

Date: _____

Mode

Most repeated value in your data.

it's denoted by

$$\hat{X}$$

1, 2, 3, 4

Yaha mode? ayyga?

$$\hat{X} = 1$$

Example: — Page (49)

Find Mode?

Subtract km/h then 30.0 column
2 column key sath + km/h
33.0 - 32.9 ko 2 py dekh kr key sath

Mileage Rating	class Boundary	No. of cars (f)
30.0 — 32.9	29.95 — 32.95	2
33.0 — 35.9	32.95 — 35.95	4
36.0 — 38.9	35.95 — 38.95	14
39.0 — 41.9	38.95 — 41.95	8
42.0 — 44.9	41.95 — 44.95	2
		30

$$\Sigma f = 30$$

STA 301

Lec # 11

Mean Deviation?

Two techniques

ky sath nikal saki h.

- 1 Simple Mean Average ky sath
- 2 Midain ky sath

Day	No of fatalities (x)	$d = x - \bar{x}$	$ d = x - \bar{x} $
Sunday	4	0	0
Monday	6	+2	2
Tuesday	2	-2	2
Wednesday	0	-4	4
Thursday	3	-1	1
Friday	5	+1	1
Saturday	8	4	4
$\Sigma n = 28$ $\Sigma(x - \bar{x}) = 0$ $\Sigma d = 14$			

How To find Mean?

$$\bar{x} = \frac{\Sigma x}{n} = \frac{28}{7} = 4$$

$$d = x - \bar{x} = 4 - 4 = 0$$

d for difference $|d|$ for
- wali ko + plus aur
plus wali ko plus hi
- wali sam term plus
ho Jagn gi.

$$\text{Mean deviation} = \frac{\sum |d|}{n}$$

Putting values

$$n=7, |d|=14$$

$$\frac{14}{7} = 2$$

$\bar{X}$ ki Jagn $\bar{X}$ bar bi use
kr skly ho Midain.

assignment :- find Midain

Put value in eq - ①

$$\text{Mean deviation} = \frac{\sum |x - \bar{X}|}{n} \quad \text{--- ①}$$

coefficient of M.D

$$= \frac{M.D}{\text{Midain}} \quad \text{or} \quad \frac{M.D}{\text{Mean}}$$

$$\frac{M.D}{\bar{X}} \quad \text{or} \quad \frac{M.D}{\bar{X}}$$

Find Mean deviation in group data (mean frequency) $\sum f$ in n given mai. x_a frequency nhi thi ya bi x_{ad} rakhi n .

for group data

$$M.D = \frac{\sum f |x - \bar{x}|}{\sum f} = \frac{\sum f |d|}{\sum f}$$

$$M.D = \frac{\sum f |x - \bar{x}|}{\sum f} = \frac{\sum f |d|}{\sum f}$$

Group data ke formula mai book wala hi get ki n dikha hawa h. waha $\sum f$ anah

Variance and Standard Deviations

Example:-

<u>X</u>	<u>$X - \bar{X} =$</u>	<u>$(X - \bar{X})^2$</u>
4	0	0
6	2	4
2	-2	4
0	-4	16
3	-1	1
5	+1	1
8	+4	16

$$\sum (X - \bar{X}) = 0 \quad \sum (X - \bar{X})^2 = 42$$

Sub sy phly humay Mean nikal na h.

$$\bar{X} = \frac{\sum X}{n} = \frac{28}{7} = 4$$

her ik term may sy isy - krty jana h.

origin of oo kehta h ky
if $\sum (X - \bar{X}) = 0$ phr
ya thk h

As we know that

$$\text{Variance} = S^2$$

$$= \frac{\sum (x - \bar{x})^2}{n}$$

putting value

$$= \frac{42}{7} = 6$$

$$\sqrt{S^2} = (\sqrt{6})^2$$

variance nikalna h tu
uski bad lagana h square
root yahan

Standard Deviation
Taking square root on B.S

$$\text{Standard Deviation} = S$$

$$= \sqrt{6}$$

$$= 2.45$$

is group data ky liye n ki
jaga f aur oper ef multiply
ho jay ga.

For group data

$$\text{variance} = s^2$$

$$= \frac{\sum f(x - \bar{x})^2}{\sum f}$$

Taking square root B.S

$$s^2 = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

$$s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

Example.

Standard Deviation?

class interval	No of Bulbs f	Midwin x	f(x)	$f(x - \bar{x})^2$	$f(x - \bar{x})$
life in hundred of hours					
0 — 5	4	2.5	10	478.5	1914.0
5 — 10	9	7.5	67.5	284.76	2562.5
10 — 20	38	15.0	570	87.89	3339.5
20 — 40	33	30.0	990	31.64	1044.0
40 — andover	16	50.0	800	656.69	1056.0
✓	$\Sigma f = 100$	$= 103$	2437.5		1747.0

Ka matlb h oper wali rakam
ko add kro $20 + 40 = 60$
60 jo h woh andover h.

As we know that

$$s^2 = \frac{\Sigma f(x - \bar{x})^2}{\Sigma f} \quad (1)$$

phly Mean nikalyn gy.

$$\text{formula } \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{2437.5}{100}$$

$$= 24.375$$

phr original X mai sy
 $\bar{X}$ ko - krna h.

$$= \frac{17467.06}{100}$$

$$S^2 = 174.6706$$

Taking square root

$$\sqrt{S^2} = \sqrt{174.6706}$$

$$= 13.21$$

ya hamary pass varius nikla h.

coefficient of standard deviation

$$= \frac{S.D}{\text{Mean}} \times 100\%$$

multiply by hundred krna h.

