# MTH645 Mega For Final Term Fall (2022)

# Prepared by Everblue

Lec (23-45)

Subjective (5 files)

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# MTH 645

## * Final Term Solved Papers. *

**QNO:01** Define Strictly monotone T-Norm ?.

**Sol :-** T-Norm $\Delta$ is called strictly monoton t-norm if $x \Delta y \leq x \Delta z$ Whenever $x > 0$ and $y < z$

**QNO:02 :-** $x \nabla y = x + y - xy$ find $\nabla$-implication.

**Sol :-** As we know that $\nabla$-implication can be define as

$$x \Rightarrow y = \eta(x) \nabla y \quad \because \eta(x) = (1-x)$$

$$= (1-x) \nabla y \quad -(1)$$

But we have given that $x \nabla y = x + y - xy$

Apply on Eq (1)

$$\Rightarrow (1-x) \nabla y = (1-x) + y - (1-x)y$$

$$= 1 - x + y - y + xy$$

$$= 1 - x + xy$$

$$\boxed{x \Rightarrow y = 1 - x + xy}$$ Ans

Q.No:03 Define nilpotent t-norm ?

Sol:- A is called nilpotent t-norm if for $b \neq 1$ we get $b^{(n)} = 0$ for some positive integer "n" where 'n' depends on 'b'

Q.No:04 :- What is idempotent element ?

Sol :- Let $a \in (0,1)$ is called idempotent element if $a \Delta a = 0$

for example $0 \Delta 0 = 0 \Rightarrow 0$ is idempotent elmt

$1 \Delta 1 = 1 \Rightarrow 1$ is idempotent.

Q.No:05 Define picture fuzzy sets ?

Sol :- Let 'X' be a non empty set i.e $X \neq 0$ than picture fuzzy set "P" on "X" can be define As.

$$P = \{ \langle x, q_p(x), \gamma_p(x), \beta_p(x) \rangle | x \in X \}$$

Wher $q_p(x), \gamma_p(x), \beta_p(x) \in [0,1]$ and fullfill the condition $0 \leq q_p(x) + \gamma_p(x) + \beta_p(x) \leq 1$

Q.No:05 Write five (5) name of fuzzy Relation ?

Sol :- i) Reflexive relation ii) Symatric relation

iii) Anti Symatric relation.

iv) Translative relation

v) Equilance relation.

Q No:06 Write five name of fuzzy set

Sol:
i) intuitionistic fuzzy set
ii) Picture fuzzy set
iii) Heisctent fuzzy set
iv) normal fuzzy set
v) Go empty fuzzy set

Q No:07 $A = \{x, (0.4, 0.2, 0.3)\}$ is a picture fuzzy set on "X" than find $A^c$

Sol: Since Given that $A = \{x, (0.4, 0.2, 0.3)\}$.
Here $\alpha_A(x) = 0.4$, $\gamma_A(x) = 0.2$, $\beta_A(x) = 0.3$
we know that complement of picture fuzzy can be written as.

$$A^c = \{x, (\beta_A(x), \gamma_A(x), \alpha_A(x))\}$$

Now putting the value

$$A^c = \{x, (0.3, 0.2, 0.4)\}$$ Ans

Q.No:08 Let $X = \{x_1\}$ be universal set and $\lambda = 3$
$h = \{x_1, (0.2, 0.8)\}$ is a HFS on "X"
Evaluat $\lambda h$. ?

Sol: $h = \{x_1, (0.2, 0.8)\}$ and $\lambda = 3$.
As we know that
$\lambda h = \{1-(1-\gamma)^\lambda / \gamma \in h(x)\}$.

$\lambda h = \{1-(1-0.2)^\lambda, 1-(1-0.8)^\lambda\}$

$\because \lambda = 3$.

$3h = \{1-(0.8)^3, 1-(0.2)^3\}$.

$= \{1-0.51, 1-0.008\}$.

$= \{0.49, 0.99\}$. Ans.

Q.No:09 Prove that $\Delta_0$ is nilpotent t-norm.

Sol: Since we know that $\Delta_0$ can be defini

$$x \Delta_0 y = \begin{cases} \min(x,y) & ; \max(x,y) = 1 \\ 0 & ; \text{otherwise} \end{cases}$$

Let $\max(x,y) = 1$
Let $x \neq 1$
$\Rightarrow \max(x,x) \neq 1$
$\Rightarrow x_0 \Delta_0 x = 0$
$\Rightarrow x^{[2]} = 0$
Here we get +ve 'n' such that
$x^{[2]} = 0$
$\Rightarrow \Delta_0$ is nilpotent T-norm

Q No; 10 If $A = (0.3, 0.4, 0.2)$ & $B = (0.3, 0.5, 0.1)$
Than find (i) $A \subseteq B$ (ii) $A \cup B$ (iii) $A \cap B$.

Sol: (i) $A \subseteq B$ if $\underset{A}{\alpha}(x) \leq \underset{B}{\alpha}(x)$, $\underset{A}{\gamma}(x) \leq \underset{B}{\gamma}(x)$

$\underset{A}{\beta}(x) \geq \underset{B}{\beta}(x)$

Here $0.3 \leq 0.3$, $0.4 \leq 0.5$ $0.2 \geq 0.1$
Hence the condition is fullfilled

$\Rightarrow A \subseteq B$

(ii) $A \cup B = \{ \langle x, \max(\underset{A}{\alpha}(x), \underset{B}{\alpha}(x)), \min(\underset{A}{\gamma}(x), \underset{B}{\gamma}(x)) \rangle$

$\min(\underset{A}{\beta}(x), \underset{B}{\beta}(x)) \}$.

$A \cup B = \{(0.3), (0.4), (0.1)\}$ Ans

(iii) $A \cap B = \{ \langle x, \min(\underset{A}{\alpha}(x), \underset{B}{\alpha}(x)), \min(\underset{A}{\gamma}(x), \underset{B}{\gamma}(x)) \rangle$

$, \max(\underset{A}{\beta}(x), \underset{B}{\beta}(x)) \}$

$= \{(0.3), (0.4), (0.2)\}$. Ans

Q No; 11 Define Symetric fuzzy relation

Sol: A fuzzy binary relation R is
Symetric if $R(u,v) = R(v,u)$

Q.NO;12 Let $X=\{x_1, x_2\}$ be universal set and

$A=\{(x_1,(0.4,0.25,0.35)),(x_2,(0.15,0.45,0.4))\}$

$B=\{(x_1,(0.3,0.4,0.3)),(x_2,(0.25,0.15,0.6))\}$

are two PFS find the normalized Haming distance

Sol:- $d_h^n(A,B)=\frac{1}{n}\sum_{n=1}^{n}|m_A(x_i)-m_B(x_i)|$

Here $n=2$.

$d_n^n(A,B)=\frac{1}{2}\sum_{n=1}^{2}|m_A(x_i)-m_B(x_i)|$

$=\frac{1}{2}\{|0.4-03|+|0.25-0.4|+|0.35-0.3|+|0.15-0.25|$

$+|0.45-0.15|+|0.4-0.6|\}$

$=\frac{1}{2}\{0.1+0.15+0.05+0.10+0.30+0.2\}$

$=\frac{1}{2}[0.9]=\boxed{0.45=d_h^n(A,B)}$ Ans

Q.NO;13 Let $X=\{x_1,x_2,x_3,x_4\}$ be a fixed set

and $A=\{(x_1,0.1),(x_2,0.2),(x_3,0.3)\}$

$B=\{(x_2,0.4),(x_3,0.4),(x_4,1)\}$ are two fuzzy set find the distance b/w them using max(min) form.

Soln.

| d | $A(x_i)$ | $B(x_i)$ | $\mu_{A(x)}$ | $\mu_{B(x)}$ | $\min(\mu_{A_x}, \mu_{B_x})$ | $\max(\min(\mu_{A_x}, \mu_{B_x}))$ |
|---|---|---|---|---|---|---|
| 0 | $x_2$ | $x_1$ | 0.1 | 0.4 | 0.1 | 0.3 |
| | $x_3$ | $x_3$ | 0.3 | 0.4 | 0.3 | |
| 1 | $x_1$ | $x_2$ | 0.1 | 0.4 | 0.1 | 0.3 |
| | $x_2$ | $x_3$ | 0.1 | 0.4 | 0.1 | |
| | $x_3$ | $x_4$ | 0.3 | 1 | 0.3 | |
| 2 | $x_1$ | $x_3$ | 0.1 | 0.4 | 0.1 | 0.1 |
| | $x_2$ | $x_4$ | 0.1 | 1 | 0.1 | |
| 3 | $x_1$ | $x_4$ | 0.1 | 1 | 0.1 | 0.1 |

$d.(A,B) = 0.3 + 0.3 + 0.1 + 0.1$

$d(A,B) = 0.8$ Ans

Qn10;14 Define fuzzy sets ?

Soln:

Let "X" be a universal set and "x" be a particular element of "X" than a fuzzy set "A" define on "X" can be written as collection of order pair

$$A = \{(x, \mu_A(x)), x \in X\}$$

where $\mu_A(x)$ membership value of $x \in X$ in A.

$\mu_A(x) \in [0,1]$

Qno.15 If $h(x) = 0.6$ and $\lambda = 2$ than find $\lambda h$ ?

Sol:- As we know that

$$\lambda h(x) = \{ 1-(1-\gamma)^{\lambda} / \gamma \in h(x) \}$$

$$2h(x) = \{ 1-(1-0.6)^{2} \}$$

$$= \{ 1 - 0.16 \}$$

$$2h(x) = \{ 0.84 \}$$

Qno.16 If $A = (1, -3)$ and $B = (1, 5)$ than find the distance b/w two fuzzy set

Sol$_1$. As we know that distance b/w two point D can be written as

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(1-1)^2 + (5-(-3))^2}$$

$$= \sqrt{0^2 + 8^2}$$

$$= \sqrt{64}$$

$$D(A,B) = 8$$

Q no; 17 If $x \Delta y = xy$, $f(x) = e^{-\left(\frac{1-x}{x}\right)}$

find q.?

Sol$_2$ As we know that

$$q = f(0) = e^{-\left(\frac{1-0}{0}\right)}$$

$$= e^{-\left(\frac{1}{0}\right)}$$

$$= e^{-\infty}$$

$$= \frac{1}{e^{\infty}}$$

$$= \frac{1}{\infty}$$

$$\Rightarrow \boxed{q = 0}$$

Q.NO; 18 If T is strictly monotone than it has only trivial idempotent element

Sol: Since we have given that T is strictly monoton than by def

$x \odot y < x \odot z$ whenever $x > 0$, $y < z$.

Let T have non-trivial idempotent element

take

$x \leq x$ and $x < 1$

then by 4th property of T-norm

$x \odot x \leq x \odot 1$

$x \odot x \leq x$

$\Rightarrow x \odot x < x$ which is contradiction.

so strictly monoton

Hence T have only trivial idempotent element.

Q.NO; 19 Define Transative fuzzy Relation

Sol: A fuzzy binary relation is transative

if $R(x,y) = 1$, $R(y,z) = 1$

$\Rightarrow R(x,z) = 1$.

OR

$R(x,z) \geq R(x,y) \wedge R(y,z)$

Q#0:20 If $x \Delta y = \min(x, y)$ then find R-implication.

Sol: As we know that R-implication can be define as

$$(x \Rightarrow y) = \max\{z \in [0,1] ; x \Delta z \leq y\}.$$

Now Apply on the given condition

$$(x \Rightarrow y) = \max\{z \in [0,1] ; \min(x, z) \leq y\}.$$

$$= \begin{cases} 1 & ; x \leq y \\ y & ; x > y \end{cases}$$

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MTH # 645 (Subjective)

Q.No. 1:-

Define picture fuzzy set.

Ans:-

Let $X = \{x_1, x_2, x_3, \cdots, x_n\}$ be a fixed set, a picture fuzzy set 'p' on X is defined as

$$p = \{\langle x, \alpha_p(x), \gamma_p(x), \beta_p(x)\rangle / x \in X\}.$$

where

$\alpha_p(x)$, $\gamma_p(x)$ and $\beta_p(x) \in [0,1]$ are called acceptance, neutral and rejection membership degrees of $x \in X$ to the set p respectively, and $\alpha_p(x)$, $\gamma_p(x)$ and $\beta_p(x)$ satisfies the condition.

$$0 \leq \alpha_p(x) + \gamma_p(x) + \beta_p(x) \leq 1 \quad , \forall\ x \in X$$

Also

$$\eta_p(x) = 1 - \alpha_p(x) - \beta_p(x) - \gamma_p(x)$$

where

$\eta_p(x)$ is said to be refusal membership degree of $x \in X$ on p.

Q.No. 2:-

$f(x) = e^{-\left(\frac{x}{1-x}\right)}$, find $f^{-1}(x)$.

$x \Delta y = xy$

Sol:-

As we know that

$x \Delta_f y = f^{-1}\left((f(x) \Delta f(y)) \vee 0\right)$.

Let

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$$y = e^{-\left(\frac{x}{1-x}\right)}$$

$$\ln y = -\frac{x}{1-x}$$

$$(1-x)\ln y = -x$$

$$\ln y - x\ln y = -x$$

$$x - x\ln y = -\ln y$$

$$x(1 - \ln y) = -\ln y$$

$$x = \frac{-\ln y}{1 - \ln y}$$

$$x = \frac{\ln y}{\ln y - 1}$$

Ans

$$y = f^{-1}(x)$$

So,

$$f^{-1}(x) = \frac{\ln x}{\ln x - 1}$$

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Q.No.3:-

Write five type of fuzzy logic sets.

Sol:-

i) Intutionistic fuzzy set.
ii) Picture fuzzy set.
iii) Hesitant fuzzy set.
iv) Normal fuzzy set.
v) Empty fuzzy set.

Q.No.4:-

Similarity Measure resemblence b/w two objects.

Sol:-

Two objects are said to be similar if they have same shape.

Similarity Measure:-

Similarity measure is a process through which we can evaluate the most resemblence b/w two objects.

There are many kinds of similarity measures like.

i) SM based on distance.
ii) cosine SM
iii) exponential SM
iv) Jaccard and dice SM
v) William and steel SM. .etc.

SM Based on Distance:-

Let S be a SM

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between two fuzzy set and it is calculated as,

if A and B are two fuzzy sets then SM S(A,B) is based on distance d(A,B), written as

$$S(A,B) = 1 - d_1(A,B)$$

or

$$S(A,B) = 1 - d_e(A,B)$$

or

$$S(A,B) = 1 - d_n(A,B)$$

and S(A,B) satisfy following axioms

i) $0 \leq S(A,B) \leq 1$

ii) $S(A,B) = 1$ iff $A = B$

iii) $S(A,B) = S(B,A)$

Q. No. 5:-

If $A = \{\mu_A(x), \nu_A(x)\}$ and $B = (\mu_B(x), \nu_B(x))$ then find $A \cap B$.

Sol:-

$$A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle / x \in X\}$$

Q. No. 6:-

Let A and B are two fuzzy set in $X = \{1, 2, 3\}$ such that

$$A = (0.7, 0.3)/1 + (0.2, 0.8)/2$$

$$B = (0.2, 0.8)/1 + (0.6, 0.4)/3$$

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then find Euclidean distance of IFSc.

Sol:-

As we know that Euclidean distance for IFSc is

$$d_e(A,B) = \sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))^2}$$

So,

$$= \sqrt{(0.7-0.3)^2 + (0.3-0.8)^2 + (0.2-1)^2 + (0.8-0)^2 + (0.6-0)^2 + (0.4-1)^2}$$

$$= 1.5524$$

Q.No.7:-

if $A = (1,-3)$, $B(1,5)$ then find the distance b/w them.

