

PHY101 Short Notes

Theme 23 to 45

BY vuonlinehelp.blogspot.com

Elone Musk

Lecture 23 Electrostatics – II

Definition of Coulomb's law

Coulomb's law a statement in physics: the force of attraction or repulsion acting along a straight line between two electric charges is directly proportional to the product of the charges and inversely to the square of the distance between them.

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

In the continuous limit:

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

where:

$$E_x = \int dE_x$$

$$E_y = \int dE_y$$

$$E_z = \int dE_z$$

Charge density

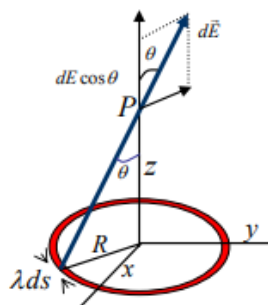
The electric charge per unit area of a surface, or per unit volume of a field or body
Meaning of Charge Density

For linear charge distribution: $dq = \lambda ds$

For surface charge distribution: $dq = \sigma dA$

For volume charge distribution: $dq = \rho dV$

As an example



$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2} = \frac{\lambda ds}{4\pi\epsilon_0 (z^2 + R^2)}$$

$$dE_z = dE \cos \theta$$

$$\cos \theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}$$

$$dE_z = \frac{z \lambda ds}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}}$$

$$E_z = \frac{z \lambda}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} \int ds$$

$$= \frac{z \lambda (2\pi R)}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}}$$

$$= \frac{qz}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}}$$

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}, \quad (z \gg R)$$

$$dq = \sigma dA = \sigma (2\pi\omega) d\omega$$

$$dE_z = \frac{z \sigma 2\pi\omega d\omega}{4\pi\epsilon_0 (z^2 + \omega^2)^{3/2}}$$

$$= \frac{\sigma z}{4\epsilon_0} (z^2 + \omega^2)^{-3/2} (2\omega) d\omega$$

$$E_z = \int dE_z = \frac{\sigma z}{4\epsilon_0} \int_0^R (z^2 + \omega^2)^{-3/2} (2\omega) d\omega$$

$$E_z = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

What if $z \ll R$?

$$E_z = \frac{\sigma}{2\epsilon_0}$$

$$E_z = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

What if $z \gg R$?

$$E_z = \frac{\sigma}{4\epsilon_0} \frac{R^2}{z^2}$$

$$= \frac{q}{4\pi\epsilon_0 z^2}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dz}{y^2 + z^2}$$

$$dE_y = dE \cos \theta, \quad dE_z = dE \sin \theta$$

$$E_y = \int dE = \int_{z=-\infty}^{z=\infty} \cos \theta dE$$

$$E_z = \int dE = \int_{z=-\infty}^{z=\infty} \sin \theta dE$$

$$E = E_y, \quad E_z = 0 \quad \text{by symmetry}$$

$$E = E_y = 2 \int_{z=0}^{z=\infty} \cos \theta dE$$

$$= \frac{\lambda}{2\pi\epsilon_0} \int_{z=0}^{z=\infty} \cos \theta \frac{dz}{y^2 + z^2}$$

$$z = y \tan \theta \Rightarrow dz = y \sec^2 \theta d\theta$$

$$E = \frac{\lambda}{2\pi\epsilon_0 y} \int_{\theta=0}^{\theta=\pi/2} \cos \theta d\theta$$

$$E = \frac{\lambda}{2\pi\epsilon_0 y}$$

This problem has cylindrical symmetry

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Flux

Flux of a particular vector field is the measure of the “flow” or penetration of the field vectors through an imaginary fixed surface

$$\Phi = EA$$

Minimum electric field lines for, θ (angle between $\vec{E}$ and $\vec{A}$) = 90°

Maximum electric field lines for, θ (angle between $\vec{E}$ and $\vec{A}$) = 0°

$$\Phi_E = EA \cos \theta = \vec{E} \cdot \vec{A}$$

$$\Phi_E = \sum \vec{E}_i \cdot \Delta \vec{A}_i$$

And in the differential limit:

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad \left(\begin{array}{l} \text{integral over} \\ \text{closed surface} \end{array} \right)$$

$$|\vec{E}_1| = |\vec{E}_2| = \dots = |\vec{E}_n| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\begin{aligned} \Phi &= \sum E \Delta A \\ &= E \sum \Delta A \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} (4\pi r^2) \\ \Phi &= \frac{q}{\epsilon_0} \end{aligned}$$

What is Gauss law explains?

Gauss Law states that the total electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity. The electric flux in an area is defined as the electric field multiplied by the area of the surface projected in a plane and perpendicular to the field.

$$\begin{aligned} \Phi &\equiv \int \vec{E} \cdot d\vec{A} \\ &= \frac{q_{\text{enclosed}}}{\epsilon_0} \end{aligned}$$

$$\epsilon_0 \int \vec{E} \cdot d\vec{A} = \epsilon_0 \int E dA = q_{\text{enclosed}}$$

$$\epsilon_0 E \int dA = q$$

$$\epsilon_0 E (4\pi r^2) = q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\begin{aligned}
 \Phi &= \int \vec{E} \cdot d\vec{A} \\
 &= \int_{\text{outer cap}} \vec{E} \cdot d\vec{A} + \int_{\text{inner cap}} \vec{E} \cdot d\vec{A} + \int_{\text{side walls}} \vec{E} \cdot d\vec{A} \\
 &= 0 + EA + 0
 \end{aligned}$$

$$\epsilon_0 \Phi = q$$

$$\epsilon_0 EA = \sigma A$$

$$E = \frac{\sigma}{\epsilon_0}$$

What is Gaussian surface and what is its use?

Gaussian surface is an enclosed surface in a three dimensional space through which the flux of a vector field is calculated (gravitational field, the electric field, or magnetic field.) Gaussian surface helps evaluate the electric field intensity due to symmetric charge distribution.

$$\epsilon_0 \int \vec{E} \cdot d\vec{A} = q$$

$$\epsilon_0 E (2\pi rh) = \lambda h$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\epsilon_0 \int \vec{E} \cdot d\vec{A} = q$$

$$\epsilon_0 (EA + EA) = \sigma A$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\epsilon_0 E (4\pi r^2) = q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{spherical shell, } r > R)$$

$$\begin{aligned}
 E &= 0 \quad (\text{spherical shell, } r < R) \int \vec{E} \cdot d\vec{A} = 4\pi r^2 E \\
 &= \frac{q'}{\epsilon_0}
 \end{aligned}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2}$$

$$E_{n2}A - E_{n1}A = \frac{\sigma A}{\epsilon_0}$$

$$E_{n2} - E_{n1} = \frac{\sigma}{\epsilon_0}$$

$$E_1 = \frac{\sigma}{2\epsilon_0}$$

$$E = 0 \text{ inside a conductor}$$

$$E = \frac{\sigma}{\epsilon_0}$$

Electric field definition

A region around a charged particle or object within which a force would be exerted on other charged particles or objects

$$U_B - U_A = W_{ab}$$

$$\Delta U = -W_{ab}$$

$$U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{s}$$

The electrostatic force is conservative and can be represented by a potential energy.

$$F = G \frac{m_1 m_2}{r^2} \quad F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}}{m} \quad \vec{E} = \frac{\vec{F}}{q}$$

Summary of Lecture 24 – ELECTRIC POTENTIAL ENERGY

ELECTRIC POTENTIAL ENERGY

Electric potential energy is a potential energy (measured in joules) that results from conservative Coulomb forces and is associated with the configuration of a particular set of point charges within a defined system.

Work done in bringing unit charge from infinity to R is,

$$\begin{aligned} &= \int_{\infty}^R E dr \\ &= \frac{1}{4\pi\epsilon_0} q \int_{\infty}^R \frac{dr}{r^2} \\ &= \frac{q}{4\pi\epsilon_0} \frac{1}{R} \end{aligned}$$

If the potential of two charges separated by an infinite distance is defined to be 0 then the stored potential energy is:

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

Units of potential: $1 \frac{\text{Joule}}{\text{Coulomb}} = 1 \text{ Volt}$

One electron-volt = energy gained by moving one
electron charge through one Volt
 $= (1.6 \times 10^{-19} \text{ C}) \times 1\text{V} = 1.6 \times 10^{-19} \text{ J}$

1 Kev = 10^3 eV

1 Mev = 10^6 eV

1 Gev = 10^9 eV

1 Tev = 10^{12} eV

Positive charges accelerate toward regions of lower potential.

Negative charges accelerate toward regions of higher potential.

- The electric potential exists only because the electrostatic force is conservative.
- The electric potential is similar but differs from the gravitational potential in one way.

- The electric potential determines the electric field and vice versa.
- The electric potential is a scalar quantity.

$$V = V_1 + V_2 + \dots + V_N$$

$$V = \sum_{i=1}^N V_i$$

$$= \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{r_i}$$

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}}$$

Dipole Moment

p = charge x distance

$$p = q d$$

$$r_2 - r_1 = d \cos \theta$$

$$V_p = V_1 + V_2$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_1} + \frac{-q}{r_2} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{r_2 - r_1}{r_1 r_2}$$

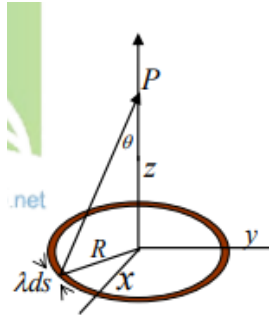
For $r \gg d$

$$r_2 - r_1 \approx d \cos \theta$$

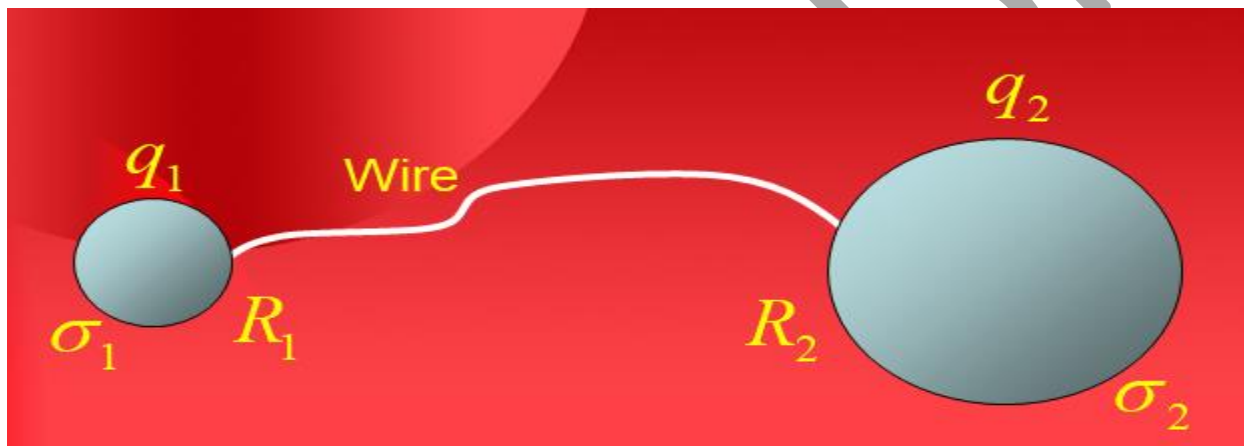
$$r_1 r_2 \approx r^2$$

$$V \approx \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

Note that $V = 0$ at $\theta = \frac{\pi}{2}$



Q: Find the ratio of charges on the two spheres. Are the electric fields on the two surfaces equal?



$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{R_2}$$

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} \quad (\text{recall that } q=4\pi R^2\sigma)$$

$$E_{\text{small sphere}} > E_{\text{large sphere}}$$

Summary of Lecture 25 – CAPACITORS AND CURRENTS

CAPACITANCE

Two conductors isolated from one another and from their surroundings, form a capacitor.

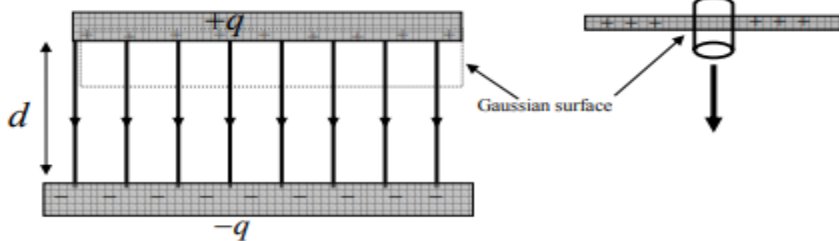
charge $\propto$ potential difference

$$Q \propto V$$

$$Q = CV \quad \Rightarrow \quad \text{CAPACITANCE}$$

$$C = \frac{Q}{V}$$

Parallel-Plate Capacitor



$$\Phi \equiv \int \vec{E} \cdot d\vec{A}$$

$$= \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

$$= EA$$

$$\Phi = \frac{Q}{\epsilon_0} \quad (\text{Gauss's Law})$$

$$E = \frac{Q}{\epsilon_0 A}$$

Any arrangement with two plates can form a capacitor: cylindrical or spherical.
The capacitance depends on geometry.

UNITS AND SIZES FOR CAPACITANCES

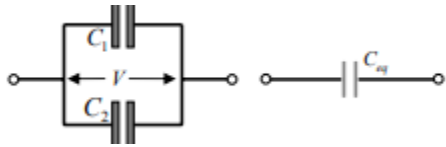
$$1 \text{ farad} = 1 \frac{\text{coulomb}}{\text{volt}}$$

$$1 \text{ microfarad} = 10^{-6} \text{ farad}$$

$$1 \text{ nanofarad} = 10^{-9} \text{ farad}$$

$$1 \text{ picofarad} = 10^{-12} \text{ farad}$$

Capacitors in Parallel



For each capacitor,

$$q_1 = C_1 V, \quad q_2 = C_2 V$$

Where V is the potential between terminals a and b

$$q = q_1 + q_2$$

$$C_{eq} V = C_1 V + C_2 V$$

$$\Rightarrow C_{eq} = C_1 + C_2$$

$$C_{eq} = \sum C_n \quad (\text{For } n \text{ capacitors})$$

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}$$

$$V = V_1 + V_2$$

$$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_n} \quad (\text{For } n \text{ capacitors})$$

ENERGY STORED IN ELECTRIC FIELD

$$dU = V dq = \frac{q}{C} dq$$

$$U = \int dU = \int_0^Q \frac{q}{C} dq$$

$$U = \frac{Q^2}{2C}$$

$$U = \frac{1}{2} CV^2$$

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad}$$

$$C = \frac{\epsilon_0 A}{d}$$

$$u = \frac{\epsilon_0}{2} \left(\frac{V}{d} \right)^2 = \frac{1}{2} \epsilon_0 E^2$$

Coulomb's Law With Dielectrics

According to Coulomb's law the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of charges. The electrostatic force of attraction or repulsion between two point charges is inversely proportional to the square of distance between them.

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{r^2}$$

ϵ_r = relative permittivity

$$E_{\text{medium}} = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q}{r^2}$$

$$E_{\text{medium}} = \frac{E_{\text{vacuum}}}{\epsilon_r}$$

Summary of Lecture 26 – ELECTRIC POTENTIAL ENERGY

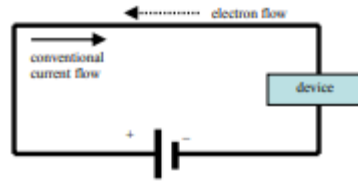
ELECTRIC CURRENT

An electric current is a stream of charged particles, such as electrons or ions, moving through an electrical conductor or space. It is measured as the net rate of flow of electric charge through a surface or into a control volume. 2. The direction of current flow is the direction in which positive charges move.

$$i = \frac{dq}{dt}$$

constant current: $q = i \times t$

else: $q = \int i dt$



CURRENT: UNITS AND SIZES

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{\text{second}}$$

$$1 \text{ milliampere} = 1 \text{ ma} = 10^{-3} \text{ A}$$

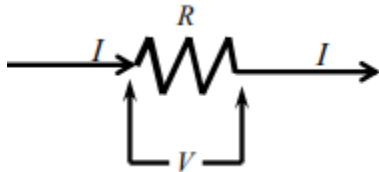
$$1 \text{ microampere} = 1 \mu\text{A} = 10^{-6} \text{ A}$$

$$1 \text{ nanoampere} = 1 \text{ nA} = 10^{-9} \text{ A}$$

$$1 \text{ picoampere} = 1 \text{ pA} = 10^{-12} \text{ A}$$

How does resistance affect current?

As resistance increases for a set voltage, the current no longer increases proportionally. As the resistance increases, the current decreases provided all other factors are kept constant. Materials with low resistance, metals for example, are called electrical conductors and allow electricity to flow easily.



$$I \equiv \frac{dq}{dt} \text{ defines current}$$

$$R \equiv \frac{V}{I} \text{ defines resistance, where } V = V_a - V_b$$

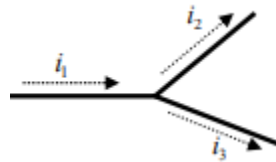
$$R \equiv \frac{V}{I} \text{ is not Ohm's Law !!}$$

OHM'S LAW

Ohm's law states that the current through a conductor is proportional to the voltage across the conductor. $V=IR$ where V is the voltage across the conductor and I is the current flowing through it.

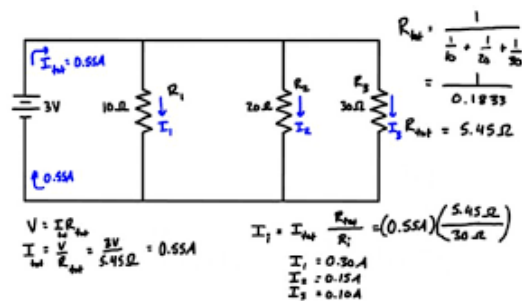
Charge is always conserved

Charge is always conserved, and therefore current is conserved as well. This means that when a current splits into two currents the sum remains constant $i_1 = i_2 + i_3$.



Resistors CURRENT

The current through the circuit is the same for each resistor in a series circuit and is equal to the applied voltage divided by the equivalent resistance: $I = V/R_S = 9V/90\Omega = 0.1A$. Note that the sum of the potential drops across each resistor is equal to the voltage supplied by the battery.



$$V_1 = iR_1, \quad V_2 = iR_2$$

$$V = V_1 + V_2$$

$$iR_{eq} = iR_1 + iR_2$$

$$\Rightarrow R_{eq} = R_1 + R_2$$

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}$$

$$i = i_1 + i_2$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Defining Relation: $Q = VC$ or $C = Q/V$

- Terms in science are usually very precise. But sometimes a word can be misleading. But because it is in such common use, we learn to tolerate it.
- Electromotive force is an example. It is because of EMF that current goes around a circuit.

- Electromotive force is actually not force. It is the energy that every unit of charge is given so that it can go around a closed circuit.

What Produces EMF?

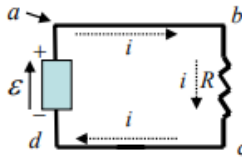
- Chemical processes (battery)
- Changing magnetic fields (generator)
- Light (solar cells)
- Heat (thermoelectricity)
- Sound (piezoelectricity)

KIRCHOFF'S LAW:

The sum of the potential differences encountered in moving around a closed circuit is zero.

$$V_a - iR + \mathcal{E} = V_a \quad (\text{a is any point})$$

$$-iR + \mathcal{E} = 0$$



INTERNAL RESISTANCE

Internal resistance refers to the opposition to the flow of current offered by the cells and batteries themselves resulting in the generation of heat. Internal resistance is measured in Ohms. The relationship between internal resistance (r) and emf ($\mathcal{E}$) of cell s given by. $\mathcal{E} = I(r + R)$.

$$V_a + \mathcal{E} - ir - iR = V_a$$

$$\mathcal{E} - ir - iR = 0$$

$$\Rightarrow i = \frac{\mathcal{E}}{r + R}$$

$$\frac{q}{C} + iR = 0 \Rightarrow \frac{1}{C} \frac{dq}{dt} + \frac{di}{dt} R = 0$$

$$\frac{di}{dt} = -\frac{1}{RC} i \quad \text{has solution: } i = i_0 e^{-\frac{t}{RC}}$$

REVIEW OF EXPONENTIAL FUNCTION

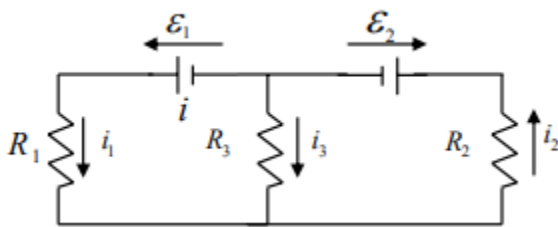
$$e^x \equiv 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

From which it follows that,

$$\frac{d}{dx} e^x = 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \dots = e^x$$

$$\text{Similarly, } \frac{d}{dx} e^{-x} = -e^{-x}$$

MULTILOOP CIRCUITS



$$i_1 + i_3 = i_2$$

$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$$

$$-i_3 R_3 - i_2 R_2 + \mathcal{E}_2 = 0$$

$$i_1 = \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$i_2 = \frac{\mathcal{E}_1 R_3 - \mathcal{E}_2(R_1 + R_3)}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$i_3 = \frac{-\mathcal{E}_1 R_2 - \mathcal{E}_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$\mathcal{E} = \frac{dW}{dq} \quad 1 \text{ volt} = \frac{1 \text{ joule}}{\text{coulomb}}$$

POWER DISSIPATED IN A RESISTOR

$$dW = \mathcal{E} dq = \mathcal{E} i dt$$

$$\mathcal{E} i dt = i^2 R dt \quad \left(\text{because } i = \frac{\mathcal{E}}{R} \right)$$

$$\therefore \text{Power} \equiv P = \frac{dW}{dt} = i^2 R$$

Other forms of $P = i^2 R$ are:

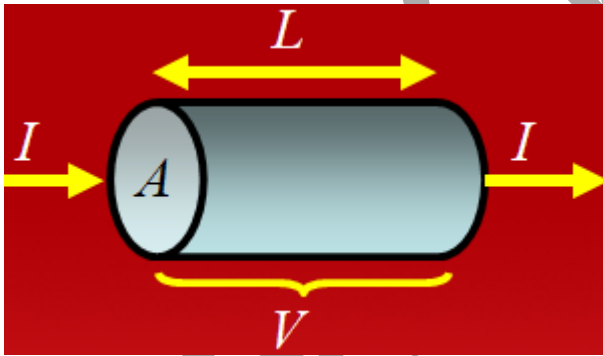
$$P = \frac{V^2}{R}$$

$$P = IV$$

$$\text{UNITS : } 1 \text{ volt-ampere} = 1 \frac{\text{joule}}{\text{coulomb}} \cdot \frac{\text{coulomb}}{\text{second}}$$

$$= 1 \frac{\text{joule}}{\text{second}} = 1 \text{ watt}$$

Consider a wire through which charge is flowing



$$q = (nAL)e$$

$$t = \frac{L}{v_d}$$

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$v_d = \frac{i}{nAe} = \frac{j}{ne}$$

$$j = nev_d$$

Velocity of the charges in terms of the measured current $v_d = \frac{i}{nAe}$ the current density

$$j = \frac{i}{A}$$

$$i = \int \vec{j} \cdot d\vec{A}$$

$$i = \frac{dq}{dt}$$

constant current: $q = i \times t$

else: $q = \int i dt$

Summary of Lecture 27 – THE MAGNETIC FIELD

Magnetic field

The magnetic field exerts a force upon any charge that moves in the field. The greater the size of the charge, and the faster it moves, the larger the force.