Coefficient of variation =

$$= \frac{S \cdot S^2}{\bar{X}} \times 100\%$$

is krny liy S^2 krny h.

Important

Example :-

Standard Deviation

Mean 195

$$S = 4, \bar{X} = 195$$

for country No 1

$$\frac{4}{195} \times 100\% = 2.05\%$$

for country No 2

$$S = 28, \bar{X} = 75$$

$$\frac{28}{75} \times 100\% = 37.32\%$$

assignment 2nd Example :-

DONE



Lec # 12

(Chebyshev's Theorem)

only statement yad rakhni h.

not included in statement only read

$$(\bar{X} - 5 \quad \bar{X} + 5)$$

mean standard deviation

islo hum usy bi likh sakte h.

$$](\bar{X} - 5 \quad \bar{X} + 5)[$$

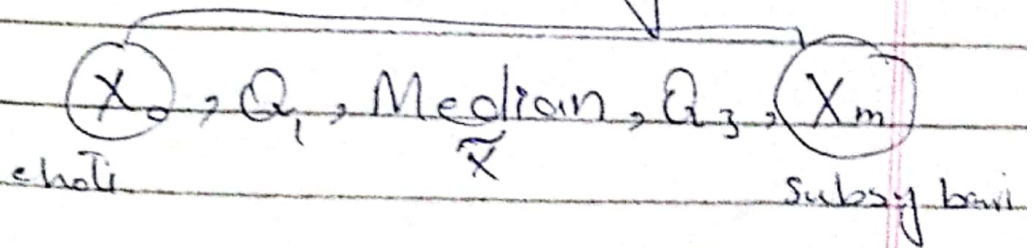
Five number Summary

py keh sakte ky bates

kary ka objective is

ka relation apes mai kya
h. Five number theory
ky ander.

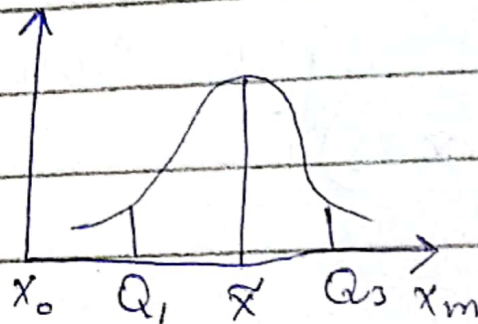
Five no konsy ?



The Symmetric Curve..

To gadi py jar rahi ho
road pey lea kisi speed sy
To ap uniform jar rahi
ho symmetric jarahi ho

asi speed jo change na ho

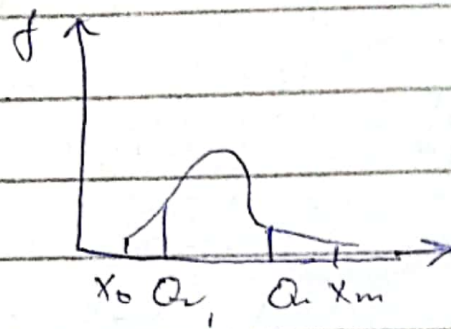


$$\bar{x} = \tilde{x} = \text{Mid-Range?}$$

Ager ya sub same h to x is
Symmetric h.

$$2 = 2 = 2 = 2 = 2$$

Right skewed
OR
Positively Skewed

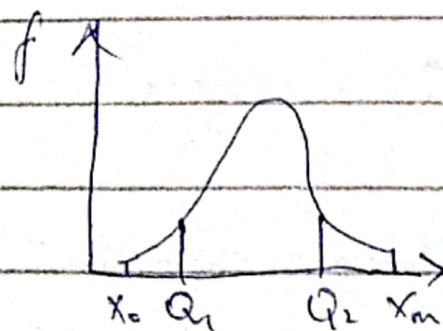


median sub sy char
ay.

MCQ — Median < Mid-Quantile ^{Range} < mid-Range
 $5 < 6 < 7$

Left skewed or negatively skewed

you book
mai nhi



Median > Mid-Quantile range > mid-Range

STA 301

Lee #13

Five number Summary

پانچ ہنری 4 number
Summary پرکی h.

is mai addition Mode ko

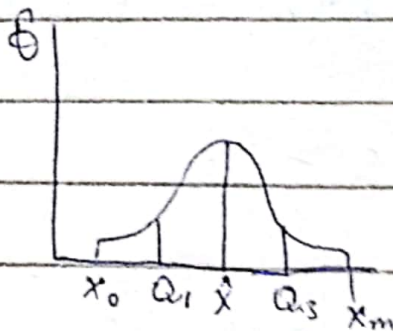
urdu mai Aada keshly

h. most repeated values ko

keshly h.

Example: 1, 2, 3, 4, (5, 5) Mode

The Symmetric Curve



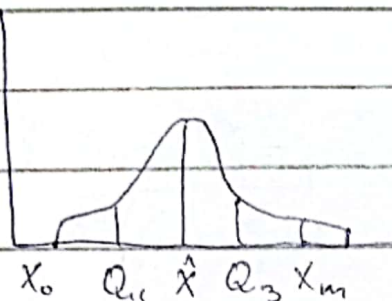
$$x_0 = 1, Q_1 = 2, \hat{x} = 3, Q_3 = 4, x_m = 500$$

Mean

$$\bar{X} = \tilde{X} = \text{Midpoint} = \text{MidQuartile Range} = \hat{x}$$

$$\text{Mean} = \text{Midain} = \text{Midpoint} = \text{MidQuartile Range} = \hat{x}$$

The positively skewed
or
Right skewed

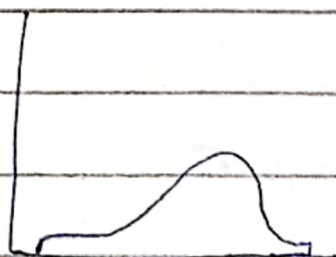


$\tilde{X} < \text{mid-quartile range} < \text{mid range}$

$$X_0 < Q_1 < \hat{X} < Q_3 < X_m$$

The negatively curve
or

left skewed



S.D

Person

coefficient of skewness

$$\frac{\text{Mean} - \text{Mode}}{S.D} \quad \text{or}$$

Person coeff. of skewness

$$\frac{3(\text{Mean} - \text{Mode})}{S.D}$$

Handwritten notes and arrows pointing to the formulas above.

