

2.4. If the series $A + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$ converges uniformly to $f(x)$ in $(-L, L)$, show that for $n = 1, 2, 3, \dots$,

$$(a) \quad a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, \quad (b) \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, \quad (c) \quad A = \frac{a_0}{2}.$$

$$(a) \quad \text{Multiplying} \quad f(x) = A + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right) \quad (1)$$

by $\cos \frac{m\pi x}{L}$ and integrating from $-L$ to L , using Problem 2.3, we have

$$\begin{aligned} \int_{-L}^L f(x) \cos \frac{m\pi x}{L} dx &= A \int_{-L}^L \cos \frac{m\pi x}{L} dx \\ &\quad + \sum_{n=1}^{\infty} \left\{ a_n \int_{-L}^L \cos \frac{m\pi x}{L} \cos \frac{n\pi x}{L} dx + b_n \int_{-L}^L \cos \frac{m\pi x}{L} \sin \frac{n\pi x}{L} dx \right\} \\ &= a_m L \quad \text{if } m \neq 0 \end{aligned} \quad (2)$$

$$\text{Thus} \quad a_m = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{m\pi x}{L} dx \quad \text{if } m = 1, 2, 3, \dots$$

(b) Multiplying (1) by $\sin \frac{m\pi x}{L}$ and integrating from $-L$ to L , using Problem 2.3, we have

$$\begin{aligned} \int_{-L}^L f(x) \sin \frac{m\pi x}{L} dx &= A \int_{-L}^L \sin \frac{m\pi x}{L} dx \\ &\quad + \sum_{n=1}^{\infty} \left\{ a_n \int_{-L}^L \sin \frac{m\pi x}{L} \cos \frac{n\pi x}{L} dx + b_n \int_{-L}^L \sin \frac{m\pi x}{L} \sin \frac{n\pi x}{L} dx \right\} \\ &= b_m L \end{aligned}$$

$$\text{Thus} \quad b_m = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{m\pi x}{L} dx \quad \text{if } m = 1, 2, 3, \dots$$

(c) Integrating (1) from $-L$ to L , using Problem 2.2, gives

$$\int_{-L}^L f(x) dx = 2AL \quad \text{or} \quad A = \frac{1}{2L} \int_{-L}^L f(x) dx$$

Putting $m = 0$ in the result of part (a), we find $a_0 = \frac{1}{L} \int_{-L}^L f(x) dx$ and so $A = \frac{a_0}{2}$.

The above results also hold when the integration limits $-L, L$ are replaced by $c, c + 2L$.

Note that in all parts above, interchange of summation and integration is valid because the series is assumed to converge uniformly to $f(x)$ in $(-L, L)$. Even when this assumption is not warranted, the coefficients a_m and b_m as obtained above are called *Fourier coefficients* corresponding to $f(x)$, and the corresponding series with these values of a_m and b_m is called the *Fourier series* corresponding to $f(x)$. An important problem in this case is to investigate conditions under which this series actually converges to $f(x)$. Sufficient conditions for this convergence are the *Dirichlet conditions* established below in Problems 2.18–2.23.

Long question

$$= \frac{2}{\pi} - \frac{4}{\pi} \left(\frac{\cos 2x}{2^2 - 1} + \frac{\cos 4x}{4^2 - 1} + \frac{\cos 6x}{6^2 - 1} + \dots \right)$$

2.12. Expand $f(x) = x$, $0 < x < 2$, in a half-range (a) sine series, (b) cosine series.

(a) Extend the definition of the given function to that of the odd function of period 4 shown Fig. 2-12 below. This is sometimes called the *odd extension* of $f(x)$. Then $2L = 4$, $L = 2$.