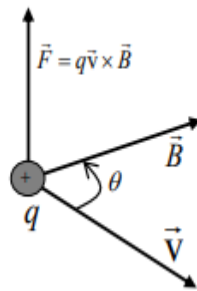
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$F = qvB \sin \phi$$

$$B = \frac{F}{qv} \quad (\text{if } \phi = \frac{\pi}{2})$$

$$1 \text{ tesla} = 1 \frac{\text{newton}}{\text{coulomb} \cdot \text{meter/second}}$$

$$1 \text{ tesla} = 1 \frac{\text{newton}}{\text{ampere} \cdot \text{meter}} = 10^4 \text{ gauss}$$



Typical Values Of Magnetic Field

1. Earth's surface 10^{-4} T
2. Bar magnet 10^{-2} T
3. Powerful electromagnet 1 T
4. Superconducting magnet 5 T

The Lorentz Force

Lorentz force, the force exerted on a charged particle q moving with velocity v through an electric field E and magnetic field B . The entire electromagnetic force F on the charged particle is called the Lorentz force (after the Dutch physicist Hendrik A. Lorentz) and is given by $F = qE + qv \times B$.

$$qE = qvB \Rightarrow v = \frac{E}{B} \quad \text{velocity selector !!}$$

But a magnetic field cannot do any work on a particle !!

$$\begin{aligned}
 dW &= \vec{F} \cdot d\vec{r} = q(\vec{v} \times \vec{B}) \cdot d\vec{r} \\
 &= q(\vec{v} \times \vec{B}) \cdot \frac{d\vec{r}}{dt} dt \\
 &= q(\vec{v} \times \vec{B}) \cdot \vec{v} dt \\
 &= 0
 \end{aligned}$$

CIRCULATING CHARGES

Magnetic forces can cause charged particles to move in circular or spiral paths. Particle accelerators keep protons following circular paths with magnetic force. The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude.

$$qvB = m \frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

Orbital radius of a charged particle

with speed v is $r = \frac{mv}{qB}$

Mass Spectrometer

Mass spectrometry is an analytical tool useful for measuring the mass-to-charge ratio (m/z) of one or more molecules present in a sample. These measurements can often be used to calculate the exact molecular weight of the sample components as well.

$$F = qvB$$

$$= \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

MAGNETIC FORCE ON A CURRENT

The force on a current-carrying wire in a magnetic field is $F = IlB \sin \theta$. Its direction is given by RHR-1.

$$\vec{F} = Ne\vec{v}_d \times \vec{B}$$

$$\vec{F} = nALe\vec{v}_d \times \vec{B}$$

$$nALe\vec{v}_d = \vec{I}$$

$$\vec{F} = \vec{I} \times \vec{B}$$

$$\tau = F \frac{a}{2} + F \frac{a}{2}$$

$$= IBb \frac{a}{2} + IBb \frac{a}{2}$$

$$= I \times (\text{area of loop}) \times B$$

$$= \mu B$$

$\mu = IA$ is called the

MAGNETIC MOMENT

Recall: $\vec{\tau} = \vec{p} \times \vec{E}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\mu = NIA$$

$$U = \mu B \cos \theta = \vec{\mu} \cdot \vec{B}$$

$$\int \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$$

AMPERE'S LAW

Ampere's law states that “The magnetic field created by an electric current is proportional to the size of that electric current with a constant of proportionality equal to the permeability of free space.”

$$\begin{aligned}
 \int \vec{B} \cdot d\vec{s} &= B \int ds \\
 &= B(2\pi r) \\
 &= \mu_0 I_{\text{enclosed}} \\
 B &= \frac{\mu_0 I}{2\pi r} \int \vec{B} \cdot d\vec{s} = B \int ds \\
 &= B(2\pi r) \\
 &= \mu_0 I_{\text{enclosed}} \\
 B &= \frac{\mu_0 I}{2\pi r}
 \end{aligned}$$

Magnetic Field of a Solenoid

The magnetic field inside a solenoid is proportional to both the applied current and the number of turns per unit length. There is no dependence on the diameter of the solenoid, and the field strength doesn't depend on the position inside the solenoid, i.e., the field inside is constant.

$$\begin{aligned}
 \int \vec{B} \cdot d\vec{s} &= \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} \\
 &+ \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s} \\
 I &= I_0 nh \\
 Bh &= \mu_0 I_0 nh \\
 B &= \mu_0 I_0 n \\
 B(2\pi r) &= \mu_0 I \left(\frac{\pi r^2}{\pi R^2} \right) \\
 B &= \frac{\mu_0 Ir}{2\pi R^2}
 \end{aligned}$$

Summary of Lecture 28 – ELECTROMAGNETIC INDUCTION

Summary of Lecture 29 – ALTERNATING CURRENT

SKIP

Summary of Lecture 30 – ELECTROMAGNETIC WAVES

The Lorentz Force

Lorentz force, the force exerted on a charged particle q moving with velocity v through an electric field E and magnetic field B . The entire electromagnetic force F on the charged particle is called the Lorentz force (after the Dutch physicist Hendrik A. Lorentz) and is given by

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}.$$

Basic Equations Of Electromagnetism

1. Gauss' law of electricity

$$\int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

2. Gauss' law of magnetism

$$\int \vec{B} \cdot d\vec{A} = 0$$

3. Faraday's law of induction

$$\int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \oint \vec{B} \cdot d\vec{S}$$

4. Ampere's law

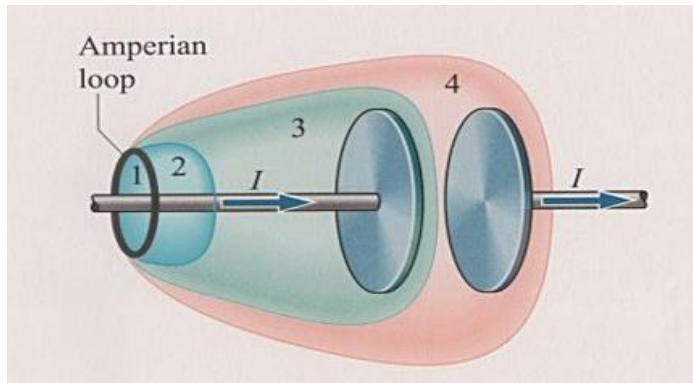
$$\int \vec{B} \cdot d\vec{s} = \mu_0 I$$

Maxwell's Displacement Current

According to Faraday's law of electromagnetic induction, a time-varying magnetic field induces an emf, According to Maxwell, an electric field sets up a current and hence a magnetic field. Such a current is called displacement current.

If the surface is chosen as 1, 2 or 4, the

enclosed current = I



James Clerk Maxwell realized around 1865 that the equations of electricity & magnetism were inconsistent.

$$\left[\int \vec{B} \cdot d\vec{s} \right]_{(1,2,4)} \neq \left[\int \vec{B} \cdot d\vec{s} \right]_3$$

Maxwell generalized Ampere's law,

$$\int \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d)$$

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\text{As } I = \frac{dQ}{dt} = \frac{d}{dt}(\epsilon_0 EA)$$

$$= \epsilon_0 \frac{d(EA)}{dt}$$

$$= \epsilon_0 \frac{d\phi_E}{dt} \equiv I_D$$

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

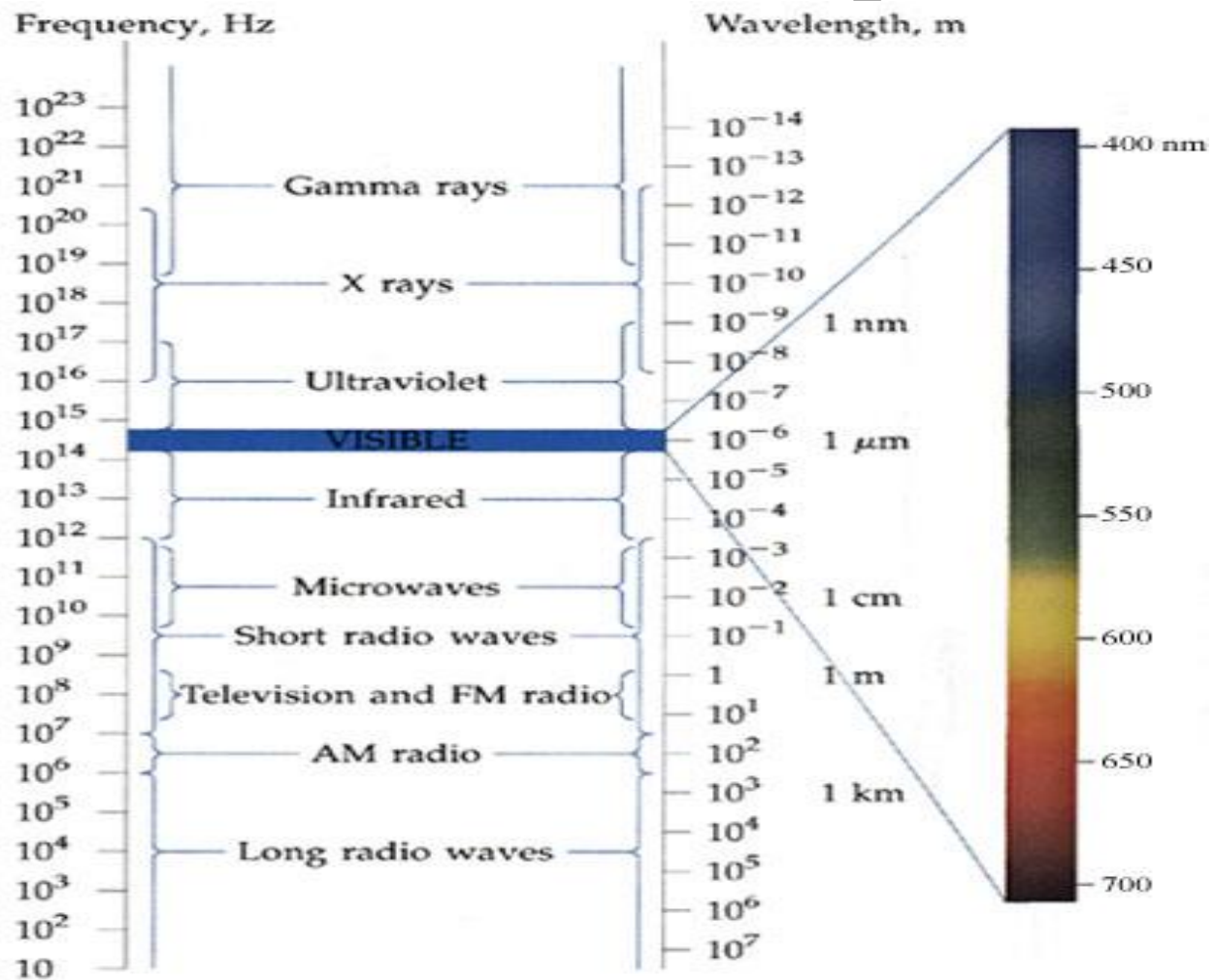
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

Lorentz Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Example: Red light has $\lambda = 700 \text{ nm}$. Calculate the frequency, ν .

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{7.00 \times 10^{-7} \text{ m}} = 4.29 \times 10^{14} \text{ Hz}$$

Comparison the electromagnetic waves



Dipole radiation pattern

- Oscillating electric dipole generates e-m radiation that is linearly polarized in the direction of the dipole
- Radiation pattern is doughnut shaped & outward traveling
 - zero amplitude above and below dipole

- maximum amplitude in-plane

Amplitude Modulation (AM)

Amplitude Modulation (AM) uses changes in the signal strength to convey information.

Frequency Modulation

Frequency Modulation (FM) uses changes in the wave's frequency to convey information

Receiving radio waves

Radio waves are received by another antenna attached to a radio receiver. When radio waves strike the receiving antenna they push the electrons in the metal back and forth, creating tiny oscillating currents which are detected by the receiver.

Converting back to sound: AM

- AM is easy: just pass the AC signal from the antenna into a diode
 - or better yet, a diode bridge
 - then use capacitor to smooth out bumps
- but not so much as to smooth out audio bumps

POLARIZED LIGHT

Polarization is a property applying to transverse waves that specifies the geometrical orientation of the oscillations. In a transverse wave, the direction of the oscillation is perpendicular to the direction of motion of the wave.

Unpolarized Light

Most sources – a candle, the sun, any light bulb – produce light that is unpolarized.

- No definite direction of the electric field
- No definite phase between orthogonal components

- The atomic or molecular dipoles that emit the light are randomly oriented in the source

Polarization

A typical plane electromagnetic wave:

$$E_x = E_0 \sin(kz - \omega t) \quad E_y = 0 \quad E_z = 0$$

$$B_x = 0 \quad B_y = \frac{E_0}{c} \sin(kz - \omega t) \quad B_z = 0$$

This wave is an example of a linearly polarized wave.

The direction of polarization is the direction of the oscillation of the electric field vector, in this case x.

Polarization

A wave traveling in the z direction can be polarized at an arbitrary angle θ in the x-y plane.

$$E_x = E_0 \cos \theta \cdot \sin(kz - \omega t)$$

$$E_y = E_0 \sin \theta \cdot \sin(kz - \omega t)$$

$$E_z = 0$$

Linear Polarization

Linear polarization: the electric field of light is confined to a single plane along the direction of propagation (Figure 1). Circular polarization: the electric field of the light consists of two linear components that are perpendicular to each other, equal in amplitude, but have a phase difference of $\pi/2$.

Summary of Lecture 31 – LIGHT

Speed of Light

Speed of light, speed at which light waves propagate through different materials. In particular, the value for the speed of light in a vacuum is now defined as exactly 299,792,458 meters per second.

1675: Roemer

Studies timing of eclipse of moons of Jupiter with Earth at A Then at C, eclipse is 16.6 minutes late. Estimated $c \approx 2.3 \cdot 10^8$ m/s

$c = \text{Round Trip Distance} / \text{Total Time}$

$$\frac{2L}{c} = \frac{\theta}{\omega}$$

$$c = \frac{2L\omega}{\theta}$$

$$c = 3 \times 10^8 \text{ m/s}$$

1860: Foucault

Change mirror rotation speed, no signal unless mirror rotates 1/8 turn in ΔT
 $c = 2.98 \cdot 10^8$ m/s Light travels VERY FAST – around 300,000 kilometres per second. At this speed it can go around the world 8 times in one second.

Normal Daylight

- Note the hump at short wavelengths - the sky is blue
- Other bumps came from solar emission spectra and atmospheric adsorption.

Tungsten Light bulb

- Most light sources are not anywhere near white
- It is a major research effort to develop light sources with particular properties

Helium Neon Laser

- Lasers emit light at a single wavelength, hence they appear colored in a very “pure” way

White

- Note that color and intensity are technically two different things
- However, in common usage we use color to refer to both

White = grey = black in terms of color

Visible Electromagnetic Spectrum

The visible light spectrum is the segment of the electromagnetic spectrum that the human eye can view. More simply, this range of wavelengths is called visible light. Typically, the human eye can detect wavelengths from 380 to 700 nanometers.



Seeing colour

The colour an object appears depends on the colours of light it reflects.

A white hat would reflect all seven colours:

Colour

White light is not a single colour

Red Orange Yellow Green Blue Indigo Violet

The Law of Reflection

The law of reflection states that, on reflection from a smooth surface, the angle of the reflected ray is equal to the angle of the incident ray.

Angle of incidence = Angle of reflection

Clear vs. Diffuse Reflection

Regular reflection occurs when light reflects off a very smooth surface and forms a clear image. Diffuse reflection occurs when light reflects off a rough surface and forms a blurry image or no image at all.

$$\text{Refractive index} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in material}}$$

$$n = \frac{c}{v}$$

Some indices of refraction for various substances at 590 nm:

Substance	Refractive index
Air	1.00029
Water	1.33
Ethyl alcohol	1.36
Fused quartz	1.46
Glycerine	1.47
Glass	1.45-1.70
Oil	1.50
Zircon	1.92
Diamond	2.42

FERMAT'S PRINCIPLE

Fermat's principle states that “light travels between two points along the path that requires the least time, as compared to other nearby paths.” From Fermat's principle, one can derive (a) the law of reflection [the angle of incidence is equal to the angle of reflection] and (b) the law of refraction [Snell's law].

Principle of Least Time

Fermat's principle - light travels in straight lines and will take the path of least time. A light ray traveling from one fixed point to another fixed point follows a path such that, compared with nearby paths, the time required is minimum.

$$L = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d-x)^2}, t = L / c$$

$$0 = \frac{dt}{dx} = \frac{1}{c} \frac{dL}{dx}$$

$$= \frac{1}{2c} (a^2 + x^2)^{-1/2} (2x)$$

$$+ \frac{1}{2c} [b^2 + (d-x)^2]^{1/2} (2)(d-x)(-1)$$

$$\therefore \frac{x}{\sqrt{a^2 + x^2}} = \frac{d-x}{\sqrt{b^2 + (d-x)^2}}$$

$$\sin \theta_1 = \sin \theta_1' \quad \text{Law of Reflection}$$

$$t = \frac{L_1}{v_1} + \frac{L_2}{v_2}, n = \frac{c}{v} \Rightarrow t = \frac{n_1 L_1 + n_2 L_2}{c} = \frac{L}{c}$$

$$L = n_1 L_1 + n_2 L_2 \quad (\text{Optical path length})$$

$$L = n_1 \sqrt{a^2 + x^2} + n_2 \sqrt{b^2 + (d-x)^2}$$

$$0 = \frac{dt}{dx} = \frac{1}{c} \frac{dL}{dx} = \frac{n_1}{2c} (a^2 + x^2)^{-1/2} (2x) \\ + \frac{n_2}{2c} [b^2 + (d-x)^2]^{1/2} (2)(d-x)(-1)$$

$$n_1 \frac{x}{\sqrt{a^2 + x^2}} = n_2 \frac{d-x}{\sqrt{b^2 + (d-x)^2}}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{Law of Refraction}$$

TOTAL INTERNAL REFLECTION

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

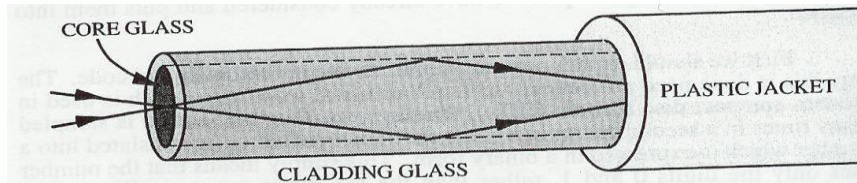
$$\theta_c = \sin^{-1} \frac{n_2}{n_1}$$

Critical Angles & Mirages

The angle a light ray in water makes with the normal is always less than the angle the light ray in air makes with the normal. This there must be a certain angle at which light rays traveling in water will, after refraction, pass directly along the surface. The angle at which this happens is called the critical angle.