STA 301

Lec #14

Pearson coefficient skewness:

~~Formula~~ Bowley's coeff of skewness

$$\frac{Q_3 + Q_2 - 2\bar{X}}{Q_3 - Q_1} \text{ or } \frac{Q_3 + Q_2 - 2\bar{X}}{Q_3 - Q_1}$$

Example := (108)

Children Manual work

C M W

$$= \frac{11 + 6 - 2(8.50)}{2.5}$$

$$= \frac{11 - 11}{2.5} = \frac{0}{2.5} = 0$$

C.N.M.W

yourself

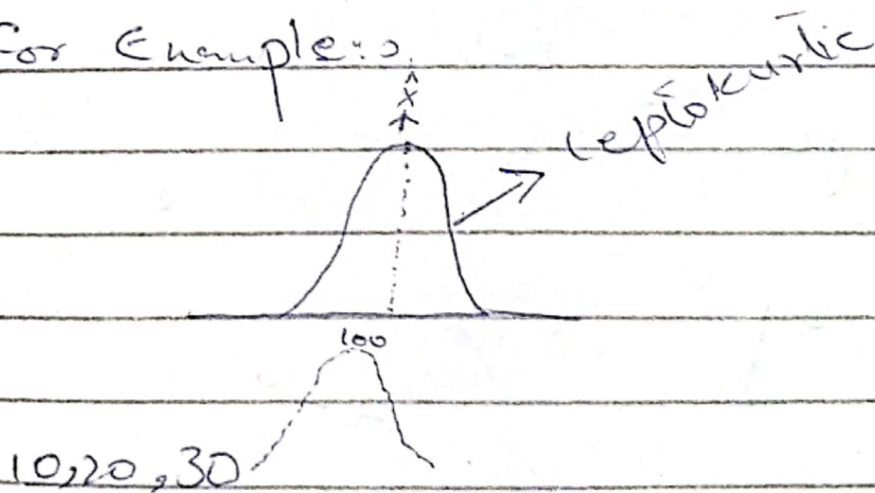
Important Topic

Kurtosis.

Jo Bara kam kr rahi hain wo Kurtosis h.

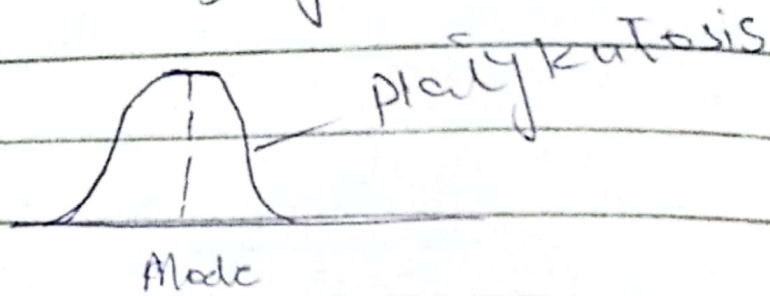
Sub sy phly ap data mein ap ky pass ik data hai h ap uska Mode nikty ho. Mode ki value boht Bari datyn h. value Jo hti h uska utar cherkho yake dum peak a jati h.

for Example:



10, 20, 30, 100, 5, 2, 1

asi value Jo oper Jasy
 thory time ley lig ruk
 Jasy h Jasy



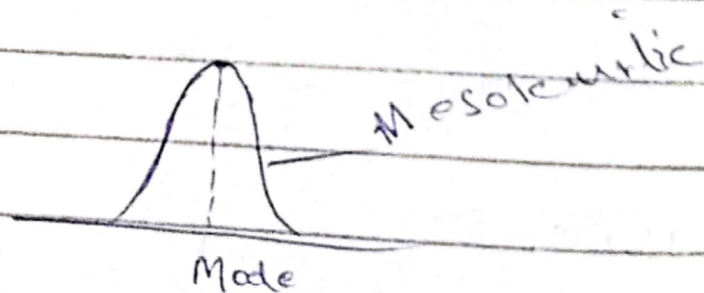
Jasy

1, 2, 3, 4, 4, 4, 3, 2, 1

Mesokurtic:

Behren sequence
 hagi, Jasy counting hagi
 h.

Jasy



1, 2, 3, 4, 5, 6, 7, 8, 9
 mode

Imp

Find Kurtic = ?

$$\text{Kurtic} = \frac{Q \cdot D}{P_{10} - P_{10}} = \text{quarto quartal} \cdot D$$

P_{10} ki value examiner khod dyga.



Moments:

Ungroup data
frequency na ho to samj
lay key ya ungroup data
h.

r = Subkrip

Formula:

$$m_r = \frac{\sum (X - \bar{X})^r}{n}$$

Find 1st four moments ?