Sol:-

As we know that

$$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

So,

$$D(A,B) = \sqrt{(1-1)^2 + (5+3)^2}$$

$$= \sqrt{64}$$

$$= 8$$

Q.No.8:-

If $h(x) = 0.6$ and $\lambda = 2$ then find $\lambda h = ?$

Sol:-

we know that, if $h$ is a hesitant fuzzy set, then

$$\lambda h(x) = \{1 - (1-\gamma)^{\lambda} \mid \gamma \in h(x)\}$$

So,

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2 $h(x) = \{1 - (1 - 0.6)^2\}$

$= \{1 - 0.16\}$

$= \{0.84\}$

Q.No.9:-

Define nilpotent t-norm?

Sol:-

$\Delta$ is said to be a nilpotent t-norm for $b \neq 1$

$[b]^n = 0$

for some +ve integer $n$.

where $n$ depends on $b$.

Q.No.10:-

Define idempotent element?

Sol:-

$a \in (0,1)$ is said to be idempotent element if

$a \Delta a = a$.

e.g

$1 \Delta 1 = 1$

$0 \Delta 0 = 0$

Q.No.11:-

If $h_1 = (0.7, 0.8, 0.9)$ and $h_2 = (0.5, 0.6, 0.8)$ be two HFSs. then find Hamming distance b/w them.

Sol:-

$D_h(h_1, h_2) = |0.7 - 0.5| + |0.8 - 0.6| + |0.9 - 0.8|$

$= 0.2 + 0.2 + 0.1$

$= 0.5$

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Q. No. 12:-

If $A = \{x, (0.4, 0.2, 0.3)\}$ is a PFS on $x$, determine $A^c$.

Sol:-

As we know that if

$P = \{(x, \alpha_p(x), \gamma_p(x), \beta_p(x)) \mid x \in X\}$ is a Picture fuzzy set. Then

$$P^c = \{\langle x, \beta_p(x), \gamma_p(x), \alpha_p(x)\rangle \mid x \in X\}$$

So

$$A^c = \{x, (0.3, 0.2, 0.4)\}.$$

Q. No. 13:-

Let $X = \{x_1\}$ be a Universal set. $\lambda = 3$ and $h = \{x_1, (0.2, 0.8)\}$ is a HFS on $X$. Evaluate $\lambda h$.

Sol:-

As we know that

$$\lambda h(x) = \{1 - (1-\gamma)^\lambda \mid \gamma \in h(x)\}$$

So,

$$\beta h(x) = \{1-(1-0.2)^3, \; 1-(1-0.8)^3\}$$

$$= \{0.488, \; 0.992\}.$$

Q. No. 14):-

Find $\nabla$- implication where

$$x \nabla y = x + y - xy.$$

Sol:-

As we know that

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∇-implication is map from $[0,1]^2 \longrightarrow [0,1]$ of the form

$$x \Rightarrow y = \eta(x) \nabla y$$

where

$$\eta(x) = 1 - x$$

So,

$$x \Rightarrow y = (1-x) \nabla y$$
$$= (1-x) + y - (1-x)y$$
$$= 1 - x + y - y + xy$$
$$= 1 - x + xy$$

Q No. 15

Prove that $\Delta_0$ is Nilpotent t-norm.

Sol:-

As we know that

$$x \Delta_0 y = \begin{cases} \min(x,y) & \text{if } \max(x,y) = 1 \\ 0 & \text{elsewhere} \end{cases}$$

Let

$$\max(x,y) = 1$$

suppose

$$x \neq 1 \Rightarrow \max(x,x) \neq 1$$

$$\max(x,x) = 0$$

$$\Rightarrow x \Delta_0 x = 0$$

$$\Rightarrow [x]^2 = 0$$

So,

$\Delta_0$ is Nilpotent t-norm.

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Q. No. 16):-

If $A = (0.3, 0.4, 0.2)$ and $B = (0.3, 0.5, 0.1)$ are PFS then Find

i) $A \subseteq B$ ii) $A \cup B$ iii) $A \cap B$

Sol:-

i) $A \subseteq B$

As we know

$A \subseteq B$ iff

$$\alpha_A(x) \leq \alpha_B(x), \quad \gamma_A(x) \leq \gamma_B(x)$$

$$\beta_A(x) \geq \beta_B(x),$$

here

$$0.3 = 0.3, \quad 0.4 < 0.5$$

and

$$0.2 > 0.1$$

So,

$A \subseteq B$.

~~Q. No. 16~~:- ii) $A \cup B = ?$

$$A \cup B = \{ x, \max(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \min(\beta_A(x), \beta_B(x)) \mid x \in X \}.$$

So,

$$= \{x, \{\max(0.3, 0.3), \min(0.4, 0.5), \min(0.2, 0.1)\}\}$$

$$= \{x, (0.3, 0.4, 0.1)\}.$$

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iii) $A \cap B = ?$

$A \cap B = \{ x_1 (\min(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \max(\beta_A(x), \beta_B(x)) \mid x \in X \}$

So,

$= \{ x_1, (\min(0.3, 0.3), \min(0.4, 0.5), \max(0.1, 0.2$

$= \{ x_1, (0.3, 0.4, 0.2) \}$

Q. No. 17):-

Let $X = \{x_1, x_2\}$ be a universal set $A = \{(x_1, (0.4, 0.25, 0.35)), (x_2, (0.15, 0.45, 0.4))\}$

$B = \{(x_1, (0.3, 0.4, 0.3)), (x_2, (0.25, 0.15, 0.6))\}$ are two PFS on X Find Normalized Hamming distance.

Sol:-

As we know that Normalized Hamming distance for PFS is

$$d_n(A,B) = \frac{1}{n} \sum_{i=1}^{n} \left[ |\alpha_A(x_i) - \alpha_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)| + |\beta_A(x_i) - \beta_B(x_i)| \right]$$

So,

$$= \frac{1}{2} \left[ |0.4 - 0.3| + |0.25 - 0.4| + |0.35 - 0.3| + |0.15 - 0.25| + |0.45 - 0.15| + |0.4 - 0.6| \right]$$

$$= 0.45$$

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Q. No. 18:-

Let $X = \{x_1, x_2, x_3, x_4\}$ be a fixed set and $A = \{(x_1, 0.1), (x_2, 0.1), (x_3, 0.3)\}$, $B = \{(x_2, 0.4), (x_3, 0.4), (x_4, 1)\}$ are fuzzy sets on $X$, evaluate the distance b/w A and B by using max (min) form.

Sol:-

| d | $x_A$ | $x_B$ | $m_A(x_i)$ | $m_B(x_i)$ | $\min(m_A(x_i), m_B(x_i))$ | $\max(\min(m_A(x_i), m_B(x_i)))$ |
|---|---|---|---|---|---|---|
| 0 | $x_2$ | $x_2$ | 0.1 | 0.4 | 0.1 | |
| | $x_3$ | $x_3$ | 0.3 | 0.4 | 0.3 | 0.3 |
| 1 | $x_1$ | $x_2$ | 0.1 | 0.4 | 0.1 | |
| | $x_2$ | $x_3$ | 0.1 | 0.4 | 0.1 | |
| | $x_3$ | $x_4$ | 0.3 | 1 | 0.3 | 0.3 |
| 2 | $x_1$ | $x_3$ | 0.1 | 0.4 | 0.1 | 0.1 |
| | $x_2$ | $x_4$ | 0.1 | 1 | 0.1 | |
| 3 | $x_1$ | $x_4$ | 0.1 | 1 | 0.1 | 0.1 |

So

$$d(A,B) = 0.3 + 0.3 + 0.1 + 0.1$$
$$= 0.8$$

Q. No. 19):-

Define strictly monotone t-norm.

Sol:-

T-norm $\Delta$ is called strictly monotone t-norm if

$$x \Delta y < x \Delta z$$

whenever $x > 0$ and $y < z$ where $x$, $y$ and $z$ fuzzy Numbers.

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Q. No. 20:-

Define fuzzy set?

Sol:-

A fuzzy set consists of two components a regular set and membership function associated with it.

$$A = \{ (x, \mu_A(x)) \mid x \in U \}.$$

Q. No. 21:-

Define symmetric fuzzy relation?

Sol:-

A fuzzy binary relation R is symmetric if

$$R(U,V) = 1 \text{ and } R(V,U) = 1$$

$$\Rightarrow R(U,V) = R(V,U)$$

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Theorem:-

If R is a fuzzy relation on a set U, then R is a fuzzy equivalence relation iff each $\alpha$-cut, $R_\alpha$ is an equivalence relation on U.

Proof:-

Let $R_\alpha$ be a fuzzy equivalence relation. Then

1) $R(u,u) = 1 \geq \alpha$

$\Rightarrow (u,u) \in R_\alpha$

So $R_\alpha$ is a reflexive relation

2)

If $R(u,v) = 1 = R(v,u)$

If $(u,v) \in R_\alpha$

$\Rightarrow R(u,v) = 1 \geq \alpha$

Also $R(v,u) = 1 \geq \alpha$

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$\Rightarrow (V,U) \in R_\alpha$

$\Rightarrow R_\alpha$ is symmetric.

3)

If $(V,U), (V,W) \in R_\alpha$

$\Rightarrow R(U,V) = 1 \geq \alpha$

and

$R(V,W) = 1 \geq \alpha$

$\Rightarrow R(U,W) \geq R(U,V) \wedge R(V,W) = 1 \geq \alpha$

$\Rightarrow R(U,W) \in R_\alpha$

So

$R_\alpha$ is transitive.

since $R_\alpha$ is reflexive, symmetric and transitive.

So

$R_\alpha$ is equivalence relation.

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# MTH645 Current papers Questions (Fall 2021) Final Term

Solved by

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## Question #1

If $f(x) = e^{-\left(\frac{1-x}{x}\right)}$, Find "a" or $f^{-1}(x) = ?$ (2,3 or 5 Marks)

## Solution:

Let $y = e^{-\left(\frac{1-x}{x}\right)} \Rightarrow \ln y = \ln e^{-\left(\frac{1-x}{x}\right)}$

$\Rightarrow \ln y = -\left(\frac{1-x}{x}\right)\ln e \quad \because \ln e = 1$

$\Rightarrow \ln y = -\left(\frac{1-x}{x}\right)$

$\Rightarrow x \ln y = x - 1$

$\Rightarrow 1 = x - x\ln y = x(1 - \ln y)$

$\Rightarrow x = \frac{1}{1-\ln y} = f^{-1}(y)$

$\Rightarrow \boxed{f^{-1}(x) = \frac{1}{1-\ln x}}$ Ans.

$a = f(0) = e^{-\left(\frac{1-0}{0}\right)} = e^{-\frac{1}{0}} = e^{-\infty} = \frac{1}{e^{\infty}}$

$a = f(0) = \frac{1}{\infty} = 0$

$\boxed{a = 0}$ Ans

Note: The question can be asked by further changing the value of $f(x)$.

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## Question # 2

If $\Delta$ is strictly monotone then it has only trivial idempotent element. (5 Marks)

### Solution:

Let $\Delta$ has non trivial idempotent element $x$ and $y$.

Let $x, y \in (0,1)$

$x \Delta y < x \Delta z \; ; \; x > 0 \; , \; y < z$

Take $x \leq x$ and $x < 1$

$\Rightarrow x \Delta x < x \Delta 1 = x$

$\Rightarrow x \Delta x < x$

which is contradiction.

So $\Delta$ has only trivial idempotent element.

## Question # 3

Define Fuzzy Set. (2 Marks)

### Solution:

A fuzzy set consist of two components a regular set and a membership function associated with it.

$$A = \{(x, \mu_A(x)) \mid x \in U\}$$

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## Question # 4

Define $\nabla$-Implication (Weber Implication).

**Solution:** (2 Marks)

A $\nabla$-Implication is a map $\Rightarrow [0,1]^2 \to [0,1]$ of the form $(x \Rightarrow y) = \eta(x) \nabla y$ ; $\eta(x) = 1 - x$

$\nabla$ is t-conorm. (Eta)

## Question # 5

Define Symmetric Relation.

**Solution:** (2 Marks)

A fuzzy binary relation "R" is symmetric if $R(x,y) = 1 \Rightarrow R(y,x) = 1$

## Question # 6

Define Antisymmetric Relation. (2 Marks)

**Solution:**

A fuzzy binary relation R is antisymmetric if $R(x,y) = \& R(y,x) = 1 \Rightarrow x = y$

## Question # 7

Define Reflexive Relation. (2 Marks)

**Solution:**

A fuzzy binary relation R is reflexive if $R(x,x) = 1$

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## Question # 8

Find Hamming distance between two fuzzy sets A and B. If $X = \{1, 2, 3, 4\}$

$A = \{(1, 0.5), (2, 1.0), (3, 0.3)\}$ and

$B = \{(2, 0.4), (3, 0.4), (4, 1.0)\}$

Solution: (5 Marks)

$d_h(1,2) = |0.5 - 0.4| = 0.1$

$d_h(1,3) = |0.5 - 0.4| = 0.1$

$d_h(1,4) = |0.5 - 1.0| = 0.5$

$d_h(2,2) = |1.0 - 0.4| = 0.6$

$d_h(2,3) = |1.0 - 0.4| = 0.6$

$d_h(2,4) = |1.0 - 1.0| = 0$

$d_h(3,2) = |0.3 - 0.4| = 0.1$

$d_h(3,3) = |0.3 - 0.4| = 0.1$

$d_h(3,4) = |0.3 - 1.0| = 0.7$

$D_h = 0.1 + 0.1 + 0.5 + 0.6 + 0.6 + 0 + 0.1 + 0.1 + 0.7$

$D_h = 2.8$ Ans

Note: The question can be asked by further changing the sets X, A and B.

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## Question #9

Define Nilpotent t-norm. (2 Marks)

**Solution:**

$\Delta$ is nilpotent t-norm if for $b \neq 1$, $b^{[n]} = 0$ for some positive integers "n", where n depends on b.

## Question #10

Define strictly monotone t-norm. (2 Marks)

**Solution:**

A t-norm $\Delta$ is called strictly monotone t-norm if $x \Delta y < x \Delta z$, whenever $x > 0$ and $y < z$.

## Question # 11

Define Transitive Relation. (2 Marks)

**Solution:**

A fuzzy binary relation R is transitive if $R(x,y) = 1$, $R(y,z) = 1 \Rightarrow R(x,z) = 1$

where $x, y, z \in$ fuzzy numbers.

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## Question#12

Write down the types of fuzzy sets. (5 Marks)

**Solution:**

(i) Intutionistic Fuzzy Sets
(ii) Picture Fuzzy Sets
(iii) Hasitent Fuzzy Sets
(iv) Normal Fuzzy Sets
(v) Empty Fuzzy Sets.

## Question#13

Define $\alpha$-cut in fuzzy binary relation. and give example. (2 or 3 Marks)

**Solution:**

$\alpha$-cut in fuzzy binary relation is

$$R_\alpha = R^{-1}([\alpha, 1]) = [(u,v) : R(u,v) \geq \alpha]$$

Example: $\alpha = 0.5$, $R = [((x_1,y_1), 0.3), ((x_2,y_2), 0.7), ((x_3,y_3), 0.9)]$

$$R_\alpha = R_{0.5} = [((x_2,y_2), 0.7), ((x_3,y_3), 0.9)]$$

## Question#14

Define zero divisor. (2 Marks)

**Solution:**

A fuzzy number $a \in (0,1)$ is called zero divisor of t-norm $\Delta$, if there exist $b \in (0,1)$ such that $a \Delta b = 0$

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## Question # 15

If $f(x) = e^{x-5}$, $f^{-1}(x) = 5 + \ln x$ and $a = \frac{1}{e^5}$, $x \Delta y = xy$ then find the value of generating t-norm $x \Delta_f y$.