Optical Fibre Cable

Optical fibers are used most often as a means to transmit light between the two ends of the fiber and find wide usage in fiber-optic communications, where they permit transmission over longer distances and at higher bandwidths (data transfer rates) than electrical cables.



Radiation Pressure

The electric field of an electromagnetic wave that strikes a surface acts on an electron, giving it a velocity v . The magnetic field then exerts a force on the moving charge in the direction of propagation of the incident light. That is the origin of the “light pressure”.

Determining Radiation Pressure

- This is an apparatus for measuring radiation pressure
- About $5 \times 10^{-6} \text{ N/m}^2$
- In practice, the system is contained in a vacuum
- The pressure is determined by the angle at which equilibrium occurs

Radiant Flux

LONG

- Total amount of energy passing through a surface or region of space per unit time
 - Typically denoted by Φ (Phi)
 - Also called power
 - Measured in watts (W) or joules/second (J/s)
 - These are metric quantities (rather than eg. calories/second)

- Typically used for a light's total output
 - You talk about a 60W light bulb
 - Only part of the story, also need distribution of power over space
- A camera integrates, over the time the shutter is open, the power arriving at the film

Elone Musk

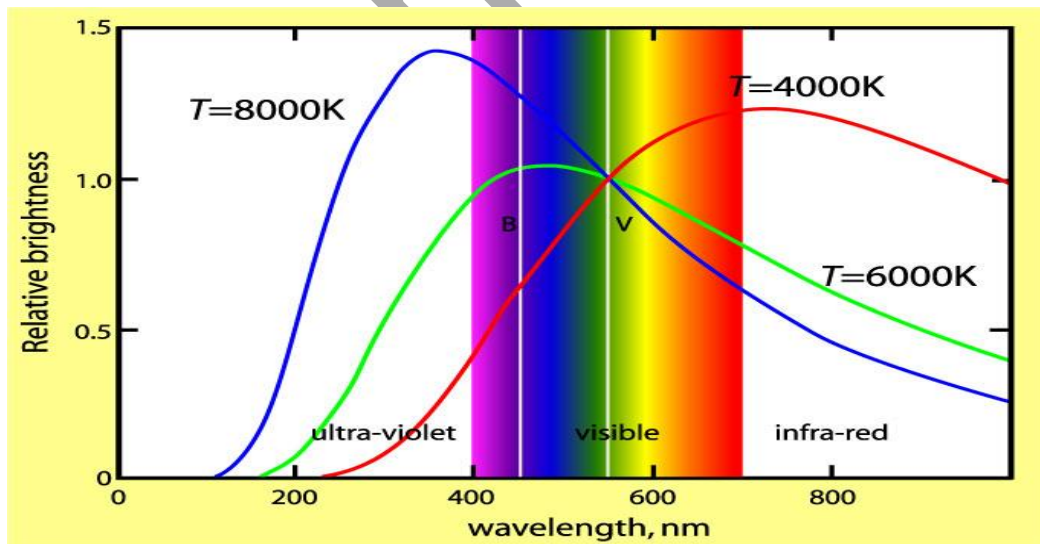
1. emission – matter releases energy as light
2. absorption – matter takes energy from light
3. transmission – matter lets light to pass through it
4. reflection – matter repels light in another direction

BLACK BODY RADIATION

Blackbody radiation is a term used to describe the relationship between an object's temperature, and the wavelength of electromagnetic radiation it emits. A black body is an idealized object that absorbs all electromagnetic radiation it comes in contact with.

We will do this experiment next week when we study temperature. We will use a domestic video camera to see the infra-red light.

- The filament of a normal light bulb is heated to about 2500 C to make it give off 'white' light
- When something is at about 800 C: it is red hot
- When colder, it gives off only infra-red light. IR light is absorbed by molecules in our skin.



Wien's Law: $\lambda_{\text{max}} T = 2.90 \times 10^{-3} \text{ m K}$

A dense object at any temperature emits at all wavelengths, so the spectrum is smooth without gaps. An ideal object is called a blackbody.

- Since the earth's ambient temperature is about 300 K, Wien's law shows that the peak occurs at a wavelength of about 9.7 μm (thermal IR energy).
- The sun's energy peak is at 0.5 μm, so when the sun is present, earth features can be observed by virtue of reflected solar energy.

Wien's Law

- Wien's law allows astronomers to determine the temperature of a star.
- The wavelength at which a star is brightest is related to its temperature
- Hotter objects radiate more strongly at shorter wavelengths
- Blue has a shorter wavelength than red, so hotter objects look bluer.

Energy and Radiation

Radiation is energy that comes from a source and travels through space at the speed of light. This energy has an electric field and a magnetic field associated with it, and has wave-like properties. You could also call radiation "electromagnetic waves".

- All heated matter radiates energy
- Hotter objects radiate more energy:

$$P = \sigma T^4$$

(power radiated per unit area at temperature T)

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

This is called the Stefan-Boltzmann Law for black-body radiation.

Spectra

A band of colours, as seen in a rainbow, produced by separation of the components of light by their different degrees of refraction according to wavelength.

- Each chemical element produces a very distinct pattern of colors called a spectrum
- Spectra that has all colors = Continuous
- Spectra that has only certain frequencies = Line Spectra of which there are 2 types:
 - Absorption: Some material in front of a continuous source absorbs some energy.
 - Emission: Some material giving off energy.

ABSORPTION OF LIGHT

Light absorption is a process by which light is absorbed and converted into energy. An example of this process is photosynthesis in plants. However, light absorption doesn't occur exclusively in plants, but in all creatures/inorganic substances.

Atmospheric absorption

Atmosphere absorbs radiation at most wavelengths

Atmosphere protects us too!

Molecular absorption bands in constituents of the Earth's atmosphere due to:

- oxygen
- nitrogen
- ozone
- water
- Carotene: makes carrots orange, tomatoes red, sarson yellow – must absorb blue light
- Long, organic molecular chain resonances in optical light
- Chlorophyll makes leaves green and must absorb red and blue

Polarization

Polarization (also polarization) is a property applying to transverse waves that specifies the geometrical orientation of the oscillations. In a transverse wave, the direction of the oscillation is perpendicular to the direction of motion of the wave.

$$E_x = E_0 \sin(kz - \omega t) \quad E_y = 0 \quad E_z = 0$$
$$B_x = 0 \quad B_y = \frac{E_0}{c} \sin(kz - \omega t) \quad B_z = 0$$

This wave is an example of a *linearly* polarized wave.

The direction of polarization is the direction of the oscillation of the electric field vector, in this case x.

A wave traveling in the z direction can be polarized at an arbitrary angle θ in the x-y plane.

$$E_x = E_0 \cos \theta \cdot \sin(kz - \omega t)$$
$$E_y = E_0 \sin \theta \cdot \sin(kz - \omega t)$$
$$E_z = 0$$

Polarization by Scattering

Scattered light is preferentially polarized perpendicular to the plane of the scattering.

- For example, assume the incident unpolarized light is moving in the z-direction.
- Scattered light observed along the x-direction will be polarized along the y-direction.
- Scattered light observed along the y-direction will be polarized along the x-direction.

Polarizing Material

A Polarizing material will only allow the passage of that component of the electric field parallel to the polarization direction of the material.

Polarizer & Unpolarized Light

Unpolarized light is the superposition of many waves, each with random polarization direction q , relative to fixed axis of polarizer. Each wave is attenuated by factor $\cos^2 q$. Average attenuation is $\langle \cos^2 q \rangle = \frac{1}{2}$

BIREFRINGENCE

Birefringence is formally defined as the double refraction of light in a transparent, molecularly ordered material, which is manifested by the existence of orientation-dependent differences in refractive index.

Dipoles of the molecules orient along an externally applied electric field.

Changing the field direction will

- change the birefringence
- change the polarization of transmitted light

Then pass through polarization analyzer to change the intensity

- Digital displays, LCD monitors, etc.

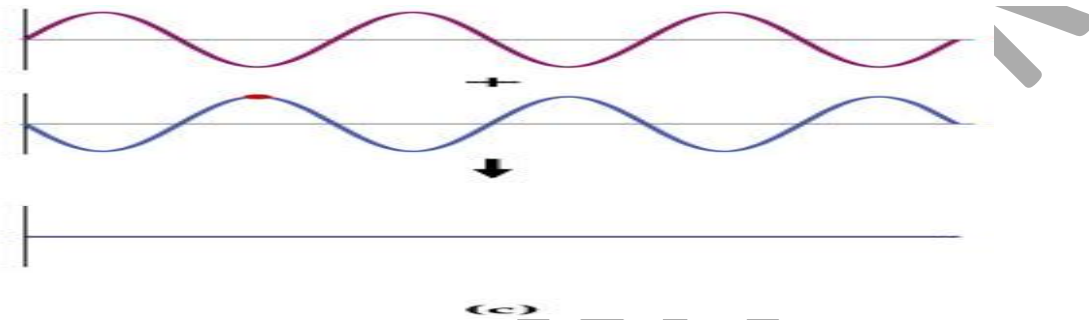
Summary of Lecture 33 – INTERFERENCE AND DIFFRACTION

Constructive Interference

Constructive interference occurs when the maxima of two waves add together (the two waves are in phase), so that the amplitude of the resulting wave is equal to the sum of the individual amplitudes. Equivalently, the minima of the waves would be aligned.

Destructive Interference

Destructive interference occurs when the maxima of two waves are 180 degrees out of phase: a positive displacement of one wave is cancelled exactly by a negative displacement of the other wave. The amplitude of the resulting wave is zero.



Interference for Light

- Can't produce coherent light from separate sources. ($f \approx 10^{14}$ Hz)
- Need two waves from single source taking two different paths
 - Two slits
 - Reflection (thin films)
 - Diffraction

Two-Slit Interference

An incoherent light source illuminates the first slit. This creates a uniform but coherent illumination of the second screen (from side-to-side on the screen, the light wave has the same oscillating phase). The two waves from the two slits S1 and S2 create a pattern of alternating light and dark fringes on the third screen.

Path difference:

$$\Delta l = d \sin \theta$$

Conditions for bright fringes (constructive interference):

$$d \sin \theta = m\lambda \quad (m = 0, \pm 1, \pm 2...)$$

Conditions for dark fringes (destructive interference):

$$d \sin \theta = (m - \frac{1}{2})\lambda \quad (m = 0, \pm 1, \pm 2...)$$

m is called the order number

Interference in an Air Wedge

Constructive interference: $\frac{1}{2} + \frac{2d}{\lambda} = m \quad (m = 1, 2, 3...)$

Destructive interference: $\frac{1}{2} + \frac{2d}{\lambda} = m + \frac{1}{2} \quad (m = 0, 1, 2...)$

Interference In Thin Films

Thin-film interference is the phenomenon that is a result of lightwave being reflected off two surfaces that are at a distance comparable to its wavelength. When light waves that reflect off the top and bottom surfaces interfere with one another we see different coloured patterns.

Constructive interference: $2nd = (m + \frac{1}{2})\lambda \quad (m = 0, 1, 2, ...)$

Destructive interference: $2nd = m\lambda \quad (m = 0, 1, 2, ...)$

Diffraction

The bending of light around objects into what would otherwise be a “shadowed” region is known as diffraction.

Diffraction occurs when light passes through very small apertures or near sharp edges.

We can get an interference pattern with a single slit provided its size is approximately λ (neither too small nor too large)

Single-Slit Diffraction

Conditions for dark fringes in single-slit interference $W \sin \theta = m\lambda$ $m = \pm 1, \pm 2, \pm 3 \dots$

Resolution

The resolving power, or resolution, of a mass spectroscopy is a measure of its ability to separate adjacent masses that are displayed as peaks on the detector. If two peaks due to mass m and $(m + \Delta m)$ can just be separated, the resolving power is $m/\Delta m$.

First dark fringe:

$$\sin \theta = 1.22 (\lambda/D)$$

D = diameter of the aperture

Rayleigh's criterion: $\theta_{\min} = 1.22 \frac{\lambda}{D}$

- Two objects can be barely resolved if the diffraction maximum of one object lies in the diffraction minimum of the second object.
- The larger diameter D , the smaller the angular diameter separation (greater resolution)

Summary of Lecture 34 – THE PARTICLE NATURE OF LIGHT

Energy carried by EM waves is shared equally by the electric and magnetic fields

$$\text{Average power per unit area} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{c B_{\max}^2}{2\mu_0}$$

➤ Electromagnetic waves transport linear momentum as well as energy

- For complete absorption of energy U , $p = \frac{U}{c}$
- For complete reflection of energy U , $p = \frac{2U}{c}$
- Radiation pressures can be determined experimentally

Determining Radiation Pressure

- This is an apparatus for measuring radiation pressure
- About $5 \times 10^{-6} \text{ N/m}^2$
- In practice, the system is contained in a vacuum
- The pressure is determined by the angle at which equilibrium occurs

OBSERVED:

The photoelectric effect does not occur for all frequencies; it does not occur at all when ν is below a certain value.

CLASSICAL:

The photoelectric effect should occur in all case (any frequency or wavelength).

Planck:

Energy can only be absorbed or released from atoms in certain amounts called quanta.

$$E = h\nu$$

$$\left(\text{ or } E = \hbar\omega \text{ where } \hbar = \frac{h}{2\pi} \text{ and } \omega = 2\pi\nu \right)$$

h is Planck's constant $6.626 \times 10^{-34} \text{ Joule-seconds}$.

The Energy of EM Radiation

The energy (E) carried by electromagnetic radiation is related to the frequency (ν). X-rays have a higher frequency and higher energy than radio waves for example of this phenomenon is the oscillating charge or current in a radio antenna.

The Bohr Atom

In 1913, Niels Bohr proposed a theory for the hydrogen atom, based on quantum theory that some physical quantities only take discrete values. Electrons move around a nucleus, but only in prescribed orbits, and if electrons jump to a lower-energy orbit, the difference is sent out as radiation.

$$F = ma = \frac{(e)(e)}{r_n^2} = m \frac{v^2}{r_n}$$

$$\frac{e^2}{r_n^2} = \frac{mv^2}{r_n}$$

$$L = mvr_n = n \frac{h}{2\pi} \equiv \hbar$$

$$v = \frac{n\hbar}{mr_n}$$

$$r_n = \frac{e^2 m r_n^2}{n^2 \hbar^2}$$

$$r_n = \frac{n^2 \hbar^2}{me^2}$$

$$r_1 = 0.529 \times 10^{-10}$$

$$r_n = n^2 r_1$$

$$PE = -eV = -\frac{e^2}{r}$$

$$E_n = \frac{1}{2}mv^2 - \frac{e^2}{r_n}$$

$$E_n = -\frac{e^4 m}{n^2 \hbar^2}$$

$$E_1 = -13.6 \text{ eV}$$

LASER

Long

LASER stand for

Light Amplification by the Stimulated Emission of Radiation

- Spontaneous emission: electron at high energy level drops to the lower level and gives off a photon.
- Stimulated emission: a photon with energy equal to the energy difference between the two energy levels *enhances* the probability that electron will drop to the lower level. The photon given off has the same energy, phase and direction as the incident photon.

Laser Cavities

Two flat mirrors, the “flat-flat” laser cavity, is difficult to align and maintain aligned.

Two concave curved mirrors, the “stable” laser cavity, is easy to align and maintain aligned.

Animation of Compton scattering:

Blue light comes in, scatters from an electron, and goes out as red light.

How many photons do we see?

- Sunny day (outdoors):
 - 10^{15} photons per second enter eye (2 mm pupil)
- Moonlit night (outdoors):
 - 5×10^{10} photons/sec (6 mm pupil)

- Moonless night (clear, starry sky)
 - 10^8 photons/sec (6 mm pupil)
- Light from dimmest naked eye star (mag 6.5):
 - 1000 photons/sec entering eye

Elone Musk

Summary of Lecture 35 – GEOMETRICAL OPTICS

Spherical lenses

LONG

Spherical lenses also sometimes referred to as singlets are optical lenses that feature a spherical surface with a radius of curvature that is consistent across the entire lens. They are constructed such that the light entering them diverges or converges, depending on the lens design.

Types of Spherical Single Thin Lens

The two main types of lenses are:

- Convex Lens (Converging)
- Concave Lens (Diverging)

Spherical Concave or Spherical Convex Mirrors

Spherical mirrors whose inner side is reflecting are called concave mirrors. Spherical mirrors whose outer side is reflecting are called convex mirrors.

Spherical Concave (muaqqir) Mirrors

A concave mirror has a real focus: $f = R/2$

Spherical Convex Mirrors

A convex mirror has a virtual focus: $f = -R/2$

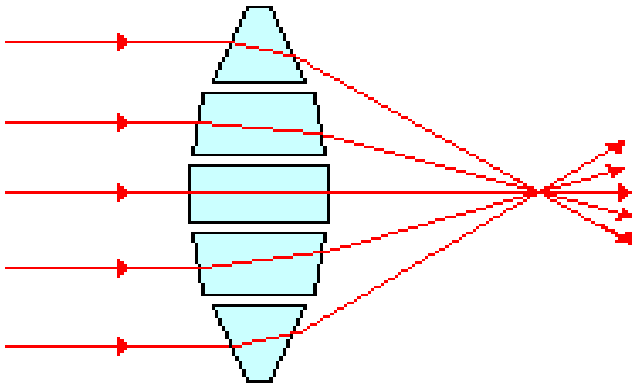
Focusing light with lens

The eye focuses light in a similar way to when you use a magnifying glass to concentrate the Sun's rays onto a piece of paper. The distance from the magnifying lens to the piece of paper is the focal length. For the eye, light from distant objects is focused onto the retina at the back of the eye.

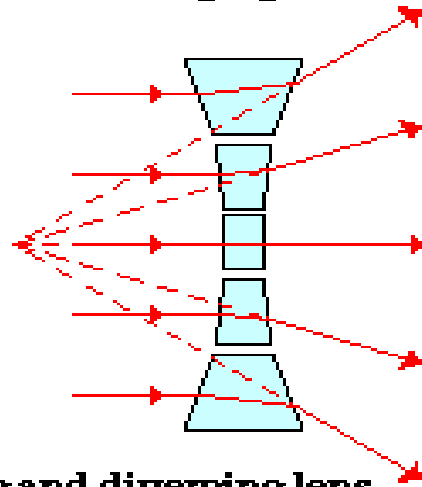
Divergent lens

A diverging lens is a lens that diverges rays of light that are traveling parallel to its principal axis. The fact that a double concave lens is thinner across its middle is an indicator that it will diverge rays of light that travel parallel to its principal axis. A double concave lens is a diverging lens.

Converging Prisms



Diverging Prisms



A set of prisms acting as a converging and diverging lens.

Lensmaker's Formula

Lens maker's formula is the relation between the focal length of a lens to the refractive index of its material and the radii of curvature of its two surfaces. It is used by lens manufacturers to make the lenses of particular power from the glass of a given refractive index.

Lensmaker's Formula $\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

Diopter

- If the focal length of a lens is expressed in meters, the diopter of the lens is defined as $D = 1/f$
- *The refractive index of a lens is n . The diopters due to the first interface diffraction, D_1 and the second interface diffraction, D_2 are*

$$D_1 = (n-1)/R_1$$

$$D_2 = (1-n)/R_2$$

The total diopter of the lens is

$$D = D_1 + D_2$$

MAGNIFICATION

Magnification is the process of enlarging the apparent size, not physical size, of something. This enlargement is quantified by a calculated number also called "magnification". When this number is less than one, it refers to a reduction in size, sometimes called minification or de-magnification

Magnification formula

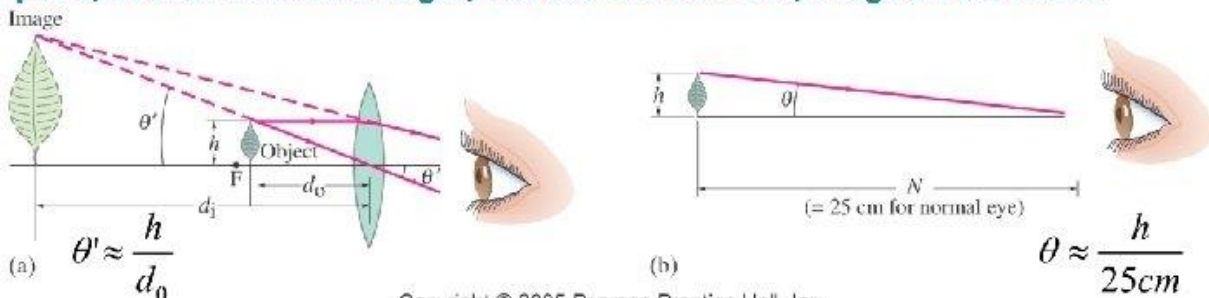
The level of magnification is proportional to the ratio of v and u . An image that is double the size of the object would have magnification $m=2$.

The Simple Magnifier

A simple magnifier is a converging lens and produces a magnified virtual image of an object located within the focal length of the lens. Angular magnification accounts for magnification of an image created by a magnifier.

5) Magnifying glass (Simple magnifier)

A converging lens that allows one to focus on objects closer than the near point, so that it makes a larger, and therefore clearer, image on the retina.