4 Power Tak isy multiply
kry jao gy.

x	$(x - \bar{x})$	$(x - \bar{x})^2$	$(x - \bar{x})^3$	$(x - \bar{x})^4$
32	-8	64	-512	4096
36	-4	16	-64	256
36	-4	16	-64	256
37	-3	9	-27	81
49	-1	1	-1	1
41	1	1	1	1
45	5	25	125	625
46	6	36	216	1296
48	8	64	512	4096
$\Sigma x = 360$	$\Sigma = 0$	$\Sigma = 232$	$\Sigma = 186$	$\Sigma = 10768$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{360}{9} = 40 \checkmark$$

as we know that

$$m_r = \frac{\Sigma (x - \bar{x})^r}{n}$$

put $r = 1, 2, 3, 4$ and $n = 9$

$$m_1 = \frac{\Sigma (x - \bar{x})^1}{9}$$

$$m_1 = \frac{\Sigma (x - \bar{x})}{9}$$

$$= \frac{0}{9}$$

$$m_2 = \frac{\sum (x - \bar{x})^2}{n}$$

$$m_2 = \frac{232}{9}$$

$$= 25.78$$

$$r_3 = 3$$

$$m_3 = \frac{\sum (x - \bar{x})^3}{n}$$

$$= \frac{186}{9}$$

$$= 20.67$$

$$r_4 = 4$$

$$m_4 = \frac{\sum (x - \bar{x})^4}{n}$$

$$= \frac{10708}{9}$$

$$= 1187.78$$

In Group data

$$m_r = \frac{\sum f(x - \bar{x})^r}{\sum f}$$

1st moment ka sum hamisha 00 hota h.

In group data key liye frequency hogi aur iss mai frequency ko sub key saath multiply kr dein gaun. 4 moment key saath

Moment about Mean.

correction:-

'm' key oper Jo 'm' lagi hoti h. osy prime bolty h. 4 prime kesy nikalany thy ya sikha phly. bad meri sikha moment about Mean kesy nikaly h. aik bat xad rakha prime sy Jo m_1 m_2 m_3 m_4 nikalaty h. wo uncorrected hoty h. ab osi ko correct krna h.

objectiv $\leftarrow$ 1 mp

Correction:

m_1 aur m_2 Same
hi rehly h. m_2 hr m_4
change haly rehly h.
moment about mean m_1 to
hota h. wo always 00
hota h.

$$m_1 = 0$$

uncorrected

$$m_2 = \overset{\text{m.s.d.}}{2.64} - \overset{\text{u.c.}}{m_3}$$

$$2.64 - 0.08$$
$$= 2.56$$

distance =

$$m_3 = 0.08$$

$$m_4 = \overset{\text{m.s.d.}}{28.30} - \overset{\text{u.c.}}{m_2} + m_1$$
$$28.30 - 1.32 + 0.06$$

$$27.04$$

Moment Ratio

$$b_1 = \frac{(m_3)^2}{(m_2)^3}$$

β_1

and

$$b_2 = \frac{m_4}{(m_2)^2}$$

Note:-

1. Δ

if $b_2 = 3$ then Normal distribution
if $b_2 < 3$ platykurtic
if $b_2 > 3$ leptokurtic

coefficient

Lec #15

Regression line y on x

not py nhi h

$$\hat{y} = a + bX$$

↓ Dependent Variable
 stop of B
 Independent variable
 x -intercept

$\hat{y}$ Dependent variable h. Kis
 $Py \times Py \times$ Independent
 variable h.

$$y = mx + c \rightarrow \underline{x\text{-intercept}}$$

$m = \text{stop}$
 isiko expand kr ky brai gyi h.

x barej h. b pami h. a consider
 krto khad etc. ya server ka
 server Independent h. aur
 depend kon kr rahi h x a $\hat{y}$
 $\hat{y}$ kya h. gadam. jitna zyada
 mud itna hi jy jy ga
 itni fasal kati jaygi gi.

Regression line X on Y

$$\hat{X} = a + bY$$

Examples Pg #120

Find Regression line.

Sol:→

X	Y	X^2	XY
1	1	1	1
2	1	4	2
3	2	9	6
4	2	16	8
5	4	25	20
$\Sigma X = 15$	$\Sigma Y = 10$	$\Sigma X^2 = 55$	$\Sigma XY = 37$

Let

$\hat{Y} = a + bX$ be the regression line Y on X

$\hat{Y}$ cap ko khatam krdo , smitien.

obj $\Sigma e = 0$ Sum apply hot kr a py to wo constant chodta h.

Normal equation

$$y = a + bx \quad \text{--- eq A}$$

$$\sum y = \sum a + b \sum x$$

$$\sum y = an + b \sum x \quad \text{--- eq (1)}$$

and

multiply Both side $\times$ eq A

$$xy = ax + bx^2$$

sum aplay

$$\sum xy = a \sum x + b \sum x^2 \quad \text{--- (11)}$$

putting the value in eq (1)

$$10 = 5a + 15b \quad \text{--- B.}$$

$$37 = 15a + 55b \quad \text{--- C}$$

(eq B) $\times 3$ on both side - (eq C)

$$30 = 15a + 45b$$

$$\pm 37 = 15a \pm 55b$$

$$\hline -7 = -10b$$

$$b = \frac{7}{10}$$

$$b = 0.7$$

put in eq $\rightarrow 13$

$$10 = 5a + 15(0.7)$$

$$10 = 5a + (10.5)$$

$$10 - 10.5 = 5a$$

$$-0.5 = 5a$$

$$\frac{-0.5}{5} = a$$

$$\boxed{-0.1 = a}$$

putting the value y on x

$$y' = a + bx$$

$$\boxed{y' = -0.1 + 0.7x}$$

most Imp

Standard Error

$$S_{y \cdot x} = \sqrt{\frac{\sum y^2 - a \sum y - b \sum xy}{n-2}}$$

wahi Example lay ley chally
h.

$$a = -0.1, b = 0.7$$

putting values

$$S_{y \cdot x} = \sqrt{\frac{26 - (-0.1)(10) - (0.7)(37)}{5-2}}$$

$$= \sqrt{\frac{26 + 1 - 25.9}{3}}$$

$$= \sqrt{\frac{1.1}{3}}$$

$$= 0.61$$

Correlation

Relation x aur y ke beech

Covariance

$$r = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}}$$

$$\sqrt{\text{Var}(X) \text{Var}(Y)}$$

Where

$$\text{Cov}(X, Y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{n}$$

and

$$\text{Var}(X) = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

and

$$\text{Var}(Y) = \frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2$$

find?