Short

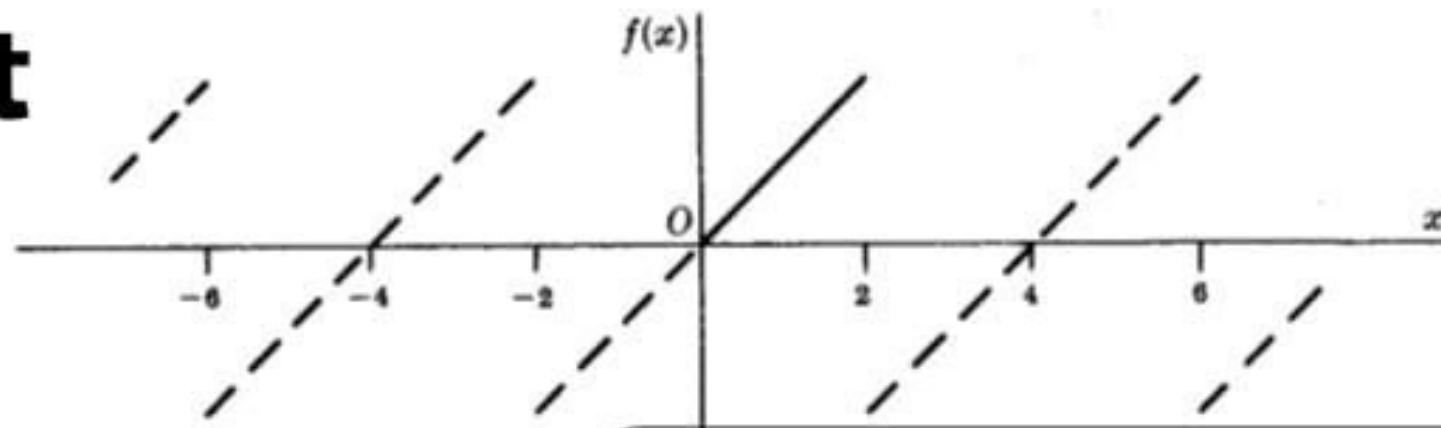


Fig. 2-12? **Odd function find bn**

Thus $a_n = 0$ and

$$\begin{aligned} b_n &= \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx = \frac{2}{2} \int_0^2 x \sin \frac{n\pi x}{2} dx \\ &= \left\{ (x) \left(\frac{-2}{n\pi} \cos \frac{n\pi x}{2} \right) - (1) \left(\frac{-4}{n^2 \pi^2} \sin \frac{n\pi x}{2} \right) \right\} \Big|_0^2 = \frac{-4}{n\pi} \cos n\pi \end{aligned}$$

Then

$$\begin{aligned} f(x) &= \sum_{n=1}^{\infty} \frac{-4}{n\pi} \cos n\pi \sin \frac{n\pi x}{2} \\ &= \frac{4}{\pi} \left(\sin \frac{\pi x}{2} - \frac{1}{2} \sin \frac{2\pi x}{2} + \frac{1}{3} \sin \frac{3\pi x}{2} - \dots \right) \end{aligned}$$

LAPLACE'S EQUATION

2.30. Suppose that the square plate of Problem 2.29 has three sides kept at temperature zero, while the fourth side is kept at temperature u_1 . Determine the steady-state temperature everywhere in the plate.

Choose the side having temperature u_1 to be the one where $y = 1$, as shown in Fig. 2-16. Since we wish the steady-state temperature u , which does not depend on time t , the equation is obtained from (1) of Problem 2.29 by setting $\partial u / \partial t = 0$; i.e. Laplace's equation in two dimensions:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (1)$$

The boundary conditions are

$$u(0, y) = u(1, y) = u(x, 0) = 0, \quad u(x, 1) =$$

$$\text{and } |u(x, y)| < M.$$

Mcqs

To solve this boundary value problem let $u = XY$ in (1) to obtain

$$X''Y + XY'' = 0 \quad \text{or} \quad \frac{X''}{X} = -\frac{Y''}{Y}$$

Setting each side equal to $-\lambda^2$ yields

$$X'' + \lambda^2 X = 0 \quad Y'' - \lambda^2 Y = 0$$

Let $\{\phi_n(x)\}$ be a set of functions which are mutually orthonormal in (a, b) .
 that if $\sum_{n=1}^{\infty} c_n \phi_n(x)$ converges uniformly to $f(x)$ in (a, b) , then