## Solution: (5 Marks)

$f(x) = e^{x-5}$, $f^{-1}(x) = 5 + \ln x$, $a = \frac{1}{e^5}$, $x \Delta y = xy$

We know that:

$$x \Delta_f y = f^{-1}\left((f(x) \Delta f(y)) \vee a\right)$$

$$= f^{-1}\left((f(x) \cdot f(y)) \vee a\right)$$

$$= f^{-1}\left((e^{x-5} \cdot e^{y-5}) \vee \frac{1}{e^5}\right)$$

$$= f^{-1}\left((e^{x+y-10}) \vee \frac{1}{e^5}\right)$$

$$= f^{-1}\left(e^{x+y-10} \vee e^{-5}\right)$$

$$= 5 + \ln(e^{x+y-10} \vee e^{-5})$$

$$= 5 + (\ln e^{x+y-10} \vee \ln e^{-5})$$

$$= 5 + ((x+y-10) \vee (-5))$$

$$= (5 + x + y - 10) \vee (5 - 5)$$

$$x \Delta_f y = (x + y - 5) \vee (0)$$

$$x \Delta_f y = x + y - 5 \vee 0 \quad \underline{\text{Ans}}$$

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# Question #16

Find the max-min composition of R and S where

$$R = \begin{Bmatrix} 0.1 & 0 & 0.6 \\ 0 & 0.3 & 1 \\ 0.2 & 0.4 & 0.4 \end{Bmatrix}, \quad S = \begin{Bmatrix} 0.2 & 0.5 & 0 \\ 0.9 & 1 & 0.3 \\ 1 & 0.5 & 0 \end{Bmatrix}$$

## Solution: (5 Marks)

$\min(0.1, 0.2) = 0.1$
$\min(0, 0.9) = 0$
$\min(0.6, 1) = 0.6$ } $\max = 0.6$

$\min(0.1, 0.5) = 0.1$
$\min(0, 1) = 0$
$\min(0.6, 0.5) = 0.5$ } $\max = 0.5$

$\min(0.1, 0) = 0$
$\min(0, 0.3) = 0$
$\min(0.6, 0) = 0$ } $\max = 0$

$\min(0, 0.2) = 0$
$\min(0.3, 0.9) = 0.3$
$\min(1, 1) = 1$ } $\max = 1$

$\min(0, 0.5) = 0$
$\min(0.3, 1) = 0.3$
$\min(1, 0.5) = 0.5$ } $\max = 0.5$

$\min(0, 0) = 0$
$\min(0.3, 0.3) = 0.3$
$\min(1, 0) = 0$ } $\max = 0.3$

$\min(0.2, 0.2) = 0.2$
$\min(0.4, 0.9) = 0.4$
$\min(0.4, 1) = 0.4$ } $\max = 0.4$

$\min(0.2, 0.5) = 0.2$
$\min(0.4, 1) = 0.4$
$\min(0.4, 0.5) = 0.4$ } $\max = 0.4$

$\min(0.2, 0) = 0$
$\min(0.4, 0.3) = 0.3$
$\min(0.4, 0) = 0$ } $\max = 0.3$

$$R \circ S = \begin{bmatrix} 0.6 & 0.5 & 0 \\ 1 & 0.5 & 0.3 \\ 0.4 & 0.4 & 0.3 \end{bmatrix}$$ Ans

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Note: The question can be asked by further changing the values of R and S.

## Question # 17

Let $X = \{x_1\}$ be a universal set $h_1 = \{\langle x_1, \{0.3, 0.4\}\rangle\}$ and $h_2 = \{\langle x_1, \{0.5, 0.7\}\rangle\}$ are two hesitant fuzzy sets on X evaluate: $h_1 \oplus h_2$ (5 Marks)

### Solution:

$$h_1 \oplus h_2 = \{0.3, 0.4\} \oplus \{0.5, 0.7\}$$

$$= \{0.3+0.5-(0.3)(0.5),\ 0.3+0.7-(0.3)(0.7),\ 0.4+0.5-(0.4)(0.5),\ 0.4+0.7-(0.4)(0.7)\}$$

$$= \{0.8-0.15,\ 1.0-0.21,\ 0.9-0.2,\ 1.1-0.28\}$$

$$= \{0.65, 0.79, 0.7, 0.82\}$$

## Question # 18

Define Picture Fuzzy Set. (2 Marks)

### Solution:

Let X be a fixed set, a picture fuzzy set P on X is defined as $P = \{\langle x, \alpha_p(x), \gamma_p(x), \beta_p(x)\rangle / x \in X\}$ where $\alpha_p(x), \beta_p(x)$ and $\gamma_p(x) \in [0,1]$ are called the acceptance, neutral and rejection membership degrees of $x \in X$ to the set P respectively and fulfil the condition: $0 \le \alpha_p(x) + \gamma_p(x) + \beta_p(x) \le 1, \forall x \in X$.

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# Question #19

Let $X = \{x_1, x_2, x_3\}$ be a fixed set and $A = \{(x_1, 0.3), (x_2, 0.5), (x_3, 0.9)\}$ and $B = \{(x_1, 0.5), (x_2, 0.8), (x_3, 1)\}$ are two fuzzy sets on X evaluate the normalized Hamming distance between A and B.

## Solution: (5 Marks)

The Hamming distance with all generating values is evaluated as:

$d_h(x_1, x_1) = |0.3 - 0.5| = 0.2$

$d_h(x_1, x_2) = |0.3 - 0.8| = 0.5$

$d_h(x_1, x_3) = |0.3 - 1.0| = 0.7$

$d_h(x_2, x_1) = |0.5 - 0.5| = 0$

$d_h(x_2, x_2) = |0.5 - 0.8| = 0.3$

$d_h(x_2, x_3) = |0.5 - 1.0| = 0.5$

$d_h(x_3, x_1) = |0.9 - 0.5| = 0.4$

$d_h(x_3, x_2) = |0.9 - 0.8| = 0.1$

$d_h(x_3, x_3) = |0.9 - 1.0| = 0.1$

The Normalized Hamming Distance is

$$D_h^n(A, B) = \frac{1}{n} \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$$= \frac{1}{3}(0.2 + 0.5 + 0.7 + 0 + 0.3 + 0.5 + 0.4 + 0.1 + 0.1)$$

$$= \frac{1}{3}(2.8) = 0.93 \quad \text{Ans}$$

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# Question # 20

Let $A = (0.3, 0.4, 0.2)$ and $B = (0.3, 0.5, 0.1)$ be two PFSs, then find (i) $A \subseteq B$ (ii) $A \cup B$ (iii) $A \cap B$

## Solution: (5 Marks)

(i) $A \subseteq B = ?$

$A \subseteq B$ if $\alpha_A(x) \leq \alpha_B(x)$, $\gamma_A(x) \leq \gamma_B(x)$ and $\beta_A(x) \geq \beta_B(x)$, $\forall x \in X$

$0.3 \leq 0.3$, $0.4 \leq 0.5$ and $0.2 \geq 0.1$

Hence $A \subseteq B$.

(ii) $A \cup B = ?$

$$A \cup B = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \min(\beta_A(x), \beta_B(x))\rangle \mid x \in X\}$$

$$A \cup B = \{\langle x, \max(0.3, 0.3), \min(0.4, 0.5), \min(0.2, 0.1)\rangle\}$$

$$= \{\langle x, 0.3, 0.4, 0.1\rangle\} = (0.3, 0.4, 0.1)$$

(iii) $A \cap B = ?$

$$A \cap B = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \max(\beta_A(x), \beta_B(x))\rangle \mid x \in X\}$$

$$A \cap B = \{\langle x, \min(0.3, 0.3), \min(0.4, 0.5), \max(0.2, 0.1)\rangle\}$$

$$= \{\langle x, 0.3, 0.4, 0.2\rangle\} = (0.3, 0.4, 0.2)$$

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## Question #21

In R-implication find the values of 1 implies 1 by using formula of R-implication

### Solution: (2 Marks)

We know that $(x \Rightarrow y) = \max\{z \in [0,1] : x \Delta z \leq y\}$

$(1 \Rightarrow 1) = \max\{z \in [0,1] : 1 \Delta z \leq 1\}$

$= \max\{z \in [0,1] : z \leq 1\}$

$= \max\{z \in [0,1]\}$

$(1 \Rightarrow 1) = 1$ Ans

## Question #22

Let X be a fuzzy number and $\Delta$ be an idemponent t-norm then find the value of $(x \Delta x) \Delta x$.

### Solution: (3 Marks)

We know that $\Delta$ is an idemponent t-norm if $x \Delta x = x$, $\forall\ x \in [0,1]$

Now we find the value of $(x \Delta x) \Delta x$

$(x \Delta x) \Delta x = x \Delta x \quad \because x \Delta x = x$

$(x \Delta x) \Delta x = x$

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## Question #23

Let $X = \{x_1\}$ be a fixed set and $A = \{\langle x_1, (0.5, 0.3)\rangle\}$ and $B = \{\langle x_1, (0.6, 0.2)\rangle\}$ are two IFSs on $X$. Evaluate $A \cup B$.

### Solution: (3 Marks)

We know that

$$A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle \mid x \in X\}$$

$$A \cup B = \{\langle x_1, \max(0.5, 0.6), \min(0.3, 0.2)\rangle\}$$

$$A \cup B = \{\langle x_1, (0.6, 0.2)\rangle\}$$

## Question #24

Let $X = \{x_1\}$ be a fixed set and $A = \{\langle x_1, (0.5, 0.3)\rangle\}$ and $B = \{\langle x_1, (0.6, 0.2)\rangle\}$ are two IFSs on $X$. Evaluate $A \otimes B$.

### Solution: (3 Marks)

We know that

$$A \otimes B = \{\langle x, \mu_A(x) \times \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \times \nu_B(x)\rangle \mid x \in X\}$$

$$A \otimes B = \{\langle x_1, 0.5 \times 0.6, 0.3 + 0.2 - 0.3 \times 0.2\rangle\}$$

$$A \otimes B = \{\langle x_1, 0.3, 0.5 - 0.06\rangle\}$$

$$A \otimes B = \{\langle x_1, (0.3, 0.44)\rangle\}$$ Ans

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## Question # 25

Define the composition of RoS of fuzzy relation R and S with respect to t-norm Δ.

**Solution:** (2 Marks)

Let R and S be fuzzy relation in UxV and VxW respectively and let Δ be a t-norm. Then composition RoS of R and S with respect to Δ is the fuzzy relation on UxW with membership function

$$(R \circ S)(u,w) = \bigvee [R(u,v) \Delta S(v,w)]$$

## Question # 26

Evaluate the Euclidean distance between two HFSs $h_1 = \{\langle x_1, \{0.8, 0.9\}\rangle\}$, $h_2 = \{\langle x_1, \{0.6, 0.7\}\rangle\}$

**Solution:** (2 Marks)

$$De(h_1,h_2) = \sqrt{\sum_{i=1}^{n} (h_{1i} - h_{2i})^2}$$

$$De(h_1,h_2) = \sqrt{(0.8-0.6)^2 + (0.9-0.7)^2}$$

$$= \sqrt{(0.2)^2 + (0.2)^2}$$

$$= \sqrt{0.04 + 0.04}$$

$$= \sqrt{0.08}$$

$$De(h_1,h_2) = 0.2828$$

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## Question # 27

Let $X = \{x_1\}$ be a fixed set and $A = \{\langle x_1, (0.8, 0.2)\rangle\}$ and $B = \{\langle x_1, (0.7, 0.3)\}$ are two IFSs on X. Then find A − B.

### Solution: (3 Marks)

We know that

$$A - B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\mu_B(x), \nu_A(x))\rangle | x \in X\}$$

$$A - B = \{\langle x_1, \min(0.8, 0.3), \max(0.7, 0.2)\rangle\}$$

$$A - B = \{\langle x_1, (0.3, 0.7)\rangle\}$$ Ans

## Question # 28

Let $\Delta$ be archemidian t-norm and f is generator of t-norm, Then $(x \Rightarrow y) = f^{-1}\left[\frac{f(y)}{f(x)} \wedge 1\right]$

### Solution: (5 Marks)

$$x \Delta y = f^{-1}(f(x) \cdot f(y) \vee f(0))$$

$$(x \Rightarrow y) = \max\{z \in [0,1] : x \Delta z \leq y\}$$

$$= \max\{z \in [0,1] : f^{-1}(f(x) \cdot f(z) \vee f(0)) \leq y\}$$

Now $f^{-1}(f(x) \cdot f(z) \vee f(0)) \leq y$

$\Rightarrow f(x) \cdot f(z) \vee f(0) \leq f(y)$

$\Rightarrow f(x) \cdot f(z) \vee a \leq f(y)$

$\Rightarrow f(x) f(z) \leq f(y)$

$\Rightarrow f(z) \leq f(y)/f(x)$

$\Rightarrow z \leq f^{-1}\left(\frac{f(y)}{f(x)} \wedge 1\right)$

So

$$(x \Rightarrow y) = f^{-1}\left[\frac{f(y)}{f(x)} \wedge 1\right]$$ proved

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## Question # 29

Find Hamming distance between two Hesitent fuzzy sets $h_1$ and $h_2$. If

$h_1 = \{\langle x_1, (0.3, 0.4, 0.4, 0.65)\rangle\}$, $h_2 = \{\langle x_1, (0.5, 0.7, 0.8, 0.95)\rangle\}$

**Solution:** (5 Marks)

$$D_h(h_1, h_2) = \sum_{i=1}^{n} |h_{1i} - h_{2i}|$$

$$D_h(h_1, h_2) = |0.3-0.5| + |0.4-0.7| + |0.4-0.8| + |0.65-0.95|$$

$$= 0.2 + 0.3 + 0.4 + 0.3$$

$$D_h(h_1, h_2) = 1.2 \quad \text{Ans}$$

## Question # 30

Find Euclidean distance between two PFSs $A = (0.4, 0.5, 0.1)$ and $B = (0.2, 0.6, 0.2)$

**Solution:** (3 Marks)

$$de(A,B) = \sqrt{\sum_{i=1}^{n} \left(m_A(x_i) - m_B(x_i)\right)^2}$$

$$de(A,B) = \sqrt{(0.4-0.2)^2 + (0.5-0.6)^2 + (0.1-0.2)^2}$$

$$= \sqrt{(0.2)^2 + (-0.1)^2 + (-0.1)^2}$$

$$= \sqrt{0.04 + 0.01 + 0.01} = \sqrt{0.06}$$

$$de(A,B) = 0.245 \quad \text{Ans}$$

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## Question #31

Find Hamming distance between two fuzzy sets.

$A = \{(x_1, 0.3), (x_2, 0.5)\}$, $B = \{(x_1, 0.5), (x_2, 0.8)\}$.

### Solution: (3 Marks)

$$d_h(A,B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$d_h(x_1, x_1) = |0.3 - 0.5| = 0.2$

$d_h(x_1, x_2) = |0.3 - 0.8| = 0.5$

$d_h(x_2, x_1) = |0.5 - 0.5| = 0$

$d_h(x_2, x_2) = |0.5 - 0.8| = 0.3$

$d_h(A,B) = 0.2 + 0.5 + 0 + 0.3$

$d_h(A,B) = 1.0$ Ans

## Question #32

If $f(x) = e^{x-5}$ then find the value of "a".