Copyright © 2005 Pearson Prentice Hall, Inc.

The power of a magnifying glass is described by its angular magnification:

$$M \equiv \frac{\theta'}{\theta} = \frac{25\text{cm}}{d_o}$$

$$\text{if } d_i \rightarrow -\infty \Rightarrow d_o \rightarrow f \Rightarrow$$

$$M = \frac{25\text{cm}}{f}$$

$$\text{if } d_i = -25\text{cm} \Rightarrow \frac{1}{d_o} = \frac{1}{f} + \frac{1}{25\text{cm}} \Rightarrow M = \frac{25\text{cm}}{f} + 1$$

Note!

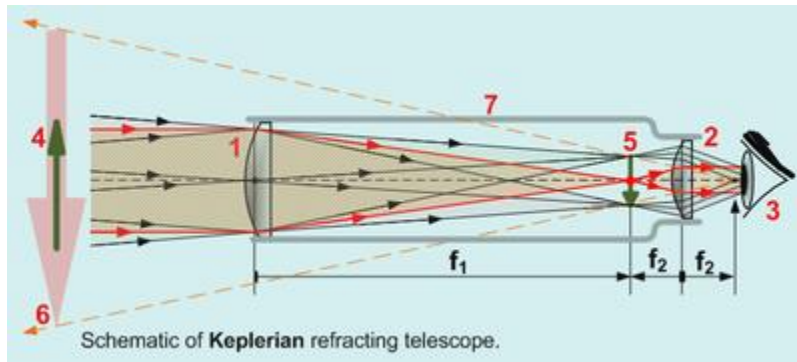
$$d_o \leq f \Rightarrow d_i < 0$$

$$d_o \rightarrow f \Rightarrow d_i \rightarrow -\infty \Rightarrow m = -\frac{d_i}{d_o} \rightarrow \infty$$

$$\frac{25\text{cm}}{f} \leq M \leq \frac{25\text{cm}}{f} + 1$$

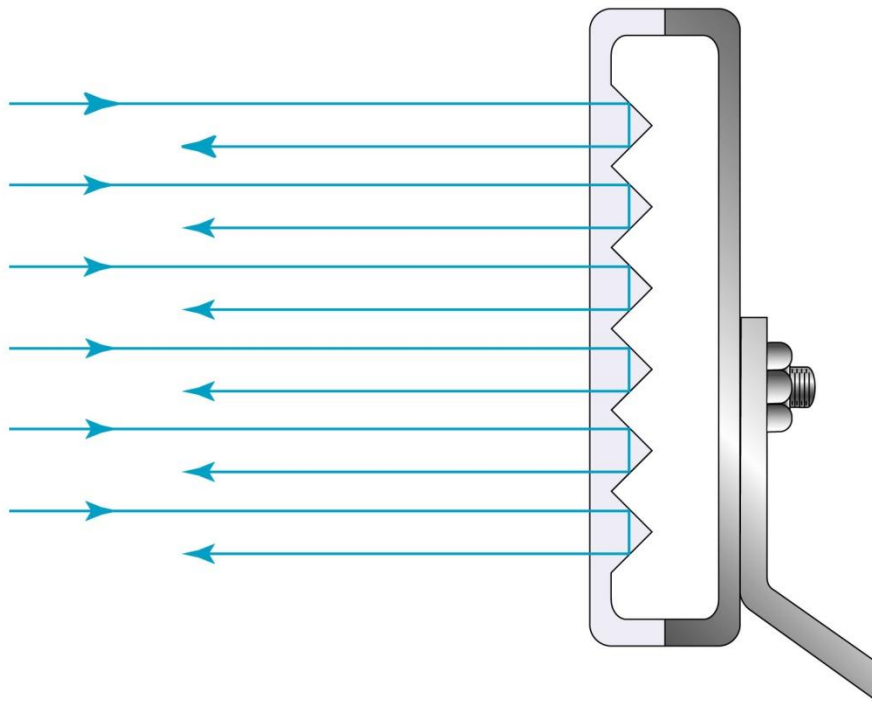
Refracting Telescope

A refracting telescope (also called a refractor) is a type of optical telescope that uses a lens as its objective to form an image (also referred to a dioptric telescope). The refracting telescope design was originally used in spyglasses and astronomical telescopes but is also used for long-focus camera lenses.



Prisms

A prism is a 3-dimensional shape with two identical shapes facing each other. These identical shapes are called “bases”. The bases can be a triangle, square, rectangle or any other polygon. Other faces of a prism are parallelograms or rectangles.



Summary of Lecture 36 – THERMAL PHYSICS I

Phlogiston

The ancient view was that heat is a color less, weightless; fluid which occupies no volume has no smell, etc. This imagined substance called phlogiston.

Heat (Q)

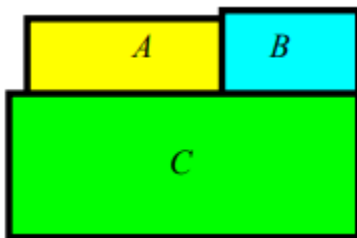
The term heat (Q) refers to energy in transit, thermal energy transferred into or out of a system at a certain temperature.

Heat is the flow of internal energy due to a temperature difference

High T  Low T

Thermal Equilibrium

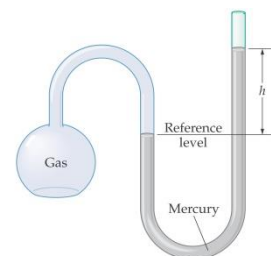
Objects that are in thermal contact, but have no heat exchange between them, are said to be in thermal equilibrium.



If systems A and B are each in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other.

Constant Volume Gas-Thermometer

- Keep the reference level fixed = fixed gas volume.
- Adjust height as temperature of gas is varied



- Pressure of gas = $\rho g h$

Temperature Scales

The three main temperature scales are Celsius, Fahrenheit, and Kelvin. Temperatures can be converted from one scale to other using temperature conversion equations. The three phases of water (ice, liquid water, and water vapor) can coexist at a single pressure and temperature known as the triple point.

Kelvin scale

The Kelvin scale is the most suited for scientific purposes, and you should be careful to use this in all heat related calculations

	°F	°C	K
Boiling point of water	212	100	373
Freezing point of water	32	0	273
Freezing point of dry ice (CO ₂)	-109	-78	195
Boiling point of nitrogen	-321	-196	77
Absolute zero	-460	-273	0

- Conversion between degrees Celsius and degrees Fahrenheit

$$T_F = \left(\frac{9}{5} F^\circ / C^\circ\right) T_C + 32^\circ F$$

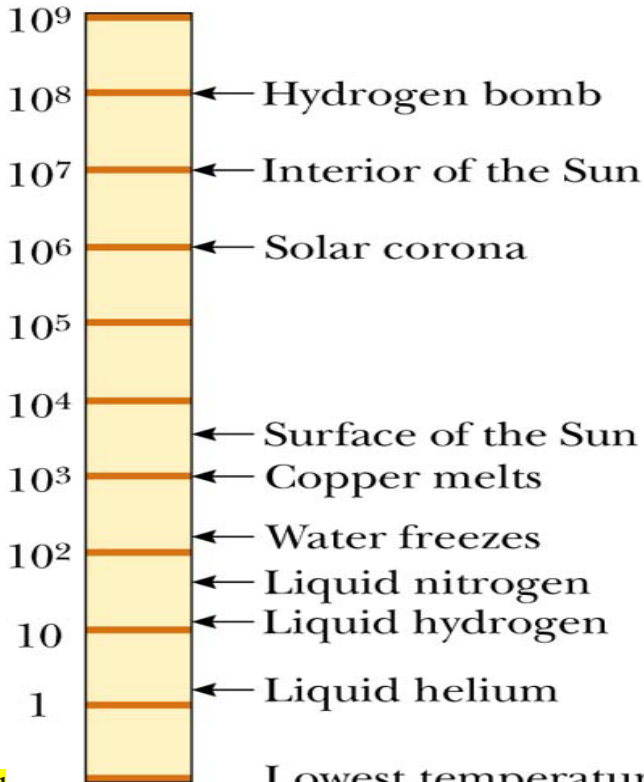
- Conversion between degrees Fahrenheit and degrees Celsius

$$T_C = \left(\frac{5}{9} C^\circ / F^\circ\right) (T_F - 32^\circ F)$$

- Conversion between Celsius and Kelvin temperatures

$$T = T_C + 273.15$$

Temperature (K)



Joule

Lowest temperature achieved $\sim 10^{-7}$ K

© 2003 Thomson - Brooks/Cole

Joule's experiments conclusively established that heat is a form of energy. Always the same amount of heat was produced by spending a given amount of mechanical work. It is immaterial what type of arrangement is used for doing mechanical work. It defines the relationship between Joule and Calories.

$$\text{Work} = \Delta(\text{Energy})$$

UNITS OF HEAT

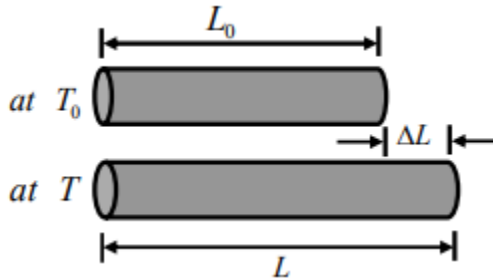
- Units: 1 calorie (1 cal) raises the temperature of 1 g of water by 1 °C
- 1 kilocalorie (1 kcal) = 1000 cal
- 1 cal = 4.186 Joule

It's called Mechanical equivalent of heat

Heat = Δ (Internal Energy)

MICROSCOPIC BASIS OF HEAT IS RANDOMNESS

Most substances expand when heated and contract when cooled.



$$L = L_0 [1 + \alpha (T - T_0)]$$

$$L - L_0 = \alpha L_0 (T - T_0)$$

$$\Delta L = \alpha L_0 \Delta T$$

$$\alpha = \frac{\Delta L / L_0}{\Delta T} \quad \text{Coefficient of linear expansion}$$

$$\beta = \frac{\Delta V / V_0}{\Delta T} \quad \text{Coefficient of volume expansion}$$

Relation between α and β

$$V' = (L + \Delta L)^3 = (L + \alpha L \Delta T)^3$$

$$= L^3 + 3\alpha L^3 \Delta T + 3\alpha^2 L^3 \Delta T^2 + \alpha^3 L^3 \Delta T^3$$

$$\approx L^3 + 3\alpha L^3 \Delta T$$

$$= V + 3\alpha V \Delta T$$

$$\Rightarrow \Delta V = 3\alpha V \Delta T$$

$$\Rightarrow \boxed{\beta = 3\alpha}$$

Thermostat

For winter, the ideal thermostat temperature is 68 degrees Fahrenheit when you're at home. Energy gov 68 degrees is a good room temperature while you're awake at home, but recommends lowering it while you're asleep or away. Lowering your thermostat 10-15 degrees for eight hours can reduce your heating bill by 5-15%.

THE STRANGE BEHAVIOUR OF WATER

Common examples include the fact that liquid water expands on cooling below 4°C, which is responsible for lakes freezing from the top rather than the bottom. In addition, the fact that water becomes less viscous when compressed, or its unusually high surface tension, allows insects to walk on water's surface.

Ice forms on the surface of bodies of water.

$$\rho_{4^{\circ}\text{C}} > \rho_{10^{\circ}\text{C}} \Rightarrow \text{sinks}$$



$$\rho_{0^{\circ}\text{C}} < \rho_{4^{\circ}\text{C}} \Rightarrow \text{floats}$$



Definition of Heat Capacity

It is defined as amount of heat required to raise the temperature of substance by one degree Celsius.

$$\text{Definition of Heat Capacity, } C = \frac{Q}{\Delta T}$$

Unit: $J/K = J/^\circ\text{C}$

Heat capacity is always positive

Heat Q and ΔT have the same sign:

- Q is positive if ΔT is positive; heat is added to a system.
- Q is negative if ΔT is negative; heat is removed from a system.

$$Q = mc\Delta T$$

- Q = heat added , m = mass , c = specific heat , ΔT = change in temperature

Water has a very large heat capacity; a lot of energy transfer (heat) is required to change its temperature.

- Water: $c = 1.0 \text{ cal / (}^\circ\text{C g)}$
- It takes one calorie to raise the temperature of 1 gm of water by 1 degree Celsius.

Specific Heats at Atmospheric Pressure

Substance	Specific heat, $c \text{ [J / (kg} \cdot \text{K)]}$
Water	4186
Ice	2090
Steam	2010
Beryllium	1820
Air	1004
Aluminum	900
Glass	837
Silicon	703
Iron (steel)	448
Copper	387
Silver	234
Gold	129
Lead	128


$$Q = kA \left(\frac{\Delta T}{L} \right) t$$

k = thermal conductivity

Q = heat transferred

A = cross sectional area

t = duration of heat transfer

L = length

ΔT = temperature difference

Summary of Lecture 37 – THERMAL PHYSICS II

Thermodynamic variables

The state of a system is specified by giving its pressure, volume, temperature, etc. These are called thermodynamic variables.

Equation of state relates P , V , T .

- $P=f(V,T)$
- $V=g(P,T)$
- $T=h(P,V)$

For an ideal gas $PV=Nk_B T$ Here N is the number of molecules in the gas, T is the temperature in degrees Kelvin, k_B and is called the Boltzman constant.

Internal Energy

Internal energy is defined as the energy associated with the random, disordered motion of molecules. It is separated in scale from the macroscopic ordered energy associated with moving objects; it refers to the invisible microscopic energy on the atomic and molecular scale.

$$E = \frac{3}{2} Nk_B T$$

First Law of Thermodynamics

The First Law of Thermodynamics states that heat is a form of energy, and thermodynamic processes are therefore subject to the principle of conservation of energy. This means that heat energy cannot be created or destroyed.

$$\Delta E = \Delta q + \Delta w$$

Where:

- $E \equiv$ Internal energy of a system
- $q \equiv$ Heat transferred to a system from the surroundings.
- $W \equiv$ Work done on a system by the surroundings.

q and w are positive if heat is added to or work done on a system; negative if reverse.

Path variables

Work and heat are path variables their values depend on the steps leading from one state to another.

BUT the internal energy does not depend on the path $\int_{\text{State1}}^{\text{State2}} dE = E_2 - E_1 = \Delta E$.

WORK DONE BY AN EXPANDING GAS

For expansion (work done by the gas on its surroundings) $dw = -P_{\text{ext}} dV$ and $w = -\int_{V_1}^{V_2} P_{\text{ext}} dV$.

Why negative?

because work is being done on the surroundings by our closed system, P_{ext} and dV are both positive.

Consider three possible cases:

1. Expansion into a vacuum

in this case, $P_{\text{ext}} = 0$,

$$\Rightarrow w = 0$$

2. Expansion against a constant external pressure

$\Rightarrow P_{\text{ext}} = \text{constant}$ (no dependence upon V)

$$\text{and } w = -P_{\text{ext}} (V_2 - V_1) = -P_{\text{ext}} \Delta V$$

3. A limiting case, such that $P_{\text{int}} = P_{\text{ext}}$ at every instant of the expansion – a reversible process.

$P_{\text{ext}} = P_{\text{int}}$ (recall $PV = NkT$)

$$P_{\text{ext}} = \frac{NkT}{V}$$

$$w = -\int_{V_1}^{V_2} \frac{NkT}{V} dV = -NkT \ln \frac{V_2}{V_1}$$

Depending upon the conditions, the amount of work produced to reach the same state differs.

The Internal Energy of An Ideal Gas

$$E_i = \frac{3}{2} NkT, \quad E_f = \frac{3}{2} NkT$$

For an isothermal process,

$$PV = Nk_B T$$

$$dW = PdV$$

$$= Nk_B \frac{T}{V} dV$$

$$W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{Nk_B T}{V} dV$$

$$W = Nk_B T \ln \frac{V_2}{V_1}$$

$$Q_V = C_V \Delta T$$

Since, $W = 0$

$$Q_V = \Delta E + W = \Delta E$$

$$\Delta E = C_V \Delta T$$

As $\Delta T \rightarrow 0$

$$dE = C_V dT$$

$$C_V = \frac{dE}{dT}$$

$$Q_p = C_p \Delta T$$

$$Q_p = \Delta U + W = \Delta U + P\Delta V$$

$$C_p \Delta T = \Delta U + P\Delta V$$

As $\Delta T \rightarrow 0$

$$C_p dT = dU + PdV$$

$$C_p dT = C_V dT + PdV$$

As $PV = Nk_B T$

$$PdV = Nk_B dT$$

$$C_p dT = C_v dT + Nk_B dT$$

$$C_p = C_v + Nk_B$$

$$U = \frac{3}{2} Nk_B T$$

$$C_v = \frac{dU}{dT} = \frac{3}{2} Nk_B$$

$$C_p = \frac{3}{2} Nk_B + Nk_B$$

$$C_p = \frac{5}{2} Nk_B$$

Heat Capacities of Gases

Rigid dumbbell model of diatomic molecule

$$K = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2 + \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2$$

$$E = 5 \times \left(\frac{1}{2} Nk_B T \right) = \frac{5}{2} Nk_B T$$

$$C_v = \frac{5}{2} Nk_B$$

The Quasi-static Adiabatic Expansion of a Gas

A quasi-static adiabatic expansion of an ideal gas produces a steeper pV curve than that of the corresponding isotherm. A realistic expansion can be adiabatic but rarely quasi-static.

$$dQ = dE + dW = 0$$

$$C_v dT + PdV = 0$$

$$C_v dT + Nk_B T \frac{dV}{V} = 0$$

$$\frac{dT}{T} + \frac{Nk_B}{C_v} \frac{dV}{V} = 0$$

$$\text{As } C_p - C_v = Nk_B$$

$$\frac{Nk_B}{C_v} = \frac{C_p - C_v}{C_v}$$

$$\text{Define } \frac{C_p}{C_v} \equiv \gamma$$

$$\frac{dT}{T} + (\gamma - 1) \frac{dV}{V} = 0$$

$$\ln T + (\gamma - 1) \ln V = \text{Constant}$$

$$\ln(TV^{\gamma-1}) = \text{Constant}$$

$$TV^{\gamma-1} = \text{Constant}$$

$$\frac{PV}{Nk_B} V^{\gamma-1} = \text{Constant}$$

$$PV^\gamma = \text{Constant}$$

$$dQ = dU + dW = 0$$

$$C_V dT + dW = 0 \Rightarrow dW = -C_V dT$$

$$W_{\text{adiabatic}} = dW = -\int C_V dT = -C_V \Delta T$$

$$W_{\text{adiabatic}} = -C_V \Delta T = C_V (T_1 - T_2)$$

$$W_{\text{adiabatic}} = C_V \left(\frac{P_1 V_1}{Nk_B} - \frac{P_2 V_2}{Nk_B} \right) = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

SECOND LAW OF THERMODYNAMICS

There can be no process whose only final result is to transfer thermal energy from a cooler object to a hotter object.

$$Q = \Delta U + W = W$$

$$W = Q_{\text{in},h} - Q_{\text{out},c}$$

$$\begin{aligned} \varepsilon &= \frac{W}{Q_{\text{in},h}} = \frac{Q_{\text{in},h} - Q_{\text{out},c}}{Q_{\text{in},h}} = 1 - \frac{Q_{\text{out},c}}{Q_{\text{in},h}} \\ &= 1 - \frac{T_C}{T_H} \end{aligned}$$

$$\varepsilon = 1 - \frac{T_C}{T_H}$$

$$\text{Coefficient of performance} = \frac{Q_c}{W}$$

It is impossible for a refrigerator working in a cycle to produce no other effect than the transfer of thermal energy from a cold object to a hot object.

$$ds = \frac{dQ}{T}$$

Entropy of an ideal gas :

$$dQ = dU + dW = dU + PdV$$

$$dQ = C_v dT + Nk_B T \frac{dV}{V}$$

$$dS = \frac{dQ}{T} = C_v \frac{dT}{T} + Nk_B \frac{dV}{V}$$

$$\Delta S = \int \frac{dQ}{T} = C_v \ln \frac{T_2}{T_1} + Nk_B \ln \frac{V_2}{V_1}$$

Summary of Lecture 38 – THERMAL PHYSICS III

ENTROPY

Entropy, the measure of a system's thermal energy per unit temperature that is unavailable for doing useful work. Because work is obtained from ordered molecular motion, the amount of entropy is also a measure of the molecular disorder, or randomness, of a system.

$$ds = \frac{dQ}{T}$$
$$\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T}$$
$$= \frac{3}{2} Nk_B \ln \frac{T_2}{T_1} + Nk_B \ln \frac{V_2}{V_1} \text{ (for an ideal gas)}$$

What must happen for a substance to change phase?

Heat must be added (or extracted) to initiate this phase transition while system remains at constant temperature.

latent heat of fusion L_F liquid $\rightleftharpoons$ solid

$Q = m L_F$ = heat absorbed (released) to melt (freeze) a mass of material

latent heat of vaporization L_V liquid $\rightleftharpoons$ gas

$Q = m L_v$ = heat absorbed (released) to vaporize (condense) a mass of material

Heat of Transformation

How much heat must be added to change ice to water to steam?