$$c_n = \int_a^b f(x) \phi_n(x) dx$$

Multiplying both sides of

$$f(x) = \sum_{n=1}^{\infty} c_n \phi_n(x)$$

Long

by $\phi_m(x)$ and integrating from a to b , we have

$$\int_a^b f(x) \phi_m(x) dx = \sum_{n=1}^{\infty} c_n \int_a^b \phi_m(x) \phi_n(x) dx$$

where the interchange of integration and summation is justified by the fact that the series converges uniformly to $f(x)$. Now since the functions $\{\phi_n(x)\}$ are mutually orthonormal in (a, b) , we

$$\int_a^b \phi_m(x) \phi_n(x) dx = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases}$$

so that (2) becomes

$$\int_a^b f(x) \phi_m(x) dx = c_m$$

as required.

We call the coefficients c_m given by (3) the *generalized Fourier coefficients* corresponding to $f(x)$ even though nothing may be known about the convergence of the series in (1). As in the case of Fourier series, convergence of $\sum_{n=1}^{\infty} c_n \phi_n(x)$ is then investigated using the coefficients (3). The conditions of convergence depend of course on the types of orthonormal functions used. In the remainder of this book we shall be concerned with many examples of orthonormal function series.

4.7. Evaluate (a) $\Gamma(-1/2)$ (b) $\Gamma(-5/2)$.

We use the generalization to negative values defined by $\Gamma(n) = \frac{\Gamma(n+1)}{n}$.

Short

(a) Letting $n = -\frac{1}{2}$, $\Gamma(-1/2) = \frac{\Gamma(1/2)}{-1/2} = -2\sqrt{\pi}$.

(b) Letting $n = -3/2$, $\Gamma(-3/2) = \frac{\Gamma(-1/2)}{-3/2} = \frac{-2\sqrt{\pi}}{-3/2} = \frac{4\sqrt{\pi}}{3}$, using (a).

Then $\Gamma(-5/2) = \frac{\Gamma(-3/2)}{-5/2} = -\frac{8}{15}\sqrt{\pi}$.

THE CONVOLUTION THEOREM FOR FOURIER TRANSFORMS

The *convolution* of the functions $f(x)$ and $g(x)$ is defined by

Mcqs
$$f * g = \int_{-\infty}^{\infty} f(u) g(x-u) du$$

An important theorem, often referred to as the *convolution theorem*, states that the Fourier transform of the convolution of $f(x)$ and $g(x)$ is equal to the product of the Fourier transforms of $f(x)$ and $g(x)$. In symbols,

Mcqs
$$\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \mathcal{F}\{g\}$$

There are other important properties. For example, we have for f , g , and h ...

$$f * g = g * f, \quad f * (g * h) = (f * g) * h, \quad f * (g + h) = f * g + f * h$$

i.e., the convolution obeys the commutative, associative and distributive laws

5.6. Solve the integral equation

$$\int_0^{\infty} f(x) \sin \alpha x \, dx = \begin{cases} 1 - \alpha & 0 \leq \alpha \leq 1 \\ 0 & \alpha > 1 \end{cases}$$

If we write

$$F_S(\alpha) = \int_0^{\infty} f(x) \sin \alpha x \, dx = \begin{cases} 1 - \alpha & 0 \leq \alpha \leq 1 \\ 0 & \alpha > 1 \end{cases}$$

then, by (10), page 81,

$$f(x) = \frac{2}{\pi} \int_0^{\infty} F_S(\alpha) \sin \alpha x \, d\alpha$$

$$= \frac{2}{\pi} \int_0^1 (1 - \alpha) \sin \alpha x \, d\alpha$$

$$= \frac{2(x - \sin x)}{\pi x^2}$$

Long question

LECTURE NO. 25

(ii): For even extension of $f(x) = |x|$ in $(-2, 2)$

This implies $2l = 4$ so $l = 2$

For odd expansion; $b_n = 0$ and

$$a_n = \frac{2}{l} \int_0^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

For given function,

$$a_n = \frac{2}{2} \int_0^2 x \cos\left(\frac{n\pi x}{2}\right) dx \Rightarrow \int_0^2 x \cos\left(\frac{n\pi x}{2}\right) dx$$