### Solution: (2 Marks)

$f(x) = e^{x-5}$

$a = f(0) = e^{0-5}$

$a = f(0) = e^{-5}$

$$\boxed{a = \frac{1}{e^5}}$$ Ans

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## Question # 33

Show that $A \subseteq B$ of two PFSs, if

$A = (0.2, 0.5, 0.3)$, $B = (0.3, 0.6, 0.1)$

**Solution:** (2 Marks)

$A \subseteq B$ if $\alpha_A(x) \leq \alpha_B(x)$, $\gamma_A(x) \leq \gamma_B(x)$

and $\beta_A(x) \geq \beta_B(x)$, $\forall x \in X$

$0.2 \leq 0.3$, $0.5 \leq 0.6$ and $0.3 \geq 0.1$

Hence $A \subseteq B$ (Proved)

## Question # 34

Let $A = \{(x_1, 0.6)\}$, $B = \{(x_1, 0.9)\}$ be two fuzzy sets on $X = \{x_1\}$. Determine normalized Hamming distance between A and B.

**Solution:** (2 Marks)

$$d_h^n(A,B) = \frac{1}{n}\sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$$= \frac{1}{1}(|0.6 - 0.9|)$$

$$d_h^n(A,B) = 0.3 \quad \text{Ans}$$

## Question # 35

Define Idempotent element. (2 Marks)

**Solution:**

Let $a \in (0,1)$ is idempotent element if $a \Delta a = a$.

For example (i) $0 \Delta 0 = 0 \Rightarrow 0$ is idempotent element

(ii) $1 \Delta 1 = 1 \Rightarrow 1$ is idempotent element

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## Question # 36

Prove that $\Delta_0$ is nilpotent t-norm.

**Solution:** (5 Marks)

$$x \Delta_0 y = \begin{cases} \min(x,y) & ; \text{if } \max(x,y) = 1 \\ 0 & ; \text{otherwise.} \end{cases}$$

Let $\max(x,y) = 1$

$\Rightarrow x \neq 1 \quad , \max(x,x) \neq 1$

$\Rightarrow x \Delta_0 x = 0$

$\Rightarrow x^{[2]} = 0$

$\Rightarrow \Delta_0$ is nilpotent t-norm.

## Question # 37

If $h(x) = \{0.6\}$ and $\lambda = 2$ then find $\lambda h(x)$.

**Solution:** (2 or 3 Marks)

We know that

$$\lambda h(x) = \{1 - (1-x)^{\lambda}\}$$

$x = 0.6 \quad , \lambda = 2$

$$\lambda h(x) = \{1 - (1-0.6)^2\}$$

$$= \{1 - (0.4)^2\} = \{1 - 0.16\}$$

$$\lambda h(x) = \{0.84\} \quad \text{Ans}$$

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## Question # 38

If $A = \{\langle x, (0.4, 0.2, 0.3)\rangle\}$ is a PFS on $X$, Determine $A^c$.

**Solution:** (2 Marks)

We know that, If $A = \{\langle x, (\alpha_A(x), \gamma_A(x), \beta_A(x))\rangle\}$ is a PFS then $A^c = \{\langle x, (\beta_A(x), \gamma_A(x), \alpha_A(x))\rangle\}$

$A = \{\langle x, (0.4, 0.2, 0.3)\rangle\}$

$A^c = \{\langle x, (0.3, 0.2, 0.4)\rangle\}$ Ans

## Question # 39

Let $X = \{x_1\}$ be a universal set, $\lambda = 3$ and $h = \{\langle x_1, (0.2, 0.8)\rangle\}$ is a HFS on $X$, Evaluate $\lambda h(x_1)$.

**Solution:** (2 or 3 Marks)

We know that

$\lambda h(x) = \{1 - (1 - \gamma_i)^\lambda\}$

$\lambda h(x_1) = \{1 - (1 - 0.2)^3, 1 - (1 - 0.8)^3\}$

$\lambda h(x_1) = \{1 - (0.8)^3, 1 - (0.2)^3\}$

$\lambda h(x_1) = \{1 - 0.512, 1 - 0.008\}$

$\lambda h(x_1) = \{0.488, 0.992\}$ Ans

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## Question # 40

Find the $\nabla$-implication where $x \nabla y = x + y - xy$

**Solution:** (3 Marks)

$$(x \Rightarrow y) = \eta(x) \nabla y$$

$$= (1-x) \nabla y \qquad \because \eta(x) = 1-x$$

$$= (1-x) + y - (1-x)y$$

$$= 1 - x + y - y + xy$$

$$(x \Rightarrow y) = 1 - x + xy \quad \text{Ans}$$

## Question # 41

Let $X = \{x_1, x_2\}$ be a universal set, $A = \{\langle x_1, (0.4, 0.25, 0.35)\rangle, \langle x_2, (0.15, 0.45, 0.4)\rangle\}$ and $B = \{\langle x_1, (0.3, 0.4, 0.3)\rangle, \langle x_2, (0.25, 0.15, 0.6)\rangle\}$ are two PFSs on $X$. Find Normalized Hamming distance.

**Solution:** (5 Marks)

$$d_h^n(A,B) = \frac{1}{n} \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

$$d_h^n(A,B) = \frac{1}{2}\Big[|0.4-0.3| + |0.25-0.4| + |0.35-0.3| + |0.15-0.25| + |0.45-0.15| + |0.4-0.6|\Big]$$

$$= \frac{1}{2}[0.1 + 0.15 + 0.05 + 0.10 + 0.30 + 0.20]$$

$$d_h^n(A,B) = \frac{1}{2}[0.9] = 0.45 \quad \text{Ans}$$

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## Question #42

Let $X=\{x_1, x_2, x_3, x_4\}$ be a fixed set and $A=\{(x_1, 0.1), (x_2, 0.1), (x_3, 0.3)\}$ and $B=\{(x_2, 0.4), (x_3, 0.4), (x_4, 1)\}$ are two fuzzy sets ~~Solve~~ on $X$, Evaluate the distance between $A$ and $B$ by using max and min.

### Solution: (5 Marks)

Distance between fuzzy sets $A$ and $B$

| d | $x_A$ | $x_B$ | $m_A(x_i)$ | $m_B(x_i)$ | $\min(m_A(x_i), m_B(x_i))$ | $\max[\min(m_A(x_i), m_B(x_i))]$ |
|---|---|---|---|---|---|---|
| 0 | $x_2$ | $x_2$ | 0.1 | 0.4 | 0.1 | 0.3 |
| | $x_3$ | $x_3$ | 0.3 | 0.4 | 0.3 | — |
| 1 | $x_1$ | $x_2$ | 0.1 | 0.4 | 0.1 | 0.3 |
| | $x_2$ | $x_3$ | 0.1 | 0.4 | 0.1 | — |
| | $x_3$ | $x_2$ | 0.3 | 0.4 | 0.3 | — |
| | $x_3$ | $x_4$ | 0.3 | 1 | 0.3 | — |
| 2 | $x_1$ | $x_3$ | 0.1 | 0.4 | 0.1 | 0.1 |
| | $x_2$ | $x_4$ | 0.1 | 1 | 0.1 | — |

Therefore $d(A,B) = 0.3+0.3+0.1 = 1.0$ Ans

## Question #43

If $x \Delta y = \min(x,y) = x \wedge y$ then find $(x \Rightarrow y)$.

### Solution: (5 Marks)

We know that

$$(x \Rightarrow y) = \begin{cases} \max\{z \in [0,1] : x \Delta z \le y\} \end{cases}$$

If $x \Delta y = \min(x,y)$

$$(x \Rightarrow y) = \max\{z \in [0,1] : \min(x,z) \le y\}$$

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$$(x \Rightarrow y) = \begin{cases} 1 & ; x \leq y \\ y & ; x > y \end{cases}$$

## Question # 44

Define Similarity Measures based on distance.

**Solution:** (2 or 3 Marks)

Let S be a Similarity Measures between two fuzzy sets is calculated as:

Suppose that A and B two fuzzy sets then the similarity measures S(A,B) based on distance d(A,B) is written as:

$$S(A,B) = 1 - d_h(A,B)$$

or $$S(A,B) = 1 - d_e(A,B)$$

or $$S(A,B) = 1 - d_h^n(A,B)$$

or $$S(A,B) = 1 - d_e^n(A,B)$$

Similarity Measures satisfied the condition.

$$0 \leq S(A,B) \leq 1$$

Solved By Mahar Afaq Safdar Muhammadi

دعاؤں میں یاد رکھیے گا۔ جزاک اللہ خیرا 🤲

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## Question#45

$H_1(x) = \{0.125, 0.035, 0.245, 0.063\}$ and $H_2(x) = \{0.225, 0.365, 0.415\}$ be two HFSs then Compute (i) $H_1 \cup H_2$ (ii) $H_1 \cap H_2$ (5 Marks)

### Solution:

$$H_1 \cup H_2 \geq \max\{\min\{0.125, 0.035, 0.245, 0.063\}, \min\{0.225, 0.365, 0.415\}\}$$
$$\geq \max\{0.035, 0.225\}$$
$$\geq 0.225$$
$$= \{0.225, 0.245, 0.365, 0.415\}$$

$$H_1 \cap H_2 \leq \min\{\max\{0.125, 0.035, 0.245, 0.063\}, \max\{0.225, 0.365, 0.415\}\}$$
$$\leq \min\{0.245, 0.415\}$$
$$\leq 0.245$$
$$= \{0.035, 0.063, 0.125, 0.225, 0.245\}$$

## Question#46

Define Normal Fuzzy Set. (2 Marks)

### Solution:

A fuzzy set is said to be normal fuzzy set if there exist atleast one element whose membership is one.

For example $A = \{(x_1, 0), (x_2, 0.1), \boxed{(x_3, 1)}, (x_4, 0.5)\}$

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## Question #47

Define Empty Fuzzy Set.

### Solution: (2 Marks)

A fuzzy set is said to be empty fuzzy set if all elements of universal set maps on zero. i.e. the membership value of all elements are zero.

For Example: $A = \{(x_1, 0), (x_2, 0), (x_3, 0)\}$

## Question # 48

Name of any five similarity measure. (5 Marks)

### Solution:

(i) Similarity measures based on distance.
(ii) Cosine similarity measures
(iii) Exponential similarity measures
(iv) Jaccart and Dice Similarity measures.
(v) William and steel similarity measures.

## Question # 49

List the most used distance formulae b/w FSs. (2 marks)

### Solution:

(i) Hamming distance. $d_h(A,B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$

(ii) Euclidean distance. $de(A,B) = \sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))}$

(iii) Normalized Hamming distance $d_h^n(A,B) = \frac{1}{n} \cdot \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$

(iv) Normalized Euclidean distance $de^n(A,B) = \frac{1}{n} \sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))}$

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# Question#50

Written in matrix form.

$$R=\left\{\frac{(x_1,y_1)}{0.3}, \frac{(x_1,y_2)}{0.7}, \frac{(x_1,y_3)}{0.4}, \frac{(x_2,y_1)}{0.1}, \frac{(x_2,y_2)}{0.5}, \frac{(x_2,y_3)}{0.9}\right\}$$

## Solution: (3 Marks)

$$R=\begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.3 & 0.7 & 0.4 \\ 0.1 & 0.5 & 0.9 \end{bmatrix} \end{array}$$

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Lecture # 23

Example 3:- Generating T-Norms:-

$x \Delta y = xy$, $f(x) = e^{-\left(\frac{1-x}{x}\right)}$

So find

$x \Delta_f y = ?$

Sol:-

As we know that

$x \Delta_f y = f^{-1}(f(x) \Delta f(y) \vee a)$

first we find $f^{-1}(x)$

For this

let

$y = e^{-\left(\frac{1-x}{x}\right)}$

$\ln y = -\left(\frac{1-x}{x}\right)$

$x \ln y = x - 1$

$x \ln y - x = -1$

$(\ln y - 1)x = -1$

$x = \frac{-1}{\ln y - 1}$

$x = \frac{1}{1 - \ln y}$

So

$f^{-1}(x) = \frac{1}{1 - \ln x}$

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Now we find the value of a

For this

$$f(0) = e^{-\left(\frac{1-0}{0}\right)}$$

$$f(0) = e^{-\infty}$$

$$= \frac{1}{e^{\infty}}$$

$$= \frac{1}{\infty}$$

$$= 0$$

So

$$a = 0$$

Now

$$x \odot_p y = f^{-1}\left[(f(x) \Delta f(y)) \vee a\right]$$

$$= f^{-1}\left((f(x) f(y)) \vee 0\right)$$

$$= f^{-1}\left(f(x) f(y)\right)$$

$$= f^{-1}\left(e^{-\left(\frac{1-x}{x}\right)} e^{-\left(\frac{1-y}{y}\right)}\right)$$

$$= f^{-1}\left(e^{-\frac{(1-x)}{x} - \frac{1-y}{y}}\right)$$

$$= \frac{1}{1 - \ln\left(e^{-\left(\frac{1-x}{x}\right) - \left(\frac{1-y}{y}\right)}\right)}$$

$$= \frac{1}{1 + \frac{1-x}{x} + \frac{1-y}{y}}$$

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$$= \frac{1}{\frac{xy + y(1-x) + x(1-y)}{xy}}$$

$$= \frac{xy}{xy + y - xy + x - xy}$$

$$x \Delta_f y = \frac{xy}{x + y - xy}$$

$$x \Delta_f y = \Delta_4 \quad \underline{AR}$$

## Lecture # 24

Example 4:-

$$f(x) = e^{x-1}, \quad x \Delta y = xy$$

$$x \Delta_f y = ?$$

Sol:-

Let

$$y = e^{x-1}$$

$$\ln y = x - 1$$

$$x = 1 + \ln y$$

So

$$f^{-1}(x) = 1 + \ln x$$

Now

$$f(0) = e^{0-1}$$

$$= e^{-1}$$

$$= \frac{1}{e}$$

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$\Rightarrow \quad a = \frac{1}{e}$

So $x \omega_f y = f^{-1}[(f(x) \Delta f(y)) \vee a]$

$= f^{-1}[(f(x) f(y)) \vee a]$

$= f^{-1}[(f(x) f(y)) \vee \frac{1}{e}]$

$= f^{-1}[(e^{x-1} \cdot e^{y-1}) \vee \frac{1}{e}]$

$= f^{-1}[e^{x-1+y-1} \vee \frac{1}{e}]$

$= f^{-1}[e^{x+y-2} \vee \frac{1}{e}]$

$= 1 + \ln(e^{x+y-2} \vee e^{-1})$

$= 1 + \ln(e^{x+y-2}) \vee \ln e^{-1}$

$= 1 + [(x+y-2) \vee (-1)]$

$= (1+x+y-2) \vee (1-1)$

$= (x+y-1 \vee 0)$

$x \omega_f y = \Delta_1$

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Example: 5 :-

$$f(x) = \frac{x}{2-x}, \quad x \oplus y = xy$$

$x \oplus f y = ?$

Sol:-

Let $y = \frac{x}{2-x}$

$$y(2-x) = x$$

$$2y - xy = x$$

$$2y = x + xy$$

$$2y = (1+y)x$$

$$\Rightarrow x = \frac{2y}{1+y}$$

So

$$f^{-1}(x) = \frac{2x}{1+x}$$

Now

$$f(0) = \frac{0}{2-0}$$

$$f(0) = 0$$

So,

$$a = 0$$

$$\Rightarrow x \oplus f y = f^{-1}[(f(x) \oplus f(y)) \vee a]$$

$$= f^{-1}[(f(x) f(y)) \vee a]$$

$$= f^{-1}\left[\frac{x}{2-x} \cdot \frac{y}{2-y} \vee 0\right]$$

$$= f^{-1}\left(\frac{xy}{(2-x)(2-y)}\right)$$

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$$= f^{-1}\left(\frac{xy}{4-2y-2x+xy}\right)$$

$$= \frac{2\left(\frac{xy}{4-2y-2x+xy}\right)}{1+\left(\frac{xy}{4-2y-2x+xy}\right)}$$

$$= \frac{2xy}{4-2y-2x+xy} \times \frac{4-2y-2x+xy}{4-2y-2x+xy+xy}$$

$$= \frac{2xy}{4-2y-2x+2xy}$$

$$= \frac{2xy}{2(2-y-x+xy)}$$

$$= \frac{xy}{2-(x+y-xy)}$$

$x \oplus y = \Delta_2$

# Lecture # 25

**Nilpotent T-norm:-**

$\Delta$ is nilpotent if for $b \neq 1$

$b^{(n)} = 0$, for some $^{+ve}$ integer $n$

where $n$ depends on $b$.