Pearson's coefficient of correlation - ~~cor~~
OR

Regression correlation OR - ~~cor~~
Find small r

$$r = \frac{\sum xy - (\sum x)(\sum y)/n}{\sqrt{\left[\sum x^2 - \frac{(\sum x)^2}{n}\right] \left[\sum y^2 - \frac{(\sum y)^2}{n}\right]}}$$

MCQ

Note

Lies b/w $-1 < r < 1$

Case: 1

positive correlation

$$0 < r < 1$$

Example: 0.5, 0.9, 0.1

Case II

Negative correlation

$$-1 < r < 0$$

Ex: $-0.1, -0.8$

Case III

No correlation

$$r = 0$$

Example $\rightarrow$

X	Y	X^2	Y^2	XY
5	11	25	121	55
12	16	144	256	192
14	15	196	225	210
16	20	256	400	320
18	17	324	289	306
21	19	441	361	399
22	25	484	625	550
23	24	529	576	552
25	21	625	441	525
156	168	324	3294	3

$$r = \frac{\sum XY - \frac{(\sum X)(\sum Y)}{n}}{\sqrt{\left[\sum X^2 - \frac{(\sum X)^2}{n}\right] \left[\sum Y^2 - \frac{(\sum Y)^2}{n}\right]}}$$

$$\sqrt{\left[\sum X^2 - \frac{(\sum X)^2}{n}\right] \left[\sum Y^2 - \frac{(\sum Y)^2}{n}\right]}$$

putting values.

$$\underline{3109 - (136)(168)/9}$$

$$z = 0.88 > 0$$

Positive correlation

set:

A set is any well defined collection of list of distinct objects

~~Two types~~ finite, Infinite
 ex: Star Stud

Sub set

A has element B
 B is element of A
 A sub set of B

$$A \subseteq B$$

Sub set Super set

chole phy aur Banna bad
 mai lekhna h.

Set Types

- 1 - Simple
- 2 - nouns of study of car
- 3 - set better form $A = \{x | x \in N\}$

union of set

$$A = \{1, 2\}, B = \{1, 3, 4\}$$

$$\text{Ans } [1, 2] \cup [1, 3, 4]$$

$$= \{1, 2, 3, 4\}$$

Intersection of set

$$A \cap B = \{1, 2\} \cap \{1, 3, 4\}$$

$$= \{1\}$$

set Difference

$$A - B = \{1, 2\} - \{1, 3, 4\}$$

$$= \{2\}$$

complementation of set

$$\text{if } A = \{1, 2\}, U = \{1, 3, 4\}$$

$$\text{compliment } A^c = U - A = \{1, 3, 4\} - \{1, 2\} \\ = \{3, 4\}$$

Laws

Commutative law

$$A \cup B = B \cup A, A \cap B = B \cap A$$

Associative law

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

idempotent law

$$A \cup A = A$$

$$A \cap A = A$$

I

Identity law

union bare set 'a' h

$$A \cup S = S$$

$$A \cap S = A$$

$$A \cap \emptyset = A$$

$$A \cap \mathbb{R} = \emptyset$$

3 Terko

van diagram
ke space
mulin

X, S — space
Jo Boleh Base
nalar h

complement law

$$A \cup \bar{A} = S$$

Enu

$$U = \{1, 2, 3, 5, 6\}$$

$$A = U - A$$

$$= \{1, 2, 5, 6\} - \{1, 2, 7\} = \{5, 6\}$$

$$A \cup A = \{5, 6\} \cup \{1, 2, 7\}$$

$$= \{1, 2, 5, 6\}$$

$$U = U - U$$

$$A \cap \bar{A} = \emptyset$$

$$(\bar{A}) = A$$

$$\bar{S} = \emptyset$$

$$\bar{\emptyset} = S$$

De Morgan's law

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$(\overline{A \cap B}) = \bar{A} \cup \bar{B}$$

partition of set

$$\text{if } A = \{1, 2, 3, 4, 5\}$$

$$A_1 = \{1, 2\}, A_2 = \{2, 4, 5\}$$

$$A_1 \cup A_2 = \{1, 2, 3, 4, 5\} = A$$

power set

$$A = \{H, T\}$$

$$P(A) = \{\emptyset, \{H\}, \{T\}, \{H, T\}\}$$

$$A = \{1, 2, 3\} \text{ your set}$$

Cartesian product.

$$2 \times 6 = 12$$

Example:

$$A = \{H, T\}$$

$$B = \{1, 2, 3, 4, 5, 6\}$$

$$A \times B = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), \\ (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

Exemp

$$3 \times 9 \times 2 = 24$$

Time in x kb

↔

6

17/02/22

SIA 301

Leet #17

Probability

Counting Rules:

Multiple choice

$$\text{time} \rightarrow 3 \times 2 \times 4 = 24$$

Jinso ko
Rank
Karyo ko

Permutations

Formula:-

Giving
thing

$$P_r^n = \frac{n!}{(n-r)!}$$

r is n

Example:-

4 choices

3 choices

2 choices

$$\Rightarrow 2 \times 3 \times 4 = 24$$

Permutation

$$M = 4$$

choices = 3

$$P_3^4 = \frac{4!}{(4-3)!}$$

$$\frac{4!}{1!}$$

$$\frac{4 \times 3 \times 2 \times 1}{1}$$

$$24$$

Note - do)

$$P_n^n = \frac{n!}{(n-n)!}$$

$$= \frac{n!}{0!}$$

$$1! = 1$$

$$= \frac{n!}{1}$$

$$0! = 1$$

$$P_n^n = n!$$

$$P_{10}^{10} = 10!$$

Example:-

persons = 3

chairs = 3

$$P_3^3 = \frac{3!}{(3-3)!} = \frac{3 \times 2 \times 1}{0!} = \frac{6}{1} = 6$$

Formula:- why use

Arrangement or Permutation

$$P = \frac{n!}{n_1! n_2! \dots n_n!}$$

Example:-

Committee form? ^{9 8 7 6 5 4 3 2 1}

$$= \frac{9!}{1! \times 1! \times 2! \times 1! \times 2! \times 2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1 \times 1 \times 2 \times 1 \times 2 \times 2}$$

CS
9/4/24
m 2/4

find Arrangement name Sufyan ♥

$$P = \frac{6!}{1 \times 1! \times 1! \times 1! \times 1! \times 1!}$$

$$P = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$P = 720$$

start/choice

Combination:-

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Relation b/w P_r^n and C_r^n

$$C_r^n = \frac{n!}{r!(n-r)!}$$

$$r! C_r^n = \frac{n!}{(n-r)!}$$

$$\Rightarrow \frac{n!}{(n-r)!} = r! C_r^n$$

objective

$$P_r^n = r! C_r^n$$

Notes:-

C_r^n is called binomial coefficient formula's

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

Example:-

A, B, C selected

(A, B), (B, A)

(A, C), (C, A)

(B, C), (C, B)

properties:-

(I) $\binom{n}{r} = \binom{n}{n-r}$ ——— objective

Example:-

$$\binom{3}{2} = \binom{3}{3-2}$$

$$= \binom{3}{1}$$

$$\frac{3!}{2!(3-2)!} = \frac{3!}{1!(3-1)!} = \frac{6}{2} = 3$$

(II) $\binom{n}{n-r} + \binom{n}{r} = \binom{n+1}{r}$

combination kisi kahi check
kiya jata h na oo kay sath
to ans 1 aata h.

$$\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{n!} = 1$$

object

$$C_0^n = C_{n-1}^n$$

$$C_1^n = \frac{n!}{1!(n-1)!}$$

$$= \frac{n(n-1)!}{1(n-1)!}$$

IV — $C_1^n = n$

$$C_2^3 = \frac{3!}{2!(3-1)!}$$

$$= \frac{3 \times 2 \times 1}{2! (2!)}$$

$$= 6$$

Example: $\therefore$

$$n = 10, r = 3$$

$$C_r^n = \frac{10!}{3! (10-3)!}$$

$$= 120$$

$${}^5C_5 = \frac{5!}{5!(5-5)!}$$

$$= 2598960$$

آزادانه تجربہ Random Experiment :-

An experiment which produce different result even though it is performed a large numbers of times under essentially similar conditions is called R.E

for example :-

The Tossing of a coin.

properties :-

The experiment can be repeated any number of time.

A random Trial consists of at least possible outcomes.

Nothing can be said

Which certainly about the outcomes of random experiment.

Sample Space

A set consists of all possible outcomes that may result from a random experiment is called sample space.

It is denoted by S

For Example::

The Tossing of a coin

$$S = \{\text{Head, Tail}\}$$

and when a die is rolled

$$S = \{1, 2, 3, 4, 5, 6\}$$

Events

Any subset of the sample space is called an event

For Example::

$$A = \{\text{Head}\}$$

$$B = \{6\}$$

Simple event / Elementary

An event consist only one sample point is called sample Event.

for Example

$$A = \{\text{Head}\}$$

Compound event / complimentary

An event consist more than one event is called

Examp

$$A = \{1, 2, 3\}$$

Mutually Exclusive Events

Two events is said to be mutually Exclusive events or disjoint if they can not occur together.

Example:-

A single coin is tossed then Head and Tail does not occur together.

Meaning this is only

Exhaustive Events:-

if Two events A and B constitute the entire sample space

Example:-

When a die is rolled

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3\}$$

$$B = \{4, 5, 6\}$$

$$(A \cup B) = S$$

Partition of the Sample Space

if Sample Space (S)

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A_1 = \{1, 2\}$$

$$A_2 = \{3\}, A_3 = \{4, 5, 6\}$$

$$A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6\} = S$$

Equally likely events:

Two events are said to be equally likely if they have the same probability of occurrence.

for Example:

A coin is Tossed

$S = \{H, T\}$



Lec #18

Mutually Exclusive Events

Exhaustive Event

partition of the sample space

Equally likely Events:-

Subjective or

Personalistic probability

It refers to the degree of belief of an individual that the event will occur based on whatever evidence is available to the individual.