By integrating and applying limits, we have

$$a_n : \quad \text{Short} \quad (1) \left\{ \frac{-4}{n^2 \pi^2} \cos\left(\frac{n\pi x}{2}\right) \right\} \Big|_0^2$$

$$a_n = \quad \therefore \Rightarrow \frac{4}{n^2 \pi^2} [(-1)^n - 1]$$

For $n = 0$,

$$a_0 = \frac{2}{l} \int_0^l x dx \Rightarrow \frac{2}{2} \int_0^2 x dx \Rightarrow \left[\frac{x^2}{2} \right]_0^2 \Rightarrow 2$$

$$\frac{a_0}{2} = 1$$

Now putting values in Fourier Series,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right]$$

$$f(x) = 1 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right) + 0 \Rightarrow 1 + \sum_{n=1}^{\infty} \frac{4}{n^2 \pi^2} [(-1)^n - 1] \cos\left(\frac{n\pi x}{2}\right)$$

$$f(x) = 1 + \frac{4}{\pi^2} \left[-2 \cos\left(\frac{\pi x}{2}\right) - \frac{2}{9} \cos\left(\frac{3\pi x}{2}\right) - \frac{2}{25} \cos\left(\frac{5\pi x}{2}\right) - \dots \right]$$

Heat Flux across the Plane:

Suppose planes 'I' and 'II' are parallel and at distance ' Δn ' apart. Let

$u = \text{temp of plane I}$ And $u + \Delta u = \text{temp of plane II}$

Heat flows from higher temp plane to lower temp plane.

Heat flux is the amount of heat that flows through unit area per unit time. Mathematically

Mcqs

Heat flux $\propto \frac{\text{difference of temperature}}{\text{distance between planes}}$

$$\text{Heat flux from I to II} = k \frac{(u) - (u + \Delta u)}{\Delta n} \Rightarrow -k \frac{\Delta u}{\Delta n}$$

Where ' k ' is constant of proportionality and is called thermal conductivity.

Thermal conductivity is defined as; Measure of how well a material can conduct or transfer heat.

Note: As in this case heat flows from plane I to II (lower temp) negative value of heat flux. $\therefore \Delta u > 0$ (Heat value will be positive) (higher temp to low temp) and vice versa.

Mcqs

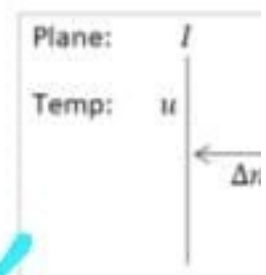
For limiting case: If $\Delta n \rightarrow 0$ and $\Delta u \rightarrow 0$ then

$$\text{Heat flow across plane I} = -k \frac{\partial u}{\partial n}$$

In vector form

$$\text{Heat flow across plane I} = \left\{ -k \frac{\nabla u}{\nabla n} \right\}$$

Note: As $\Delta n \rightarrow 0$ (distance between planes approaches to zero), so we can write it as plane 'I' only.



LECTURE NO. 06

Partial Differential Equations (Definition and Related Terms):

Definition: It is an equation containing unknown functions of two or more variables and partial derivatives with respect to these variables. Example:

Mcqs

Dirichlet Condition:

Fourier series corresponding to ' $f(x)$ ' is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right]$$

Question is; Series converge or diverge? If converge then converges to $f(x)$ or not.

Suppose that:

Fourier series satisfy all three conditions.