To change:

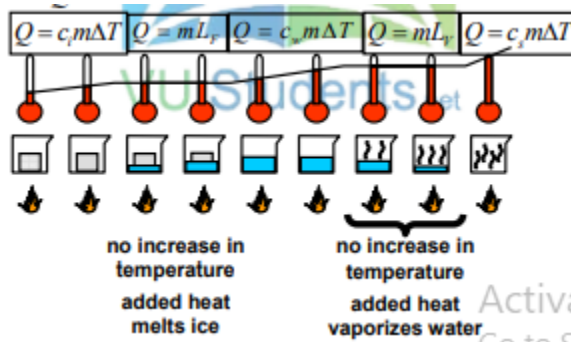
temp. of ice by 1 oC 0.5 cal / g $\Delta Q = c_i m \Delta T$

ice to water 80 cal / g $\Delta Q = m L_F$

temp. of water by 1 oC 1.0 cal / g $\Delta Q = c_w m \Delta T$

water to steam 540 cal / g $\Delta Q = m L_v$

temp. of steam by 1 oC 0.5 cal / g $\Delta Q = c_s m \Delta T$

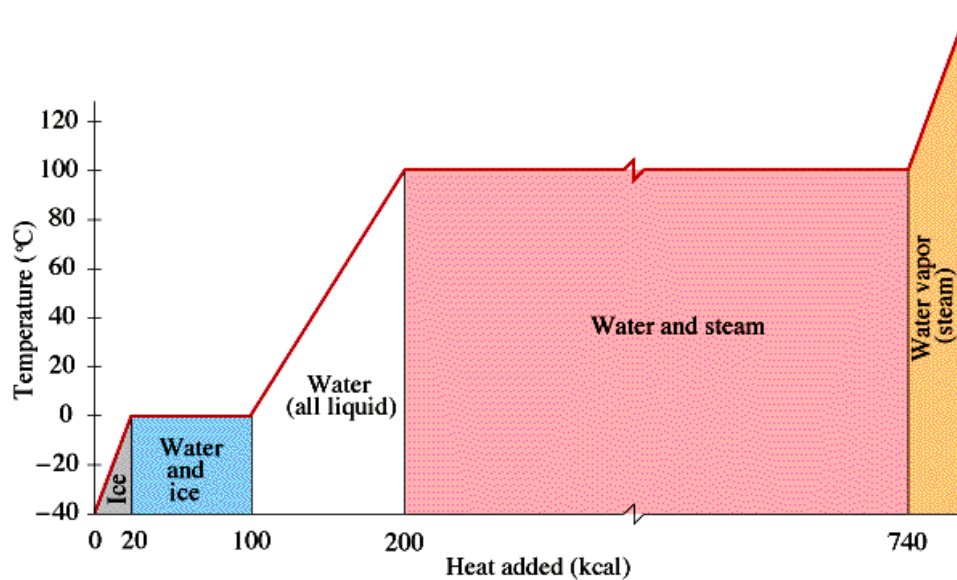


Animate this slide from left to right.

That is: start from first (thermometer, beaker, flame) then show 2nd, 3rd, 4th till last .

Temperature as a function of the heat

Temperature as a function of the heat added when bringing 1 kg of ice from -40°C to steam at 120°C .



$$Q = c_i m \Delta T$$

- Water $\rightarrow$ Steam
- Steam $\rightarrow$ Dissociates into atoms

- Atoms $\rightarrow$ Ionization
- Ions $\rightarrow$ nuclei + electrons

Evaporation

Water molecules with high enough KE can break free from the puddle and escape.

$\Rightarrow$ Water evaporates

How does a water molecule get enough energy to escape?

From random intermolecular collisions.



Water molecules left behind in the puddle are slow ones.

$\Rightarrow$ average KE is reduced

$\Rightarrow$ the puddle cools off

Hot day: Lots of molecules have high KE $\Rightarrow$ rapid evaporation

Humid day: Plenty of water from air joins puddle $\Rightarrow$ slow evaporation

Boiling -- Evaporation below the surface of a liquid

- bubbles form if $P_{\text{vapor}} > P_{\text{liquid}}$
- high altitude $\rightarrow$ lower pressure $\rightarrow$ low boiling temp.

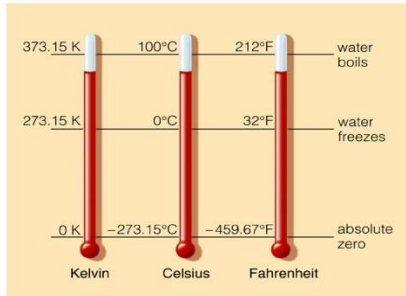
Statistical Thermodynamics

The *microscopic* or *molecular* approach

- The goal is the understanding and prediction of macroscopic phenomena and the calculation of macroscopic properties from the properties of individual molecules.

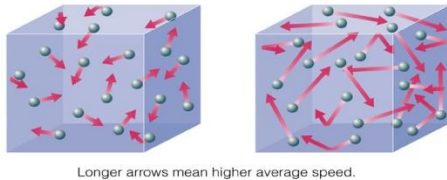
Heat vs. Temperature

- Temperature is the average energy related to the speed of atoms in an object



Show random movement in slow motion.

- Heat is the amount of energy transferred from one object to another



Show random movement in fast motion

$$N = \sum_i n(v_i)$$

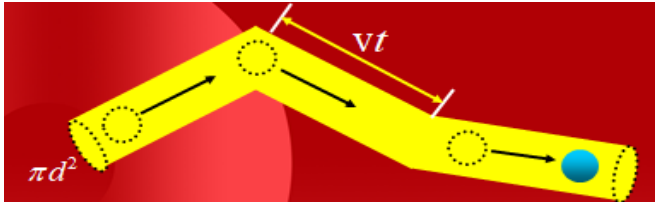
$$\bar{v} = \frac{\sum_i v_i n(v_i)}{\sum_i n(v_i)}$$

$$P(v_i) \equiv \frac{n(v_i)}{\sum_i n(v_i)} = \frac{n(v_i)}{N}$$

$$\bar{v} = \sum_i v_i P(v_i)$$

Mean Free Path λ

Mean free path, average distance an object will move between collisions. The actual distance a particle, such as a molecule in a gas, will move before a collision, called free path, cannot generally be given because its calculation would require knowledge of the path of every particle in the region.



$$d = r_1 + r_2$$

Diameter d

$$\text{Area} = \pi d^2$$

$$\lambda(n_v \pi d^2 vt) \approx vt$$

$$\therefore \lambda \approx \frac{1}{n_v \pi d^2}$$

$$\lambda = \frac{1}{\sqrt{2} n_v \pi d^2} \quad (\text{more accurate})$$

When Are Gases Not Ideal ?

- (1) near the *boiling point* (where liquid turns to gas)
- (2) at very *high pressure* or *high density*

How about Pressure (P) and Temperature (T)?

For an ideal gas in a constant volume, the pressure is directly proportional to the temperature: $P \propto T$

Increasing the temperature T makes gas molecules move faster, striking the walls of the container harder and more often, thus giving an increase in pressure P .

And finally volume (V) and temperature (T)?

For an ideal gas at constant pressure, the volume is directly proportional to the temperature: $V \propto T$

This is where the *Absolute* or *Kelvin* scale comes from: at 0°K ideal gas would have zero volume .

Molecules

Molecules are made up of one or more atoms. If they contain more than one atom, the atoms can be the same (an oxygen molecule has two oxygen atoms) or different

(a water molecule has two hydrogen atoms and one oxygen atom). Biological molecules, such as proteins and DNA, can be made up of many thousands of atoms.

A atomic mass mass of one atom (from Periodic Table)

m molecular mass mass of one molecule

N # of molecules total # of molecules in gas

N is typically a big, inconvenient number ($\sim 10^{24}$), so instead of talking about number of molecules, we talk about number of moles.

1 mole: amount of gas containing 6.02×10^{23} molecules

Moles

M molar mass mass of one mole of a gas $M = A$ grams

N_A Avogadro's # # of molecules in one mole $N_A = 6.02 \times 10^{23}$

n number of moles $n = N / N_A$

The Ideal Gas Law:

$$PV = nRT \quad PV = Nk_B T$$

- where n = number of moles in the gas
- and $R = 8.315 \text{ J/(mol K)}$ (*universal gas constant*)
- where N = number of molecules in the gas and $k_B = 1.38 \times 10^{-23} \text{ J/K}$ (*Boltzmann's constant*)

Use *absolute pressure* and *absolute temperature* (Kelvin)!

The Kinetic Theory of Gases

The simplest kinetic model is based on the assumptions that the gas is composed of a large number of identical molecules moving in random directions, separated by distances that are large compared with their size.

Pressure:

Related to the outward force per unit area exerted by the gas on the container wall

$$\text{Newton's 2nd Law: } F = \frac{\Delta p}{\Delta t}$$

Calculating the Pressure Exerted by a Gas

$$\text{Molecules that hit the wall} = \frac{1}{2} \frac{N}{V} v_x \Delta t A$$

$$\Delta p = (2mv_x) \times \left(\frac{1}{2} \frac{N}{V} v_x \Delta t A \right)$$

$$= \frac{N}{V} mv_x^2 \Delta t$$

$$P = \frac{F}{A} = \frac{1}{A} \frac{\Delta p}{\Delta t} = \frac{N}{V} mv_x^2$$

$$PV = Nmv_x^2 = 2N \left(\frac{1}{2} mv_x^2 \right)_{av}$$

The Molecular Interpretation of Temperature

$$PV = Nk_B T = 2N \left(\frac{1}{2} mv_x^2 \right)_{av}$$

$$\left(\frac{1}{2} mv_x^2 \right)_{av} = \frac{1}{2} k_B T$$

$$(v_x^2)_{av} = (v_y^2)_{av} = (v_z^2)_{av}$$

$$(v^2)_{av} = (v_x^2)_{av} + (v_y^2)_{av} + (v_z^2)_{av} = 3(v_x^2)_{av}$$

$$K_{av} = \left(\frac{1}{2} mv^2 \right)_{av} = \frac{3}{2} k_B T$$

$$K = N \left(\frac{1}{2} mv^2 \right)_{av} = \frac{3}{2} Nk_B T$$

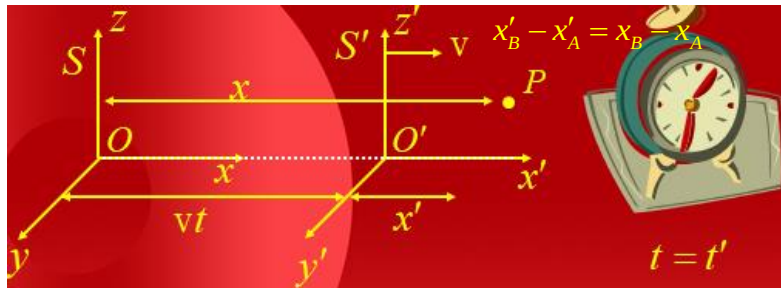
$$(v^2)_{av} = \frac{3k_B T}{m}$$

$$v_{rms} = \sqrt{(v^2)_{av}} = \sqrt{\frac{3k_B T}{m}}$$

Summary of Lecture 39 – SPECIAL RELATIVITY I

Galilean coordinate transformations

The Special Theory of Relativity was created by Albert Einstein, when he was a very young man in 1905.



$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$$

It is assumed that time can be defined independently of any particular frame of reference these are called Galilean coordinate transformations.



A rod is at rest in S-frame.

length in S-frame = $x_B - x_A$

length in S'-frame = $x'_B - x'_A$

Galilean transformation gives:

$$x'_B - x'_A = x_B - x_A - v(t_B - t_A)$$

Since the two end points, A and B are measured at the same instant $t_B = t_A$,

$$x'_B - x'_A = x_B - x_A$$

length in S-frame = length in S'-frame

Lengths, mass and time are all independent of the relative motion of the observer.

Newtonian Relativity

As Huygens had done, Newton presented the relativity principle as a fundamental principle, “Law 3”: The motions of bodies included in a given space are the same

among themselves whether that space is at rest or moves uniformly in a straight line without circular motion (1684b, p. 40r.).

$$x' = x - vt$$

differentiate wrt time:

$$\frac{dx'}{dt} = \frac{dx}{dt} - v$$

$$t = t' \Rightarrow \frac{dx'}{dt} = \frac{dx'}{dt'}$$

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

$$\frac{dy'}{dt'} = \frac{dy}{dt}, \quad \frac{dz'}{dt'} = \frac{dz}{dt}$$

$$\frac{dx'}{dt'} = u'_x = \text{the x-component of the velocity measured in S'-frame}$$

$$\frac{dx}{dt} = u_x = \text{the x-component of the velocity measured in S-frame}$$

Therefore :

$$u'_x = u_x - v \quad u'_y = u_y \quad u'_z = u_z$$

In general

$$\mathbf{u}' = \mathbf{u} - \mathbf{v}$$

To obtain the acceleration transformation differentiate the velocity relations.

$$\frac{du'_x}{dt'} = \frac{d}{dt}(u_x - v) = \frac{du_x}{dt} \quad \text{Q } v = \text{constant}$$

$$\frac{du'_y}{dt'} = \frac{du_y}{dt}, \quad \frac{du'_z}{dt'} = \frac{du_z}{dt}$$

i.e.,

$$a'_x = a_x, \quad a'_y = a_y, \quad a'_z = a_z$$

In general

$$\mathbf{a}' = \mathbf{a},$$

Acceleration of a particle is the same in all inertial frames of reference

Newton's second law: $\vec{F} = m\vec{a}$ in S-frame

But in S'-frame: $\vec{F}' = m'\vec{a}'$

But: $m' = m$ $\vec{a} = \vec{a}'$

$\therefore \vec{F}' = m'\vec{a}' = m\vec{a} = \vec{F}$

Newton's laws of motion are exactly the same in all inertial frames.

Light is electromagnetic wave

People considered a hypothetical medium called “ether” for the propagation of light waves.

Speed of light in ether's rest frame is $c = 2.907925 \times 10^8 \text{ m/sec}$

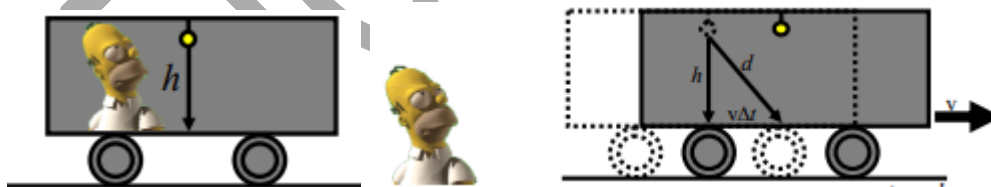
For a frame moving with velocity $\vec{v}$ with respect to ether frame, velocity of light is given by the Galilean velocity transformation.

$$\vec{c}' = \vec{c} + \vec{v}$$

Postulates of Special Relativity Theory

- 1) Laws of physics are same in all inertial systems
- 2) The speed of light in vacuum has the same value in all inertial systems, independent of the relative motion of source and observer

The relativity of simultaneity

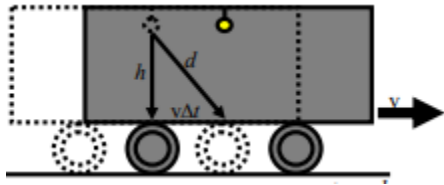


Two events that are simultaneous in one inertial system are not, in general, simultaneous in another.

Time dilation

Let's consider a light ray that leaves the bulb and strikes the floor of the train directly below

Time it take the light to reach at floor = $\Delta t' = \frac{h}{c}$.



With respect to the ground observer, train is moving with speed v .

Time it take the light to reach at floor = $\Delta t = \frac{d}{c}$

but $d = \sqrt{h^2 + (v\Delta t)^2}$

$$\Delta t = \frac{\sqrt{h^2 + (v\Delta t)^2}}{c}$$

Solving for Δt , we have $\Delta t = \frac{h}{c} \frac{1}{\sqrt{1 - (v/c)^2}}$

Therefore $\Delta t' = \Delta t \sqrt{1 - (v/c)^2}$

or

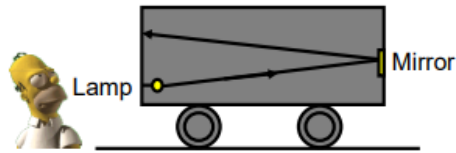
$$\Delta t' = \frac{\Delta t}{\gamma} \quad \text{where } \gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$

for $v < c$, $\gamma > 1 \Rightarrow \Delta t' < \Delta t$

Moving clocks run slowly this is called time dilation.

Lorentz contraction

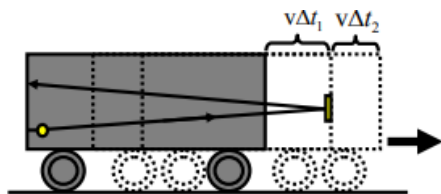
Lorentz - Fitzgerald contraction also called space contraction, in relativity physics, the shortening of an object along the direction of its motion relative to an observer. Dimensions in other directions are not contracted.



How long does the signal take to complete the round trip?

$$\Delta t' = \frac{2\Delta x'}{c} \quad (\text{w.r.t car observer})$$

$\Delta x'$ = length of the car with respect to car observer



Δx = length of the car according to ground observer

Δt_1 = time for the signal to reach the front end

Δt_2 = return time

$$\therefore \Delta t_1 = \frac{\Delta x + v\Delta t_1}{c} \quad \text{and} \quad \Delta t_2 = \frac{\Delta x - v\Delta t_2}{c}$$

solving for Δt_1 and Δt_2 :

$$\Delta t_1 = \frac{\Delta x}{c - v} \quad \Delta t_2 = \frac{\Delta x}{c + v}$$

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{2\Delta x / c}{(1 - v^2 / c^2)} \quad (\text{w.r.t ground observer})$$

from time dilation formula:

$$\Delta t' = \sqrt{1 - v^2 / c^2} \Delta t$$

$$\Delta t' = \sqrt{1 - v^2 / c^2} \left(\frac{2\Delta x / c}{(1 - v^2 / c^2)} \right) = \frac{2\Delta x / c}{\sqrt{1 - v^2 / c^2}}$$

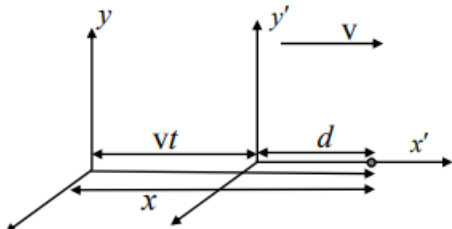
but

$$\Delta t' = \frac{2\Delta x'}{c} \quad \therefore \Delta x' = \frac{\Delta x}{\sqrt{1 - v^2 / c^2}}$$

Moving objects are shortened Lorentz contraction

The Lorentz transformations

In physics, the Lorentz transformations are a six-parameter family of linear transformations from a coordinate frame in spacetime to another frame that moves at a constant velocity relative to the former. The respective inverse transformation is then parameterized by the negative of this velocity.



$$z' = z$$

$$y' = y$$

$$x = d + vt \text{ but } d = \frac{x'}{\gamma} \quad \therefore x' = \gamma(x - vt)$$

because x' is moving with respect to O -frame

$$\therefore x = \gamma(x' + vt') = \gamma(\gamma(x - vt) + vt')$$

$$\therefore t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$x' = \gamma(x - vt)$$

$$x = \gamma(x' + vt')$$

$$y' = y$$

$$y = y'$$

$$z' = z$$

$$z = z'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Time dilation

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \Rightarrow \Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = \gamma\Delta t \quad \text{Q } \Delta x = 0$$

Lorentz contraction

$$x' = \gamma(x - vt) \Rightarrow \Delta x' = \gamma(\Delta x - v\Delta t) = \gamma\Delta x \quad \text{Q } \Delta t = 0$$

Einstein velocity addition rule

The relative velocity of any two objects never exceeds the velocity of light. Applying the Lorentz transformation to the velocities, expressions are obtained for the relative velocities as seen by the different observers. They are called the Einstein velocity addition relationships.

Suppose a particle moves a distance dx in a time dt . Its velocity u is then

$$u = \frac{dx}{dt} \text{ (in S-frame)}$$

In S' -frame, meanwhile, it has moved a distance

$$dx' = \gamma(dx - vdt)$$

in time

$$dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

The velocity in S' -frame is therefore

$$u' = \frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{dx/dt - v}{1 - \frac{v}{c^2}dx/dt}$$

$$\therefore u' = \frac{u - v}{1 - \frac{uv}{c^2}} \quad \text{or} \quad u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

Summary of Lecture 40 – SPECIAL RELATIVITY II

LORENTZ TRANSFORMATION

$$x' = \gamma(x - vt) \quad x = \gamma(x' + vt')$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The invariant interval

Invariant interval in the theory of relativity, a quantity that characterizes the relation between the spatial distance and the time interval that separate two events. From the mathematical standpoint it is the “distance” between two events in four-dimensional space-time. Prove that the following interval is invariant under Lorentz transformations

$$I = (c\Delta t)^2 - (\Delta x)^2$$

Proof :

$$I' = (c\Delta t')^2 - (\Delta x')^2$$

$$= c^2 \gamma^2 \left(\Delta t - \frac{v}{c^2} \Delta x \right)^2 - \gamma^2 (\Delta x - v\Delta t)^2$$

$$I' = \gamma^2 \left((c\Delta t)^2 + (\Delta x)^2 \frac{v^2}{c^2} - 2v\Delta t\Delta x - (\Delta x)^2 - (v\Delta t)^2 + 2v\Delta t\Delta x \right)$$

$$= \frac{1}{(1 - v^2/c^2)} \left((c^2 - v^2)(\Delta t)^2 - (\Delta x)^2(1 - v^2/c^2) \right)$$

$$= (c\Delta t)^2 - (\Delta x)^2 = I$$

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

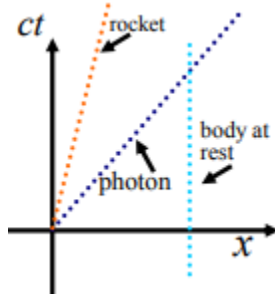
If $I > 0$, we call the interval **timelike**.

