \{(0.3), (0.4), (0.1)\} \text{ Ans}$$

$$(iii) A \cap B = \{x, \min(\frac{d_A(x)}{d_B(x)}, \frac{d_B(x)}{d_A(x)}), \max(\frac{\gamma_A(x)}{\gamma_B(x)}, \frac{\gamma_B(x)}{\gamma_A(x)})\}$$

$$= \{(0.3), (0.4), (0.2)\} \text{ Ans}$$

Q No, 11 Define Symmetric fuzzy relation

Sol: A fuzzy binary relation R is
 Symmetric if $R(u, v) = R(v, u)$

QNO, 12 Let $X = \{x_1, x_2\}$ be universal set and
 $A = \{(x_1, (0.4, 0.25, 0.35)), (x_2, (0.15, 0.45, 0.4))\}$
 $B = \{(x_1, (0.3, 0.4, 0.3)), (x_2, (0.25, 0.15, 0.6))\}$
 are two PFS find the normalized Hamming distance

Soln: $d_h^n(A, B) = \frac{1}{n} \sum_{i=1}^n |m_A(x_i) - m_B(x_i)|$

Here $n = 2$.

$$d_h^n(A, B) = \frac{1}{2} \sum_{i=1}^2 |m_A(x_i) - m_B(x_i)|$$

$$= \frac{1}{2} \left(|0.4 - 0.3| + |0.25 - 0.4| + |0.35 - 0.3| + |0.15 - 0.25| \right. \\ \left. + |0.45 - 0.15| + |0.4 - 0.6| \right)$$

$$= \frac{1}{2} (0.1 + 0.15 + 0.05 + 0.10 + 0.30 + 0.2)$$

$$= \frac{1}{2} (0.9) = \boxed{0.45 = d_h^n(A, B)} \text{ Ans}$$

QNO, 13 Let $X = \{x_1, x_2, x_3, x_4\}$ be a finite set

and $A = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3)\}$

$B = \{(x_2, 0.4), (x_3, 0.4), (x_4, 1)\}$ are two
 fuzzy set find the distance b/w them
 using max(min) form.

Soln.	d	$A(x_i)$	$B(x_i)$	$\mu_{A(x)}$	$\mu_{B(x)}$	$\min(\mu_{A(x)}, \mu_{B(x)})$	$\max(\min(\mu_{A(x)}, \mu_{B(x)}))$
	0	x_2	x_1	0.1	0.4	0.1	0.3
		x_3	x_3	0.3	0.4	0.3	
	1	x_1	x_2	0.1	0.4	0.1	0.3
		x_2	x_3	0.1	0.4	0.1	
		x_3	x_4	0.3	1	0.3	
	2	x_1	x_3	0.1	0.4	0.1	0.1
		x_2	x_4	0.1	1	0.1	
	3	x_1	x_4	0.1	1	0.1	0.1

$$d(A, B) = 0.3 + 0.3 + 0.1 + 0.1$$

$$d(A, B) = 0.8 \quad \text{Ans}$$

Qn 10, 14 Define fuzzy sets ?

Soln:

Let 'X' be a universal set and 'x' be a particular element of 'X' then a fuzzy set 'A' define on 'X' can be written as collection of order pair

$$A = \{ (x, \mu_A(x)), x \in X \}$$

where $\mu_A(x)$ membership value of $x \in X$ in A.

$$\mu_A(x) \in [0, 1]$$

Qno, 15 if $h(x) = 0.6$ and $n = 2$
then find λh ?

Sol: As we know that

$$\lambda h(x) = \{ 1 - (1-x)^n \mid x \in h(x) \}$$

$$\lambda h(x) = \{ 1 - (1-0.6)^2 \}$$

$$= \{ 1 - 0.16 \}$$

$$\lambda h(x) = \{ 0.84 \}$$

Qno, 16 if $A = (1, -3)$ and $B = (1, 5)$ then
find the distance b/w two fuzzy set

Solⁿ 1. As we know that distance b/w two point D can be written as

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(1-1)^2 + (5-(-3))^2}$$

$$= \sqrt{0^2 + 8^2}$$

$$= \sqrt{64}$$

$$D(A, B) = 8$$

Qno: 17 if $xy = xy$, $f(x) = e^{-\left(1 - \frac{x}{x}\right)}$
find q.7.

Solⁿ 2 As we know that

$$q \Rightarrow f(0) = e^{-\left(1 - \frac{0}{0}\right)}$$

$$= e^{-\left(\frac{1}{0}\right)}$$

$$= e^{-\infty}$$

$$= \frac{1}{e^{\infty}}$$

$$= \frac{1}{\infty}$$

$$\Rightarrow \boxed{q = 0}$$

Qno: 18

If T is strictly monotone then
It has only trivial idempotent element

Sol:

Since we have given that
 T is strictly monotone then by def

$x \otimes y < x \otimes z$ whenever $x > 0, y < z$.

Let T have non-trivial idempotent element
take

$$x \leq x \text{ and } x < 1$$

then by 4th property of T -norm

$$x \otimes x < x \otimes 1$$

$$x \otimes x < x$$

$\Rightarrow x \otimes x < x$ which is contradiction.
to strictly monotone

Hence T have only trivial idempotent element.

Qno: 19

Define Transitive fuzzy Relation

Sol:

A fuzzy binary relation is transitive

$$\text{if } R(x, y) = 1, R(y, z) = 1$$

$$\Rightarrow R(x, z) = 1.$$

OR

$$R(x, z) \geq R(x, y) \wedge R(y, z)$$

Q40:20 If $x \odot y = \min(x, y)$ Then

find R-implication.

Solⁿ As we know that R-implication can be define as

$$(x \Rightarrow y) = \max\{z \in [0, 1] : x \odot z \leq y\}.$$

now apply on the given condition

$$(x \Rightarrow y) = \max\{z \in [0, 1] : \min(x, z) \leq y\}.$$

$$= \begin{cases} 1 & ; x \leq y \\ y & ; x > y \end{cases}$$