MTH647

- i) $f(x)$ is defined and single valued on $(-l, l)$
- ii) $f(x)$ is periodic on $(-l, l)$ with period ' $2l$ '
- iii) $f(x)$ & $f'(x)$ are piecewise continuous, then the series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right] = \begin{cases} f(x) & \text{if 'x' is point of continuity} \\ \frac{f(x+0) + f(x-0)}{2} & \text{if 'x' is point of discontinuity} \end{cases}$$

Mcqs

Finding a Fourier Series by Integration:

Theorem: Fourier series of $f(x)$ can be integrated by term from a to x and the resulting series will converge uniformly to $\int_0^x f(x)$ provided that;

(i): $f(x)$ is piecewise continuous in $(-l, l)$

(ii): $a, x \in (-l, l)$

As we know

$$F_c(\alpha) = \int_0^{\infty} f(u) \cos \alpha u \, du$$

For given function, it can be written as

$$F_c(\alpha) = \int_0^{\infty} e^{-mu} \cos \alpha u \, du \quad \dots (1)$$

Using integral formula, which is given as

$$\int e^{px} \cos qx \, dx = \frac{e^{px}}{p^2 + q^2} (p \cos qx + q \sin qx)$$

Here $p \rightarrow -m$, $q \rightarrow \alpha$ and $x \rightarrow u$

So, by applying integral formula, equation (1) becomes

$$F_c(\alpha) = \left[\frac{e^{-mu}}{m^2 + \alpha^2} (-m \cos \alpha u + \alpha \sin \alpha u) \right]_0^{\infty}$$

By applying limits, we get

$$F_c(\alpha) = \frac{m}{m^2 + \alpha^2} = F_c\{e^{-mx}\}$$

Now its inverse, cosine transform;

$$f(x) = \frac{2}{\pi} \int_0^{\infty} F_c(\alpha) \cos \alpha u \, d\alpha$$

For given function, it can be written as (Putting $F_c(\alpha)$ calculated value)

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \frac{m}{m^2 + \alpha^2} \cos \alpha u \, d\alpha$$

As $f(x) = e^{-mx}$,

$$e^{-mx} = \frac{2m}{\pi} \int_0^{\infty} \frac{\cos \alpha u}{m^2 + \alpha^2} \, d\alpha$$

$$\frac{\pi e^{-mx}}{2m} = \int_0^{\infty} \frac{\cos \alpha u}{m^2 + \alpha^2} \, d\alpha$$

This is our required result.

LECTURE NO. 71

Integral Equations Solution by Fourier Sine Transform:

Solve the integral equation

$$\int_0^{\infty} f(x) \sin \alpha x \, dx = \begin{cases} 1 - \alpha & 0 \leq \alpha \leq 1 \\ 0 & \alpha > 1 \end{cases}$$

Solution: The given function can be written as

$$F_s\{f(\alpha)\} = F_s(\alpha) = \int_0^{\infty} f(x) \sin \alpha x \, dx = \begin{cases} 1 - \alpha & 0 \leq \alpha \leq 1 \\ 0 & \alpha > 1 \end{cases}$$

Its F^{-1} for given function,

$$F_s^{-1}\{f(x)\} = f(x) = \frac{2}{\pi} \int_0^{\infty} F_s(\alpha) \sin \alpha x \, d\alpha$$

$$f(x) = \frac{2}{\pi} \int_0^1 (1 - \alpha) \sin \alpha x \, d\alpha$$

Integrating and applying limits, we have

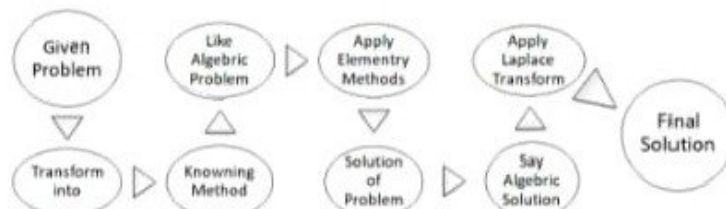
$$\begin{aligned} &= \frac{2}{\pi} \left[(1 - \alpha) \left(-\frac{\cos \alpha x}{x} \right) \Big|_0^1 + \int_0^1 \frac{\cos \alpha x}{x} (-1) \, d\alpha \right] \\ &= \frac{2}{\pi} \left[0 - (1) \left(-\frac{1}{x} \right) - \frac{1}{x} \left[\frac{\sin \alpha x}{x} \right]_0^1 \right] \Rightarrow \frac{2}{\pi} \left[\frac{1}{x} - \frac{\sin x}{x^2} \right] \\ &f(x) = \frac{2}{\pi} \left(\frac{x - \sin x}{x^2} \right) \end{aligned}$$

This is our required result.