If $I < 0$, we call the interval **spacelike**.

If $I = 0$, we call the interval **lightlike**.

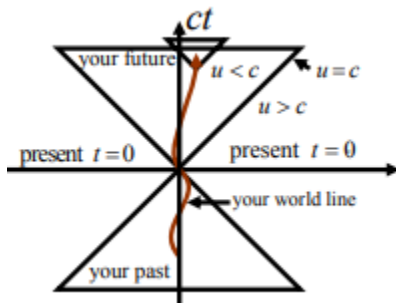
Space-time

A space time diagram is a graphical illustration of the properties of space and time in the special theory of relativity. Space time diagrams allow a qualitative understanding of the corresponding phenomena like time dilation and length contraction without mathematical equations.



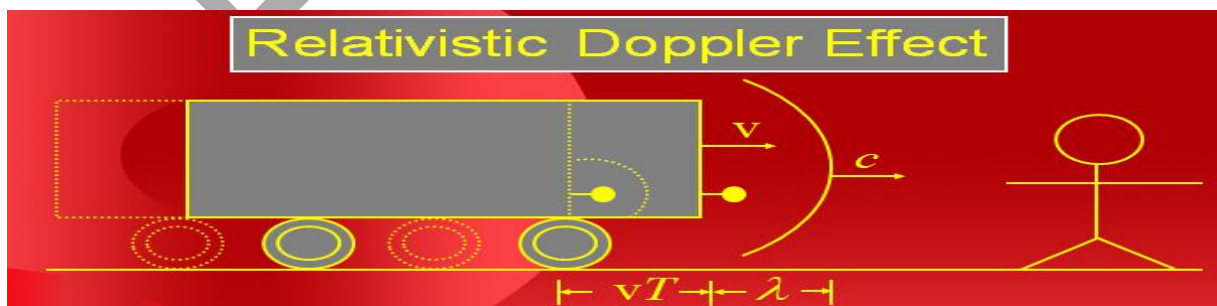
World-line

The trajectory of a body as it moves through space- time is called its world-line.



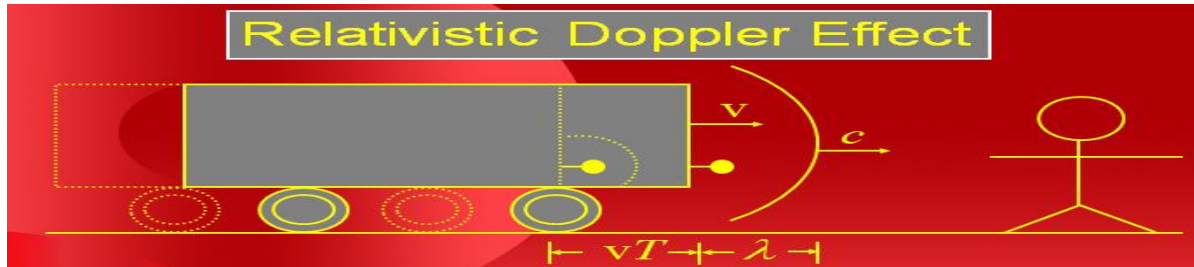
If interval between two events is time like, their time ordering is absolute. If the interval is space like their ordering depends on the inertial system from which they are observed.

Relativistic Doppler Effect



$$\nu_0 = \frac{1}{T_0} = \text{frequency measured in source rest frame}$$

ν = frequency measured with respect to observer = ?



During time T the crests ahead of the source move a distance cT , and the source moves a shorter distance vT in the same direction

$$\therefore \lambda = (c - v)T \quad \Rightarrow \quad \nu = \frac{c}{\lambda} = \frac{c}{(c - v)T}$$

T_0 = time measured in rest frame of source

T = time measured by observer

$$T = \frac{T_0}{\sqrt{1 - v^2 / c^2}} = \frac{cT_0}{\sqrt{c^2 - v^2}}$$

$$\nu = \frac{c}{(c - v)T} = \left(\frac{c}{c - v} \right) \frac{\sqrt{c^2 - v^2}}{c} \nu_0$$

$$\therefore \nu = \sqrt{\frac{c + v}{c - v}} \nu_0$$

If source moves away from the observer just change the sign of v !

$$\nu = \sqrt{\frac{c - v}{c + v}} \nu_0$$

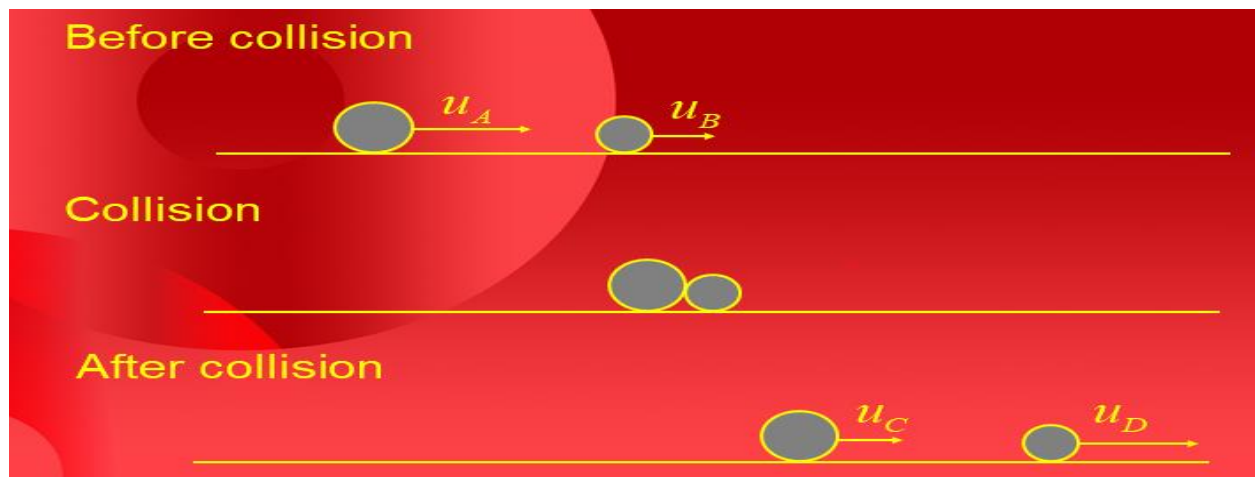
Newtonian definition of momentum:

Momentum, product of the mass of a particle and its velocity Momentum is a vector quantity; i.e., it has both magnitude and direction. Isaac Newton's second

law of motion states that the time rate of change of momentum is equal to the force acting on the particle. See Newton's laws of motion.

$$\dot{\vec{p}} = m\dot{\vec{u}}$$

Consider the collision of two particles



In S-frame

Conservation of momentum implies that

$$m_A u_A + m_B u_B = m_C u_C + m_D u_D$$

and

$$m_A + m_B = m_C + m_D$$

S'-frame is moving with speed v with respect to S-frame

Using Galilean velocity addition rule

$$u' = u - v \quad \text{or} \quad u = u' + v$$

Conservation of momentum becomes

$$m_A (u'_A + v) + m_B (u'_B + v) = m_C (u'_C + v) + m_D (u'_D + v)$$

$$\Rightarrow m_A u'_A + m_B u'_B = m_C u'_C + m_D u'_D$$

Now using Einstein velocity addition rule

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} \quad u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

Conservation of momentum becomes

$$\begin{aligned} m_A \left(\frac{u'_A + v}{1 + u'_A v / c^2} \right) + m_B \left(\frac{u'_B + v}{1 + u'_B v / c^2} \right) \\ = m_C \left(\frac{u'_C + v}{1 + u'_C v / c^2} \right) + m_D \left(\frac{u'_D + v}{1 + u'_D v / c^2} \right) \end{aligned}$$

Can we find some suitable new definition of momentum?

The new definition must have two properties

- 1) It must yield a law of conservation of momentum that satisfies the principle of relativity.
- 2) At low speeds, the new definition must reduce to $\vec{p} = m\vec{u}$.

Relativistic momentum

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

The relativistic formula for the momentum of a particle of mass m moving with velocity $\vec{u}$ is

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1 - u^2 / c^2}} = \gamma m\vec{u}$$

$$\begin{aligned} m_A (u'_A + v) + m_B (u'_B + v) &= m_C (u'_C + v) + m_D (u'_D + v) \\ \Rightarrow m_A u'_A + m_B u'_B &= m_C u'_C + m_D u'_D \end{aligned}$$

Now using Einstein velocity addition rule

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} \quad u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

Conservation of momentum becomes

$$m_A \left(\frac{u'_A + v}{1 + u'_A v / c^2} \right) + m_B \left(\frac{u'_B + v}{1 + u'_B v / c^2} \right) \\ = m_C \left(\frac{u'_C + v}{1 + u'_C v / c^2} \right) + m_D \left(\frac{u'_D + v}{1 + u'_D v / c^2} \right)$$

Thus momentum is conserved in both S and S' frames

In the limit of low speeds: $\vec{p} = \frac{m\vec{u}}{\sqrt{1 - u^2/c^2}} \rightarrow m\vec{u}$ as $u/c \rightarrow 0$

Relativistic Energy

Relativistic energy is conserved as long as we define it to include the possibility of mass changing to energy. Total Energy is defined as: $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$. Rest energy is $E_0 = mc^2$, meaning that mass is a form of energy. If energy is stored in an object, its mass increases.

$K = \frac{1}{2} mu^2$ Is not consistent with relativistic mechanics (it does not satisfy the law of conservation of energy in relativity)

$$K = \int F dx = \int \frac{dp}{dt} dx = \int \frac{dx}{dt} dp = \int u dp$$

$$dp = m d \left(\frac{u}{\sqrt{1 - u^2 / c^2}} \right)$$

$$= \frac{m du}{\sqrt{1 - u^2 / c^2}} + \frac{m(u^2 / c^2) du}{(1 - u^2 / c^2)^{3/2}} = \frac{m u du}{(1 - u^2 / c^2)^{3/2}}$$

$$K = \int_0^u u dp = m \int_0^u \frac{u du}{(1 - u^2 / c^2)^{3/2}} = \frac{mc^2}{\sqrt{1 - u^2 / c^2}} - mc^2$$

$$K = \frac{mc^2}{\sqrt{1 - u^2 / c^2}} - mc^2$$

$$\text{or } K = E - E_0$$

Where total relativistic energy E is defined as $E = \frac{mc^2}{\sqrt{1 - u^2 / c^2}}$

and rest energy E_0 is defined as $E_0 = mc^2$

In an isolated system of particles, the total relativistic energy remains constant.

For low speeds

$$K = \frac{mc^2}{\sqrt{1 - u^2 / c^2}} - mc^2$$

$$\frac{1}{\sqrt{1 - u^2 / c^2}} = (1 - u^2 / c^2)^{-1/2} = 1 + \frac{u^2}{2c^2} + \dots$$

$$K = mc^2 \left(1 + \frac{u^2}{2c^2} + \dots \right) - mc^2 \rightarrow \frac{1}{2} mu^2$$

as $u/c \rightarrow 0$

From

$$E = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \gamma mc^2 \quad \text{and} \quad \vec{p} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}} = \gamma m\vec{u}$$

$$\text{We get } \frac{\vec{p}}{\gamma m} = \frac{\vec{p}c^2}{E} \text{ or } pc = E \left(\frac{u}{c} \right)$$

$$\begin{aligned} (pc)^2 &= E^2 \left(\frac{u}{c} \right)^2 = \gamma^2 m^2 c^4 (1 - 1/\gamma^2) \\ &= \gamma^2 m^2 c^4 - m^2 c^4 = E^2 - m^2 c^4 \end{aligned}$$

$$E^2 = p^2 c^2 + m^2 c^4$$

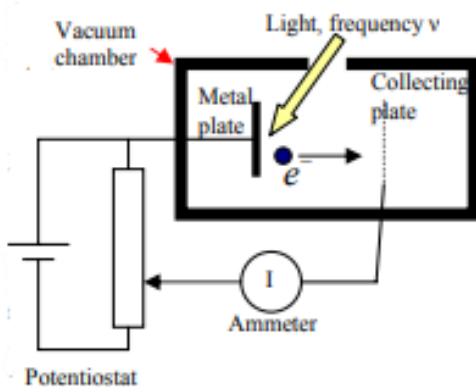
A particle may have energy and momentum even when it has no mass

$$E = pc \quad \text{or} \quad c = \frac{pc^2}{E}$$

Summary of Lecture 41 – WAVES AND PARTICLES

PHOTOELECTRIC EFFECT

Photoelectric effect, phenomenon in which electrically charged particles are released from or within a material when it absorbs electromagnetic radiation. The effect is often defined as the ejection of electrons from a metal plate when light falls on it.



Classical expectations

As intensity of light increases, KE of ejected electrons should increase. Electrons should be emitted for any ν , so long as E is sufficiently large.

Actual results:

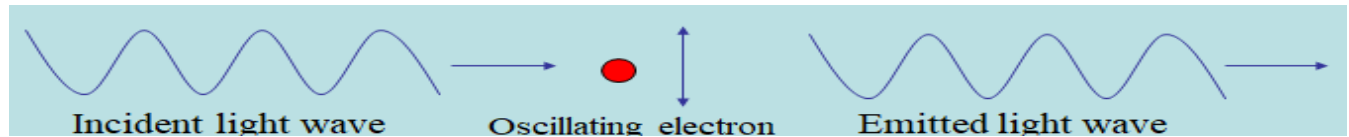
Maximum KE is independent of intensity, but dependent on ν . For $\nu < \nu_0$ (i.e. for frequencies below a cut-off frequency) no electrons are emitted.

Einstein's interpretation (1905):

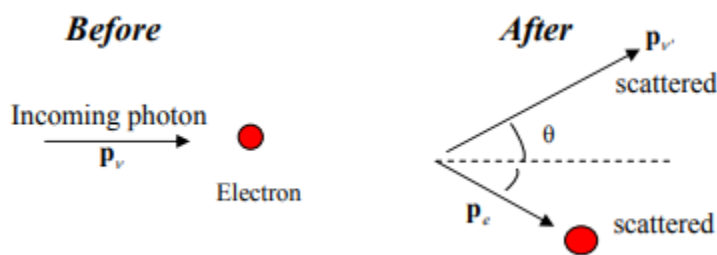
Light comes in packets of energy (photons) $E = h\nu$ an electron absorbs a single photon to leave the material.

The maximum KE of an emitted electron is then $h = 6.63 \times 10^{-34} \text{ Js}$ Planck constant: universal constant of nature $K_{\text{max}} = h\nu - W$ Work function: minimum energy needed for electron to escape from metal (depends on material, but usually 2-5eV).

Classical picture: Re-radiation is at same frequency and wavelength as incident radiation. Change in wavelength of scattered light is completely unexpected classically



Compton's explanation: "billiard ball" collisions between particles of light (X-ray photons) and electrons in the material.



Conservation of energy

$$h\nu + m_e c^2 = h\nu' + (p_e^2 c^2 + m_e^2 c^4)^{1/2}$$

Conservation of momentum

$$\mathbf{p}_\nu = \frac{h}{\lambda} \hat{\mathbf{i}} = \mathbf{p}_{\nu'} + \mathbf{p}_e$$

From this Compton derived the change in wavelength

$$\begin{aligned} \lambda' - \lambda &= \frac{h}{m_e c} (1 - \cos \theta) \\ &= \lambda_c (1 - \cos \theta) \geq 0 \end{aligned}$$

$$\lambda_c = \text{Compton wavelength} = \frac{h}{m_e c} = 2.4 \times 10^{-12} \text{ m}$$

SUMMARY OF PHOTON PROPERTIES

Relation between particle and wave properties of light

Energy and frequency $E = h\nu$

Relativistic formula relating energy and momentum: $E^2 = p^2c^2 + m^2c^4$

For light $E = pc$ and $c = \lambda \nu$

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

Also commonly written as :

$$E = \hbar\omega \quad p = \hbar k \quad \omega = 2\pi\nu \quad k = \frac{2\pi}{\lambda} \quad \hbar \equiv \frac{h}{2\pi}$$

Evidence for particle-nature of light

- Photoelectric effect
- Compton effect

Evidence for wave-nature of light

- Diffraction and interference
- Light exhibits diffraction and interference phenomena that are only explicable in terms of wave properties.
- Light is always detected as packets (photons); if we look, we never observe half a photon.
- Number of photons proportional to energy density (i.e. to square of electromagnetic field strength).

MATTER WAVES

MATTER WAVES the set of waves that represent the behavior of an elementary particle, or some atoms and molecules, under certain conditions. The de Broglie wavelength, λ , is given by $\lambda = h/mv$, where h is the Planck constant, m the mass, and v the velocity of the particle. Also called: matter waves.

Estimate some de Broglie wavelengths:

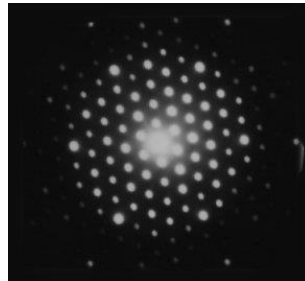
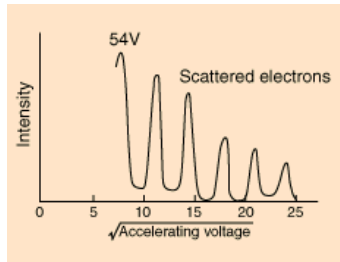
- Wavelength of 0.5 kg cricket ball moving at 2 m/sec: $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{0.5 \times 2} = 6.63 \times 10^{-34} \text{ m}$

- Wavelength of electron with 50eV kinetic energy:

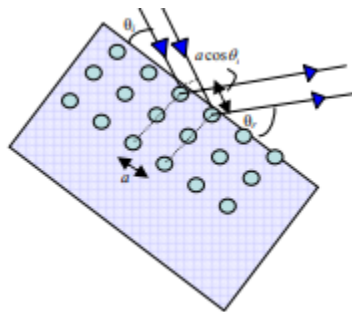
$$K = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} \Rightarrow \lambda = \frac{h}{\sqrt{2m_e K}} = 1.7 \times 10^{-10} \text{ m}$$

ELECTRON DIFFRACTION

The Davisson-Germer experiment (1927) At fixed angle, find sharp peaks in intensity as a function of electron energy. At fixed accelerating voltage (fixed electron energy) find a pattern of sharp reflected beams from the crystal.

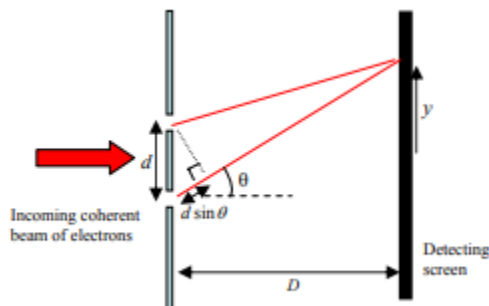


Interpretation: similar to scattering of X-rays from crystals



Path difference: $a(\cos \theta_r - \cos \theta_i)$

Constructive interference when $a(\cos \theta_r - \cos \theta_i) = n\lambda$ Electron scattering dominated by surface layers Note the identical scattering planes are oriented perpendicular to the surface. Note θ_i and θ_r are not necessarily equal.