**strict T-norm:-**

$\Delta$ is strict if for $b \neq 0$

$b^{(n)} > 0$, for some +ve integer $n$.

**Examples:-**

(1)

Prove that $\Delta_1$ is nilpotent t-norm.

Sol:-

As we know that

$x \Delta_1 y = \max(x + y - 1, 0)$

let $x \neq 1$

$x \Delta_1 x = \max(x + x - 1, 0)$

$= \max(2x - 1, 0)$

if $x \leq \frac{1}{2}$

$\Rightarrow x \Delta_1 x = 0$

if $1/2 < x < 1$

$\Rightarrow$

$x > 2x - 1$

$x \Delta_1 x \Delta_1 x = \max(\max(2x - 1, 0 + x - 1, 0)$

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$= \max(2x-1+x-1, 0)$

$= \max(3x-2, 0)$

So $x^{[2]} = x \Delta_1 x \Delta_1 x = \max(3x-2, 0)$

$\Rightarrow 3x-2 < 2x-1 < x$

$\Rightarrow 4x-3 < 3x-2 < 2x-1 < x$

Similarly

$nx-(n-1) < (n-1)x-(n-2) < \dots < x$

$\Rightarrow x^{[n]} = \max(nx-(n-1), 0)$

So we get +ve $n$ such that $nx-(n-1)$ becomes -ve.

$\Rightarrow x^{[n]} = 0$

$\Rightarrow \Delta_1$ is nilpotent T-norm.

(2)

Proove that $\Delta_0$ is nilpotent T-norm.

Sol:-

As

$$x \Delta_0 y = \begin{cases} \min(x,y), & \text{if } \max(x,y)=1 \\ 0 & \text{otherwise} \end{cases}$$

Let

$\max(x,y) = 1$

$\Rightarrow x \neq 1$

$\Rightarrow \max(x,x) \neq 1$

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$\Rightarrow \quad x \Delta_0 x = 0$

$\Rightarrow \quad x^{[2]} = 0$

$\Rightarrow \quad \Delta_0$ is nilpotent t-norm.

Zero divisor:-

$a \in (0,1)$ is called zero divisor of t-norm $\Delta$, if there exist $b \in (0,1)$ such that $a \Delta b = 0$

Theorem:-

If $\Delta$ is strictly monotone then it has no zero divisor.

Proof:-

Let $\Delta$ is strictly monotone we have to show that $\Delta$ has no zero divisor.

If $x > 0$ and $y < z$

then

$x \Delta y < x \Delta z$

let $a \in (0,1)$ is zero divisor of $\Delta$.

If $\exists \; b \in (0,1)$ such that

$a \Delta b = 0$

$\Rightarrow \quad a \Delta b = 0 \;, \quad a, b \in (0,1)$

Take

$\frac{b}{2} < b \quad ; \quad a \leq a$

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By 4th property of t-norm

$$a \triangle \frac{b}{2} \leq a \triangle b = 0$$

So

$$a \triangle \frac{b}{2} \leq 0$$

$$\Rightarrow \quad a \triangle \frac{b}{2} = 0 \quad \because \; a \triangle \frac{b}{2} \neq 0$$

contradiction to strictly monotone.

So

$\triangle$ has no zero divisor.

## Lecture # 26

**Strictly Monotone T-norm:-**

T-norm $\triangle$ is called strictly monotone t-norm if

$$x \triangle y < x \triangle z$$

whenever

$$x > 0 \quad \text{and} \quad y < z$$

where $x, y$ and $z$ are fuzzy no.

**Cancellation Law in T-norm:-**

**1st definition:-**

If $x \triangle y = x \triangle z$

$\Rightarrow \quad x = 0 \quad$ or $\quad y = z$

**2nd definition:-**

If $x \triangle y = x \triangle z$

and $x = 0 \Rightarrow y = z$

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Theorem:-

$\Delta$ is strictly monotone iff it satisfies the cancellation law.

proof:-

Let $\Delta$ is strictly monotone and we have to show that cancellation law holds.

Take $x \Delta y = x \Delta z \rightarrow (1)$

Suppose on contrary

$x \neq 0$ and $y \neq z$

$\Rightarrow x > 0$

Let $y < z$

So

$x \Delta y < x \Delta z \rightarrow (2)$

By (1) and (2)

$x \Delta y \leq x \Delta z$

which is contradiction to the cancellation law.

$\Rightarrow$ if $\Delta$ if strictly monotone then cancellation law holds.

Conversely

If it($\Delta$) satisfies cancellation law then it($\Delta$) is strictly monotone we have to show that

If $x \Delta y < x \Delta z \rightarrow (1)$

whenever $x > 0$ and $y < z$

Suppose on contrary

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Let $\Delta$ is not strictly monotone.

$\Rightarrow x \odot y \geq x \Delta z$

whenever

$x > 0$ and $y < z$

Take

$x \leq x$ and $y < z$

By 4th axiom of t-norm

$\Rightarrow x \odot y \leq x \Delta z \rightarrow (2)$

By (1) and (2)

$x \Delta y = x \Delta z$

which is contradiction to cancellation law.

So

$x \odot y < x \Delta z$

whenever

$x > 0$ and $y < z$

So $\Delta$ is strictly monotone.

which complete the theorem.

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Idempotent Element:-

Let $a \in (0,1)$ is idempotent element if $a \odot a = a$.

e.g.

$0 \odot 0 = 0$ 0 is idempotent

$1 \odot 1 = 1$ 1 is idempotent.

⇒ 0 and 1 are trivial idempotent element.

Theorem:-

If T is strictly monotone then it has only trivial idempotent element.

proof:-

Let T has non-trivial idempotent element $x$ and $y$.

Let $x, y \in (0,1)$

Take $x \odot y < x \odot z$

whenever $x > 0$, $y < z$

take $x \leq x$ and $x < 1$

$\Rightarrow x \odot x \leq x \odot 1$

$\Rightarrow x \odot x \leq x$

which is contradiction.

So T is trivial idempotent element.

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# Lecture # 27

## Binary Fuzzy Relations:-

$$R : U \times U \rightarrow [0,1]$$

### 1) Reflexive Relation:-

A fuzzy binary relation R is Reflexive if

$$R(x,x) = 1$$

### 2) Symmetric Relation:-

A fuzzy binary Relation R is Symmetric if

$$R(x,y) = 1 \Rightarrow R(y,x) = 1$$

### 3) Transitive Relation:-

A fuzzy binary relation R is transitive if

$$R(x,y) = 1, \quad R(y,z) = 1 \Rightarrow R(x,z) = 1$$

### Equivalence Relation:-

R is equivalence relation if R is reflexive, symmetric and Transitive.

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Anti-symmetric Relation:-

R is anti-symmetric binary relation if

$R(x,y) = R(y,x) = 1$

$\Rightarrow x = y$

In general

Reflexive

R is reflexive, if

$R(x,x) = 1$

Symmetric

R is symmetric, if

$R(u,v) = R(v,u)$

Transitive

R is transitive, if

$R(u,w) \geq R(u,v) \wedge R(v,w)$

Equivalence

R is equivalence if above three Properties hold.

anti-symmetric

R is anti-symmetric if

$R(u,v) > 0$ and $R(v,u) > 0$

$\Rightarrow u = v$

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α - cut in Relation :-

$$R_\alpha = R^{-1}([\alpha, 1]) = [(U,V) : R(U,V) \geq \alpha]$$

e.g.

$\alpha = 0.5$

$$R = [((x_1, y_1), 0.3), ((x_1, y_2), 0.7), ((x_2, y_1), 0.9)]$$

So

$$R_\alpha = R_{0.5} = [(x_1, y_2), 0.7), ((x_2, y_1), 0.9)]$$

Theorem :-

If R is a fuzzy relation on a set U, then R is a fuzzy equivalence relation iff each α-cut, $R_\alpha$ is an equivalence relation on U.

Proof :-

Let $R_\alpha$ be a fuzzy equivalence relation. Then

1) $R(U,U) = 1 \geq \alpha$

$\Rightarrow (U,U) \in R_\alpha$

So $R_\alpha$ is a reflexive relation

2)

If $R(U,V) = 1 = R(V,U)$

If $(U,V) \in R_\alpha$

$\Rightarrow R(U,V) = 1 \geq \alpha$

Also $R(V,U) = 1 \geq \alpha$

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$\Rightarrow (v,u) \in R_\alpha$

$\Rightarrow R_\alpha$ is symmetric.

3)

If $(v,u), (v,w) \in R_\alpha$

$\Rightarrow R(u,v) = 1 \geq \alpha$

and

$R(v,w) = 1 \geq \alpha$

$\Rightarrow R(u,w) \geq R(u,v) \wedge R(v,w) = 1 \geq \alpha$

$\Rightarrow R(u,w) \in R_\alpha$

So

$R_\alpha$ is transitive.

since $R_\alpha$ is reflexive, symmetric and transitive.

So

$R_\alpha$ is equivalence relation.

Conversely:-

Let $R_\alpha$ is an equivalence relation. we have to show that R is equivalence relation.

1)

$\Rightarrow (u,u) \in R_\alpha$

$\Rightarrow R(u,u) \geq \alpha$ (Put $\alpha = 1$)

$\Rightarrow R(u,u) \geq 1$

$\Rightarrow R(u,u) = 1$

So R is reflexive relation.

2)

$(u,v), (v,u) \in R_\alpha$

$\Rightarrow R(u,v) \geq \alpha$

and

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$R(V,U) \geq \alpha \quad \forall \ \alpha \in (0,1]$

take $\alpha = 1$

$\Rightarrow R(U,V) \geq 1$

and

$R(V,U) \geq 1$

$\Rightarrow R(U,V) = 1$ and $R(V,U) = 1$

$\Rightarrow R(U,V) = 1 = R(V,U)$

So R is Symmetric Relation

3)

If $(U,V), (V,W) \in R_\alpha$

$\Rightarrow (U,W) \in R_\alpha$

$\Rightarrow R(U,V) \geq \alpha$ and $R(V,W) \geq \alpha$

$\Rightarrow R(U,W) \geq \alpha$

$\Rightarrow R(U,V) \wedge R(V,W) \geq \alpha$

Take $\alpha = 1$

$R(U,V) \wedge R(V,W) \geq 1$

$\Rightarrow R(U,V) \wedge R(V,W) = 1$

and

$R(U,W) = 1$

$R(U,W) \geq R(U,V) \wedge R(V,W)$

$\Rightarrow$ R is transitive relation

so

R is equivalence Relation.

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# Lecture # 28

## Composition of Relations:-

Let R and S be fuzzy relations on $U \times V$ and $V \times W$ respectively.

Let $\Delta$ be a t-norm. The composition $R \circ S$ of R and S with respect to $\Delta$ is the fuzzy relation on $U \times W$ with membership function

$$(R \circ S)(u,w) = \bigvee_{v \in V} \{R(u,v) \Delta S(v,w)\}$$

$x \Delta y = x \wedge y \Rightarrow$ max-min composition

$x \Delta y = xy \Rightarrow$ max-product composition.

Example:-

Find the max-min and max-product composition of R and S.

Sol:-

$$R = \begin{bmatrix} 0.1 & 0.0 & 0.6 \\ 0.2 & 0.3 & 1 \\ 0.0 & 0.4 & 0.5 \end{bmatrix}, \quad S = \begin{bmatrix} 0.2 & 0.5 & 0 \\ 0.9 & 1 & 0.3 \\ 1 & 0.5 & 0.6 \end{bmatrix}$$

max-min composition.

$$R \circ S = \begin{bmatrix} 0.6 & 0.5 & 0.4 \\ 1 & 0.5 & 0.4 \\ 0.5 & 0.5 & 0.4 \end{bmatrix}$$

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## max-Product composition

$$R \circ S = \begin{pmatrix} 0.1 & 0 & 0.6 \\ 0.2 & 0.3 & 1 \\ 0 & 0.4 & 0.5 \end{pmatrix} \begin{pmatrix} 0.2 & 0.5 & 0 \\ 0.9 & 1 & 0.3 \\ 1 & 0.5 & 0.4 \end{pmatrix}$$

$$= \max \begin{pmatrix} (0.02, 0, 0.6) & (0.05, 0, 0.3) & (0, 0, 0.24) \\ (0.04, 0.27, 1) & (0.1, 0.3, 0.5) & (0, 0.09, 0.4) \\ (0, 0.36, 0.5) & (0, 0.4, 0.25) & (0, 0.12, 0.2) \end{pmatrix}$$

$$R \circ S = \begin{pmatrix} 0.6 & 0.3 & 0.24 \\ 1 & 0.5 & 0.4 \\ 0.5 & 0.4 & 0.2 \end{pmatrix}$$

Example:-

$R \circ R \leq R \iff R$ is transitive.

$$\text{Max}(\text{Min}(R(u,v), R(v,w)) \leq R(u,w)$$

Example:-

$A = [x_1, x_2]$, $B = (y_1, y_2, y_3)$

$A \times B = \{(x_1, y_1), (x_1, y_2), (x_1, y_3), (x_2, y_1), (x_2, y_2), (x_2, y_3)\}$

$= \{((x_1, y_1), 0.2), ((x_1, y_2), 0.3), ((x_1, y_3), 0), ((x_2, y_1), 1), ((x_2, y_2), 0.5), ((x_2, y_3), 0.7)\}$.

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| | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | 0.2 | 0.3 | 0.0 |
| $x_2$ | 1.0 | 0.5 | 0.7 |

$\tilde{R} = A \times B \rightarrow [0,1]$

complete Relation of AxB

| | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | 1 | 1 | 1 |
| $x_2$ | 1 | 1 | 1 |

Null relation of AxB

| | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|
| $x_1$ | 0 | 0 | 0 |
| $x_2$ | 0 | 0 | 0 |

Union:-
Just take a max of the element of R and S we get R∪S.

Intersection:-
Just take a min of the element of R and S we get R∩S.

complement:-
For com plement Just minus 1 from all membership value.

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α-cut:-

$$\tilde{S} = \begin{pmatrix} 0 & 0.7 & 0.1 & 0.8 \\ 0.9 & 0.2 & 0.7 & 0.1 \\ 0 & 0.3 & 0.9 & 1 \end{pmatrix}$$

α-cut of S.

Let $\alpha = 0.4$.