LECTURE NO. 72

Laplace Transforms:

Objectives/Aim/Steps:



62 sy aagy kafi mcqs aay 6:40 PM ✓

Convulation sy related 6:40 PM ✓



Fourier transform ka even or odd function 2
mcqss thy 6:40 PM ✓



62 sy aagy zrur dekh ly paper aa rha aagy sy

6:41 PM ✓

Question to ni lkn mcqs boht zyada thy

6:41 PM ✓



Recommend Fourier series ki
Schams series ki Jo sir ny do the
announcement Mai

Wh zrrur read kry udr sy mcqs thy

6:47 PM ✓



Baqi Mai mcqs soch rhi Jo Jo yad ata type krti
ho

6:48 PM ✓



Hr function ka period ___hota hai

- 1)Unique
- 2)Finite
- 3)Infinite
- 4)Does not exis

Edited 6:49 PM ✓

Fourier transform of even Function
Has specific value
Does not exist

Baqi search kr ly Kya answer is ka asy option
thy

Edited 6:51 PM ✓

Fourier transform of odd function MCQS

6:52 PM ✓

Second order partial differential equation of Linear Equation is

PDE of Linear term

PDE of Linear function

PDE is homogeneous

Yh 3 mcqs thy

6:58 PM ✓

If function is orthogonal it's mean
Pora function zero hai
1 term zero ho ge
Specific value ho ge
Asy mcqs thy shyd

7:05 PM ✓

You

 Long question



Asa question tha lkn last pr tha function
converge to their average

Thora sa change tha

Edited 7:09 PM ✓

Legendre's duplication formula why it's
duplication formula

MCQS tha

7:11 PM ✓

baqi quiz 1 sy 4 5 mcqs thy SL system ky
Fourier series generalization form wly
SL system ka key context Kya hai aik yh tha

7:13 PM ✓

$\sum_{n=1}^{\infty} (c_n)^2$ will converge $\Rightarrow c_n \rightarrow 0$ as

$$\lim_{n \rightarrow \infty} c_n = 0 \Rightarrow \lim_{n \rightarrow \infty} \int_a^b f(x) \Phi_n(x) dx$$

result.

LECTURE NO. 40

em:

problem of the form:

$$\frac{d}{dx} \left[p(x) \frac{dy}{dx} \right] + [q(x) + \lambda r(x)] y = 0 \quad a \leq x \leq b$$

$\alpha_1 y(a) + \alpha_2 y'(a) = 0$, $\beta_1 y(b) + \beta_2 y'(b) = 0$ are given constants and $p(x), q(x)$ and $r(x)$ are continuous and never independent of x .

sequence of eigen value λ_n and corresponding eigen function $y_n(x)$

$\lambda y = 0$; B.V.P: $y(0) = y(1) = 0$

$$\frac{d}{dx} \left[1 \cdot \frac{dy}{dx} \right] + [(0 + \lambda \cdot 1) y] = 0 \quad 0 \leq x \leq 1$$

can be written as,

$$y(0) = 1 y(0) + 0 y'(0) = 0 = \alpha_1 y(a) + \alpha_2 y'(a)$$

$$y(1) = 1 y(1) + 0 y'(1) = 0 = \beta_1 y(b) + \beta_2 y'(b)$$

Sturm-Liouville System, we have

$$\alpha_1 = 1, \alpha_2 = 0, \beta_1 = 1, \beta_2 = 0, p(x) = 1, q(x) = 0, r(x) = 1$$