Classical or Prior Probability
According to P.S Laplace:-

If an experiment can result in " n " mutually exclusive equally likely and exhaustive events and m of which are

favourable to the occurrence
of an event "A"

Formula:-

$$P(A) = \frac{\text{No of favourable outcome}}{\text{No of Total outcomes}} = \frac{m}{n}$$

$$P(A) = \frac{m}{n}$$

$$n = 10$$

$$\{1, 2, \dots, 10\}$$

$$\begin{matrix} \textcircled{1} & \textcircled{2} \\ (2, 4) & = m \end{matrix}$$

$$= \frac{2}{10} = \frac{1}{5}$$

Heart (♥) Club (♣) Diamond (♦) Spade (♠)

	Red (13)	Black (13)	Red (13)	Black (13)
— A	A	A	A	A
Jack (1)	1	1	1	1
King (1)	1	1	1	1
Queen (1)	1	1	1	1

Red card : 26

Red Joker = 1

Black card : 26

Black Joker = 1

Total card : 52

Face card : 12

find probability face card

$$P(A) = \frac{12}{52} = \frac{3}{13}$$

find probability Joker card

$\frac{2}{52}$ Joker of red card

playing card 52 hole b.

4 card 10 by nongy.

Example 2

Sol. is

Total playing cards = 52

$$\Rightarrow n = 52$$

Red card = 26

$$m = 26$$

$$P(A) = \frac{m}{n} = \frac{26}{52} = \frac{1}{2}$$

No of 10s cards: 4

$$m = 4$$

$$P(B) = \frac{m}{n} = \frac{4}{52} = \frac{1}{13}$$

Example 3

Imp Number of sample size

$$\text{Number of sample} = \frac{1}{8} 2^3 = 8$$

$$\frac{8}{2} = 4 \quad \frac{4}{2} = 2 \quad \frac{2}{2} = 1$$

$$\frac{8}{2} = 4, \quad \frac{4}{2} = 2, \quad \frac{2}{2} = 1$$

H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$S = \{ H, H, H, HHT, HTH, H\bar{T}, \\ T\bar{H}H, THT, T\bar{T}H, T\bar{T}\bar{T} \}$$

$$n(S) = 8$$

At least one head appears

$$A = \{ HHH, HHT, HTH, T\bar{H}H, \\ HTT, THT, T\bar{T}H, \bar{T}\bar{T}\bar{T} \}$$

$$n(A) = 7$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

fall factorial = combination

Examples 3

Imp Sol sample space = $n(S)$

1 2 3 4
5 6 7 8
9 10 11 12

4 choice

$$C^{12}_4 = 495 \quad \text{fall factorial}$$

OR

$$n(A) = C^3_0 \times C^9_4 + C^3_1 \times C^9_3$$

$$= 1 \times (126) + (3)(84)$$

$$= 126 + 252$$

$$= 378$$

Notes

$$P(A) = \frac{n(A)}{n(S)}$$

$$(i) - 0 \leq P(A) \leq 1$$

$$(ii) - P(S) = 1$$

sure events

$$= \frac{378}{495}$$

$$= 0.76$$

Relative frequency or posterior,
If an experiment is repeated
a large number of times say
"n" under uniform conditions
and if the event A occurs m
times then

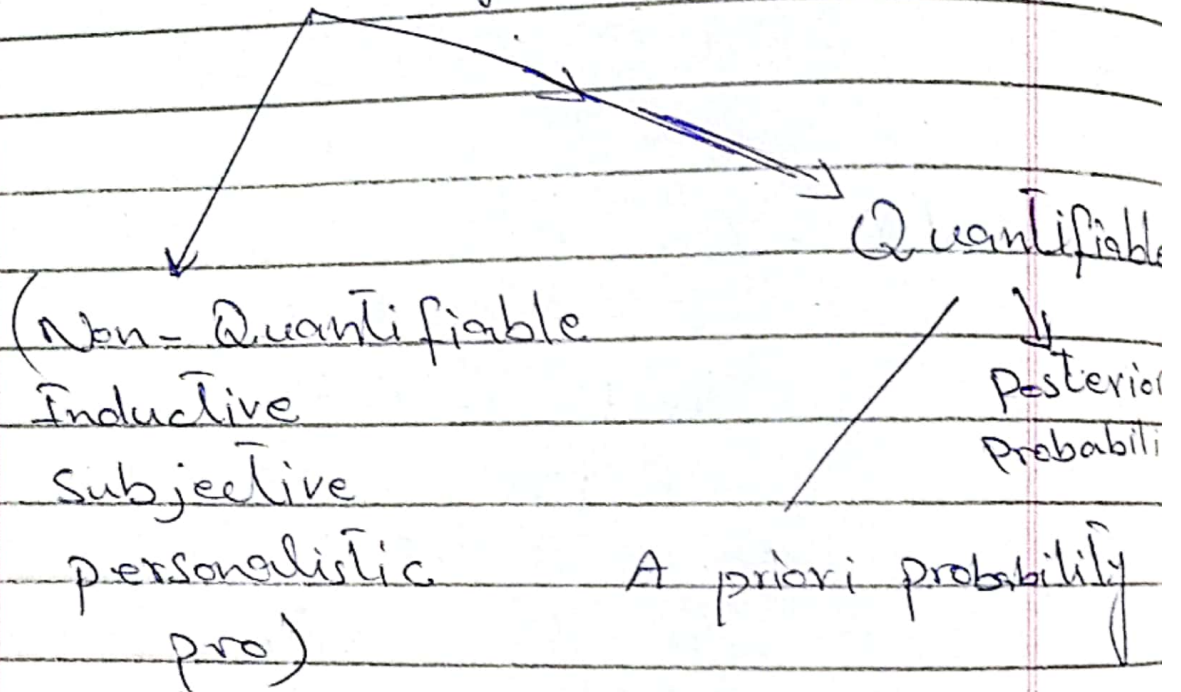
$$P(A) = \lim_{n \rightarrow \infty} \frac{m}{n}$$