سیٹ اس میں جو ویلیو 0.4 سے برابر یا بڑی ہوگی اوھر 1 لکھ دیں گے اور جو کم ہوگی 0.4 سے اوھر 0 لکھ دیں گے

$$[S]^{0.4} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

## Lecture # 29

Fuzzy Implication:-

A fuzzy implication is a map from.

$$\Rightarrow : [0,1]^2 \to [0,1]$$

Satisfying

| $\Rightarrow$ | 1 | 0 |
|---|---|---|
| 1 | 1 | 0 |
| 0 | 1 | 1 |

$1 \Rightarrow 1 = 1$ $\quad$ $0 \Rightarrow 0 = 1$

$1 \Rightarrow 0 = 0$

$0 \Rightarrow 1 = 1$

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R-implication:-

An R-implication is a map $\Rightarrow : [0,1]^2 \to [0,1]$ such that

$$(x \Rightarrow y) = \max\{z \in [0,1] : x \triangle z \le y\}$$

$$= \vee\{z \in [0,1] : x \triangle z \le y\}.$$

$$(1 \Rightarrow 1) = \max\{z \in [0,1] : 1 \triangle z \le 1\}$$

$$= \max\{z \in [0,1] \quad z \le 1\}.$$

$$= \max[z \in [0,1]]$$

$$(1 \Rightarrow 1) = 1$$

$$(1 \Rightarrow 0) = \max\{z \in [0,1] : 1 \triangle z \le 0\}$$

$$= \max\{z \in [0,1] : z \le 0\}.$$

$$= \max[0] = 0$$

$$1 \Rightarrow 0 = 0$$

$$(0 \Rightarrow 0) = \max\{z \in [0,1] : 0 \triangle z \le 0\}$$

$$= \max\{z \in [0,1]\}$$

$$(0 \Rightarrow 0) = 1$$

$$0 \Rightarrow 1 = \max\{z \in [0,1] : 0 \triangle z \le 1\}$$

$$0 \Rightarrow 1 = 1$$

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As

$$x \Rightarrow y = \max\{z \in [0,1] : z \circ x \le y\}$$

if $x \le y$ : $z \le 1$

$$\Rightarrow x \circ z \le y \circ 1 = y$$

$$\Rightarrow x \circ z \le y$$

Example:-1

$$x \circ y = \min(x, y)$$

$$= x \wedge y$$

$$(x \Rightarrow y) = \max\left[z \in [0,1],\ \min(x, z) \le y\right]$$

$$= \begin{cases} 1 & , \quad x \le y \\ y & , \quad x > y \end{cases}$$

Example 2:-

$$x \circ y = xy$$

$$x \Rightarrow y = \max\{z \in [0,1] : xz \le y\}$$

$$= \begin{cases} 1 & x \le y \\ y/x & x > y \end{cases}$$

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Example 3:-

$$x \Delta y = \max(0, x+y-1)$$

$$x \Rightarrow y = \max\{z \in [0,1] : x \Delta z \leq y\}$$

$$= \max\{z \in [0,1] : \max(0, x+z-1) \leq y\}$$

$$= \max\{z \in [0,1] : x+z-1 \leq y\}$$

$$= \max\{z \in [0,1] : z \leq y-x+1\}$$

$$(x \Rightarrow y) = \begin{cases} 1 & ; \ x \leq y \\ y-x+1 & ; \ x > y \end{cases}$$

Theorem:-

Let $\Delta$ be a archimedian T-norm and $f$ is generater of t-norm then

$$x \Rightarrow y = f^{-1}\left(\frac{f(y)}{f(x)} \wedge 1\right)$$

Prove:-

$$x \Delta y = f^{-1}(f(x) f(y) \vee f(0))$$

Also

$$x \Rightarrow y = \max\{z \in [0,1] : x \Delta z \leq y\}$$

$$= \max\{z \in [0,1] : f^{-1}(f(x) f(z) \vee f(0)) \leq y\}$$

$$f : [0,1] \to [a,1]$$

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$\Rightarrow \quad 0 \leq a < 1$

$f^{-1}( f(z) f(x) \vee f(0) ) \leq y$

$f(z) f(x) \vee f(0) \leq f(y)$

$f(z) f(x) \vee a \leq f(y)$

$f(z) f(x) \leq f(y)$

$f(z) \leq \frac{f(y)}{f(x)}$

$z \leq f^{-1}\left(\frac{f(y)}{f(x)} \wedge 1\right)$

So

$x \Rightarrow y = f^{-1}\left(\frac{f(y)}{f(x)} \wedge 1\right)$

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# Lecture # 30

## ∇- implication:-

A ∇- implication is a map $\Rightarrow [0,1]^2 \to [0,1]$ of the form

$$x \Rightarrow y = \eta(x) \nabla y$$

here

$$\eta(x) = 1 - x$$

$\nabla$ is t-norm.

### Example 1:-

$$x \nabla y = \max(x, y)$$
$$= V(x, y)$$
$$= x \vee y$$

$$x \Rightarrow y = \eta(x) \nabla y$$
$$= (1 - x) \nabla y$$
$$= \max(1 - x, y)$$

### Example 2:-

$$x \nabla y = x + y - xy$$

$$x \Rightarrow y = \eta(x) \nabla y$$
$$= (1 - x) \nabla y$$
$$= 1 - x + y - (1 - x) y$$
$$= 1 - x + y - y + xy$$

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$= 1 - x + xy$

Example 3:-

$x \nabla y = 1 \wedge (x + y)$

$x \Rightarrow y = \eta(x) \nabla y$

$= (1 - x) \nabla y$

$= 1 \wedge (1 - x + y)$

$= \min(1, 1 - x + y)$

for table:-

$0 \Rightarrow 0 = \eta(0) \nabla (0)$

$= (1 - x) \nabla (0)$

$= (1 - 0) \nabla 0 = 1 \nabla 0$

$= 1$

$1 \Rightarrow 0 = \eta(x) \nabla y$

$= (1 - 1) \nabla 0$

$= 0 \nabla 0$

$= 0$

Example 4:-

$g(x) = e^{-x^2}$, $\eta(x) = \sqrt{1 - x^2}$

Then

$$x \Rightarrow y = \begin{cases} 1 & , x \leq y \\ \sqrt{1 - x^2 + y^2} & , x > y \end{cases}$$

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Sol:-

$$x \Rightarrow y = x(x) \nabla y$$

$$= g^{-1}\left[g(x)(x)\, g(y) \vee g(y)\right]$$

$$= g^{-1}\left[g(\sqrt{1-x^2})\; e^{-y^2} \vee e^{-1}\right]$$

$$x \Rightarrow y = g^{-1}\left(e^{-(1-x^2)} \cdot e^{-y^2} \vee e^{-1}\right)$$

$$= g^{-1}\left(e^{-1+x^2-y^2} \vee e^{-1}\right)$$

since $y = e^{-x^2}$

$$\ln y = -x^2$$

$$x^2 = -\ln y$$

$$x^2 = \ln y^{-1}$$

$$x = \sqrt{\ln y^{-1}}$$

so

$$g^{-1}(x) = \sqrt{\ln x^{-1}}$$

So

$$x \Rightarrow y = \ln\left(e^{(-1+x^2-y^2)^{-1}}\right)^{1/2} \vee \left(\ln e^{-1}\right)^{-1/2}$$

$$= \ln\left(e^{(-1+x^2-y^2)^{-1}}\right)^{1/2} \vee \left((\ln e^{-1})^{-1}\right)^{1/2}$$

$$= \sqrt{(-1+x^2-y^2)^{-1}} \vee \sqrt{1}$$

$$= \sqrt{1-x^2+y^2} \vee 1$$

$$= \begin{cases} 1 & x \le y \\ \sqrt{1-x^2+y^2} & x > y \end{cases}$$

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# Lecture # 31

Distance b/w two points:-

let $A(x_1, y_1)$ and $B(x_2, y_2)$ then the distance D b/w two point is

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

e.g $A(-2, 1)$ and $B(2, 4)$

$$D = \sqrt{(4)^2 + (3)^2} = 5$$

Distance b/w two fuzzy sets:-

let $X = \{x_1, x_2, \ldots, x_n\}$ be a discourse set, the fuzzy sets A on X is represented in terms of a function $m : X \to [0,1]$ such as

$$A = \{ \langle x_i, m_A(x_i) \rangle \mid x_i \in X \}$$

where $m_A(x_i)$ is called a membership function and always bounded in the closed interval $[0,1]$.

Definition:-

let A and B be two fuzzy sets on $X = \{x_1, x_2, \ldots, x_n\}$ then the most widely used distance for Fuzzy sets are

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1) Euclidean distance $d_e(A,B)$

$$d_e(A,B) = \sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))^2}$$

2) Normalized Euclidean distance $d_e^n(A,B)$

$$d_e^n(A,B) = \frac{1}{n} \sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))^2}$$

3) Hamming distance

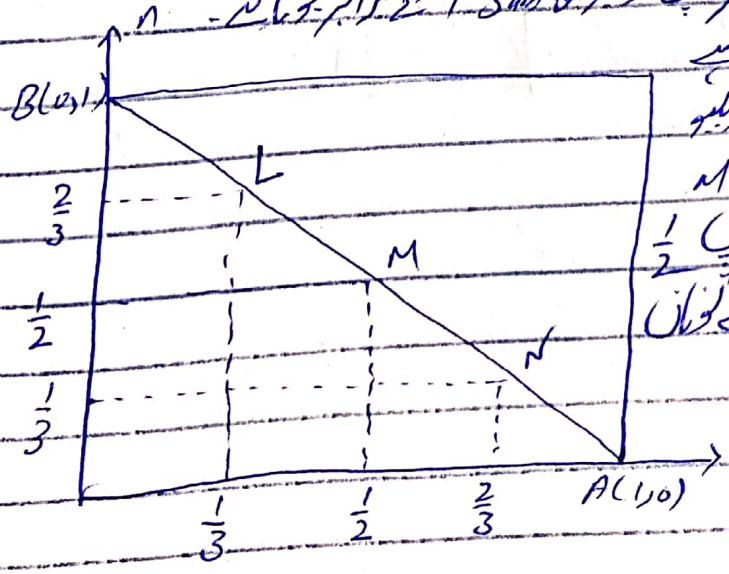
$$d_h(A,B) = \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

4) Normalized Hamming distance $d^n(A,B)$

$$d^n(A,B) = \frac{1}{n} \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

Example:-

Suppose that $A = (1,1)$, $B = (1,0)$, $L = (1, \frac{1}{3})$, $M = (1, \frac{1}{2})$, $N = (1, \frac{2}{3})$ are fuzzy sets on $X = \{1\}$, then the graphical representation is




ممبرشپ اور ان ممبرشپ ویلیو کا sum 1 کے برابر ہوتا ہے۔
L کی ممبرشپ ویلیو $\frac{1}{3}$ ہے
اس لیے ان ممبرشپ ویلیو $\frac{2}{3}$ ہوگی۔ اسی طرح M کی $\frac{1}{2}$ ہے تو ان ممبرشپ $\frac{1}{2}$ ہوگی۔ N کی $\frac{2}{3}$ ہے تو ان ممبرشپ $\frac{1}{3}$ ہوگی۔

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1) Hamming distance :-

- B/w L and M

$$d(L,M) = \left|\frac{1}{3} - \frac{1}{2}\right| = \left|\frac{2-3}{6}\right| = \left|\frac{-1}{6}\right| = \frac{1}{6}$$

- B/w M and N

$$d(M,N) = \left|\frac{1}{2} - \frac{2}{3}\right| = \left|\frac{3-4}{6}\right| = \left|\frac{-1}{6}\right| = \frac{1}{6}$$

2) Normalized Hamming distance :-

B/w M and N

$$d^n(A,B) = \frac{1}{1} \cdot \left|\frac{1}{2} - \frac{2}{3}\right| = \frac{1}{1} * \left|\frac{-1}{6}\right| = \frac{1}{6}$$

3) Euclidean distance :-

B/w M and N

$$de(M,N) = \sqrt{\left(\frac{1}{2} - \frac{2}{3}\right)^2} = \sqrt{\left(\frac{-1}{6}\right)^2} = \frac{1}{6}$$

4) Normalized euclidean distance :-

B/w M and N

$$de(M,N) = \frac{1}{1} \cdot \sqrt{\left(\frac{1}{2} - \frac{3}{2}\right)^2} = \frac{1}{6}$$

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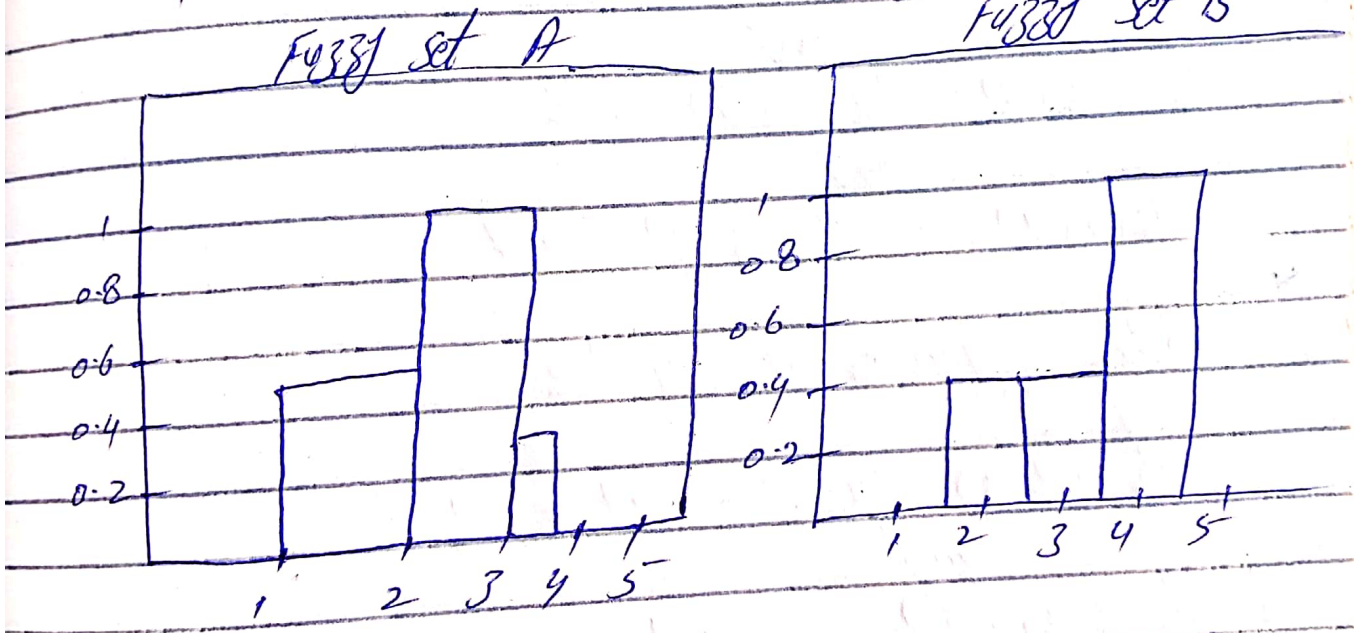
# Lecture # 32

Example 2:-

Find and plot the Hamming distance $d_h(A,B)$ b/w two fuzzy sets A and B on the discourse set $X = \{1, 2, 3, 4\}$ such that-

$A = \{(1, 0.5), (2, 1), (3, 0.3)\}$ and

$B = \{(2, 0.4), (3, 0.4), (4, 1)\}$




First we find the difference b/w generating values, that is

$|1-2| = 1$, $|1-3| = 2$, $|1-4| = 3$, $|2-2| = 0$

$|2-3| = 1$, $|2-4| = 2$, $|3-2| = 1$, $|3-3| = 6$

and $|3-4| = 1$

we get generating values

(0, 1, 2, 3)

Now,

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Hamming distance

(1) with generating value 0 is

$d_h(2,2) = |1 - 0.4| = 0.6$

and

$d_h(3,3) = |0.3 - 0.4| = 0.1$

* with generating value "1"

$d_h(1,2) = |0.5 - 0.4| = 0.1$

and

$d_h(2,3) = |1 - 0.4| = 0.6$

and

$d_h(3,4) = |0.3 - 1| = 0.7$

* with generating value 2

$d_h(1,3) = |0.5 - 0.4| = 0.1$

and

$d_h(2,0.5) = 0.1$

* Hamming distance with generating value 3

$d_h(1,4) = |0.5 - 1| = 0.5$

So

$D = 0.6 + 0.1 + 0.1 + 0.6 + 0.7 + 0.1 + 0.1 + 0.5$

$= 2.7$

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2) Euclidean distance

* generating value '0'

$$de(2,2) = \sqrt{|1-0.4|^2 + |0.3-0.4|^2}$$

$$= \sqrt{|0.6|^2 + |0.1|^2}$$

$$= 0.608$$

* generating value "1"

$$de(1,2) = \sqrt{|0.5-0.4|^2 + |1-0.4|^2 + |0.3-1|^2}$$

$$= \sqrt{|0.1|^2 + |0.6|^2 + |0.7|^2}$$

$$= 0.9273$$

* generating value "2"

$$de(1,3) = \sqrt{|0.5-0.4|^2} = 0.1$$

$$de(2,0.5) = 0.1$$

* generating value "3"

$$de(1,4) = \sqrt{|0.5-1|^2} = 0.5$$

So,

$$D = 0.608 + 0.9273 + 0.1 + 0.5$$

$$= 2.13$$

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# Lecture # 33

Distance b/w two fuzzy sets by using "max" and "min" operations:-

$$d(A,B) = \max\left[\min\left(m_A(x_i), m_B(x_i)\right)\right]$$

Example:-

$X = \{1, 2, 3, 4\}$

$A = \{(1, 0.5), (2, 1), (3, 0.3)\}$

$B = \{(2, 0.4), (3, 0.4), (4, 1)\}$.