LECTURE NO. 41

motivation for S-L System:

$$g(x) \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left[k(x) \frac{\partial u}{\partial x} \right] + h(x) u$$

$$0 \leq x \leq l, t > 0, u(0, t) = 0, u(l, t) = 0, u(x, 0) = f(x)$$

here $k = 8$ and $c = -3$,
hence

$$u(x, y) = 8e^{-3(4x+y)} = 8e^{-12x-3y}$$

LECTURE NO. 15

Heat Conduction Equation and its Physical Interpretation:

Question:

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2} \quad \dots (1) \quad \therefore 0 < x < 3, t > 0$$

Given that (boundary values)

$$u(0, t) = u(3, t) = 0 \quad \dots (i),$$

$$u(x, 0) = 5\sin 4\pi x - 3\sin 8\pi x + 2\sin 10\pi x \quad \dots (ii) \therefore u(x$$

Solution: Let

$$u(x, t) = X(x) \cdot T(t) = XT \quad \dots (z)$$

Taking derivative w.r.to 't'

$$\frac{\partial u}{\partial t} = XT'$$

Taking 1st derivative w.r.to 'x'

$$\frac{\partial u}{\partial x} = X'T$$

Taking 2nd derivative w.r.to 'x'

$$\frac{\partial^2 u}{\partial x^2} = X''T$$

By putting values equation 1 becomes

$$\begin{aligned} XT' &= 2X''T \\ \frac{X''}{X} &= \frac{T'}{2T} = -\lambda^2 \text{ (say)} \end{aligned}$$

So,

$$\begin{aligned} \frac{X''}{X} &= -\lambda^2 \\ X'' + \lambda^2 X &= 0 \dots (a) \end{aligned}$$

$$\text{Let } X = e^{mx}$$

$$X'' = m^2 e^{mx} = m^2 X$$

$$(a) \Rightarrow m^2 X + \lambda^2 X = 0$$

$$m = \pm \lambda i \text{ \& } X \neq 0$$

$$\therefore X(x) = \alpha_1 e^{\lambda i x} + \alpha_2 e^{-\lambda i x}$$

$$X(x) = \alpha_1 [\cos \lambda x + i \sin \lambda x] + \alpha_2 [\cos \lambda x - i \sin \lambda x]$$

$$X(x) = (\alpha_1 + \alpha_2) \cos \lambda x + i(\alpha_1 - \alpha_2) \sin \lambda x$$

$$X(x) = A_1 \cos \lambda x + B_1 \sin \lambda x$$

Here we were solving both functions side by side. Now

Putting values in equation (z), we get

$$u(x, t) = XT = e^{-2\lambda^2 t} (A_1 \cos \lambda x + B_1 \sin \lambda x) \quad \dots (1)$$

1TH647

Calculating (i) (boundary value) by putting values in equation 2,

$$u(0, t) = 0 = e^{-2\lambda^2 t} (A \cos \lambda(0) + B \sin \lambda(0)) = e^{-2\lambda^2 t} \cdot A \Rightarrow A = 0$$

Equation 2 becomes

$$u(x, t) = XT = B \cdot e^{-2\lambda^2 t} \sin \lambda x \quad \therefore A = 0$$

 Forwarded

MTH 647 paper today

All Mcqs from conceptual and from handouts

1. Prove eign function orthogonal
2. Form general solve PDE
3. Term by term differentiation
4. Solve heat equation form given function

Overall paper was very easy

6:34 PM ✓

 Forwarded

SL system sy NAI tha Kuch

Gemma beta sy mcqs thy Kuch

Question nai tha

Choice p Kuch b nai chora tha men NY

Zaida paper Fourier series or Fourier integral
sy tha

Or orthogonal sy b thy mcqs or question

10:23 PM ✓

$$\text{Heat flow across plane } l = \left\{ -k \frac{\nabla u}{\nabla l} \right\}$$

(distance between planes approaches to zero), so we can write

LECTURE NO. 06

Partial Equations (Definition and Related Terms):

A partial differential equation containing unknown functions of two or more variables. Example

$$\frac{\partial^2 u}{\partial x \partial y} = 2x - y \quad \text{order} = 2$$

Order of highest derivative involved in PDE.