(149) Pg

SIA 301

Let #19

probability



Axiom 1:- (most Imp)

For any event E ,

$$0 < P(E) < 1$$

~~Axiom~~ Type

$$P(E_1) \geq 0$$

Impossible event

$$P(E_1) \geq 1$$

Sure event

Anim 2:

Sure event

$$P(S) = 1$$

lolo

$$S = \{1, 2, 3, 4, 5, 6\} \quad n(S) = 6$$

$$A = \{1, 3, 5\} \quad n(A) = 3$$

$$B = \{2, 4, 6\} \quad n(B) = 3$$

Find probability even and odd numbers

Even or odd

$$\frac{1}{6}$$

$$\frac{n(B)}{n(S)} + \frac{n(A)}{n(S)} = \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1$$

Anim 3:

$$P(A \text{ or } B) = P(A \cup B)$$

$$P(A) + P(B)$$

(150) P8

Example:

Find probability
liveborn or stillborn key
b/w

	Liveborn	stillborn	Total
Male	359,881 $n(A)$	8,609 $n(B)$	368,490
Female	340,454 $n(C)$	7,796 $n(D)$	348,250
Total	700,335	16,405	716,740

$n(S)$

live born or still born

1) for Male:

$$= P(A \text{ or } B) = P(A) + P(B)$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$

$$= \frac{359881}{716740} + \frac{8609}{716740}$$

$$= 0.5021 + 0.0120$$

$$= 0.5141$$

addition law

still born (male or female)

$$P(B \text{ or } D) = P(B) + P(D)$$

$$= \frac{P(B)}{n(S)} + \frac{P(D)}{n(S)}$$

$$= \frac{8607}{716740} + \frac{7796}{716796}$$

$$= 0.0120 + 0.0109$$

$$= 0.0229$$

Law of complementation

$$P(\bar{A}) = 1 - P(A)$$

$$n(S) = 16$$

$$A = \{TTTT\}$$

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{16}$$

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{16} = \frac{16-1}{16} = \frac{15}{16}$$

phly ki Terhai 16 ko denider
kna h 8 Py

$$\frac{16}{8} = 8$$

H H H H
H H H T
H H T H
H ~~H~~ T T
H ~~H~~ H T

find propability
only Tail apor
end wala

H T ~~H~~ H
H T T T

$$\frac{1}{16}$$

T H H H
T H H T

only Head apor

T H ~~H~~ H
T H T T

$$\frac{1}{16}$$

T T H H

T T H T

T T T H

T T T ~~T~~

Again lastly

Again

3/4
hengyn

Addition Law:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example:

Sol:

$$n(S) = 52$$

$$\text{No of club cards} = n(A) = 13$$

$$\text{No of face card} = n(B) = 12$$

$$n(A \cap B) = 3$$

P(Club or face)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

$$= \frac{13 + 12 - 3}{52}$$

$$= \frac{22}{52} \Rightarrow \frac{11}{26}$$

objective

Corollary - 1 :

if A and B

mutually exclusive

then

$$A \cap B = \emptyset$$

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) \quad \text{Ans}$$

pair of die roll

Example:-

S ually

$$(2, 3) | (2, 3) = 2 + 3 = 5$$

$$(4, 1) | (1, 4) = 1 + 4 = 5$$

$$11 \Rightarrow (5, 6) | (6, 5)$$

$$11 = 6 + 5$$

$$n(S) = 36, \quad n(A) = 4, \quad n(B) = 2$$

$$P(5 \text{ or } 11) = P(A \text{ or } B) = P(A) + P(B)$$

$$\frac{n(A)}{n(B)} = \frac{n(B)}{n(B)}$$

$$= \frac{4}{36} + \frac{2}{36}$$

$$= \frac{6}{36} = \frac{1}{6}$$

Corollary - 2.3:
if $A_1, A_2, \dots, A_k$
are khatam
mutually exclusive
Then

$$P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_k) \\ = P(A_1) + P(A_2) + \dots + P(A_k)$$

1/35

twice

5P

$$\text{Probability } (A) = 2$$

$$P(C) = 2$$

find A or B probability?

Sol: =

let

$$P(C) = p \text{ --- (1)}$$

and

$$P(B) = 2p \text{ --- (2)}$$

$$P(A) = 2(2p)$$

$$= 4p \text{ --- (3)}$$

Since A, B and C
are mutually exclusive
then

$$P(A) + P(B) + P(C) = 1$$

$$p + 2p + 4p = 1$$

$$7p = 1$$

$$p = 1/7$$

put in (1) (2) and (3)

$$p = 1/7$$

$$P(B) = 2(1/7) = 2/7$$

$$= 4(1/7) = 4/7$$

$$P(A \text{ or } B)$$

$$P(A) + P(B)$$

$$\frac{4}{7} + \frac{2}{7} \Rightarrow \frac{4+2}{7} = \frac{6}{7}$$

in which - produced

Conditional probability:

if A and B are two events defined on a sample space "S" the probability that B occurs given A has already occurred is called conditional probability.

It is denoted by given by

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$$

Example:-

1	3	5
2	4	6

$$S = \{1, 2, 3, 4, 5, 6\}$$

New sample space

$$S_1 = \{2, 4, 6\}$$

$$n(S) = 3$$

$$P(6) = \frac{1}{3}$$

formulas

$$(I) \rightarrow P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$(II) \rightarrow P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Axioms

$$0 < P(A/B) < 1$$

Sho sure event

$$P(S/B) = 1$$

$$P\left(\frac{A_1 \cup A_2}{B}\right) = P\left(\frac{A_1}{B}\right) + P\left(\frac{A_2}{B}\right)$$

Bhinder

Ref.