We expect two peaks for particles, for waves an interference pattern

FRINGE SPACING IN DOUBLE-SLIT EXPERIMENT

$$d \sin \theta = n\lambda$$

Maxima when:

$D \gg d$ so use small angle approximation

$$\theta \approx \frac{n\lambda}{d}$$

$$\Rightarrow \Delta\theta \approx \frac{\lambda}{d}$$

Position on screen : $y = D \tan \theta \approx D\theta$

So separation between adjacent maxima : $\Delta y \approx D\Delta\theta$

$$\Rightarrow \Delta y = \frac{\lambda D}{d}$$

DOUBLE-SLIT EXPERIMENT WITH HELIUM ATOMS

Path difference : $d \sin \theta$

Constructive interference : $d \sin \theta = n\lambda$

Separation between maxima : $\Delta y = \frac{\lambda D}{d}$

Experiment : $d = 8\mu\text{m}$ and $D = 64\text{cm}$

Measured separation : $\Delta y = 8.2\mu\text{m}$

de Broglie wavelength of He atoms : $\lambda = 1.03 \times 10^{-10}\text{m}$

Predicted separation : $\Delta y = 8.4 \pm 0.8\mu\text{m}$

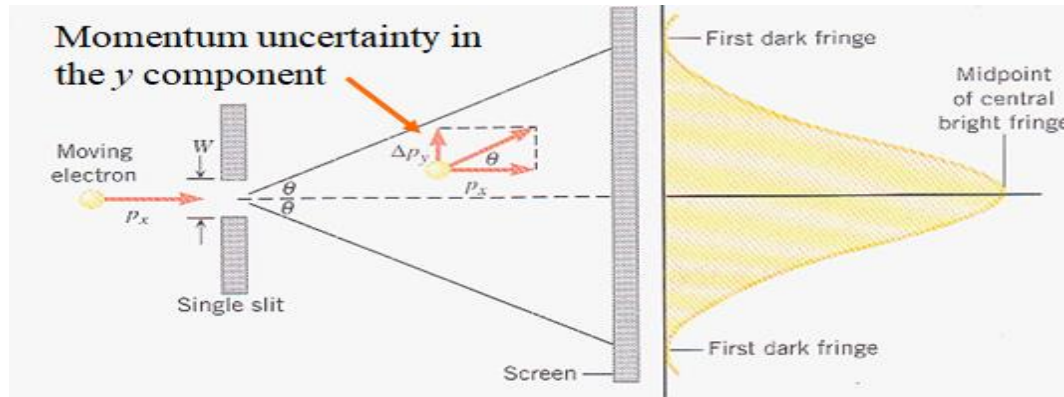
DOUBLE-SLIT EXPERIMENT INTERPRETATION

- Wavelength of matter wave unconnected to any internal size of particle. Instead it is determined by the momentum.
- The flux of particles arriving at the slits can be reduced so that only one particle arrives at a time. Interference fringes are still observed!!
 - i. Wave-behaviour can be shown by a single atom.
 - ii. Each particle goes through both slits at once.
 - iii. A matter wave can interfere with itself.

- If we try to find out which slit the particle goes through the interference pattern vanishes!

- We cannot see the wave/particle nature at the same time.

Consider electrons passing through a slit:



Now apply the formula for single slit diffraction, $\sin \theta = \lambda / W$ where λ is the de Broglie wavelength.

$$\sin \theta = \frac{\lambda}{W} \Rightarrow \theta \approx \frac{\lambda}{W}$$

But, on the other hand,

$$\tan \theta = \frac{\Delta p_y}{p_x} \Rightarrow \theta \approx \frac{\Delta p_y}{p_x}$$

$$\frac{\Delta p_y}{h / \lambda} = \frac{\lambda}{W}$$

W = uncertainty in y position = Δy

$$\therefore \Delta p_y \Delta y \approx h$$

HEISENBERG MICROSCOPE AND THE UNCERTAINTY PRINCIPLE

No such microscope exists at present, but it could be constructed in principle. Heisenberg imagined using this microscope to see an electron and to measure its position. He found that the electron's position and momentum did indeed obey the uncertainty relation he had derived mathematically. This microscope is an imaginary device to measure the position (y) and momentum (p) of a particle.

HEISENBERG MICROSCOPE

Heisenberg conducted a thought experiment as well. He considered trying to measure the position of an electron with a gamma ray microscope. The high-energy photon used to illuminate the electron would give it a kick, changing its momentum in an uncertain way.

Photons transfer momentum to the particle when they scatter. Magnitude of p is the same before and after the collision.

Uncertainty in photon y-momentum = Uncertainty in particle y-momentum

$$-p \sin(\theta/2) \leq p_y \leq p \sin(\theta/2)$$

Small angle approximation

$$\Delta p_y = 2p \sin(\theta/2) \approx p\theta$$

De Broglie relation gives $p = h/\lambda$ and so $\Delta p_y \approx \frac{h\theta}{\lambda}$

From before $\Delta y \approx \frac{\lambda}{\theta}$ hence

$$\Delta p_y \Delta y \approx h$$

HEISENBERG UNCERTAINTY PRINCIPLE

$$\Delta x \Delta p_x \geq \hbar/2$$

$$\Delta y \Delta p_y \geq \hbar/2$$

$$\Delta z \Delta p_z \geq \hbar/2$$

We cannot have simultaneous knowledge of 'conjugate' variables such as position and momenta. But arbitrary precision is possible in principle for position in one direction and momentum in another.

The thought experiment seems to imply that, while prior to experiment we have well defined values, it is the act of measurement which introduces the uncertainty by disturbing the particle's position and momentum.

Nowadays it is more widely accepted that quantum uncertainty (lack of determinism) is intrinsic to the theory.

The Energy-Time Uncertainty Principle

For energy and time, the uncertainty principle is $\Delta E \Delta t \geq \hbar / 2$ where ΔE is the uncertainty in energy and Δt is the uncertainty in time. These small limits are fundamentally important on the quantum-mechanical scale.

$$\Delta E \Delta t \geq \hbar / 2$$

ΔE Energy uncertainty of a particle when the particle is in a certain state and Δt Time interval during which the particle is in the state.

$$\Delta E \Delta t \geq \hbar / 2$$

CONCLUSIONS

Light and matter exhibit wave-particle duality

Relation between wave and particle properties given by the de Broglie relations

$$E = h\nu \quad p = \frac{h}{\lambda}$$

Evidence for particle properties of light Photoelectric effect, Compton scattering

Evidence for wave properties of matter Electron diffraction, interference of matter waves (electrons, neutrons, His atoms, C60 molecules)

Heisenberg uncertainty principle limits simultaneous knowledge of conjugate variables.

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\Delta y \Delta p_y \geq \hbar / 2$$

$$\Delta z \Delta p_z \geq \hbar / 2$$

Basic Ideas of Quantum Mechanics

- Atoms or molecules can only exist in certain energy states.
- Atoms or molecules emit or absorb energy when they change their energy state.

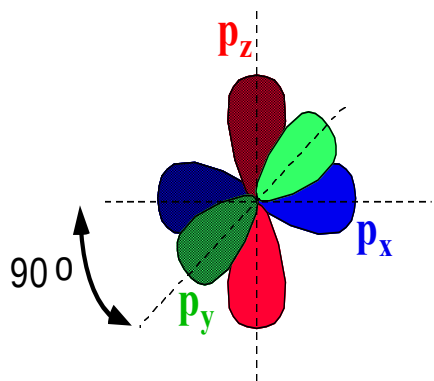
Equal to the difference between the two energy states

- The allowed energy states of atoms or molecules can be described by a set of quantum numbers.

The Wave Function Ψ

- In 1926 Schrödinger proposed a wave equation that describes the manner in which matter wave's change in space and time.
- Schrödinger's wave equation is a key element in quantum mechanics.
- Schrödinger's wave equation is solved for the wave function, Ψ
- The wavefunction $\Psi(x, t)$ of a particle depends upon its position and time.
- The probability of finding the particle at position x at time t is proportional to $|\Psi(x, t)|^2$.

p orbitals



A “p” orbital the three p orbitals lay 90° apart in space.

d Orbitals

s orbitals ($l = 0$) are spherical.

p orbitals ($l = 1$) are “dumbbell” shaped.

d ($l = 2$) have 2 planar nodes

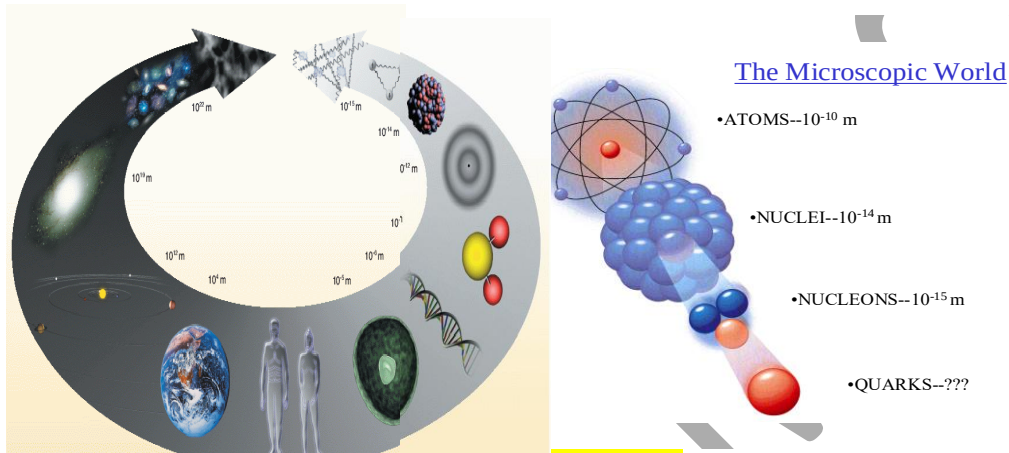
Typical d orbital planar node

IN GENERAL the number of NODES= value of angular quantum number (l)

Summary of Lecture 42 – QUANTUM MECHANICS

Quantum mechanics

Quantum mechanics is a fundamental theory in physics that provides a description of the physical properties of nature at the scale of atoms and subatomic particles. It is the foundation of all quantum physics including quantum chemistry, quantum field theory, quantum technology, and quantum information science.



Quantum Mechanics of Two-State Systems

$|\uparrow\rangle$ is a Dirac "ket" that denotes an electron with spin up.

$|\downarrow\rangle$ is a Dirac "ket" that denotes an electron with spin down.

$|\Psi\rangle = \sqrt{\frac{2}{3}}|\uparrow\rangle + \sqrt{\frac{1}{3}}|\downarrow\rangle$ denotes an electron with $|\Psi\rangle$

$$P(\uparrow) = \frac{2}{3} \text{ and } P(\downarrow) = \frac{1}{3}$$

More generally: $|\Psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$ denotes an electron with $P(\uparrow) = |c_1|^2$ and $P(\downarrow) = |c_2|^2$.

Stern-Gerlach Experiment

The Stern–Gerlach experiment demonstrated that the spatial orientation of angular momentum is quantized. Thus an atomic-scale system was shown to have intrinsically quantum properties.

$$|\Psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$$

$$P(\uparrow) = |c_1|^2 \text{ and } P(\downarrow) = |c_2|^2.$$

Quantum Amplitudes

$|\Psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$ is called a *quantum state*. c_1 and c_2 are called *quantum amplitudes*. If a_1 and a_2 are the amplitudes of the two possibilities for a particular event to take place, then amplitude for the total event is:

$$A = a_1 + a_2.$$

The probability for the event to occur is given by $P = |A|^2 = |a_1 + a_2|^2$

In daily experience we add probabilities:

$$P = P_1 + P_2$$

But in quantum mechanics we add amplitudes:

$$P = |a_1 + a_2|^2 = a_1^* a_1 + a_2^* a_2 + a_1^* a_2 + a_2^* a_1$$

$$= P_1 + P_2 + a_1^* a_2 + a_2^* a_1$$

The Wave Function Ψ

- In 1926 Schrödinger proposed a wave equation that describes the manner in which matter waves change in space and time.
- Schrödinger's wave equation is a key element in quantum mechanics.
- Schrödinger's wave equation is solved for the wave function, Ψ

- The wavefunction $\Psi(x,t)$ of a particle is the amplitude to be at position x at time t .
- The probability of finding the particle at position x between x and $x + dx$ (at time t) is $|\Psi(x,t)|^2 dx$.
- $\Psi(x,t)$ is determined by solving the "Schrodinger equation".

Double slit –probabilities

High peaks represent high $|\psi|^2$ – high probability

DOUBLE-SLIT EXPERIMENT INTERPRETATION

- Wavelength of matter wave unconnected to any internal size of particle. Instead it is determined by the momentum.
- The flux of particles arriving at the slits can be reduced so that only one particle arrives at a time. Interference fringes are still observed.
 - Wave-behaviour can be shown by a single atom.
 - Each particle goes through both slits at once.
 - A matter wave can interfere with itself.
- If we try to find out which slit the particle goes through the interference pattern vanishes!
 - We cannot see the wave/particle nature at the same time.

The Wave Function Ψ

• In 1926 Schrödinger proposed a wave equation that describes the manner in which matter wave's change in space and time.

• Schrödinger's wave equation is a key element in quantum mechanics.

• Schrödinger's wave equation is solved for the wave function, Ψ
Identical particles are indistinguishable.

If particles A and B are interchanged then

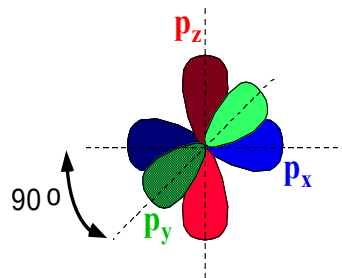
$$|\Psi(1,2)|^2 = |\Psi(2,1)|^2 \Rightarrow$$

either $\Psi(1,2) = +\Psi(2,1)$ BOSONS

or $\Psi(1,2) = -\Psi(2,1)$ FERMIONS

Note: $\Psi(1,1) = -\Psi(1,1) \therefore \Psi(1,1) = 0$

p orbitals



A “p” orbital the three p orbitals lay 90° apart in space.

d Orbitals

s orbitals ($l = 0$) are spherical.

p orbitals ($l = 1$) are “dumbbell” shaped.

d ($l = 2$) have 2 planar nodes

Typical d orbital planar node

IN GENERAL the number of NODES= value of angular quantum number (l)

- The necessity for quantum mechanics was thrust upon us by a series of observations.
- The theory of QM developed over a period of 30 years, culminating in 1925-27 with a set of postulates.
- QM cannot be deduced from pure mathematical or logical reasoning.
- QM is not intuitive, because we don’t live in the world of electrons and atoms.
- h is small. Hence we do not see quantum effects in daily life.
- QM is based on observation. Like all science, it is subject to change if inconsistencies with further observation are revealed.

Summary of Lecture 43 – INTRODUCTION TO ATOMIC PHYSICS

Lavoisier

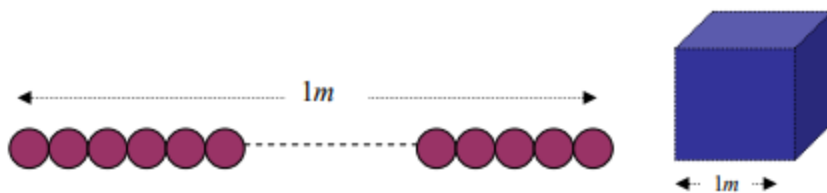
- Demonstrated that burning wood caused no change in mass
- Law of Conservation of Matter

Dalton (1803)

- Atoms are building blocks of elements
- Similar atoms in each element.
- All atoms have the same mass.
- Different from atoms of other elements
- Two or more different atoms bond in simple ratios to form compounds

Avogadro's Hypothesis

Avogadro's Hypothesis: - Equal volumes of all gases, under the same conditions of temperature and pressure, contain equal numbers of molecules. $N_0 = 6.023 \times 10^{26}$ kg.mole⁻¹ is called Avogadro's Number.



In a cube of $1m$ side we then have $(1/2r)^3$ atoms.

In 1kg.atom we have 6×10^{26} atoms and each atom occupies a volume $(A/\rho) \text{ m}^3$.

(where A = atomic weight and ρ = density).

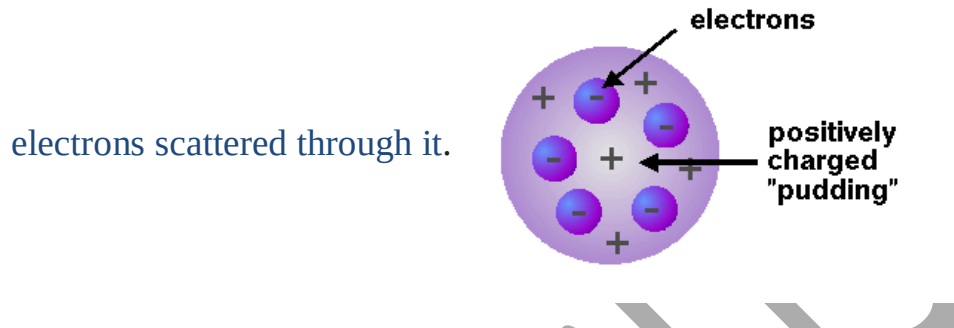
$$\therefore N_0 = (1/2r)^3 \times A/\rho$$

$$\Rightarrow r = \frac{1}{2} \left(\frac{A}{\rho N_0} \right)^{1/3} \quad r_{Ag} \approx r_{Be} \approx 10^{-10} \text{ m}$$

ATOMS ARE THEREFORE MOSTLY THE SAME SIZE !!

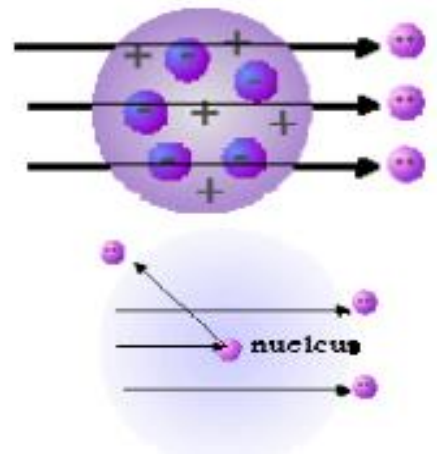
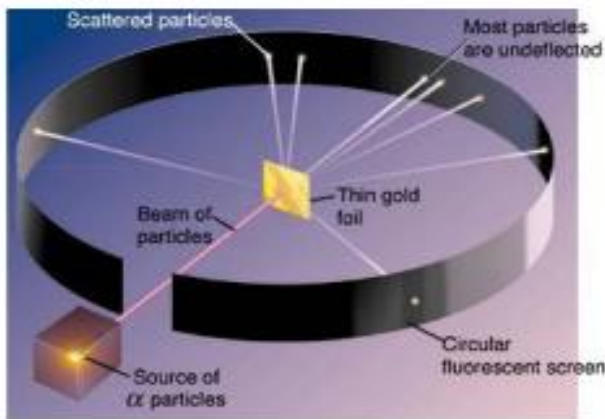
J.J. Thomson (1895)

- Plum pudding model
- Atom made of a positively charged material with the negatively charged



Rutherford's a Scattering Experiment (1911)

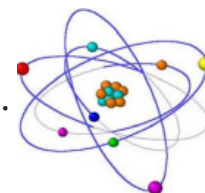
Rutherford's nuclear model. Rutherford overturned Thomson's model in 1911 with his famous gold-foil experiment, in which he demonstrated that the atom has a tiny, massive nucleus. Discovery of the Atomic Nucleus



Rutherford's Solar System Model of the Atom

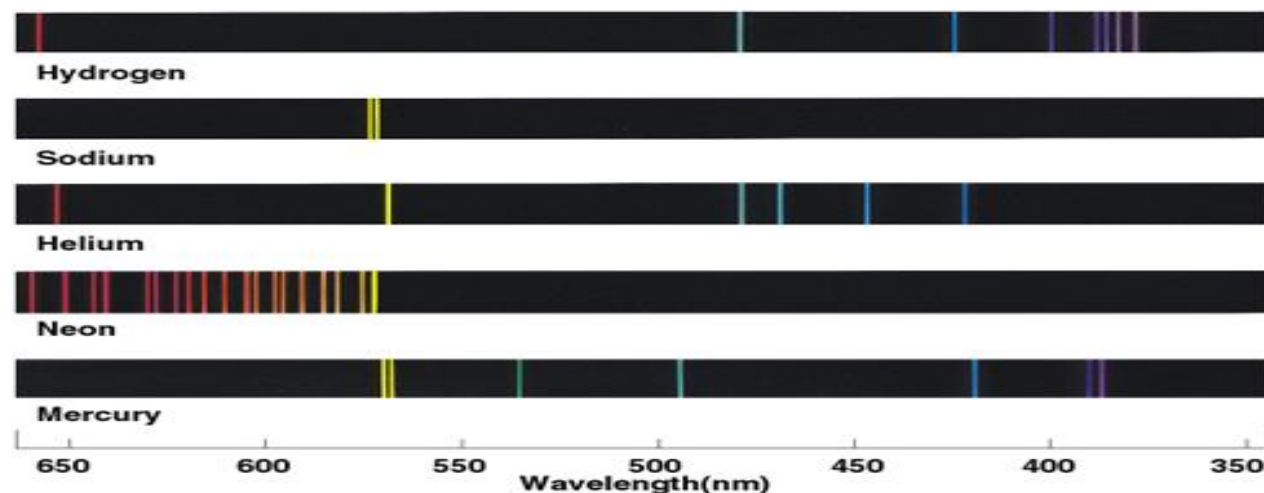
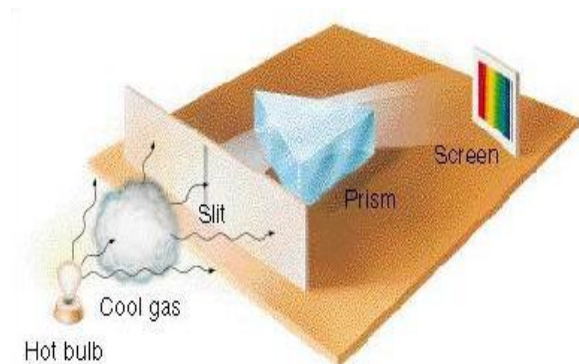
Physicist Ernest Rutherford envisioned the atom as a miniature solar system, with electrons orbiting around a massive nucleus, and as mostly empty space, with the

nucleus occupying only a very small part of the atom.