Functions which satisfy the given DE. Example:

$$u = x^2 y - \frac{1}{2} x y^2$$

$$\frac{\partial u}{\partial y} = x^2 - xy \quad \text{1st derivative w.r.t. } y$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right) = 2x - y \quad \text{2nd derivative w.r.t. } x \text{ and } y$$

$y - \frac{1}{2} x y^2 + F(x) + G(y)$ is also a solution. Here $F(x)$ and $G(y)$ are arbitrary functions.

Particular Solution: It is obtained from the general solution by particular choice of $F(x)$ and $G(y)$.

$$u = x^2 y - \frac{1}{2} x y^2 + 2 \sin x + 3y^4 - 5$$

Particular Solution: It cannot be obtained from the general solution by choosing arbitrary functions $F(x)$ and $G(y)$.

 **Short**

LECTURE NO. 07

$$y = \sqrt{x} \Rightarrow y = \frac{1}{4} (\ln x)^2 \quad \text{(General Solution—Here 'c' is arbitrary constant)}$$

$y = \sqrt{x}$ is a solution of given DE. As it is not obtained from general solution.

$$(\nexists c \in \mathbb{R} \text{ such that } y = 0)$$

LECTURE NO. 08

Their Classification:

A linear partial differential equation of order 2 in two independent variables is said to be elliptic, parabolic or hyperbolic according to the nature of the discriminant Δ .

$$A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + F u = G$$

where A, B, C, D, E, F and G may depend on x or y only but not on both x and y .

Now, by the superposition theorem we can arrive at the possible solu

$$u(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} e^{-\kappa(m^2+n^2)\pi^2 t} \sin m\pi x \sin n\pi y$$

Letting $t = 0$ and using the condition $u(x, y, 0) = f(x, y)$, we arriv

$$f(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin m\pi x \sin n\pi y$$

As in Problem 2.24 we then find that

$$B_{mn} = 4 \int_0^1 \int_0^1 f(x, y) \sin m\pi x \sin n\pi y \, dx \, dy$$

Thus the formal solution to our problem is given by

48 / 197

LAPLACE'S EQUATION

2.30. Suppose that the square plate of Problem 2.29 has three sides kept at temperature zero while the fourth side is kept at temperature u_1 . Determine the steady-state temperature everywhere in the plate.

Choose the side having temperature u_1 to be the one where $y = 1$, as shown in Fig. 2-16. Since we wish the steady-state temperature u , which does not depend on time t , the equation is obtained from (1) of Problem 2.29 by setting $\partial u / \partial t = 0$; i.e. Laplace's equation in two dimensions:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (1)$$

The boundary conditions are

$$u(0, y) = u(1, y) = u(x, 0) = 0, \quad u(x, 1) = u_1$$

and $|u(x, y)| < M$.

To solve this boundary value problem let $u = XY$ in (1) to obtain

$$X''Y + XY'' = 0 \quad \text{or} \quad \frac{X''}{X} = -\frac{Y''}{Y}$$

Setting each side equal to $-\lambda^2$ yields

$$X'' + \lambda^2 X = 0 \quad Y'' - \lambda^2 Y = 0$$

from which

$$X = a_1 \cos \lambda x + b_1 \sin \lambda x \quad Y = a_2 \cosh \lambda y + b_2 \sinh \lambda y$$

Then a possible solution is

$$u(x, y) = (a_1 \cos \lambda x + b_1 \sin \lambda x)(a_2 \cosh \lambda y + b_2 \sinh \lambda y)$$

From $u(0, y) = 0$ we find $a_1 = 0$. From $u(x, 0) = 0$ we find $a_2 = 0$. Thus a solution satisfying all these conditions is

$$u(x, y) = B \sin m\pi x \sinh m\pi y$$

To satisfy the last condition, $u(x, 1) = u_1$, we must first use the principle of superposition to write the solution

$$u(x, y) = \sum_{m=1}^{\infty} B_m \sin m\pi x \sinh m\pi y$$