First we find the generating values.

$|1-2| = 1$, $|1-3| = 2$, $|1-4| = 3$

$|2-2| = 0$, $|2-3| = 1$, $|2-4| = 2$

$|3-2| = 1$, $|3-3| = 0$, $|3-4| = 1$

Generating values is $(0, 1, 2, 3)$.

Now

| d | $x_B$ | $x_A$ | $m_B(x_i)$ | $m_A(x_i)$ | $\min(m_A(x_i), m_B(x_i))$ | $\max[\min(m_A(x_i), m_B(x_i))]$ |
|---|---|---|---|---|---|---|
| 0 | 2 | 2 | 0.4 | 1.0 | 0.4 | 0.4 |
| | 3 | 3 | 0.4 | 0.3 | 0.3 | |
| 1 | 2 | 1 | 0.4 | 0.5 | 0.4 | |
| | 3 | 2 | 0.4 | 1.0 | 0.4 | 0.4 |
| | 2 | 3 | 0.4 | 0.3 | 0.3 | |
| | 4 | 3 | 1.0 | 0.3 | 0.3 | |
| 2 | 3 | 1 | 0.4 | 0.5 | 0.4 | |
| | 4 | 2 | 1.0 | 1.0 | 1.0 | 1.0 |
| 3 | 4 | 1 | 1.0 | - | | |

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So

$$d(A,B) = 0.4 + 0.4 + 1$$
$$= 1.8$$

Similarly

If we want to find $d(B,D)$, then obvious

$$d(A,B) = d(B,A)$$

Some important facts:-

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe set, then $D(A,B)$ the distance measure b/w two Intuition Fuzzy sets A and B satisfy

1) $D_h(A,B) = 2d_h(A,B)$
2) $D_h^n(A,B) = 2d_h^n(A,B)$

Proof 1)

$$D_h(A,B) = 2d_h(A,B)$$

Let $A(\mu_A(x), \nu_A(x))$ and $B(\mu_B(x), \nu_B(x))$ be two IFSs then

$$D_h(A,B) = \sum_{i=1}^{n} |\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)|$$

$$= \sum_{i=1}^{n} |\mu_A(x) - \mu_B(x)| + |1 - \mu_A(x) - (1 - \mu_B(x))|$$

$$= \sum_{i=1}^{n} |\mu_A(x) - \mu_B(x)| + |-\mu_A(x) + \mu_B(x)|$$

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$$= \sum_{i=1}^{n} |\mu_A(x) - \mu_B(x)| + |\mu_A(x) - \mu_B(x)|$$

$$= 2 \sum_{i=1}^{n} |\mu_A(x) - \mu_B(x)|$$

$$D_h(A,B) = 2 d_h(A,B)$$

2) Similarly

$$D_h^n(A,B) = \frac{1}{n} \sum_{i=1}^{n} \left( |\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| \right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left( |\mu_A(x) - \mu_B(x)| + |1 - \mu_A(x) - (1 - \mu_B(x))| \right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left( |\mu_A(x) - \mu_B(x)| + |-\mu_A(x) + \mu_B(x)| \right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left( |\mu_A(x) - \mu_B(x)| + |\mu_A(x) - \mu_B(x)| \right)$$

$$= 2 \frac{1}{n} \sum_{i=1}^{n} |\mu_A(x) - \mu_B(x)|$$

$$D_h^n(A,B) = 2 d_h^n(A,B)$$

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# Lecture # 34

## Properties of Distance:-

**Theorem:-**

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe set, then $d(A,B)$, the distance b/w two Fuzzy sets A and B satisfy

(1) $0 \leq d(A,B) \leq 1$

(2) $d(A,B) = 0$ iff $A = B$

(3) $d(A,B) = d(B,A)$

(4) $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$

for any $A, B, C \in FS_c(X)$

# Lecture # 35

Same as Lecture # 34

# Lecture # 36

## Extensions of Fuzzy set:-

### Intuitionistic Fuzzy set (IFSs)

An IFSs A on X is given below

$$A = \{\langle x_i, (\mu_A(x), \nu_A(x)) \rangle \mid x_i \in X\}$$

where

$\mu_A(x): X \to [0,1]$ and

$\nu_A(x): X \to [0,1]$

satisfies the following conditions

$0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\forall x \in X$

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The number $\mu_A(x), \nu_A(x) \in (0,1)$ denote the degree of membership and non-membership of $x$ to $A$, respectively.

Some operations on IFSs:-
Let $A(\mu_A(x), \nu_A(x))$ and $B(\mu_B(x), \nu_B(x))$ be two IFSs then

1) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ $\forall x \in X$.

2) $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ $\forall x \in X$.

3) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle | x \in X\}$

4) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle | x \in X\}$.

5) $A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x), \nu_A(x) \times \nu_B(x) \rangle | x \in X\}$.

6) $A \otimes B = \{\langle x, \mu_A(x) \times \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \times \nu_B(x) \rangle | x \in X\}$.

7) $A - B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) \rangle | x \in X\}$

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8) $A^c = \{\langle x, (\nu_A(x), \mu_A(x)\rangle > x \in X\}$

Question:-
Let A and B be two IFSs in a non-empty set X, then

1) $A-B = A \cap B^c$
2) $A-B = B-A$ iff $A=B$
3) $A-B = B^c - A^c$

Sol:- Let $A(\mu_A(x), \nu_A(x))$ and $B=(\mu_B(x), \nu_B(x))$ be IFSs

1)
$$A-B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)\rangle / x \in X\}$$

$$B^c = \{\langle x, (\nu_B(x), \mu_B(x)) / x \in X\}$$

$$A \cap B^c = \{\langle x, (\mu_A(x), \nu_A(x)) \cap (\nu_B(x), \mu_B(x))\rangle / x \in X\}$$

$$= \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle / x \in X\}$$

$$A \cap B^c = A-B$$

2) Since
$$A-B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle / x \in X\}$$
$$B-A = \{\langle x, \min(\mu_B(x), \nu_A(x)), \max(\nu_B(x), \mu_A(x))\rangle / x \in X\}$$
So
$$A-B = B-A$$

3) Since
$$A-B = \{\langle x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle / x \in X\}$$
and
$$A^c = \{\langle x, (\nu_A(x), \mu_B(x))\rangle / x \in X\}$$

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$B^c = \{\langle x, (\nu_B(x), \mu_B(x))\rangle / x \in X\}$

now

$B^c - A^c = \{\langle x, (\nu_B(x), \mu_B(x)) - (\nu_A(x), \mu_A(x))\rangle / x \in X\}$

$= \{\langle x, \min(\nu_B(x), \mu_A(x)), \max(\mu_B(x), \nu_A(x))\rangle / x \in X\}$

$= A - B$

So

$A - B = B^c = A^c$

## Lecture # 37

Hamming and Euclidean distance b/w Two Intuitionistic fuzzy sets:-

Example:-

Let A and B be two IFSs in $X = \{1, 2, 3, 4, 5, 6, 7\}$ such that

$A = (0.7, 0.3)/1 + (0.2, 0.8)/2 + (0.6, 0.4)/4 + (0.5, 0.5)/5 + (1, 0)/6$

$B = (0.2, 0.8)/1 + (0.6, 0.4)/4 + (0.8, 0.2)/5 + (1, 0)/7$

Find Hamming distance b/w A and B.

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Solution:- Hamming distance:-

$$d_h(A,B) = \sum_{i=1} |m_A(x_i) - m_B(x_i)|$$

$$= \sum_{i=1}^{7} |m_A(x_i) - m_B(x_i)|$$

$$d_h(A,B) = |0.7-0.2| + |0.3-0.8| + |0.2-0| + |0.8-0| + |0.6-0.6| + |0.4-0.4| + |0.5-0.8| + |0.5-0.2| + |1-0| + |0-0| + |0-1| + |1-0|.$$

$$= 6$$

- Normalized Hamming distance is

$$d_h'(A,B) = \frac{1}{n} \sum_{i=1}^{7} |m_A(x_i) - m_B(x_i)|$$

$$= \frac{1}{7}(6)$$

$$= \frac{6}{7}$$

$$= 0.86.$$

Euclidean distance:-

$$d_e(A,B) = \sqrt{\sum_{i=1}^{n} (m_A(x_i) - m_B(x_i))^2}$$

So

$$d_e(A,B) = \sqrt{\sum_{i=1}^{7} (m_A(x_i) - m_B(x_i))^2}$$

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$$= \sqrt{(0.7-0.2)^2 + (0.3-0.8)^2 + (0.2-0)^2 + (0.8-0)^2 + (0.6-0.6)^2 + (0.4-0.4)^2 + (0.5-0.8)^2 + (0.5-0.2)^2 + (1-0)^2 + (0-1)^2 + (0-1)^2 + (1-0)^2}$$

$$= \sqrt{5.21} = 2.283$$

Normalized Euclidean distance

$$d_e^n(A,B) = \frac{1}{n} \sqrt{\sum_{i=1}^{n} (m_A(x_i) + m_B(x_i))^2}$$

$$= \frac{1}{7} (2.283)$$

$$= 0.326.$$

اس example میں A میں 6 پوائنٹ ہیں جو (1,0) کی ہے جب کہ B میں اسکے corresponding کوئی ویلیو نہیں تو ہم نے (0,1) لے لی۔
اسی طرح B میں 7 پوائنٹ ہیں جو (1,0) کی اور A میں کوئی نہ تھی تو ہم نے ہر n میں (0,1) لے لی۔

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# Lecture # 38

## Picture fuzzy set:-

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set, a picture fuzzy set $P$ on $X$ is defined as

$$P = \{\langle x, \alpha_P(x), \gamma_P(x), \beta_P(x)\rangle \mid x \in X\}$$

where

$\alpha_P(x), \beta_P(x), \gamma_P(x) \in [0,1]$ are called the acceptance membership, neutral and rejection membership degrees of $x \in X$ to the set $P$, respectively and $\alpha_P(x), \gamma_P(x)$ and $\beta_P(x)$ fulfil the condition

$$0 \leq \alpha_P(x) + \gamma_P(x) + \beta_P(x) \leq 1, \quad \forall x \in X.$$

Also

$$\eta_P(x) = 1 - \alpha_P(x) - \gamma_P(x) - \beta_P(x)$$

then $\eta_P(x)$ is said to be a degree of refusal membership of $x \in X$ in $P$.

For our convenience, the picture fuzzy sets over a fixed set $X$ is written as $PFS_S(X)$.

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Some operations on PFSs :-

Let A and B be two PFSs on X, then

1) $A \subseteq B$ iff $\alpha_A(x) \leq \alpha_B(x)$, $\gamma_A(x) \leq \gamma_B(x)$ and $\beta_A(x) \geq \beta_B(x)$, $\forall x \in X$

2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

3) $A \cup B = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \min(\beta_A(x), \beta_B(x)) \mid x \in X\}$

4) $A \cap B = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\gamma_A(x), \gamma_B(x)), \max(\beta_A(x), \beta_B(x)) \mid x \in X\}$.

5) $A^c = \{\langle x, \beta_A(x), \gamma_A(x), \alpha_A(x)\rangle \mid x \in X\}$.

Lecture # 39

Example:-

Let $P_1 = \{(x_1, (0.6, 0.2, 0.1)), (x_2, (0.4, 0.1, 0.4))\}$

$P_2 = \{(x_1, (0.7, 0.3, 0)), (x_2, (0.5, 0.2, 0.3))\}$

i) $P_1 \subseteq P_2$ - ?

we know that

$P_1 \subseteq P_2$ if and only if

$\alpha_{P_1} \leq \alpha_{P_2}$, $\gamma_{P_1} \leq \gamma_{P_2}$, $\beta_{P_1} \geq \beta_{P_2}$

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here

$\alpha_{P_1}(x) \leq \alpha_{P_2}(x)$ , $\gamma_{P_1}(x) \leq \gamma_{P_2}(x)$

and

$\beta_{P_1}(x) \geq \beta_{P_2}(x)$

so

$P_1 \subseteq P_2$

Now we see that

$P_2 \subseteq P_1 = ?$

here

$\alpha_{P_2} \nleq \alpha_{P_1}$ , $\gamma_{P_2} \nleq \alpha_{P_1}$

and

$\beta_{P_2} \neq \beta_{P_1}$

so

$P_2 \nsubseteq P_1$

2)

$(P_1 = P_2) = ?$

we know that

$A = B$

iff

$A \subseteq B$ and $B \subseteq A$

here

$P_1 \subseteq P_2$ but $P_2 \nsubseteq P_1$

so

$P_1 \neq P_2$.