Absorption Spectrum of a Gas

An absorption spectrum occurs when light passes through a cold, dilute gas and atoms in the gas absorb at characteristic frequencies; since the re-emitted light is unlikely to be emitted in the same direction as the absorbed photon, this gives rise to dark lines (absence of light) in the spectrum.



BOHR'S HYPOTHESIS

- Electrons can only take discrete energies (energy is related to radius of the orbit)
- Electrons can jump between different orbits due to the absorption or emission of photons
- Dark lines in the absorption spectra are due to photons being absorbed
- Bright lines in the emission spectra are due to photons being emitted

- The electron has a wavelength and forms standing waves in its orbit around the nucleus.
- An integral number of electron wavelengths must fit into the circumference of the circular orbit.

$$n\lambda = 2\pi r$$

$$p = mv = \frac{h}{\lambda}$$

$$= \frac{h}{(2\pi r / n)} = \frac{n\hbar}{r} \quad n = 1, 2, 3, \dots$$

$$L = rmv = n\hbar$$

Bohr Orbits

In the Bohr model of the atom, electrons travel in defined circular orbits around the nucleus. The orbits are labeled by an integer, the quantum number n . Electrons can jump from one orbit to another by emitting or absorbing energy.

$$\begin{aligned} \frac{mv_n^2}{r} &= k \frac{e^2}{r^2} \\ L_n &= n\hbar \\ v_n &= \frac{n\hbar}{mr_n} \\ r_n &= \left(\frac{\hbar^2}{mke^2} \right) n^2 \quad n = 1, 2, 3, \dots \end{aligned}$$

The Bohr Radius

The Bohr radius, symbolized a_0 , is the mean radius of the orbit of an electron around the nucleus of a hydrogen atom at its ground state (lowest-energy level). The value of this radius is a physical constant; a_0 is approximately equal to 5.29177×10^{-11} meter (m).

- For $n = 1$ the electron orbit is closest to the nucleus, $r_n = a_0 \equiv 0.53 \times 10^{-8} \text{ cm}$ (the Bohr radius).
- For higher n , $r_n = a_0 n^2$.
- Note that the speed of the electron is smaller in orbits farther from the nucleus,

$$v_n = \frac{ke^2}{n\hbar}$$

Energy of a Bohr Orbit

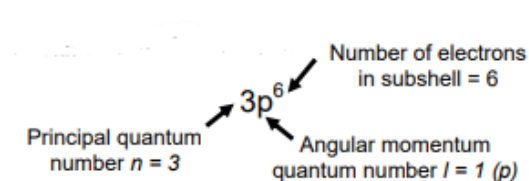
Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantized, with energy for transitions between orbits given by $\Delta E = hf = E_i - E_f$, where ΔE is the change in energy between the initial and final orbits and hf is the energy of an absorbed or emitted photon.

$$\begin{aligned}
 E &= K + U = \frac{1}{2}mv^2 + U \\
 &= \frac{1}{2}\left(\frac{ke^2}{r}\right) - \frac{ke^2}{r} = -\frac{ke^2}{2r} \\
 \therefore E_n &= -\left(\frac{ke^2}{2}\right)\left(\frac{mke^2}{\hbar^2}\right)\frac{1}{n^2} = \\
 &= -\left(\frac{mk^2e^4}{2\hbar^2}\right)\frac{1}{n^2} = -(13.6 \text{ eV})\frac{1}{n^2}
 \end{aligned}$$

- The principal quantum number $n = 1, 2, 3 \dots$
- The orbital angular momentum quantum number l , where $L = \sqrt{l(l+1)}\hbar$ and $l = 0, 1, 2, 3 \dots, n-1$.
- The magnetic quantum number m_l where $m_l = -l, \dots, -2, -1, 0, 1, 2, \dots, l$ ($L_z = m_l\hbar$)
- The electron spin quantum number m_s where $m_s = -\frac{1}{2}, \frac{1}{2}$

Electronic Configurations

Electron configuration is how the electrons are distributed among the various atomic orbitals in an atom.



Subatomic particles in some atoms

O	P	Zn
8 p^+	15 p^+	30 p^+
8 n	16 n	35 n
8 e^-	15 e^-	30 e^-

Summary of Lecture 44 – INTRODUCTION TO NUCLEAR PHYSICS

CONSTITUENTS OF THE NUCLEUS

The nucleus is a collection of particles called protons, which are positively charged, and neutrons, which are electrically neutral. Protons and neutrons are in turn made up of particles called quarks. The chemical element of an atom is determined by the number of protons, or the atomic number, Z , of the nucleus.

PROTON: mass = $1.672 \times 10^{-27} \text{ kg}$
charge = $1.6 \times 10^{-19} \text{ C}$

NEUTRON: mass = $1.675 \times 10^{-27} \text{ kg}$
charge = 0

ELECTRON: mass = $9.109 \times 10^{-31} \text{ kg}$
charge = $-1.6 \times 10^{-19} \text{ C}$

UNITS IN NUCLEAR PHYSICS

- $E = mc^2$ implies $m = E / c^2$

So measure mass in MeV / c^2 :

$$M_p \approx 938 \text{ MeV} / c^2, M_n \approx 940 \text{ MeV} / c^2$$

$$M_e \approx 0.5 \text{ MeV} / c^2$$

- Measure distance in units of fermis

$$1 \text{ fm} = 10^{-13} \text{ cm}$$

- Atomic mass number $A = Z + N$ = the number of protons + neutrons, A determines the mass of the nucleus
- Symbol for a nucleus whose chemical symbol is X

Nuclear Radii

The nuclear radius is the distance from the centre of the nucleus at which the density of nuclear material decreases to one-half of its maximum value at the centre.

What energy should electrons have in order to see a nucleus ?

$$\lambda = h / p \text{ (De Broglie)}$$

Must have $\lambda \approx 1 \text{ fm}$.

$$\therefore E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$
$$\approx \text{few MeV}$$

$$r \approx r_0 A^{1/3} \quad \text{where } r_0 = 1.2 \text{ fm}$$

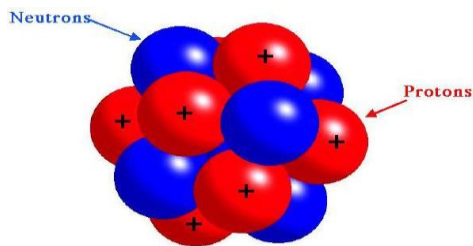
$$\text{note: } V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi r_0^3 A$$

$$\therefore \frac{A}{V} = \frac{3}{4\pi r_0^3}$$

This is the number of nucleons per cubic fermi.

NUCLEAR FORCES

"The fundamental strong interaction holds the constituent quarks of a hadron together, and the residual force holds hadrons together with each other, such as the proton and neutrons in a nucleus."



Yukawa Force

1935 – Hideki Yukawa The strong, short-range force between nucleons, as calculated on the assumption that this force is due to the exchange of a particle of finite mass (Yukawa meson), just as electrostatic forces are interpreted in quantum electrodynamics as being due to the exchange of photons

Calculating the range of the nuclear force from pion exchange

$$\Delta E \Delta t \approx \hbar / 2$$

$$\Delta E = \text{Minimum energy of pion} \approx mc^2$$

$$\therefore \Delta t \approx \frac{\hbar}{2mc^2}$$

$$\text{distance travelled by pion} \approx c \Delta t \approx \frac{\hbar}{2mc}$$

$$\text{Put } \frac{\hbar}{2mc} \approx 1.2 \text{ fm (range of nuclear force)}$$

$$\Rightarrow mc^2 \approx 124 \text{ MeV} / c^2$$

Pion

a meson that is a combination of up and down quarks and antiquarks, that may be positive, negative, or neutral, and that has a mass about 270 times that of the electron.

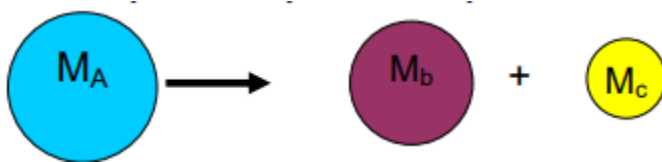
Three: p^+, p^- mass = $139.6 \text{ MeV} / c^2$

and p^0 mass = $135.0 \text{ MeV} / c^2$

$$p + p \rightarrow p + p + \pi^0$$

$$p + p \rightarrow p + n + \pi^+$$

Pions are mediators of the strong force. The most famous formula of physics: $E = mc^2$ is the basis for nuclear energy.



$$Q = (M_A - M_{a1} - M_{a2})c^2 .$$

$$= \text{energy released}$$

Binding Energy

Nuclear binding energy. Nuclear binding energy. Nuclear binding energy is the energy required to hold an atom's protons and neutrons together in the nucleus.

Energy required holding neutrons and protons together therefore keeps the nucleus intact.

$$\Delta N = -\lambda N \Delta t$$

This becomes $\frac{dN}{dt} = -\lambda N$

with solution: $N(t) = N_0 e^{-\lambda t}$

λ = decay constant

The greater λ , the more radioactive the sample.

Half Life Of Nuclei

If N_0 is the number of parent nuclei present at time $t=0$ then:

$$\ln \frac{N}{N_0} = -\lambda t \quad \text{where } N_0 = N(t=0)$$

We can also define the half-life as the time it takes for half the original sample to decay.

Thus $N=N_0/2$ so

$$\ln \frac{N_0}{2N_0} = -\lambda t$$

- There are NO completely stable nuclides above $Z=82$.
- No naturally occurring nuclides above $Z=92$
- Above these limits the nuclei decay or fall apart in some fashion to get below these limits

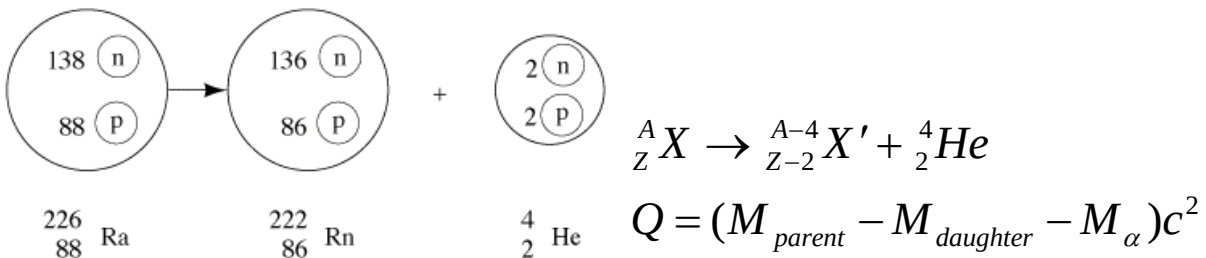
Typical values of half life, $t_{1/2}$

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Isotope	Half Life
Polonium ²¹⁴ 84Po	1.64x10 ⁻⁴ s
Krypton ⁸⁹ 36Kr	3.16 minutes
Strontium ⁹⁰ 38Sr	28.5 years
Radium ²²⁶ 88Ra	1600 years
Carbon ¹⁴ 6C	5730 years
Uranium ²³⁸ 92U	4.5x10 ⁹ years

- $^{222}_{86}\text{Rn}$ Decays to $^{218}_{84}\text{Po}$ by emitting an alpha particle (^4_2He) with a half-life of 3.8 days.
- If we started with 20,000 atoms of Rn-222, then in 3.8 days we would have 10,000 atoms of Rn-222 and 10,000 atoms of Po-218
- In 7.6 days we would have 5000 atoms of Rn-222, in 11.4 days, 2500 Rn-222's, etc

Alpha Decay



- The unstable nuclei simply cannot overcome the proton repulsion indefinitely.
- We saw from the binding energy graph that helium-4 nuclei are very stable.
- Inside big nuclei, two protons and two neutrons seem to form a little gang that hangs together.

Beta Decay

$$n \rightarrow p + e^{-} + \bar{\nu}$$

$${}^A_Z X \rightarrow {}^A_{Z+1} X' + e^{-} + \bar{\nu}$$

$${}^A_Z X \rightarrow {}^A_{Z-1} X' + e^{+} + \nu$$

- Involves the weak nuclear force
- Neutron turns into a proton plus electron plus anti-neutrino
- Proton turns into a neutron plus positron plus neutrino

Summary of Lecture 45 – PHYSICS OF THE SUN

BASIC FACTS ABOUT THE SUN

long

1. Mass: 2×10^{30} kg = 333,000 Earth's
2. Diameter: 1,392,000 km = 109 Earth's
3. Age: 4.6 billion years
4. Rotation Period: 25 days at equator, 36 at poles (at surface)
5. Temperature: 15 million K at core, 5770 at surface
6. Density: 8 x gold at the core, average ~ 1.5 water
7. Composition: 72% H, 25% He, "metals"
8. Luminosity: 3.83×10^{27} joules per second

Solar Power

- Energy output every second? 3.83×10^{24} kilowatts
- Equal to 8×10^{16} of the largest power plants on Earth, which produce ~ 5000 Megawatts of power.
- Every second the sun puts out as much energy as 2.5×10^9 (2 billion) such power plants would put out every year

What Powers the Sun?

Earth is very old (billions of years)

What fuel supply could last that long? Chemical burning (e.g. coal)?

Could only last a few million years

Nuclear Fusion

- Conversion of mass directly into energy $E = mc^2$

- Happens when atomic nuclei fuse
- The only plausible source of solar energy Needs extreme temperature, density, pressure

Nuclear Fusion in the Sun

- Stellar core temperatures are greater than the ~8 million K required for hydrogen fusion
- Amount of repulsion increases with more positive charge, so fusion of larger nuclei requires increasingly higher temperatures & densities (100 million K for helium fusion)
- Thus stars fuse hydrogen first.
- Reaction efficiency ~0.008. Sun can last ~10 billion years on core hydrogen fusion



Convection Zone:

Energy is transported through convection: hot gas rises, irradiates energy, and becomes cold. Cold gas sinks to the bottom. $\Rightarrow$ *Solar Granulation*

Photosphere

The photosphere is the lowest layer of the solar atmosphere. It is essentially the solar "surface" that we see when we look at the Sun in "white" (i.e. regular or visible) light. When we observe sunspots and faculae (bright little cloud-like features) we are observing them in the photosphere.

Corona



- Outermost layer of Sun – a low density cloud of plasma; visible during solar eclipses (as above)
- Hotter than most of the inner layers. Average temperature 1 million K, but can reach 3 million K

- Each second, the sun turns 4 million tons of hydrogen into energy.
- Energy generation is from fusion:
 $4\text{ H} \rightarrow 1\text{ He} + 25\text{ MeV} + 2\text{ neutrinos}.$

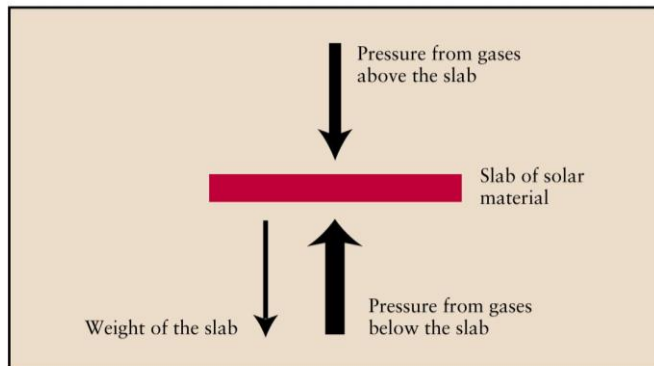
Nuclear Fusion in the Sun

- Reaction efficiency ~ 0.008 .
- Sun can last ~ 10 billion years on core hydrogen fusion

Hydrostatic Equilibrium

Balance between outward gas pressure and inward gravitational pressure

- Gas pressure
 - collisions among the very hot and densely packed particles
- Gravitational pressure
 - mass pressing inward, increasing toward the core



The Equilibrium between Gravity and Pressure

This pressure counteracts the force of gravity, putting the star into what is called hydrostatic equilibrium. A star is okay as long as the star has this equilibrium between gravity pulling the star inwards and pressure pushing the star outwards.

$$dF = - \frac{GM(r) \times \rho(r) 4\pi r^2 dr}{r^2}$$

$$dF = dP \times 4\pi r^2 dr$$

$$\therefore dP = - \frac{GM(r) \rho(r) dr}{r^2}$$

$$\text{or, } \frac{dP}{dr} = - \frac{GM(r) \rho(r)}{r^2}$$

Similarly,

$$M(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$

Knowing $\rho(r)$ gives the mass distribution and pressure !

$$\text{Model: assume } \rho \approx \frac{M}{(4\pi/3)R^3} \approx 1.4 \text{ gm cm}^{-3}$$

$$P_{\text{centre}} \approx \frac{GM\rho}{R} \approx 3 \times 10^9 \text{ atmospheres}$$

Helioseismology

- Probe of Sun's interior using pulsating motions of the Sun
- Can be seen by measuring the Doppler shifts across the face of the Sun
- Some parts expanding towards Earth while adjacent regions contract away – like modes in a ringing bell

Thermal Radiation from a Solid

Calculate the power radiated by a 10-cm-diameter sphere of metal at room temperature (20°C).

$$P = \sigma T^4 \quad (\text{power radiated per area})$$

$$= (5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}) (293\text{K})^4$$

$$= 418 \text{ W/m}^2$$

$$\text{Area: } A = 4\pi r^2 = 4\pi (5 \times 10^{-2} \text{ m})^2 = 3.14 \times 10^{-2} \text{ m}^2$$

$$\begin{aligned} \text{Power radiated} &= P \times A = (418 \text{ W/m}^2) (3.14 \times 10^{-2} \text{ m}^2) \\ &= 13 \text{ Watts} \end{aligned}$$

Surface Temperature of the Earth

$$P_S = \text{Sun's flux at its surface} = \sigma T_S^4$$

$$R_S = 7 \times 10^8 \text{ metres}$$

$$P_R = \text{Sun's flux at earth} = \sigma T^4 \times \left(\frac{R_S}{R}\right)^2$$

$$R = 1.5 \times 10^{11} \text{ metres}$$

$$T_S = 5800^\circ \text{K}$$

$$J_E = \text{Earth's flux at its surface} = \sigma T_E^4$$

Power absorbed from sun = Power radiated by earth

$$P_R \times \pi R_E^2 = P_E \times 4\pi R_E^2$$

$$P_R = 4P_E$$

$$\sigma T_S^4 \times \left(\frac{R_S}{R}\right)^2 = 4\sigma T_E^4$$

$$T_E = \left(\frac{R_S}{2R}\right)^{1/2} T_S = 280\text{K}$$

Surface Temperature of the Earth

- About 30% of the sun's radiation reflects off our atmosphere!
- In fact the average surface temperature of the earth is about 290K, or about 13°C.
- The extra warming is mainly due to the "greenhouse effect".

Molecular Vibrations:

A molecular vibration is a periodic motion of the atoms of a molecule relative to each other, such that the center of mass of the molecule remains unchanged.

Energy levels of an oscillator:

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$n = 3$$

$$n = 2$$

$$n = 1$$

$$n = 0$$

The Greenhouse Effect

The greenhouse effect is the way in which heat is trapped close to Earth's surface by “greenhouse gases.” These heat-trapping gases can be thought of as a blanket wrapped around Earth, keeping the planet toastier than it would be without them.

- The main greenhouse gases are CO₂, CH₄, H₂O. (N₂, O₂ and Ar are transparent to solar and earth radiation.)
- CO₂ is now 360 ppm of the atmosphere, up from 227 ppm in 1750, before the industrial revolution. Plants turn H₂O + CO₂ into O₂ plus organics.
- Some estimates predict a 3 to 10 °C rise in the earth’s surface temperature over the next 100 years due to the increased CO₂ greenhouse effect.

Main Sequence Evolution

Stellar Evolution: Main Sequence. Once a protostar starts burning hydrogen in its core, it quickly passes through the T-Tauri stage (in a few million years) and becomes a main sequence star where its total mass determines all its structural properties.

Earth’s core

Core is collapsing and heating up He Fusion (Triple Alpha Process)

- $3\ ^4\text{He} \Rightarrow\ ^{12}\text{C} + \text{energy}$
- $10^8\ \text{K} = 100,000,000\ \text{K}$
- Massive stars do this easily Core is still ideal gas
- Low mass stars (like the Sun) struggle Core becomes degenerate

Helium Depletion in Core

Once the gas leaks into the atmosphere, it is light enough to escape the Earth's gravitational field so it bleeds off into space, never to return. We may run out of helium within 25–30 years because it's being consumed so freely.

Second Giant Phase

- He is depleted in the last fusion zone
- Reaction rate falls
- Core can no longer support its weight core shrinks
- Temperature in the core increases Carbon fusion threshold 600 million Low Mass stars cannot reach this temperature
- Envelope expands to supergiant.

For Short Notes

<https://vuonlinehelp.blogspot.com/p/notes.html>

Over YouTube channel

https://www.youtube.com/results?search_query=orange+monkey+team

PHY101 YouTube Final term Short Video 2:33:49

Video one 23 to 35

<https://youtu.be/BHvR0Fthna8>

Video Two