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3)

$$P_1 \cup P_2 = \{ x, \max(\alpha_{P_1}(x), \alpha_{P_2}(x)), \min(\alpha_{P_1}(x), \alpha_{P_2}(x)) \\ \min(\beta_{P_1}(x), \beta_{P_2}(x)) \}$$

$$= \{ (x_1, (\max(0.6, 0.7), \min(0.2, 0.3), \min(0.1, 0.2)) \\ (x_2, (\max(0.4, 0.5), \min(0.1, 0.2), \min(0.4, 0.3)) \}$$

$$P_1 \cup P_2 = \{ (x_1, (0.7, 0.2, 0.1)), (x_2, (0.5, 0.1, 0.3)) \}$$

Similarly

4) $P_1 \cap P_2 = \{ x, \min(\alpha_{P_1}(x), \alpha_{P_2}(x)), \min(\alpha_{P_1}(x), \alpha_{P_2}(x)) \\ \max(\beta_{P_1}(x), \beta_{P_2}(x)) \}$

$$= \{ (x_1, (0.6, 0.2, 0.1)), (x_2, (0.4, 0.1, 0.4)) \}$$

5)

$$P_1^c = \{ (x_1, (0.1, 0.2, 0.6)), (x_2, (0.4, 0.1, 0.4)) \}$$

$$P_2^c = \{ (x_1, (0.1, 0.3, 0.7)), (x_2, (0.3, 0.2, 0.5)) \}$$

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# Lecture # 40

**Normalized Hamming distance for PFSs:-**

Let $P_1$ and $P_2$ be two PFSs such that

$$P_1 = \{(x_i, (\alpha_{P_1}(x_i), \gamma_{P_1}(x_i), \beta_{P_1}(x_i))\}$$

$$P_2 = \{(x_i, (\alpha_{P_2}(x_i), \gamma_{P_2}(x_i), \beta_{P_2}(x_i))\}$$

$$\Rightarrow P_1 = (\alpha_{P_1}(x_i), \gamma_{P_1}(x_i), \beta_{P_1}(x_i))$$

$$P_2 = (\alpha_{P_2}(x_i), \gamma_{P_2}(x_i), \beta_{P_2}(x_i))$$

As we know Normalized Hamming distance is

$$d_n(A,B) = \frac{1}{n} \sum_{i=1}^{n} |m_A(x_i) - m_B(x_i)|$$

so, For Normalized Hamming distance for PFSs

$$= \frac{1}{n} \sum_{i=1}^{n} \Big( |\alpha_{P_1}(x_i) - \alpha_{P_2}(x_i)| + |\gamma_{P_1}(x_i) - \gamma_{P_2}(x_i)| + |\beta_{P_1}(x_i) - \beta_{P_2}(x_i)| \Big)$$

**Example:-**

Let $P_1$ and $P_2$ be two PFSs defined on a $X = \{x_1, x_2, x_3, x_4\}$ such that

$$P_1 = \{(x_1, (0.56, 0.34, 0.1)), (x_2, (0.9, 0.07, 0.03)), (x_3, (0.4, 0.33, 0.29)), (x_4, (0.09, 0.79, 0.03))\}$$

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$P_2 = \{(x_1, (0.7, 0.1, 0.09)), (x_2, (0.1, 0.66, 0.2)),$
$(x_3, (0.06, 0.81, 0.12)), (x_4, (0.72, 0.14, 0.09))\}$

So, Find

$D_h^{\wedge}(P_1, P_2) = ?$

Sol:-

here

$n = 4$

So

$$D_h^{\wedge}(P_1, P_2) = \frac{1}{4}\{|0.56 - 0.7| + |0.34 - 0.1| +$$

$$|0.1 - 0.09| + |0.9 - 0.1) + |0.07 - 0.66| + |0.03 - 0.2|$$

$$+ |0.4 - 0.06| + |0.33 - 0.8| + |0.19 - 0.12| +$$

$$|0.09 - 0.72| + |0.79 - 0.14| + |0.03 - 0.09|$$

$$= \frac{1}{4}[0.14 + 0.24 + 0.01 + 0.8 + 0.59 + 0.17 +$$
$$0.34 + 0.48 + 0.07 + 0.63 + 0.65 + 0.06]$$

$$= \frac{1}{4}[4.18]$$

$$= 1.045$$

Extended form of Normalized Hamming distance for PFSs :-

Let $P_1 = (\alpha_{P_1}(x_i), \gamma_{P_1}(x_i), \beta_{P_1}(x_i))$ and $P_2 = (\alpha_{P_2}(x_i), \gamma_{P_2}(x_i), \beta_{P_2}(x_i))$ are two PFSs defined on $X = \{x_1, x_2, x_3 \dots, x_n\}$ then

extended form of Normalized Hamming distance for PFSs is

$$D_p(P_1, P_2) = \frac{1}{3n} \sum_{i=1}^{n} \left[ |\alpha_{P_1}(x_i) - \alpha_{P_2}(x_i)| + |\gamma_{P_1}(x_i) - \gamma_{P_2}(x_i)| + |\beta_{P_1}(x_i) - \beta_{P_2}(x_i)| + \max\left[ |\alpha_{P_1}(x_i) - \alpha_{P_2}(x_i)|, |\gamma_{P_1}(x_i) - \gamma_{P_2}(x_i)|, |\beta_{P_1}(x_i) - \beta_{P_2}(x_i)| \right] \right]$$

Example :-

Let $P_1$ and $P_2$ be two PFSs defined on a $X = \{x_1, x_2, x_3\}$ such that

$P_1 = \{(x_1, (0.8, 0.1, 0)), (x_2, (0.4, 0.2, 0.3)), (x_3, (0.5, 0.3, 0))\}$

$P_2 = \{(x_1, (0.3, 0.3, 0.2)), (x_2, (0.7, 0.1, 0.1)), (x_3, (0.4, 0.3, 0.2))\}$.

$D_p(P_1, P_2) = ?$

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Sol:-

here

$n = 3$

So,

$$D_p(P_1, P_2) = \frac{1}{3 \times 3} \left[ |0.8 - 0.3| + |0.1 - 0.3| + |0 - 0.2| + \max\{|0.8 - 0.3|, |0.1 - 0.3|, |0 - 0.2|\} + |0.4 - 0.7| + |0.2 - 0.1| + |0.3 - 0.1| + \max\{|0.4 - 0.7|, |0.2 - 0.1|, |0.3 - 0.1|\} + |0.5 - 0.4| + |0.3 - 0.3| + |0 - 0.2| + \max\{|0.5 - 0.4|, |0.3 - 0.3|, |0 - 0.2|\} \right)$$

$$= \frac{1}{9} \{0.5 + 0.2 + 0.2 + 0.5 + 0.3 + 0.1 + 0.2 + 0.3 + 0.1 + 0 + 0.2 + 0.2\}$$

$$= \frac{1}{9} [2.8]$$

$$= 0.311$$

# Lecture # 41

## Hesitant Fuzzy sets (HFSs)

Let X be a reference set, then we define a hesitant fuzzy set on X in terms of a function h that when applied to X returns a subset of [0,1].

A HFS H on X is written as

$$H = \{\langle x, h(x) \rangle | \forall x \in X\}$$

where h(x) is a set of some different values in [0,1] representing the possible membership degrees of the element $x \in X$ to H.

# Lecture # 42

Examples of HFS :-

$$X = \{x_1, x_2\}$$

$$H_1 = \{(x_1, \{0.1, 0.2\}), (x_2, \{0.5, 0.6, 0.8\})$$

$$H_2 = \{(x_1, \{0.8\}), (x_2, \{0.6, 0.7\})\}.$$

Some operations on HFS :-

For any $h, h_1$ and $h_2$ in H(X), some operations on them can be described as follows.

1) Lower bound :-

$$h^-(x) = \min h(x)$$

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2) upper bound:

$$h^+(x) = \max\{h(x)\}$$

3) complement

$$h^c(x) = \{1-\gamma \mid \gamma \in h(x)\}$$

4) Union

$$(h_1 \cup h_2)(x) = \{\theta \in h_1 \cup h_2 \mid \theta \geq \max(h_1^+(x), h_2^+(x))\}$$

5) Intersection

$$(h_1 \cap h_2)(x) = \{\gamma \in h_1 \cap h_2 \mid \gamma \geq \min(h_1^+(x), h_2^+(x))\}$$

6) $h^{\lambda}(x) = \{\gamma^{\lambda} \mid \gamma \in h(x)\}$

7) $\lambda h(x) = \{1-(1-\gamma)^{\lambda} \mid \gamma \in h(x)\}$

8) $(h_1 \oplus h_2)(x) = \{\gamma_1 + \gamma_2 - \gamma_1 \cdot \gamma_2 \mid \gamma_1 \in h_1(x), \gamma_2 \in h_2(x)\}$

9) $(h_1 \otimes h_2)(x) = \{\gamma_1 \cdot \gamma_2 \mid \gamma_1 \in h_1(x), \gamma_2 \in h_2(x)\}$

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# Lecture # 4.3

Example:-

Let $X = \{x_1, x_2, x_3\}$ be a universal set, $h_1$ and $h_2$ be two HFS on X such that

$h_1 = \{(x_1, \{0.3, 0.4\}), (x_2, \{0.6, 0.8\}), (x_3, \{0.3, 0.4, 0.5, 0.7\})\}$

$h_2 = \{(x_1, \{0.5, 0.6\}), (x_2, \{0.4, 0.5\}), (x_3, \{0.2, 0.3, 0.4, 0.6\})\}$.

Then find out following operations.

i) $h_1^-(x_3) = ?$

As we know that

$h_1^-(x_3) = \min\{0.3, 0.4, 0.5, 0.7\}$

$= 0.3$

ii) $h_2^+(x_2) = ?$

As we know that

$h_2^+(x_2) = \max\{0.4, 0.5\}$

$= 0.5$

iii) $h_1^c(x_2) = ?$

As we know that

$h^c(x) = \{1-\gamma \mid \gamma \in h(x)\}$

So,

$h_1^c(x_2) = \{1-0.6, 1-0.8\}$

$= \{0.4, 0.2\}$

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iv) $h_1^{\wedge}(x_2) = ?$

$\Rightarrow {}^{\wedge}\{0.6^{\wedge}, 0.8^{\wedge}\}$

if $r = 2$, then

$h^2(x_2) = \{0.6^2, 0.8^2\}$

$= \{0.36, 0.64\}$

v) $d(h_1(x_1)) = ?$

$\Rightarrow d(h_1(x_1)) = \{1-(1-0.3)^{\wedge}, 1-(1-0.4)^{\wedge}\}$

if $r = 2$, then

$= \{1-0.7^2, 1-0.6^2\}$

$= \{1-0.49, 1-0.36\}$

$= \{0.51, 0.64\}$

vi) $(h_1 \oplus h_2)(x_2) = ?$

Since

$(h_1 \oplus h_2)(x) = \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2 / \gamma_1 \in h_1, \gamma_2 \in h_2\}$

here

$\{0.6, 0.8\} \oplus \{0.4, 0.5\} = \{0.6+0.4-(0.6)(0.4), 0.6+0.5-(0.6)(0.5), 0.8+0.4-(0.8)(0.4), 0.8+0.5-(0.8)(0.5)\}$

$(h_1 \oplus h_2)x_2 = \{0.76, 0.8, 0.88, 0.9\}$.

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(vii) $(h_1 \otimes h_2)(x_1) = ?$

Since

$(h_1 \otimes h_2)\, x = \{\gamma_1 \gamma_2 \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$

So,

$\{0.3, 0.4\} \otimes \{0.5, 0.6\}$

$= \{0.3 \times 0.5, 0.3 \times 0.6, 0.4 \times 0.5, 0.4 \times 0.6\}$

$= \{0.15, 0.18, 0.2, 0.24\}$

(viii) $(h_1 \cup h_2)\, x_3 = ?$

Since

$(h_1 \cup h_2)\, x = \{\gamma \in h_1 \cup h_2 \mid \gamma \geq \max(h_1^-(x), h_2^-(x))\}$

So,

$(h_1 \cup h_2)\, x_3 = \{\gamma \in h_1 \cup h_2 \mid \gamma \geq \max(h_1^-(x_3), h_2^-(x_3))\}$

$\geq \max(0.3, 0.2) = \{0.3, 0.4, 0.5, 0.6, 0.7\}$

ix) $(h_1 \cap h_2)(x_3) = ?$

Since

$(h_1 \cap h_2)(x_3) = \{\gamma \in h_1 \cap h_2 \mid \gamma \geq \min(h_1^+(x_3), h_2^+(x_3))\}$

So,

$\geq \min\{0.7, 0.6\} = \{0.2, 0.3, 0.4, 0.5, 0.6\}$

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Lecture # 44

Theorem:- Suppose that $h_1$ and $h_2$ are two HFS then

$$l_{h_1 \oplus h_2} = l_{h_1} \times l_{h_2}$$

and

$$l_{h_1 \otimes h_2} = l_{h_1} \times l_{h_2}$$

Example:- Let $h_1 = (0.1, 0.2, 0.7)$ and $h_2 = (0.2, 0.4)$ be two HFS then

$$h_1 \oplus h_2 = \{0.1+0.2-0.1\times0.2,\ 0.1+0.4-0.1\times0.4,\ 0.2+0.2-0.2\times0.2,\ 0.2+0.4-0.2\times0.4,\ 0.7+0.2-0.7\times0.2,\ 0.7+0.4-0.7\times0.4\}$$

$$= \{0.28, 0.36, 0.46, 0.52, 0.76, 0.82\}$$

Similarly

$$h_1 \otimes h_2 = \{0.1\times0.2,\ 0.1\times0.4,\ 0.2\times0.2,\ 0.2\times0.4,\ 0.7\times0.2,\ 0.7\times0.4\}$$

$$= \{0.02, 0.04, 0.04, 0.08, 0.14, 0.28\}$$

So,

$$l_{h_1 \oplus h_2} = 6 = 3\times2 = l_{h_1} \times l_{h_2}$$

$$l_{h_1 \otimes h_2} = 6 = 3\times2 = l_{h_1} \times l_{h_2}$$

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Distance b/w two Hesitant Fuzzy set:-

- Euclidean:-

$$D_e(h_1, h_2) = \sqrt{\sum_{i=1}^{n} (h_{1i} - h_{2i})^2}$$

- Hamming

$$D_h(h_1, h_2) = \sum_{i=1}^{n} |h_{1i} - h_{2i}|$$

Example:-1

Let $h_1 = (0.7, 0.8, 0.9)$ and $h_2 = (0.5, 0.6, 0.8)$ be two HFS. Then evaluate (i) $D_h(h_1, h_2) = ?$ (ii) $D_e(h_1, h_2)$

Sol:-

i) $D_h(h_1, h_2) = |0.7 - 0.5| + |0.8 - 0.6| + |0.9 - 0.8|$

$= 0.2 + 0.2 + 0.1$

$= 0.5$

ii)

$D_e(h_1, h_2) = (0.7 - 0.5)^2 + (0.8 - 0.6)^2 + (0.9 - 0.8)$

$= (0.2)^2 + (0.2)^2 + (0.1)^2$

$= 0.04 + 0.04 + 0.01$

$= 0.09$

Example 2)

Let $h_1 = (0.3, 0.4)$ and $h_2 = (0.6, 0.7, 0.8)$ be two HFS. the evaluate normalized Hamming distance and Euclidean distance b/w $h_1$ and $h_2$.

Sol:-

$h_1 = (0.3, 0.3, 0.4)$

and

$h_2 = (0.6, 0.7, 0.8)$

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ii) $D_h^n(h_1, h_2) = \frac{1}{3}\{|0.3-0.6| + |0.3-0.7| + |0.4-0.8|\}$

$= \frac{1}{3}(0.3 + 0.4 + 0.4)$

$= \frac{1}{3}(1.1)$

$= 0.367$

ii) $D_e^n(h_1, h_2) = \sqrt{\frac{1}{3}\{(0.3-0.6)^2 + (0.3-0.7)^2 + (0.4-0.8)^2\}}$

$= \sqrt{\frac{1}{3}(0.09 + 0.16 + 0.16)}$

$= 0.13667$

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# Lecture # 45

**Similarity:-**

Two objects are said to be similar if both have same shape.

**Similarity measures:-(SM)**

It is a process through which we can evaluate most resemblance b/w two objects.

There are many kinds of SMs, like

i) SM based on distance.
ii) Cosine SM
iii) Exponential SM
iv) Jaccard and dice SM.
v) William and steel SM. etc.

**i) SM based on distance:-**

Let S be a SM b/w two Fuzzy set is calculated as;

Suppose that A and B be two FS then SM, S(A,B) is ~~written~~ based on distance $d_f(A,B)$ is written as

$$S(A,B) = 1 - d_f(A,B)$$

or $$= 1 - d_e(A,B)$$

or $$= 1 - d_e^2(A,B)$$

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$0 \leq S(A,B) \leq 1$

Theorem:-

The S.M, $S(A,B)$ for two Fuzzy set satisfied the following axioms.

i) $0 \leq S(A,B) \leq 1$

ii) $S(A,B) = 1$ iff $A = B$

iii) $S(A,B) = S(B,A)$ (commutative)



Remember in your project